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PORTFOLIO MANAGEMENT FEES: ASSETS OR PROFITS BASED COMPENSATION?[1]

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Abstract

This paper compares assets-based portfolio management fees to profits-based fees. Whilst both forms of compensation can provide appropriate risk incentives, fund managers´ limited liability induces more excess risk-taking under a profits-based fee contract. On the other hand, an assets-based fee is more costly to investors. In Spain, where the law explicitly permits both forms of retribution, assets-based fees are observed far more frequently. Under this type of compensation, the paper provides some insights into how management fees should be determined in order to solve the principal´s trade-off between providing better risk incentives and incurring a lower cost of compensation.

Keywords: fund management, performance fees, risk incentives, cost of compensation, indirect sales of information.

JEL Classification: G23, G38

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1 Introduction

European Council Directive 85/611/EEC sets the general legal framework within which undertakings for collective investment in transferable securities may carry on their business. The directive establishes that "the law or the fund rules must prescribe the remuneration and the expenditure which a management company is empowered to charge to a unit trust and the method of calculation of such remuneration." Therefore, legal restrictions to the way companies managing mutual funds can be compensated for their services, if any, are to be found only at the national level. In Spain the law (Ley 46/1984) foresees three forms of retribution to management companies: fees can be set (within certain limits) as a function of assets under management, pro…ts, or both. Companies managing mutual funds are therefore left with a large degree of latitude when it comes to deciding on the mechanism and the value of their compensation. Investors, in turn, may choose from a wide range of mutual funds and mutual fund families, so it is not obvious a priori what compensation mechanism should be expected to take place in practice.

The analysis presented in this paper builds on the work by Ross (1973, 1974) and Leland (1978), who showed that –under certain assumptions– it is optimal for the agent to o¤er the principal an e¢cient incentive system as long as the optimal risk sharing rule is linear. In other words, the linear compensation scheme induces the agent to act in the principal's best interest. In a delegated portfolio management framework, Bhattacharya and P‡eiderer (1985) studied departures from the e¢cient solution when managers have heterogeneous forecasting abilities which are unknown to the investor, and Grinblatt and Titman (1989) identi…ed an option e¤ect of compensation schemes that only provide compensation for positive returns relative to a benchmark. More recently, Admati and P‡eiderer (1997) have shown that using benchmark portfolios to determine the manager's compensation may lead to suboptimal portfolio decisions even if negative compensations are allowed. Recently, García (2000) has studied the contracting problem when the principal and the agent have di¤erent risk endowments and/or di¤erent beliefs about the risk-return trade-o¤ of the assets under management.

De…ning the management fee as a fraction of assets under management at the beginning of the period, this paper compares the implications of two simple types of performance-based fees: a constant times the portfolio's gross return and a constant times the portfolio's net return. The …rst type of retribution can be thought of as an end-of-period assets-based compensation, whereas the second type is rather a pro…ts-based payment. It is shown that the former is generally an e¢cient albeit expensive way of inducing the manager to take any given action. On the other hand, the latter induces the manager to select undesirably high levels of risk from the investors' viewpoint. This is a consequence of the option e¤ect studied by Grinblatt and Titman (1989) which results from managers' limited liability. Contrary to their …nding however, in the absence of a benchmark return a risk averse agent does not necessarily wish to take an unbounded position in the risky asset, the reason being that for some low realizations of the risky asset's payoff, the manager receives a fee that is positive although monotically decreasing in the level of risk undertaken.

The legal provision for the case of Spain[1] enables us to study which type of retribution (assets versus profits -based) is chosen more frequently in practice. Casual observation of mutual fund compensation contracts reveals that management companies are almost invariably compensated with an assets-based rather than a profits-based fee. Next, taking this form of compensation as given, the paper studies how the value of expense ratios should be determined if the principal's trade-off between providing the agent with better incentives in terms of risk-taking and incurring a lower cost of compensating the manager is to be optimally solved. The analysis is carried out both for the case of an agent who has the same information as the principal, and for the case of an agent who owns some valuable piece of information. A striking finding is that the optimal expense ratio under the assets-based retribution is not monotonically increasing in the manager's ability to forecast future returns.

The paper is organized as follows: section 2 explores the agent's portfolio choice problem when fees are based on performance; section 3 studies the optimal determination of the compensation function parameter given that it is chosen to be a linear function of assets under management; section 4 extends the analysis of section 3 to the case when the portfolio manager is better informed than the market; and finally section 5 concludes.

2 Performance-based fee contracts

Consider the following simple performance-based fee:

$$F = k\,(R_p - Q) \tag{1}$$

where F is the fraction of the portfolio's value at the beginning of the period that the management company charges to investors, R_p is defined as the gross return of the managed fund in the evaluation period and k and Q are constants. Throughout the paper, k will also be referred to as expense ratio accordingly with the industry's usual terminology. Assuming that no further contributions are made to the managed portfolio, if $Q = 0$, then the management company's (henceforth "the manager" or "the agent") compensation equals kW, where W denotes the total value of assets under management at the end of the period. On the other hand, if $Q = 1$; then the fund manager's retribution equals $k(W - W_0)$, where W_0 denotes the total value of assets under management at the beginning of the period. Note that $(W - W_0)$ can be positive or negative, so this type of contract admits the possibility of penalizing the manager for underperformance.

Next, assume that:

[1] In Spain, companies were managing over 200,000 million euros in publicly available, open-end mutual fund assets as of the end of March 31, 2000 according to the Investment Company Institute. This makes Spain the country with the sixth largest mutual fund industry in Europe.

Assumption 1 there are two assets in the economy: a riskless asset with unit price that pays R per share at the end of the period, with $R > 1$, and a risky asset with price P that pays r per share,

Assumption 2 the risky asset's price at the end of the period is given by:

$$r = s + \epsilon; \tag{2}$$

where s belongs to the information set of all individuals, $s > RP$, and ϵ is distributed as $N(0; \sigma_\epsilon^2)$;

Assumption 3 an investor –or a group of investor's– (henceforth "the investor" or "the principal") is endowed with initial wealth W_0; and delegates all investment decisions affecting her portfolio on the manager,

Assumption 4 the investor/principal and the agent/manager have preferences that can be represented by the exponential utility function: $V(Z) = -\exp(-aZ)$; where $a > 0$ is the constant absolute risk aversion coefficient, and Z is the individual's wealth at the end of the period,

Assumption 5 neither the manager nor the investor can trade on their own portfolios, and

Assumption 6 trading takes place once at the beginning of the period. At the end of the period the fund is liquidated and the manager is compensated with FW_0:

The final value of the fund if the agent buys M of the riskless asset and X of the risky asset equals:

$$W = RM + rX = RW_0 + (r - RP)X; \tag{3}$$

which incorporates the budget constraint $W_0 = M + PX$. W is thus normally distributed with mean equal to $RW_0 + (s - RP)X$ and variance equal to $\sigma_\epsilon^2 X^2$:

An investor acting on her account would choose X so as to maximize her expected utility:

$$EV_i = -\exp\left[-a_i\left(RW_0 + (s - RP)X - \frac{a_i\sigma_\epsilon^2 X^2}{2}\right)\right]: \tag{4}$$

The optimal portfolio choice then equals:

$$X = \frac{s - RP}{a_i\sigma_\epsilon^2}; \tag{5}$$

so her expected utility is given by:

$$EV_{-} = -\exp\left(-a_{-}\left(RW_0 + \frac{(s - RP)^2}{2a_{-}\sigma_\epsilon^2}\right)\right): \tag{6}$$

Consider next the case when $F = kR_p$; so the manager is compensated with kW. Once the portfolio management contract has been signed, the agent chooses the amount X that maximizes his expected utility:

$$EV_A = -\exp\left\{-a_A\left(k(RW_0 + (s - RP)X) - \frac{a_A k^2\sigma_\epsilon^2 X^2}{2}\right)\right\}; \tag{7}$$

where a_A denotes the agent's risk aversion coefficient.

From the first order condition, the agent's decision is:

$$X_A = \frac{s - RP}{a_A k\sigma_\epsilon^2}: \tag{8}$$

On the other hand, the utility that the principal expects to obtain from her participation in the contract equals:

$$EV_P = -\exp\left\{-a_P\left((1 - k)(RW_0 + (s - RP)X) - \frac{a_P(1 - k)^2\sigma_\epsilon^2 X^2}{2}\right)\right\}; \tag{9}$$

where a_P denotes the principal's risk aversion coefficient.

Given her expected utility, the principal would like the agent to purchase an amount of the risky asset such that the previous expression is maximized:

$$X_P = \frac{s - RP}{a_P(1 - k)\sigma_\epsilon^2}: \tag{10}$$

Clearly, k can be chosen in a way that the agent's choice is optimal from the principal's perspective. This is achieved by setting:

$$k = k^* \equiv \frac{a_P}{a_P + a_A}: \tag{11}$$

This value of k is the familiar first-best efficient compensation coefficient when the principal and the agent have exponential utility functions. Substituting the agent's choice of X for k^* into (9) gives the following value for the investor's expected utility:

$$EV_P = -\exp\left\{-a_P\left(-k^*RW_0 + RW_0 + \frac{(s - RP)^2}{2a_P\sigma_\epsilon^2}\right)\right\}: \tag{12}$$

If $F = k(W/W_0 - 1)$, then the manager's compensation equals $k(W - W_0)$. Setting this kind of compensation rather than kW does not alter the value of k that achieves efficient risk sharing, which is still given by (11). The investor's expected utility in this case equals:

$$EV_P = -\exp\left\{-a_P\left[-k^*(R-1)W_0 + RW_0 + \frac{(s-RP)^2}{2a_P\sigma^2}\right]\right\}. \tag{13}$$

The profits-based fee is however a much cheaper way of compensating the manager: it increases the principal's certainty equivalent by k^*W_0. Think of a mutual fund with one hundred million euros in assets. If 2% is the value of k that ensures that the manager will take the level of risk that the investor desires, an assets-based compensation contract decreases the investor's wealth by two million euros. More generally, the value of Q does not affect the variable part of the agent's compensation, kW, and hence the agent's risk choice for a given k. It does however decrease the fixed compensation to the manager by kQW_0. So if the principal could choose, she would select k^* and a negative value of Q only bounded by the agent's participation constraint. A profits-based compensation mechanism hence dominates an assets-based one.

Unfortunately for investors, management companies are not liable for any loss that the fund may suffer. This implies that the kind of contracts that are observed in reality take the nonlinear form:

$$F = \max[0, k(R_p - Q)]. \tag{14}$$

How different are in practice the contracts considered above from their limited liability counterparts? The answer depends on the likelihood of $k(R_p - Q)$ being negative. If $Q = 0$, the manager's compensation equals $\max[0, kW]$ which provided that k is positive equals kW whenever W is nonnegative. If borrowing by funds is not allowed[2], the probability of W being negative is bounded by the probability of the risky asset's payoff being negative, which is precluded by asset-holders' limited liability. Given the normal distribution assumption however, this event is possible in our model economy. Nevertheless, if we think of the risky asset as a stock index, usual moment values would make that event very unlikely. For instance, if $r - 1$ is normally distributed with mean 15% and standard deviation 35%, then the likelihood of r being negative is just 0.05%. On the other hand, the probability of $r - 1$ being lower than zero –namely the probability of the riskiest possible portfolio's profits being negative– is as high as $1/3$. This example illustrates how under the normal distribution assumption the linear contract with $Q = 0$ can be a good approximation of (14), whereas the same cannot be said about the linear contract when $Q = 1$.

In what follows, the term "assets-based contract" will indicate that the portfolio manager is compensated with kW, whereas a "profits-based" compensation will denote:

[2] Directive 85/611/EEC, article 36 only allows borrowing "up to 10% of the value of the fund provided that the borrowing is only on a temporary basis".

$$FW_0 = k\max[0; (W - W_0)]: \qquad (15)$$

The question is whether this sort of compensation scheme induces the manager to take too much risk. It can be shown that:

Proposition 1 for any given value of k under the compensation scheme (15): (i) the agent's unconstrained choice of X exceeds his optimal response under the contract without limited liability, but (ii) the agent's unconstrained choice of X is not necessarily unbounded.

Proof. See appendix (A1). ■

The first part of the proposition is very intuitive. Figure 1 shows the manager's fee as a function of the risky asset's payoff r and for different values of X. Since the agent's payoff is bounded from below, increasing the riskiness of the managed portfolio has a positive asymmetric effect on the agent's final wealth. However, for those states in which:

$$(R - 1)W_0 > (R - 1)W_0 + (r - RP)X > 0$$

namely, whenever the fund's profit is positive but less than the riskless profit (which is the case as long as $r < RP$), the manager's payoff is monotonically decreasing in X. This possibility deters a risk averse manager from taking an unbounded position in the risky asset.

Consider what happens if the manager is compensated with a constant fraction of the portfolio's profit in excess of the riskless profit. In that case, the manager's payoff could be calculated by replacing (3) into $k\max[0; (W - W_0) - (RW_0 - W_0)]$:

$$FW_0 = k\max[0; (r - RP)X] = kX\max[0; (r - RP)]; \qquad (16)$$

so as long as $k > 0$ any utility maximizing manager would take an unbounded position in the risky asset. The reason is quite simple: if the riskless profit is taken as a benchmark, then an increase in X does not affect the manager's payoff when $r < RP$ (which is zero anyway), and does however increase the manager's payoff for those states in which $r > RP$:

In the absence of a benchmark, the appendix shows that the agent's optimal position in the risky asset is bounded if the agent's utility from choosing $X = 0$ plus the probability of r being lower than RP is nonnegative. In other words, the following is a sufficient condition for a bounded solution to the manager's problem to exist:

$$-\exp(-a_A k W_0(R - 1)) + \Phi\left(-\frac{s - RP}{\sigma}\right) \geq 0 \qquad (17)$$

Where $\Phi(\cdot)$ denotes the cumulative standard normal distribution. Intuitively, the manager refrains more from taking very risky positions the larger his expected utility from not taking any risks and the higher the probability of $(r - RP)$ being negative. Again, if the riskless profit is taken as the benchmark profit, then condition (17) becomes:

$$-1 + \Phi\left(-\frac{s - RP}{\sigma}\right) \geq 0 \qquad (18)$$

which never holds true.

It is therefore clear that profit-based fees do not necessarily result in the manager taking extraordinary amounts of risk, and are a cheaper way of compensating managers. How popular are they in the industry? Consider the case of Spain where the law explicitly distinguishes between assets-based and profit-based fees. During 1999, 290 new mutual funds registered at the Spanish Security Exchange Commission. Fund prospectuses were consulted on the internet for 239 of those newly registered mutual funds. The search showed that 228 funds were charging a fee that was proportional to the value of assets under management, and 11 also charged a fee proportional to the fund's positive net income. None of the prospectuses consulted established a fee proportional to the fund's income exclusively. This result could be interpreted as investors being reluctant to invest in funds whose managers are compensated only in those periods where the fund makes a net positive profit, perhaps deterred by fears of managers taking too much risk. On the other hand, it could simply mean that management companies expect more profit from assets-based fee contracts even though the legal upper limit is lower than in the case of a profit-based fee. Whatever the explanation, assets-based fee contracts clearly dominate in practice.

The next two sections take the assets-based form of compensation as exogenous and solve for the value of k that maximizes investors' welfare, i.e., the optimal solution to the investor's trade-off between incurring a lower cost of incentive provision and providing the manager with better incentives. The resulting expense ratio will be termed "optimal" in that particular sense, not because it corresponds to an optimal contract. To see that an assets-based contract is not optimal for the principal, simply note that it would be dominated by a profit-based contract with no manager's limited liability.

3 Optimal expense ratio without private information

This section studies the portfolio management problem when both the principal and the agent know the value of s. If the agent does not manage the principal's portfolio, he may trade on his own account. His optimal choice of X and his corresponding reservation expected utility are given by:

$$X = \frac{s - RP}{a_A \sigma^2}; \tag{19}$$

$$\hat{u} = -\exp\left(-a_A \frac{(s - RP)^2}{2a_A \sigma^2}\right): \tag{20}$$

On the other hand, substituting (8) in (7) and (9), it is possible to calculate the value of the agent and principal's expected utility in anticipation of the agent's decision:

$$EV_A = -\exp\left\{-a_A\left[kRW_0 + \frac{(s - RP)^2}{2a_A\sigma^2}\right]\right\}; \tag{21}$$

$$EV_P = -\exp\left\{-a_P\left[(1-k)RW_0 + C(k)\frac{(s - RP)^2}{a_P\sigma^2}\right]\right\}: \tag{22}$$

with:

$$C(k) \equiv \frac{(1-k)a_P}{ka_A} - \frac{1}{2}\left[\frac{(1-k)a_P}{ka_A}\right]^2: \tag{23}$$

Function $C(k)$ always takes on a value less than or equal to $1/2$. When $k = k^*$; the agent's decision is optimal for the principal too, and $C(k)$ reaches its maximum value. Values underneath $1/2$ therefore reflect the negative impact of a suboptimal incentive system on the principal's expected utility. The form of the function shows two possible sources for an agency problem in this model: different attitudes towards risk and different shares in the final portfolio. When k is set at its efficient value k^*, both effects cancel out.

The properties of $C(k)$ are analyzed in the appendix and figure 2 shows its plot.

In any case, (21) proves that for any linear compensation function the agent adjusts his response so that he always obtains his reservation certainty equivalent plus kRW_0. Therefore, the manager's participation constraint implies:

$$kRW_0 + \frac{(s - RP)^2}{2a_A\sigma^2} \geq \frac{(s - RP)^2}{2a_A\sigma^2} \Longrightarrow k \geq 0 \tag{24}$$

Taking the assets-based fee contract as exogenous, the principal's problem is that of choosing k such that her expected utility is maximized. An expense ratio equal to k^* guarantees efficient risk sharing, but is more costly than a lower value of k: If the expense ratio were chosen so that it maximizes the investor's welfare, it would be the solution to the following problem:

$$\max_{k} \left[(1-k)RW_0 + C(k)\frac{(s-RP)^2}{a_P\sigma_\varepsilon^2} \right] \quad \text{subject to } k \geq 0 \tag{25}$$

Proposition 2 When the investor can choose the terms of the linear retribution function under (1) with $Q = 0$, the solution to the principal's problem is given by k such that:

$$C'(k)\frac{(s-RP)^2}{a_P\sigma_\varepsilon^2} = RW_0 \tag{26}$$

Proof. See appendix (A3). ■

Since $RW > 0$, it follows that at the optimum $C'(k) > 0$, so $k < k^*$ from the concavity of C. The most immediate consequence is that the agent will not act in the investor's best interest. In particular, for positive values of $(s-RP)$, he will purchase a larger than optimal amount of the risky asset.

The values of the agent and the principal's expected utility under this type of compensation should therefore be:

$$EV_A = -\exp\left\{-a_A\left[kRW_0 + \frac{(s-RP)^2}{2a_A\sigma_\varepsilon^2}\right]\right\}, \tag{27}$$

$$EV_P = -\exp\left\{-a_P\left[(1-k)RW_0 + C(k)\frac{(s-RP)^2}{a_P\sigma_\varepsilon^2}\right]\right\}. \tag{28}$$

where k solves (26). Note that the solution is not efficient since $C(k) < 1/2$.

The resulting value of k depends on the model parameters as well as on the agent and principal's preferences. $C(k)$ is concave at the optimum since $C'(k)$ is positive at the solution and the region where $C(k)$ is increasing is contained within the concave region. As a consequence, the optimal k is increasing in the risk premium and decreasing in the initial portfolio size, W_0, as well as in the volatility of the risky asset's return.

4 The case of private information

Suppose that only a few individuals know the actual value of s. The rest only know that s is distributed as $N(Es, \sigma_s^2)$, which is independent of the conditional distribution of r. For those individuals who do not observe s, the unconditional distribution of r from (2) is normal with mean Es and variance σ_r^2 with:

$$\sigma_r^2 = \sigma_s^2 + \sigma_\varepsilon^2 \tag{29}$$

The realized value of s can be interpreted as a privately observed signal, $1/\sigma_\varepsilon^2$ being the signal's precision. In the extreme case that $\sigma_\varepsilon^2 = 0$, the informed individual knows the value of r exactly. It will be assumed that σ_ε^2 is a strictly positive value.

The model is completed with the assumption that individuals' actions do not affect asset prices and prices do not contain information about s. This assumption isolates the market for information from the market for the risky asset, which given the risk aversion assumption, is sufficient to guarantee that the risky asset has a positive risk premium (see Allen (1990)):

$$\mu \equiv Es - RP > 0. \tag{30}$$

An uninformed investor's problem amounts to maximizing her expected utility which follows from the unconditional distribution of r:

$$EV_i = -\exp\left[-a_i\left(RW_{0,i} + \mu X - \frac{a_i\sigma_r^2 X^2}{2}\right)\right], \tag{31}$$

the uninformed investor's optimal demand is thus:

$$X = \frac{\mu}{a_i\sigma_r^2}, \tag{32}$$

so her expected utility equals:

$$EV_i = -\exp\left[-a_i\left(RW_{0,i} + \frac{\mu^2}{2a_i\sigma_r^2}\right)\right]. \tag{33}$$

On the other, the expected utility of an informed investor who observes s is given by (6). As shown by Allen (1990), taking expectations over s it is possible to obtain the informed investor's expected utility prior to observing s:

$$EV_i = -\exp\left[-a_i\left(RW_{0,i} + \frac{\mu^2}{2a_i\sigma_r^2} + \eta\right)\right], \tag{34}$$

with

$$\eta \equiv \frac{1}{2a_i}\log\frac{\sigma_r^2}{\sigma_\varepsilon^2}. \tag{35}$$

The second term in the argument of (34) corresponds to the second term in (6) and (33). It is the value of taking an optimal position in the risk-reward trade-off given the conditioning information. The third term is the signal's ex ante value, i.e. the value of private information. Of course, η is higher the higher the signal's precision. The existence of informed agents justifies delegated portfolio management when direct sale of information is not possible or too costly. This section shows that agency contracts allow the investor to extract at least part of the value of the manager's private information.

Finally, if the informed agent has no initial endowment, his reservation expected utility equals:

$$EV_A = -\exp\left\{-a_A\left(\frac{\theta^2}{2a_A\sigma_r^2} + \eta\right)\right\}. \tag{36}$$

It will be assumed that the delegated portfolio management contract is signed before the agent obtains his private information.

Taking the assets-based fee as given, the investor and the manager's expected utility as well as their optimal portfolios for a given value s coincide with those of the model without private information. Substituting the agent's optimal demand (8) in both expected utility functions conditional on s gives again (21) and (22). Next, taking expectations over s, the ex ante expected utility of the agent and the principal are obtained as:

$$EV_A = -\exp\left\{-a_A\left(kRW_0 + \frac{\theta^2}{2a_A\sigma_r^2} + \eta\right)\right\} \quad \text{and} \tag{37}$$

$$EV_P = -\exp\{-a_P[(1-k)RW_0 + \alpha + \Omega]\}. \tag{38}$$

where:

$$\alpha \equiv \frac{C}{a_P}\frac{\theta^2}{(\sigma_\varepsilon^2 + 2C\sigma_s^2)} \tag{39}$$

$$\Omega \equiv \frac{1}{2a_P}\log\left(\frac{\sigma_\varepsilon^2 + 2C\sigma_s^2}{\sigma_\varepsilon^2}\right). \tag{40}$$

where C is the function defined by (23).

The Appendix (A4) describes the properties of α and Ω.

When C is lower than 1/2, i.e. when $k \neq k^*$, the first term α is lower than the value of the principal's optimal position in her risk-reward trade-off:

$$\frac{C(k)}{a_P}\frac{\theta^2}{(\sigma_\varepsilon^2 + 2C(k)\sigma_s^2)} < \frac{\theta^2}{2a_P\sigma_r^2} \qquad \forall k \neq k^* \tag{41}$$

The above inequality is an agency cost similar to that borne by the investor in the model with no private information. It can be eliminated by setting k such that the agent takes the position in the risky asset that is optimal for the principal.

Similarly, function Ω reaches its maximum value, η, when C equals 1/2. In that case, the principal extracts the full value of the manager's private information. This second agency cost is therefore a consequence of the manager's private information not being fully exploited in the investor's best interest. Again, the first-best expense ratio, k^* eliminates this agency cost.

As for the manager, for any value of k, he responds so that he takes his optimal position in the risk-reward trade-off and at the same time he obtains the full value of his private information as seen from (37). As in the case of no private information, his participation constraint is met as long as:

$$k \geq 0. \tag{42}$$

The principal's problem stated in terms of certainty equivalents and incorporating the agent's portfolio decision can then be formulated as follows:

$$\begin{array}{c} \max_{k} \quad [(1 - k)RW_0 + \Psi(k) + \Omega(k)] \\ \text{subject to} \quad k \geq 0 \end{array}. \tag{43}$$

Proposition 3 In the private information model the solution to the principal's problem under (1) with $Q = 0$ is given by k such that:

$$\frac{\partial(\Psi + \Omega)}{\partial k} = RW_0. \tag{44}$$

Proof. See appendix (A5). ■

From the properties of Ψ and Ω; and given that RW_0 is strictly positive, the value of k that solves (43) is in the interval $(0, \frac{a_P}{a_P + a_A})$; and therefore in the region in which $(\Psi + \Omega)$ is concave in k. The solution can be interpreted as in the case of symmetric information: when the investor must bear the cost of incentive provision, the expense ratio should solve the investor's trade off between a higher cost of incentive provision and a lower agency cost.

Note that the investor is able to capture part of the value of the manager's private information, so it is possible for her to improve her expected utility through a delegated portfolio management contract provided that the increase in her expected utility offsets the loss due to the agency cost.

Note also that the expense ratio that solves the principal's problem under the contract constraints depends on model parameters other than the risk aversion coefficients. Given that at the optimum $(\Psi + \Omega)$ is a concave function, it follows that the expense ratio decreases with the value of the investor's initial wealth. The reason, as in the model with no private information, is that a higher value of W_0 increases the marginal cost of increasing k.

A deeper comparative statics analysis is required in order to study the relationship between the optimal expense ratio and the quality of the manager's information given the highly nonlinear functional form of Ψ and Ω. First, define

$$\delta \equiv \frac{\sigma_\epsilon^2}{\sigma_r^2},$$

so σ_ϵ^2 can be replaced with $\delta\sigma_r^2$ and σ_s^2 with $(1-\delta)\sigma_r^2$. An increase in δ can be interpreted as a decrease in the precision of the manager's private information.

Next, define

$$F(\delta; k) \equiv \frac{\partial(\Psi + \Omega)}{\partial k}$$

The principal chooses k such that $F(\delta; k) = RW_0$. From the implicit function theorem and given that at the optimal choice $(\Psi + \Omega)$ is concave in k, it follows that k is increasing in δ (decreasing in precision) as long as:

$$\frac{\partial F(\delta; k)}{\partial \delta} > 0$$

The appendix (A6) shows that the slope of Ω with respect to k is decreasing in δ. On the other hand, for values of k above a cut-off value, the slope of Ψ is increasing in δ; whereas for values of k below the threshold, the slope is decreasing. If only Ω is considered, it is optimal for the investor to pay a higher k the higher the quality of the manager's information since an increase in the precision of the manager's signal increases the marginal benefit of raising k above its marginal cost and Ω is concave. This is however not true for Ψ when the value of k is high enough. In that case the investor is better off improving the incentives of a worse quality manager.

As a result of the interaction of both effects, the appendix shows that:

$$\frac{\partial F(\delta; k)}{\partial \delta} > 0 \iff C > \frac{1}{2}\frac{\delta(\zeta^2 + \sigma_r^2)}{(\zeta^2 - \sigma_r^2) + \delta(\zeta^2 + \sigma_r^2)}: \tag{45}$$

Since C is increasing in k, there exists a value $\hat{k}$ so high that condition (45) is met so k is decreasing in the quality of the manager's information. That particular value is such that:

$$C = \frac{1}{2}\frac{\delta(\zeta^2 + \sigma_r^2)}{(\zeta^2 - \sigma_r^2) + \delta(\zeta^2 + \sigma_r^2)} \tag{46}$$

Note that if $\sigma_r^2 > \zeta^2$; then the above condition requires that $C > \frac{1}{2}$; which is impossible. In that case, condition (45) is never met and k is always increasing in the precision of the manager's signal.

Assuming that $\sigma_r^2 < \zeta^2$; the precision of private information and the optimal value of k are related as follows. Starting off with a level of precision such that the optimal value of k is below $\hat{k}$; if precision increases; i.e., if δ decreases then α increases as predicted by (45). Therefore, the new optimal value k must be higher since (ª + -) is concave. On the other hand, as ± decreases, the threshold value $\hat{k}$ also decreases. There must be a value of ± such that the optimal k is exactly $\hat{k}$. At that point, a small increase in precision does not a¤ect the optimal value of k; but the threshold value $\hat{k}$ that corresponds to the new level of precision decreases so condition (45) holds true. It can be concluded that the form of the relationship between the optimal expense ratio and the quality of management in this model is concave with both an increasing and a decreasing region.

This result is a consequence exclusively of the agency cost associated with ª; which arises when the manager's investment decision departs from the investor's optimum. An increase in k improves this agency cost more the lower the manager's information quality. When this e¤ect dominates the e¤ect on the value of private information, the optimal expense ratio is decreasing in the signal precision.

5 Direct market participation

So far, it has been assumed that neither the agent nor the principal can trade on their own accounts as long as they are committed to a delegated portfolio management contract. However, if either party is allowed to trade on his own account, an excess exposure to the single risk factor through the managed portfolio could be o¤set by decreasing the number of shares directly held in the risky asset and viceversa. The implication regarding the case of the agent participating directly in the market is that an in…nite number of responses to any given compensation function are optimal from the agent's viewpoint. Consequently, the principal cannot ensure any particular action on the agent's part through the linear compensation contract.

Consider now the case when only the principal can trade directly in the market in the absence of a privately observed signal. Since she can trade on her own account, she is able to o¤set the agent's actions in such a way that any compensation scheme achieves the optimal exposure to the risk factor. If the principal cares about the cost of incentive provision and is able to o¤set inadequate risk taking by the agent, she will reduce k to zero. As a consequence, the agent will respond by taking an unbounded position in the risky asset which will be o¤set by an equally unbounded position of the principal's personal account.

Finally, if the agent owns a valuable piece of information about the risky asset's return, the principal cannot anticipate the agent's actual response, which depends on the noisy signal, and therefore is unable to o¤set the agent's departure from appropriate risk taking. Hence, being able to trade directly in the market does not help the principal in the presence of a privately observed signal. Of course, the problem could be solved if the signal were observable by the principal after the contracting stage. In that case, a contract scheme that penalizes opportunistic behavior on the investor's part would be feasible.

6 Conclusions

This paper shows in a very simple model that a profit-based fee is arguably a less efficient compensation mechanism in terms of risk sharing consistently with findings by Grinblatt and Titman (1989) or Chevalier and Ellison (1997). However, even though the incremental cost of an assets-based fee to the investor can be significantly larger, a profit-based fee does not generally induce the agent to take an unbounded position in the risky asset.

Since assets-based fees appear to be a more popular means of retribution, the paper studies how the value of these expense ratios should be determined as a function of some portfolio characteristics such as fund size or the manager's forecasting ability. In this context, it is no longer obvious that the resulting expense ratio must be monotonic in the manager's forecasting precision. This perspective can thus be seen as a new look at the negative relationship between managerial performance and expense ratios found in the mutual fund industry (see for instance Gruber (1996), Carhart (1997), Metrick and Zeckhauser (1999) and Christoffersen and Musto (1999).)

Appendix

A1. Proof of Proposition 1.

Let $g(W)$ denote the manager's compensation under (15):

$$g(W) = \begin{cases} g_1(W) = 0 & \text{if } W \le W_0 \\ g_2(W) = k(W - W_0) & \text{if } W > W_0 \end{cases}$$

The manager's expected utility can be therefore expressed as the sum of two integrals:

$$\begin{aligned} EV_A &= \int_{-\infty}^{\infty} V(g(W))f(W)dW \qquad (47) \\ &= \int_{-\infty}^{W_0} V(g_1(W))f(W)dW + \int_{W_0}^{\infty} V(g_2(W))f(W)dW; \end{aligned}$$

where $f(W)$ denotes the probability density function of W. Alternatively, (47) can be expressed as:

$$\begin{aligned} EV_A &= \int_{-\infty}^{W_0} [V(g_1(W)) - V(g_2(W))]f(W)dW + \int_{-\infty}^{\infty} V(g_2(W))f(W)dW \\ &= \int_{-\infty}^{W_0} [V(g_1(W)) - V(g_2(W))]f(W)dW \\ &\quad - \exp\left\{-a_A\left(k((R-1)W_0 + (s-RP)X) - \frac{a_A k^2 \sigma_\varepsilon^2 X^2}{2}\right)\right\}; \qquad (48) \end{aligned}$$

where the second equality follows from the fact that W is normally distributed. Note that the second term in (48) is the agent's expected utility without limited liability.

The first term in (48) can also be written in terms of the standardized normally distributed variable $z \equiv \frac{W-(RW_0+(s-RP)X)}{\sigma X}$:

$$\begin{aligned} &\int_{-\infty}^{W_0} [V(g_1(W)) - V(g_2(W))]f(W)dW \\ &= \int_{-\infty}^{-\frac{(R-1)W_0}{\sigma X} - \frac{(s-RP)}{\sigma}} (-1 + \exp(-ak(\sigma X z + (s-RP)X + (R-1)W_0)))\phi(z)dz; \end{aligned}$$

where $\phi(z)$ denotes the standard normal density. The integral's derivative with respect to X equals:

$$-\int_{-\infty}^{-\frac{(R-1)W_0}{\sigma X} - \frac{(s-RP)}{\sigma}} ak(z\sigma + (s-RP)) \cdot \exp(-ak(\sigma X z + (s-RP)X + (R-1)W_0))\phi(z)dz.$$

Since $z\sigma + (s - RP)$ is negative in the integration interval, the above expression is positive provided k is positive. It thus follows that imposing limited liability induces the agent to take a larger position in the risky asset.

In order to prove the second part of proposition 1, note that (48) can alternatively be written as:

$$\begin{aligned} EV_A &= \int_{-\infty}^{\infty} V(g_1(W)) f(W) dW + \int_{W_0}^{\infty} [V(g_2(W)) - V(g_1(W))] f(W) dW \\ &= \int_{-\infty}^{\infty} V(0) f(W) dW + \int_{W_0}^{\infty} [V(g_2(W)) - V(0)] f(W) dW \end{aligned}$$

Since $V(0) = -1$; it follows that:

$$\begin{aligned} EV_A &= -1 + \int_{W_0}^{\infty} [V(g_2(W)) + 1] f(W) dW \\ &= -1 + \int_{W_0}^{\infty} [V(g_2(W)) + 1] f(W) dW. \end{aligned}$$

Consider partitioning the above integral in two parts:

$$\begin{aligned} & \int_{W_0}^{\infty} [V(g_2(W)) + 1] f(W) dW \\ = & \int_{W_0}^{RW_0} [V(g_2(W)) + 1] f(W) dW + \int_{RW_0}^{\infty} [V(g_2(W)) + 1] f(W) dW \\ = & I1 + I2 \end{aligned}$$

where:

$$\begin{aligned} I1 &\equiv \int_{-\frac{(R-1)W_0}{\sigma X} - \frac{(s-RP)}{\sigma}}^{-\frac{(s-RP)}{\sigma}} [-\exp(-ak(\sigma X z + (s - RP)X + (R - 1)W_0)) + 1] \phi(z) dz; \\ I2 &\equiv \int_{-\frac{(s-RP)}{\sigma}}^{\infty} [-\exp(-ak(\sigma X z + (s - RP)X + (R - 1)W_0)) + 1] \phi(z) dz. \end{aligned}$$

At $X = 0$ the manager's expected utility equals $-\exp(-ak(R - 1)W_0)$. On the other hand, when $X \to \infty$:

$$\begin{aligned} I1(X \to \infty) &= 0; \\ I2(X \to \infty) &= 1 - \Phi\left(-\frac{(s - RP)}{\sigma}\right). \end{aligned}$$

Therefore, whenever $-\exp(-ak(R - 1)W_0) > -\Phi(-\frac{(s-RP)}{\sigma})$, the manager's expected utility at $X = 0$ is not lower than at $X \to \infty$. If additionally the first derivative of EV_A with respect to X evaluated at $X = 0$ is positive, then there must exist X_0 with $0 < X_0 < \infty$, such that EV_A evaluated at X_0 is strictly larger than EV_A evaluated at $X \to \infty$. Taking the first derivatives of $I1$ and $I2$ with respect to X; it can be shown that:

$$\left.\frac{\partial I1}{\partial X}\right|_{X=0} = \exp(-ak(R-1)W_0) \int_{-\infty}^{-\frac{(s-RP)}{\sigma}} ak(z\sigma + (s - RP))\phi(z)dz$$

$$\left.\frac{\partial I2}{\partial X}\right|_{X=0} = \exp(-ak(R-1)W_0) \int_{-\frac{(s-RP)}{\sigma}}^{\infty} ak(z\sigma + (s - RP))\phi(z)dz$$

Hence,

$$\begin{aligned}\left.\frac{\partial EV_A}{\partial X}\right|_{X=0} &= \left.\frac{\partial I1}{\partial X}\right|_{X=0} + \left.\frac{\partial I2}{\partial X}\right|_{X=0} \\ &= \exp(-ak(R-1)W_0) \cdot ak(s - RP)\end{aligned}$$

where the second equality follows from the fact that $\int_{-\infty}^{\infty} z\phi(z)dz = 0$, and $\int_{-\infty}^{\infty} \phi(z)dz = 1$. For $k > 0$, the expected utility at $X = 0$ increases with an increase in X.

A2. Properties of $C(k)$:

Function $C(k)$ is defined as:

$$C(k) \equiv \frac{(1-k)a_P}{ka_A} - \frac{1}{2}\left(\frac{(1-k)a_P}{ka_A}\right)^2:$$

The previous function is only defined for values of k strictly different from zero. The first derivative with respect to k is:

$$C'(k) = \frac{1 - k(\frac{a_A}{a_P} + 1)}{(\frac{a_A}{a_P})^2 k^3};$$

which takes on a positive value in the interval $(0; \frac{a_P}{a_A + a_P})$, and a negative value for values of k in the interval $(\frac{a_P}{a_A+a_P}; \infty)$:

The second derivative with respect to k is given by:

$$C''(k) = \frac{2k\frac{a_A}{a_P} + 2k - 3}{\left(\frac{a_A}{a_P}\right)^2 k^4}:$$

Therefore, $C(k)$ is concave in the interval $(0; \frac{3}{2}\frac{a_P}{a_A+a_P})$ and convex in $(\frac{3}{2}\frac{a_P}{a_A+a_P}; \infty)$.

A3. Solving the principal's problem with no private information

Problem (25) can be re-written as the following minimization program:

$$\min_{k} \left[-(1-k)RW_0 - C(k)\frac{(s-RP)^2}{a_P\sigma^2} \right] \quad \text{subject to} \quad -k \leq 0 :$$

The associated Lagrangian is:

$$L = -(1-k)RW_0 - C(k)\frac{(s-RP)^2}{a_P\sigma^2} - \lambda k$$

The first-order condition for k is:

$$RW_0 - C'(k)\frac{(s-RP)^2}{a_P\sigma^2} - \lambda = 0$$

With $\lambda = 0$; k is the solution to:

$$C'(k)\frac{(s-RP)^2}{a_P\sigma^2} = RW_0$$

A4. Properties of functions Φ and Ψ.

Function Ψ is defined as:

$$\Psi \equiv \frac{C(k)}{a_P}\frac{\zeta^2}{(\sigma^2 + 2C(k)\sigma_S^2)}:$$

The partial derivative of Ψ with respect to k is:

$$\frac{\partial \Psi}{\partial k} = \frac{\zeta^2}{a_P}\frac{\sigma^2}{(\sigma^2 + 2C\sigma_S^2)^2}C':$$

Therefore, the increasing and decreasing regions of Ψ coincide with those of C. The function reaches its maximum value when $k = \frac{a_P}{a_A + a_P}$:

The second partial derivative with respect to k equals:

$$\frac{\partial^2 \Psi}{(\partial k)^2} = \frac{\zeta^2}{a_P}\left[\frac{\sigma^2}{(\sigma^2 + 2C\sigma_S^2)^2}C'' - \frac{4\sigma^2\sigma_S^2}{(\sigma^2 + 2C\sigma_S^2)^3}[C']^2\right]:$$

Since C is assumed to be nonnegative, the term $(\sigma^2 + 2C\sigma_S^2)$ is always positive. Hence, Ψ is concave with respect to k whenever C'' takes on a negative value, i.e., when k is lower than $\frac{3}{2}\frac{a_P}{a_P + a_A}$:

On the other hand function Φ is defined as:

$$\Omega \equiv \frac{1}{2a_P}\log\left(\frac{\sigma_\varepsilon^2 + 2C\sigma_s^2}{\sigma_\varepsilon^2}\right):$$

Its first derivative with respect to k is:

$$\frac{\partial\Omega}{\partial k} = \frac{1}{a_P}\frac{\sigma_s^2}{\sigma_\varepsilon^2 + 2C\sigma_s^2}C';$$

hence Ω is increasing in k for the same values as C.

The second derivative is given by:

$$\frac{\partial^2\Omega}{(\partial k)^2} = \frac{1}{a_P}\left[\frac{\sigma_s^2}{\sigma_\varepsilon^2 + 2C\sigma_s^2}C'' - \frac{2(\sigma_s^2)^2}{(\sigma_\varepsilon^2 + 2C\sigma_s^2)^2}[C']^2\right];$$

therefore, $C'' < 0$ is a sufficient condition for Ω being concave in k:

A5. Solving the principal's problem with private information

Problem (43) can be rewritten as follows:

$$\begin{array}{c} \min\limits_{k} \quad [-(1-k)RW_0 - \Psi(k) - \Omega(k)] \\ \text{subject to} \quad -k \leq 0 \end{array}:$$

The associated Lagrangian is:

$$L = -(1-k)RW_0 - \Psi(k) - \Omega(k) - \lambda k$$

The first-order condition for k is:

$$RW_0 - \frac{\partial(\Psi + \Omega)}{\partial k} - \lambda = 0$$

With $\lambda = 0$; k is the solution to:

$$\frac{\partial(\Psi + \Omega)}{\partial k} = RW_0$$

A6. How the slope of $\Psi + \Omega$ changes with the precision of private information

First, define δ as:

$$\delta \equiv \frac{\sigma_\varepsilon^2}{\sigma_r^2};$$

so it is possible to replace σ_ϵ^2 with $\delta\sigma_r^2$ and σ_s^2 with $(1-\delta)\sigma_r^2$ in the expression for the first derivative of α with respect to k in appendix (A4). Next it is possible to calculate how the slope of α with respect to k changes when δ increases,

$$\frac{\partial(\frac{\partial\alpha}{\partial k})}{\partial\delta} = \frac{\partial^2\alpha}{\partial k\partial\delta} = \frac{\gamma^2}{\sigma_r^2 a_P}\frac{2C(1+\delta)-\delta}{(2C(1-\delta)+\delta)^3}C'.$$

From the above expression, and focusing exclusively on the region for which $C' > 0$; it can be deduced that the slope of α is increasing in δ when k is such that:

$$C > \frac{\delta}{2(1+\delta)}.$$

As for β:

$$\frac{\partial(\frac{\partial\beta}{\partial k})}{\partial\delta} = \frac{\partial^2\beta}{\partial k\partial\delta} = \frac{-C'}{a_P(2C(1-\delta)+\delta)^2}.$$

The slope of β is therefore always decreasing in δ.

Finally the joint effect of changes in δ on the slope of $\alpha+\beta$ can be seen through the following derivative:

$$\begin{aligned}\frac{\partial(\frac{\partial(\alpha+\beta)}{\partial k})}{\partial\delta} &= \frac{\partial^2\alpha}{\partial k\partial\delta}+\frac{\partial^2\beta}{\partial k\partial\delta}\\ &= \frac{C'}{a_P(2C(1-\delta)+\delta)^2}\left(\frac{\gamma^2}{\sigma_r^2}\frac{2C(1+\delta)-\delta}{(2C(1-\delta)+\delta)^3}-1\right);\end{aligned}$$

which is positive for values of k such that:

$$C > \frac{1}{2}\frac{\delta(\gamma^2+\sigma_r^2)}{(\gamma^2-\sigma_r^2)+\delta(\gamma^2+\sigma_r^2)}.$$

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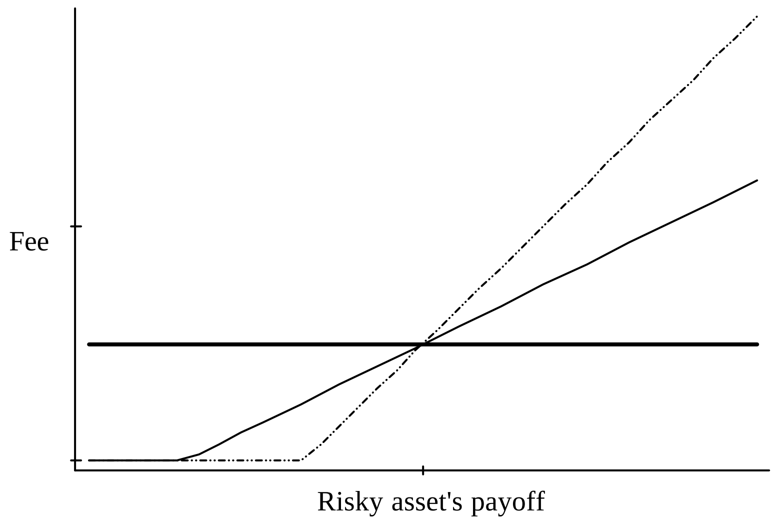

Figure 1. Plot of the fund manager's fee, $k\cdot\max(0; W - W_0)$ –where W_0 and W denote the portfolio's initial and terminal values respectively– as a function of the risky asset's payoff, r, and X, i.e., the number of shares of the risky asset, for $X = 0$ (thick line), $X = X_0 > 0$ (solid line); and $X = 2X_0$ (dashed line). All lines intersect when r equals the payoff of the investment in the riskless asset. For larger values the manager's compensation is increasing in X, whereas the converse is true for smaller values of r.

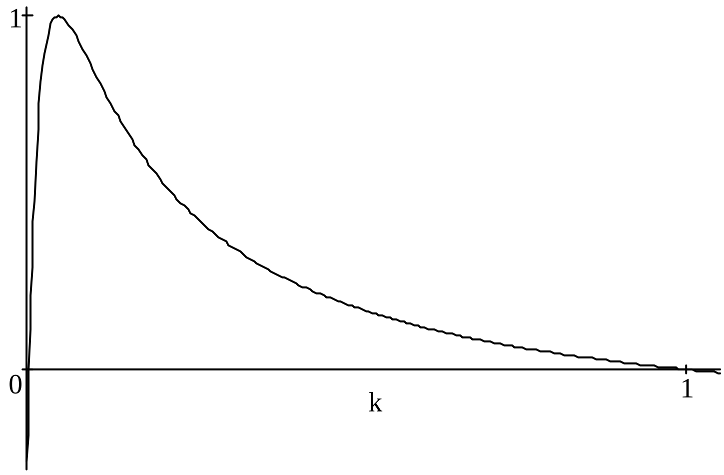

Figure 2. The figure shows the fraction of the investor's optimal certainty equivalent in excess of the certainty equivalent derived from a zero investment in the risky asset, that the investor may obtain as a function of the expense ratio k. The function is one only if $k = k^* = \frac{a_P}{a_A + a_P}$; where a_P and a_A denote the principal's and the agent's risk aversion coefficients. The function is concave for $k < \frac{3}{2}k^*$: