



UNIVERSIDAD CARLOS III DE MADRID

working  
papers

Working Paper 02-60  
Business Economics Series 22  
January 2003

Departamento de Economía de la Empresa  
Universidad Carlos III de Madrid  
Calle Madrid, 126  
28903 Getafe (Spain)  
Fax (34-91) 6249608

## ASSET PRICING AND SYSTEMATIC LIQUIDITY RISK: AN EMPIRICAL INVESTIGATION OF THE SPANISH STOCK MARKET

Miguel A. Martínez<sup>1</sup>, Belén Nieto<sup>2</sup>, Gonzalo Rubio<sup>3</sup> and Mikel Tapia<sup>4</sup>

Abstract

---

It seems reasonable to expect systematic liquidity shocks to affect the optimal behavior of agents in financial markets. Indeed, fluctuations in various measures of liquidity are significantly correlated across common stocks (Chordia, Roll and Subrahmanyam (2000)). Thus, this paper empirically analyzes whether Spanish expected returns during the nineties are associated cross-sectionally to betas estimated relative to two competing liquidity risk factors.

On one hand, we propose a new market-wide liquidity factor which is defined as the difference between returns of stocks highly sensitive to changes in the relative bid-ask spread less returns from stocks with low sensitivities to those changes. We argue that stocks with positive covariability between returns and this factor are assets whose returns tend to go down when aggregate liquidity is low, and hence do not hedge a potential liquidity crisis. Consequently, investors will require a premium to hold these assets. Similarly, note that in the case of assets that covary negatively with the liquidity factor, investors may be willing to pay a premium rather than to require an additional compensation.

On the other hand, Pastor and Stambaugh (2002) suggest that a reasonable liquidity risk factor should be associated with the strength of volume-related return reversals since order flow induces greater return reversals when liquidity is lower.

Our empirical results show that neither of these proxies for systematic liquidity risk carries a premium in the Spanish stock market.

---

Miguel A. Martínez and Gonzalo Rubio acknowledge the financial support provided by Ministerio de Ciencia y Tecnología grant BEC2001-0636. Mikel Tapia acknowledge the financial support provided by Ministerio de Ciencia y Tecnología grant BEC2002-00279.

1. Departamento de Fundamentos del Análisis Económico. II Facultad de Ciencias Económicas y Empresariales. Universidad del País Vasco. Avda. del Lehendakari Aguirre 83 48015 Bilbao. Vizcaya. E-mail: [jepuig@bs.ehu.es](mailto:jepuig@bs.ehu.es)

2. Departamento de Economía Financiera, Contabilidad y Marketing. Carretera San Vicente del Raspeig, s/n, 03690 San Vicente del Raspeig. Alicante. E-mail: [belen.nieto@ua.es](mailto:belen.nieto@ua.es)

3. Departamento de Fundamentos del Análisis Económico. II Facultad de Ciencias Económicas y Empresariales. Universidad del País Vasco. Avda. del Lehendakari Aguirre 83. 48015 Bilbao, Vizcaya. E-mail: [jemasem@bs.ehu.es](mailto:jemasem@bs.ehu.es)

4. Departamento de Economía de la Empresa, Universidad Carlos III de Madrid. Calle Madrid, 126. 28903 - Getafe Madrid. E-mail: [mtapia@emp.uc3m.es](mailto:mtapia@emp.uc3m.es)

## 1. Introduction

The key issue in asset pricing theory is the specific functional form of the stochastic discount factor. In particular, the relevant literature discusses whether the aggregate discount factor is linear or not in alternative state variables, what might be the appropriate number and economic meaning of these competing variables, and what the relevance of idiosyncratic income shocks and incomplete markets might be<sup>1</sup>.

Rather surprisingly, at the same time, asset pricing generally does not deal with the actual mechanisms of the trading process, and how those characteristics affect the price formation of financial assets. One important exception is the literature associated with the liquidity premium of infrequently traded stocks<sup>2</sup>. Closely related is the recent work of Easley, Hvidkjaer and O'Hara (2002) in which the authors study the role of information-based trading in affecting expected stock returns. They argue that stocks with a higher probability of being traded with private information require a compensation in expected returns. They also point out that asymmetric information risk is, in some sense, systematic because traders cannot diversify away the probability of information-based trading simply because they do not know with whom they are trading. In any case, it is not clear how this idiosyncratic characteristic of a given stock can be embedded in the stochastic discount factor. This remains as a clear limitation of this literature.

Interestingly, starting with the papers by Chordia, Roll and Subrahmanyam (2000), and Hasbrouck and Seppi (2001), commonality in liquidity seems to be well documented in the US stock market. In other words, fluctuations in various measures of liquidity are significantly correlated across common stocks. The issue then becomes whether systematic (market-wide) liquidity is priced in the stock market or whether a liquidity risk factor enters the stochastic discount factor as an additional state variable. Indeed, it is reasonable to expect systematic liquidity shocks to affect the optimal behavior of agents given that stocks tend to perform badly in recessions which may, of course, be easily characterized by aggregate liquidity restrictions. Hence, we may expect a higher expected return on stocks

---

<sup>1</sup> Recent surveys may be found in Campbell (2001), Cochrane (2001), and Constantinides (2002).

<sup>2</sup> Classic examples are the papers by Amihud and Mendelson (1986), and Brennan and Subrahmanyam (1996).

highly sensitive to systematic liquidity shocks. As discussed by Pastor and Stambaugh (2002) (P&S hereafter), when investors face an economic recession, and their overall wealth decreases, they may be forced to liquidate some assets to pay for their purchases. Unfortunately, this is relatively more costly when liquidity is lower, particularly when wealth has dropped and marginal utility is higher. Moreover, these effects will be even more pronounced for assets that react strongly to changes in market-wide liquidity crises. Therefore, investors will require a systematic liquidity premium to hold such highly sensitive assets.

Aggregate arguments associated with liquidity restrictions directly related to the discussion above have been put forward by Ericsson and Renault (2000), Holmström and Tirole (2001), Lustig (2001), Acharya and Pedersen (2002), and Domowitz and Wang (2002). These papers develop theoretical arguments implying a covariance between returns and some measure of aggregate liquidity. Their work may be understood as attempts to rationalize the consequences of commonality in liquidity, and to justify the need for empirical research analyzing the impact of aggregate liquidity shocks on asset pricing.

Along these lines, our empirical work analyzes whether Spanish expected returns during the nineties are associated cross-sectionally to betas estimated relative to two competing liquidity risk factors. In particular, we propose a new market-wide liquidity factor which is defined as the difference between returns of stocks highly sensitive to changes in the relative bid-ask spread less returns from stocks with low sensitivities to those changes. We argue that stocks with positive covariability between returns and this factor are assets whose returns tend to go down when aggregate liquidity is low, and hence do not hedge a potential liquidity crisis. Consequently, investors will require a premium to hold these assets<sup>3</sup>. On the other hand, P&S suggest that a reasonable liquidity risk factor should be associated with the strength of volume-related return reversals since order flow induces greater return reversals when liquidity is lower. They show that their aggregate measure seems to be priced in the US market. Unfortunately, our empirical results show that neither of these proxies for systematic liquidity risk carries a premium in the Spanish stock market.

This, of course, does not imply that aggregate liquidity is not a relevant systematic risk factor. Further research is clearly justified.

The rest of the paper is organized as follows. Section 2 briefly describes the data used in this work. Section 3 reports additional evidence on commonality and discusses both our liquidity risk factor and the market-wide measure proposed by P&S. Other results regarding general characteristics of the portfolios employed in our research are also reported. Section 4 contains the empirical results on asset pricing with market-wide liquidity risk factors, and Section 5 concludes.

## **2. Data**

We have individual daily and monthly returns for all stocks trading in the Spanish continuous market from January 1991 through December 2000. The return of the market is an equally-weighted portfolio comprised of all stocks available either in a given month or on a particular day in the sample. The monthly Treasury Bill rate observed in the secondary market is used as the risk-free rate when monthly data is needed. All individual stocks are employed to construct two alternative liquidity-based 10 sorted portfolios<sup>4</sup>, and also the traditional 10 portfolios formed according to market value. Data from portfolios are always monthly returns. For the same set of common stocks we also have daily data on the relative bid-ask spread, depth and the number of shares traded.

Moreover, two additional variables have been used to construct risk factors in different asset pricing models. In particular, for the Fama-French unconditional three factor model, we employ a size proxy and the book-to-market ratio (BM). As a measure of size for each company in a single month we use the logarithm of market value, calculated by multiplying the number of shares of each firm in December of the previous year by their price at the end of each month. To compute the book-to-market ratio for each firm, we employ the accounting information from the balance sheets of each firm at the end of each year. For the

---

<sup>3</sup> Similarly, note that in the case of assets that covary negatively with the liquidity factor, investors may be willing to pay a premium rather than to require an additional compensation.

<sup>4</sup> These are described in the next section.

years as from 1990, this information is provided by the National Security Exchange Commission. The book value for any firm in month  $t$  is given by its value at the end of the previous year, and it remains constant from January to December. The market value is given by total capitalization of each company in the previous month. These data are employed to construct the well-known SMB and HML Fama-French portfolios as is commonly done in literature. For the conditional asset pricing models used in the paper, we propose the aggregate BM ratio as the relevant state variable calculated as the arithmetic mean of the individual BM ratios. Nieto (2002), and Nieto and Rodríguez (2002) show that the aggregate book-to-market is a good predictor of future market returns and, for Spanish data, seems to be superior to the deviations (from the long run equilibrium level) of the consumption to wealth ratio proposed by Lettau and Ludvigson (2001).

### 3. Commonality and Systematic Liquidity

#### 3.1 Brief Evidence on Commonality in Liquidity

Our discussion in the previous section suggests that asset pricing and liquidity have not been properly addressed in the standard literature. We should not regress common stock returns on individual characteristics of liquidity such as the relative bid-ask spread, adverse selection, depth, or probability of information-based trading, but rather on a proxy for a liquidity factor reflecting aggregate (market-wide) liquidity restrictions.

In order to confirm that there exists commonality in liquidity in the Spanish stock market, we regress the monthly percentage change in the relative bid-ask spread for each of the 204 companies available in the sample,  $DSP_{jt}$ , on a cross-sectional equally-weighted average of the same variable representing the market-wide relative spread,  $DSP_{mt}$ ,

$$DSP_{jt} = \mathbf{a}_j + \mathbf{b}_j DSP_{mt} + \mathbf{e}_{jt} \quad (1)$$

The cross-sectional average of the 204 individual coefficients is reported in Table 1. The average sensitivity of changes in the bid-ask spread relative to changes in the aggregate

measure of liquidity is a significant 0.88. Moreover, most of the individual coefficients are positive and significantly different from zero. This indicates that individual liquidity co-moves with market liquidity, and that commonality in liquidity exists in the Spanish market.

### 3.2 Liquidity Risk Factors

#### A. The Pastor and Stambaugh Factor (OFL)

The market-wide liquidity factor proposed by P&S in a given month is obtained as the equally-weighted average of the liquidity measures of individual stocks which are calculated with daily return and volume data within that particular month. Hence, for a given month  $t$  and a security  $j$  we perform the following OLS regression using daily data as long as the stock has at least 15 observations in that month:

$$R_{jd+1t}^e = a_{jt} + b_{jt} R_{jdt} + \mathbf{I}_{jt} \text{sign}(R_{jdt}^e) \text{vol}_{jdt} + u_{jd+1t} \quad (2)$$

where  $R_{jd+1t}^e$  is the return on stock  $j$  on day  $d+1$  (in month  $t$ ) minus the market return on the same day,  $R_{jdt}$  is the return on stock  $j$  on day  $d$ , and  $\text{vol}_{jdt}$  is the euro volume for stock  $j$  on day  $d$  in month  $t$ . The key coefficient is the sensitivity of the percentage price change of stock  $j$  on day  $t+1$  to the order flow on  $t$ , constructed as the volume signed by the returns on the stock minus the return on the market. The larger the sensitivity coefficient,  $\mathbf{I}_{jt}$ , the less liquid the stock will be. The basic idea is that a financial market may be considered liquid if it is able to quickly absorb or accommodate large amounts of trading without distorting prices.

To further understand this measure, assume that there exists selling pressure by non-informational investors. The market maker (or agents sending limit orders to the book) tries to accommodate this desire to sell by lowering the price at which he is willing to buy (bid), and increasing the price at which he is willing to sell (ask). Of course to provide this liquidity service, the market maker requires an additional compensation reflected in the

lower buying price and, therefore, in terms of higher expected return<sup>5</sup>. At the same time, it is important to note that bid-ask spread increases which implies a lower liquidity on that stock. The larger the order flow (volume signed), the larger the increase in the expected return will be. This idea is captured by the regression above by noting that the selling pressure on stock  $j$  makes  $R_{jdt}^e$  negative, and therefore inducing a negative relationship between the explanatory variable and  $R_{jd+It}^e$ . Hence, a large order flow (volume signed) induces greater return reversals precisely when liquidity is lower. It is an interesting measure of liquidity because it captures not only the idea of changing bid-ask spreads (higher spread and lower liquidity), but also the amount of volume needed to change the stock price marginally.

Alternatively, the measure may be associated with the recent literature on order imbalances, defined as the number of buyer-initiated trades less the number of seller-initiated trades, and inventory control<sup>6</sup>. After a public sale (purchase) at the bid (ask), the market maker lowers (raises) the bid (ask) relative to the fundamental value. This is so because he wants to increase the probability of a subsequent public purchase (sale), and the market maker is then compensated for inventory risk because the expected midquote change is positive after a market maker sale and negative after a purchase. Therefore, a negative serial correlation in trades is induced. Moreover, given that Chordia, Roll and Subrahmanyam (2002) find a strong positive correlation coefficient between order imbalance and market returns, we should find that, subsequent (in the following day) to a selling pressure, the stock return should increase on average. This is precisely the induced-liquidity reversal suggested by the measure proposed by P&S.

Once the individual coefficients have been estimated for each stock and each month in the sample, P&S calculate the cross-sectional average of the estimates as

---

<sup>5</sup> In fact, their reasoning is based on the ideas suggested by Campbell, Grossman and Wang (1993).

<sup>6</sup> See Chordia, Roll and Subrahmanyam (2002).

$$\mathbf{I}_t = \frac{1}{N} \sum_{j=1}^N \mathbf{I}_{jt} \quad (3)$$

We obtain the same measure for our 204 stocks from January 1991 to December 2000<sup>7</sup>. It is important to note that the measure depends on the magnitude of the market. Following P&S and to avoid these effects, we calculate the scaled series  $(m_t/m_l)\mathbf{I}_t$ , where  $m_t$  is the total euro value at the end of month  $t-1$  of the stocks included in the IBEX-35 index, and month  $l$  corresponds to December 1990. The objective is to evaluate the scale of the shocks in liquidity on stock returns. Therefore, we really need the covariance between returns and unanticipated innovations in liquidity. Thus, we first take the difference of the scaled series as the measure of liquidity:

$$D\hat{\mathbf{I}}_t = \left( \frac{m_t}{m_l} \right) (\hat{\mathbf{I}}_{jt} - \hat{\mathbf{I}}_{jt-1})$$

Then, we perform the regression of  $D\hat{\mathbf{I}}_t$  on its lag as well as the lagged value of the scaled series:

$$D\hat{\mathbf{I}}_t = c + dD\hat{\mathbf{I}}_{t-1} + e \left( \frac{m_t}{m_l} \right) \hat{\mathbf{I}}_{t-1} + \mathbf{e}_t \quad (4)$$

It should be pointed out that if expected changes in liquidity are correlated with time-varying expected returns, then employing liquidity innovations avoids the potential contamination in the usual risk (covariances) measures. The final systematic liquidity factor is taken as the fitted residual of (4) scaled by  $10^9$  simply to obtain more convenient magnitudes of the liquidity market-wide factor<sup>8</sup>

$$OFL_t = \hat{\mathbf{e}}_t \times 10^9 \quad (5)$$

---

<sup>7</sup> Not all stocks are available throughout the period.



### ***B. The Martínez-Nieto-Rubio-Tapia Factor (HLS)***

The basic idea behind this factor is to form a portfolio as the difference between the returns on a long position on assets especially sensitive to changes in the relative bid-ask spread and a short position on assets with the lowest sensitivity. This is similar in spirit to the Fama-French factors, and to the market risk factor understood as the difference between the return on the risky assets (the market portfolio) and the risk-free rate.

In particular, we estimate how sensitive each asset is to variations in relative spread in the sample<sup>9</sup>:

$$R_{jt} = a_j + b_j DSP_{jt} + u_{jt} \quad (6)$$

Assets are classified in three blocks according to their sensitivities: high, medium and low sensitivity to spread variations. This ranking is changed every month according to their sensitivities over the previous 36 months in the sample. For each block (and each month), equally-weighted portfolios are formed using the assets that belong to each block. We have monthly time-series of three equally-weighted portfolios (HS, MS, LS) between January 1992 and December 2000. The liquidity factor is defined as the difference between the returns of the high and the low sensitivity portfolio returns:  $HLS = HS - LS$ .

The intuition behind our proposal is the existing negative covariance between market returns and credit (liquidity) restrictions. The 1929 and 1987 crashes, the Russian debt crisis and its effects on the Long Term Capital Management hedge fund, the recent Asian financial crisis or the strong negative shock on liquidity on September 11<sup>th</sup> are all excellent examples. It is always the case that the larger the restriction in liquidity is the lower the market return is. At the same time, the covariance between changes in the average aggregate bid-ask spread and the market return is negative<sup>10</sup>. Both results imply a positive covariance between liquidity restrictions and changes in the bid-ask spread, both in

---

<sup>8</sup> The scale factor is different from P&S given that volume is measured in euros, while in the case of P&S it is in millions of dollars.

<sup>9</sup> Similar results are obtained when we regress on changes in the average market-wide bid-ask spread. The empirical results reported in the paper are all based on expression (6).

aggregate and individually<sup>11</sup>. This reasoning justifies our classifying assets according to the slope coefficient of equation (6) to construct our systematic liquidity factor. Accordingly, the relevant covariance for analyzing the sensitivity between returns and liquidity shocks is the covariance between stock returns and changes in the bid-ask spread.

It is important to note that assets with low sensitivity to changes in the bid-ask spread (LS) are those whose returns diminish relatively little when the change in the bid-ask spread goes up. On the other hand, highly sensitive stocks (HS) tend to have returns which go down by a relatively large amount when the spread increases. This implies that the portfolio returns of assets with high sensitivity minus low sensitivity, our HLS factor, must necessarily go down when changes in the spread increase (less liquidity in the market as a whole). Hence, negative liquidity shocks imply that the returns associated with our systematic liquidity factor will tend to go down. Thus, stocks with positive covariances between their returns and the HLS factor are assets whose returns tend to decrease when market-wide liquidity is lower. These assets are not able to hedge negative liquidity shocks, and investors will require an additional premium to hold them. On average, we would expect a positive relationship between average returns and liquidity betas.

### ***3.3 Some preliminary empirical evidence***

We first calculate the usual descriptive statistics of the factors employed in this research. Table 2 reports the average characteristics of the distribution of the market return factor, the Fama-French factors, and the two liquidity-based systematic factors. The latter present left-skewed distributions with rather large excess kurtosis, at least relative to the other factors. Interestingly, the OFL factor is much more volatile than the HLS market-wide measure. The correlation coefficients between them all tend to be low, although OFL has a relatively high positive correlation with the SMB factor proposed by Fama-French. Finally, the

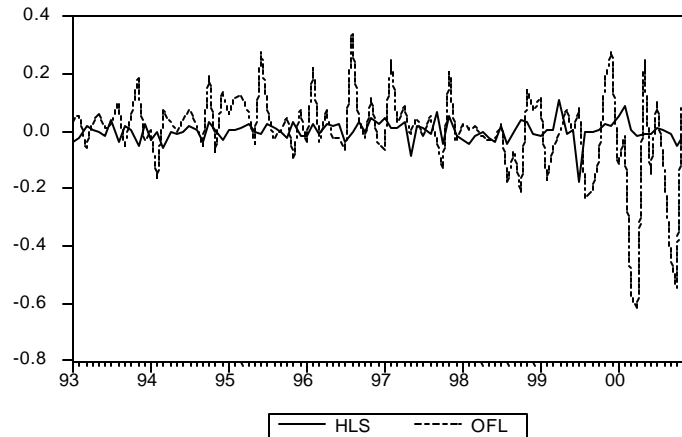
---

<sup>10</sup> The correlation coefficient between these variables over the nineties turned out to be  $-0.348$ .

<sup>11</sup> In fact, the correlation coefficient between changes in the risk-free rate and the spread over our sample period was  $0.157$ .

market return is more correlated with HLS than with OFL. Figure 1 plots the HLS and OFL factors. Note the large volatility impounded in the OFL systematic liquidity measure<sup>12</sup>.

**FIGURE 1**  
**Systematic Liquidity Factors: HLS vs. OFL: 1993-2000**



We construct 10 size sorted portfolios according to the market value of each security at the end of each year, named MV1 (smallest) to MV10 (largest), and 10 liquidity-based sorted portfolios, ranking stocks with respect to the liquidity betas they have in terms of both the HLS and OFL factors. These betas are estimated with 36 past observations, and stocks are assigned to a given portfolio at the end of every month in the sample on the basis of the estimated beta coefficient<sup>13</sup>. The average return and volatility of these portfolios are shown in Table 3. These are the portfolio returns which will be employed in testing the liquidity-based asset pricing models in next section. As expected, the smallest stocks have the largest volatility, and there is also a tendency towards lower volatility the larger the stocks included in the portfolios. The volatility of the liquidity-based portfolios is higher in the extreme ones. Both highly sensitive and relatively insensitive stocks to changes in the bid-ask spread tend to have the largest volatility. This is in itself an interesting finding that

<sup>12</sup> Note, on the other hand, that the volatility of replicating factors such as SMB, HML or HLS is always relatively low given the way these factors are constructed as portfolios of long versus short positions on financial assets.

<sup>13</sup> The month in which we classify stocks is also included in the estimation of the liquidity betas.

deserves further attention. In terms of average returns, the volatility pattern is reproduced for the HLS portfolios, but not for the OFL classification. In fact, contrary to the findings of P&S, OFL1 (low sensitivity stocks) have a much larger average return than OFL10 (high sensitivity stocks). The liquidity-based betas follow precisely the pattern expected given the ranking of the individual stocks, although they seem to be estimated with much more precision when the HLS factor is used in the estimation. Finally, large stocks have a relatively high and significant HLS beta, but much lower OFL betas.

In order to confirm the commonality of liquidity reported with individual stocks in Table 1, we perform a similar regression with our 10 size and liquidity-based sorted portfolios. The results are shown in Table 4. At the portfolio level there is even stronger evidence of co-movements between changes in the portfolio spreads and the market-wide spreads. Basically all slope coefficients of regression (1) are positive and significantly different from zero. Moreover, the average slope coefficients for the first five portfolios (portfolios 6 through 10) are 1.489 (0.584), 1.044 (0.722), and 1.140 (0.619) for the HLS, OFL and MV portfolios respectively. In other words, low sensitivity stocks tend to co-vary much more with respect to market-wide liquidity as measured by changes in the average bid-ask spread of the market as a whole. It is important to realize that this is expected given that their returns are negatively correlated with changes in the bid-ask spread. Finally, changes in the bid-ask of small stocks are much more sensitive to changes in the overall bid-ask spread than those of large stocks.

#### **4. Asset Pricing and Systematic Liquidity: The Empirical Evidence**

##### ***4.1 Alphas and Asset Pricing Models***

One way of testing the asset pricing models described in this paper is to note that if the liquidity risk factors are priced in the market, we should find systematic differences in the risk-adjusted average returns of our liquidity-beta-sorted portfolios. In other words, for a given asset pricing model, the risk-adjusted average return (alpha) of the HLS10 portfolio should be significantly higher than the alpha for the HLS1 portfolio. The same results should be observed for the OFL liquidity portfolios as long as the market prices market-

wide liquidity risk. This is the approach followed by P&S. They find that average risk-adjusted returns of stocks with high sensitivity to liquidity exceed those for stocks with low sensitivity by 7.5% on annual basis when a four-factor asset pricing model is employed in the estimation<sup>14</sup>. Indeed, P&S interpret the result as the average liquidity premium existing in the US market between 1966 and 1999. It should be pointed out that this is an extremely high liquidity premium.

Unfortunately, when we follow the same testing strategy, our results are dramatically different as reported in Table 5. We employ four alternative pricing models. The traditional CAPM, the three-factor Fama-French model, and the two CAPM liquidity-based models discussed in this paper in which we add the liquidity factor (either HLS or OFL) to the usual CAPM model. We report the differences in alphas between January 1993 and December 2000 on an annual basis. None of the models seems to indicate that there exists a liquidity premium. In fact, regardless of which model is considered, we do not find significant differences for HLS and size-sorted portfolios. This suggests that there was neither a liquidity nor a size-related premium in the Spanish market in the nineties. It is relevant to point out that adding the OFL factor to the CAPM does not seem to have any effect on the results. For examples the differences in alphas between portfolios HLS10 and HLS1 is  $-1.78$  for the CAPM, and  $-1.74$  for the CAPM once the OFL factor has been added to the model. The liquidity-based CAPM when we use our HLS systematic liquidity factor has a stronger impact on the result. The alpha is now just  $-0.02$ . In any case, as observed above, none of the differences is significantly different from zero. However, the striking result is the negative and significant difference between the alphas of the extreme portfolios when we use the ranking based on the OFL factor. This is consistent with the result already reported in Table 3. Stocks very highly sensitive to the OFL factor tend to strongly underperform stocks with low sensitivity to the OFL factor. Of course, this is very disturbing evidence for the liquidity-based model proposed by P&S.

---

<sup>14</sup> The three Fama -French factor plus a momentum factor. In any case, regardless of which model is used, they

## 4.2 Cross-Sectional Evidence

We now perform empirical tests for both the conditional and unconditional liquidity-based asset pricing models using the two systematic factors described in the paper.

### A. General Asset Pricing Framework

The fundamental equation of asset pricing is usually written as:

$$E_{t-1} \left[ M_t (1 + R_{jt}) \right] = 1; \quad j = 1, \dots, N \quad (7)$$

where  $M_t$  is the stochastic discount factor. Let  $R_{mt}$  be the return on the true mean-variance efficient portfolio. Then, we know that the discount factor under the liquidity-based unconditional pricing model is given by:

$$M_t = \mathbf{d}_0 + \mathbf{d}_1 R_{mt} + \mathbf{d}_2 L_t \quad (8)$$

where  $\mathbf{d}_0$ ,  $\mathbf{d}_1$  and  $\mathbf{d}_2$  are three constants, and  $L_t$  is the replicating liquidity portfolio as given by either HLS or OFL. On the other hand, the conditional version may be written as,

$$M_t = \mathbf{d}_{0t-1} + \mathbf{d}_{1t-1} R_{mt} + \mathbf{d}_{2t-1} L_t \quad (9)$$

where  $\mathbf{d}_{0t-1}$ ,  $\mathbf{d}_{1t-1}$  and  $\mathbf{d}_{2t-1}$  are now allowed to vary over time.

Given that the true conditional distribution is unobservable, we merely assume (as usual in conditional asset pricing literature) a linear relationship between the parameters  $\mathbf{d}_{0t-1}$ ,  $\mathbf{d}_{1t-1}$ ,  $\mathbf{d}_{2t-1}$  and a time  $t-1$  information variable,  $Z_{t-1}$ , where  $Z_{t-1}$  is a predicting variable for returns given by the (log) aggregate book-to-market ratio,  $bm_{t-1}$ :

$$\mathbf{d}_{it-1} = \mathbf{d}_i + \mathbf{d}_{i1} bm_{t-1} \quad ; \quad i = 0, 1, 2 \quad (10)$$

---

find a very similar evidence.

where  $\mathbf{d}_i$  and  $\mathbf{d}_{i1}$  are constants. Plugging (10) into (9), we transform the conditional liquidity-based asset pricing model into the so called *scaled liquidity-based asset pricing model*<sup>15</sup>, which is simply an unconditional multifactor model with constant coefficients.

The fundamental asset pricing equation is then,

$$E_{t-1} \{ [\mathbf{d}_0 + \mathbf{d}_{01}bm_{t-1} + \mathbf{d}_1R_{mt} + \mathbf{d}_{11}(bm_{t-1}R_{mt}) + \mathbf{d}_2L_t + \mathbf{d}_{21}(bm_{t-1}L_t)](1 + R_{jt}) \} = 1 \quad (11)$$

The corresponding beta coefficients are given by

$$\mathbf{b}_{jm} = \frac{\text{cov}(R_{jt}, R_{mt})}{\text{var}(R_{mt})} \quad (12)$$

$$\mathbf{b}_{jbm} = \frac{\text{cov}(R_{jt}, bm_{t-1})}{\text{var}(bm_{t-1})} \quad (13)$$

$$\mathbf{b}_{jmbm} = \frac{\text{cov}(R_{jt}, bm_{t-1}R_{mt})}{\text{var}(bm_{t-1}R_{mt})} \quad (14)$$

$$\mathbf{b}_{jL} = \frac{\text{cov}(R_{jt}, L_t)}{\text{var}(L_t)} \quad (15)$$

$$\mathbf{b}_{jLbm} = \frac{\text{cov}(R_{jt}, bm_{t-1}L_t)}{\text{var}(bm_{t-1}L_t)} \quad (16)$$

The asset pricing model (11) can be written in the traditional multi-beta representation as:

$$E(R_j) = \mathbf{g}_0 + \mathbf{g}_1\mathbf{b}_{jm} + \mathbf{g}_2\mathbf{b}_{jbm} + \mathbf{g}_3\mathbf{b}_{jmbm} + \mathbf{g}_4(6)\mathbf{b}_{jL} + \mathbf{g}_5(7)\mathbf{b}_{jLbm} \quad (17)$$

where  $\mathbf{g}_4$  and  $\mathbf{g}_5$  correspond to HLS, and  $\mathbf{g}_6$  and  $\mathbf{g}_7$  to OFL. This will be the basic model employed in our empirical exercise to test the competing liquidity-based pricing models.

### ***B. Empirical Evidence***

The results are reported for three alternative set of portfolios. In particular, as the dependent variable in the Fama-MacBeth monthly regressions, we employ the 10 portfolios constructed on the basis of sensitivity to liquidity and the traditional 10 size-sorted portfolios. The results are quite similar independently of the endogenous variable used in the test. As expected, the liquidity premium associated with the HLS factor is always positive but, unfortunately, it is never significantly different from zero. The results relative to the OFL factor are even more disappointing because, except for the size-sorted portfolios, the liquidity premium has the wrong sign. This is consistent with the differences in alphas found in Table 5.

It seems possible to conclude that in the Spanish market, at least in the nineties, there was not a significant liquidity premium. From our point of view, it may be important to further analyze the potential interpretations that current research gives to the systematic liquidity factor. It seems reasonable to expect more theory to be developed before making conclusions on a particular liquidity-based market-wide factor. None of the factors analyzed in this paper seems to be priced in the Spanish market, despite the success of the P&S factor when US market data is employed. Of course, the results should be interpreted with care given the short period of time covered by this research. Unfortunately, the Spanish continuous market did not start trading until 1989. Given the design employed in any asset pricing work, where key parameters are estimated with relatively long series of past data, we are forced to use monthly data only from 1993 to 2000 in our tests of the asset pricing models. This may be considered to be short for a paper of these characteristics. On other hand, testing a model like the one proposed by P&S with an alternative database and making comparisons with competing liquidity factors seems to be a crucial step in this type of research.

---

<sup>15</sup> See Cochrane (2001).



There are, however, other interesting results in Table 6. When the OFL portfolios are used, the market risk premium becomes positive and significant. Although not reported, the estimate of the market premium in the traditional CAPM context is 1.10 with a t-value of 0.13. This implies a tremendous amount of noise in the estimated coefficient. When the HLS factor is added, the market risk premium becomes large and significant. Moreover, under the HLS portfolios, the t-statistic associated with the estimated market risk premium becomes 1.40 when the betas with respect to the HLS factor are employed in the cross-sectional regressions.

Finally, in terms of the conditional version of the models and for the HLS factor, both the market risk premium and the liquidity premium have the correct positive sign, but we can never statistically reject their being zero. The liquidity premium, in all cases, becomes much larger in magnitude than the one obtained under the unconditional version. However, they are all estimated with too much noise and, consequently, the liquidity premium is never significantly different from zero. Interestingly, the coefficient associated with the instrument is always negative and highly significant. The book-to-market ratio predicts returns in a positive fashion. Moreover, any increase in BM suggests that market prices go down relatively more than book values. This fall represents bad news for the market. Hence, assets whose returns co-vary positively with the (lagged) BM ratio tend to pay when marginal utility is high (financial wealth low), and therefore investors should be “willing” to pay to invest in those stocks. This explains the negative sign of the BM beta, and it points toward a relevant state variable in asset pricing. It should be recalled that the BM ratio plays a similar role to the consumption-wealth ratio of Lettau and Ludvigson in the US market<sup>16</sup>.

---

<sup>16</sup> See Nieto and Rodríguez (2002).

## **5. Conclusions**

Market-wide liquidity should be a key ingredient of asset pricing models. If macroeconomic variables anticipate economic recessions, they may also anticipate lower aggregate liquidity. Indeed, the surprising success of the Fama-French HML risk factor may be explained by a state variable closely related to systematic liquidity. The HML factor is usually associated with a distress factor not theoretically identified. Taking into account that risky assets compensate not only beta risk but also the fact that they have a particularly poor performance in recession, it seems plausible to think of systematic liquidity as the missing factor. Unfortunately, none of the market-wide liquidity factors analyzed in this paper seems to be priced in the Spanish market. Thus, in our database, expected stock returns are not related cross-sectionally to betas of returns to innovations in aggregate liquidity. It seems to be the case that stocks that are more sensitive to systematic liquidity do not have statistically higher expected returns either conditionally or unconditionally. However, a predicting state variable such as the book-to-market ratio plays a relevant role in explaining average returns in Spain.

## REFERENCES

- Acharya, V., and L. Pedersen (2002). "Asset Pricing with Liquidity Risk", Working Paper, London Business School.
- Amihud, Y., and H. Mendelson (1986). "Asset Pricing and the Bid-Ask Spread", *Journal of Financial Economics* 17, 223-249.
- Brennan, M., and M. Subrahmanyam (1996). "Market Microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns", *Journal of Financial Economics* 41, 441-464.
- Campbell, J. (2001). "Asset pricing at the Millennium", *Journal of Finance* 55, 1515-1567.
- Campbell, J., Grossman, S., and J. Wang (1993). "Trading Volume and Serial Correlation in Stock Returns", *Quarterly Journal of Economics* 108, 905-939.
- Chordia, T, Roll, R., and A. Subrahmanyam (2000), "Commonality in Liquidity", *Journal of Financial Economics* 56, 3-28.
- Chordia, T, Roll, R., and A. Subrahmanyam (2002), "Order Imbalance, Liquidity, and Market Returns", *Journal of Financial Economics* 65, 111-130.
- Cochrane, J. (2001). "Asset Pricing", Princeton University Press, Princeton, NJ.
- Constantinides, G. (2002). "Rational Asset Prices", *Journal of Finance* 57, 1567-1591.
- Domowitz, I., and X. Wang (2002). "Liquidity, Liquidity Commonality and Its Impact on Portfolio Theory", Working Paper, Penn State University.

Easley, D., Hvidkjaer, S., and M. O'Hara (2002). "Is Information Risk a Determinants of Asset Returns?", forthcoming in the Journal of Finance.

Ericsson, J., and O. Renault (2000). "Liquidity and Credit Risk", Working Paper, McGill University, Montreal.

Hasbrouck, J., and D. Seppi (2001). "Common Factors in Prices, Order Flows, and Liquidity", Journal of Financial Economics 59, 383-411.

Holmström, B., and J. Tirole (2001). "LAPM: A Liquidity-Based Asset Pricing", Journal of Finance 56, 1837-1867.

Lettau, M., and S. Ludvigson (2001). "Consumption, Aggregate Wealth, and Expected Stock Returns, Journal of Finance 56, 815-849.

Lustig, H. (2001). "The Market Price of Aggregate Risk and the Wealth Distribution", Working Paper, Stanford University.

Nieto, B. (2002). "La Valoración Intertemporal de Activos: Un Análisis Empírico para el Mercado Español de Valores", forthcoming in Investigaciones Económicas.

Nieto, B., and R. Rodríguez (2002). "Consumption-Wealth Ratio, Book-To-Market Ratio, and Asset Pricing", Working Paper, Universidad de Alicante, X Foro de Finanzas, Sevilla.

Pastor, L., and R. Stambaugh (2002). "Liquidity Risk and Expected Stock Returns", forthcoming in the Journal of Political Economy.

**TABLE 1**  
**MARKET-WIDE COMMONALITY IN LIQUIDITY (Relative Spread)**  
**1991-2000**  
**(average figures for 204 stocks)**

$$DSP_{jt} = a_j + b_j DSP_{mt} + e_{jt}$$

where  $DSP_{jt}$  is the percentage change from month t-1 to t in liquidity as proxied by the relative spread of stock j, and  $DSP_{mt}$  is the concurrent change in a cross-sectional average of the same variable or the market-wide (equally-weighted) relative spread

	Avg. Alpha	Avg. Beta	Avg. R <sup>2</sup>	Avg. Adj R <sup>2</sup>
Coefficient (avg. t-stat)	0.0886 (1.031)	0.8815 (2.639)	0.1377	0.1091
% Positive	-	94.1	-	-
% + significant	-	63.2	-	-

**TABLE 2**  
**SUMMARY STATISTICS FOR RISK FACTORS**  
**1993-2000**

The numbers represent the average annualized average returns, volatilities, skewness, excess kurtosis and correlation coefficients of alternative risk factors employed in the paper. *RM* is the equally-weighted market portfolio, *SMB* is the Fama-French size related factor, *HML* is the Fama-French book-to-market related factor, *HLS* is the liquidity factors associated with sensitivities to the bid-ask spread, and *OFL* is the liquidity factor based on the order flow inducing greater return reversals when liquidity is lower. The figures are obtained from monthly returns from January 1993 to December 2000.

-----					
PANEL A					
Risk Factor	Average Return	Volatility	Skewness	Excess Kurtosis	
-----					
RM	19.53	18.75	0.765	1.242	
SMB	-0.69	13.15	0.656	0.224	
HML	1.48	11.00	0.493	0.140	
HLS	-0.04	12.38	-1.009	6.043	
OFL	-1.17	25.20	-1.389	4.122	
-----					
PANEL B					
Correlation Coefficients					
-----					
	RM	SMB	HML	HSL	OFL
RM	1.000	0.066	-0.097	0.160	0.069
SMB		1.000	0.094	0.139	0.218
HML			1.000	-0.042	0.094
HLS				1.000	0.067
OFL					1.000
-----					

**TABLE 3**  
**SUMMARY STATISTICS FOR PORTFOLIOS**  
**1993-2000**

The summary statistics represent the time-series annualized averages of returns, volatilities, and factor betas of three differently sorted portfolios according to (i) the sensitivities of returns with changes in the relative bid-ask spread (HLS); (ii) the sensitivities of returns to fluctuations in aggregate liquidity as measured by order flow inducing greater return reversals when liquidity is lower (OFL); market capitalization (MV) from January 1993 to December 2000. HLS1, OFL1 and MV1 represent low sensitivity, and small market value respectively.

Portfolios	Average Return	Volatility	HLS Beta (t-stat)	OFL Beta (t-stat)
HLS1	20.84	26.43	-0.583 (-2.75)	-
HLS2	17.14	19.85	-0.074 (-0.45)	-
HLS3	19.68	16.74	-0.053 (-0.38)	-
HLS4	14.00	15.97	0.049 ( 0.40)	-
HLS5	17.23	19.19	0.117 ( 0.73)	-
HLS6	15.72	18.78	0.219 ( 1.41)	-
HLS7	21.62	19.94	0.376 ( 2.33)	-
HLS8	19.06	21.16	0.446 ( 2.56)	-
HLS9	16.62	21.07	0.606 ( 3.70)	-
HLS10	21.77	26.60	0.855 ( 4.21)	-
OFL1	27.13	25.07	-	-0.067 (-1.45)
OFL2	20.57	18.71	-	-0.022 (-0.63)
OFL3	17.00	19.60	-	-0.011 (-0.31)
OFL4	20.94	18.30	-	0.011 ( 0.33)
OFL5	19.96	18.17	-	0.016 ( 0.47)
OFL6	12.49	16.66	-	0.017 ( 0.57)
OFL7	18.38	21.65	-	0.030 ( 0.73)
OFL8	15.92	20.69	-	0.043 ( 1.12)
OFL9	23.45	21.92	-	0.042 ( 1.03)
OFL10	13.93	24.03	-	0.093 ( 2.12)
MV1	26.59	33.48	0.092 (0.33)	0.023 (0.33)
MV2	22.33	24.93	0.192 (0.93)	0.019 (0.41)
MV3	12.81	21.11	0.299 (1.72)	0.002 (0.06)
MV4	18.75	21.88	0.097 (0.53)	0.012 (0.28)
MV5	20.45	18.69	0.231 (1.50)	0.033 (0.94)
MV6	15.20	17.91	0.180 (1.21)	0.005 (0.14)
MV7	17.59	16.64	0.146 (1.06)	0.019 (0.61)
MV8	19.77	17.91	0.320 (2.20)	0.020 (0.59)
MV9	20.30	18.46	0.362 (2.43)	0.034 (1.00)
MV10	19.78	19.33	0.523 (3.45)	0.063 (1.78)

**TABLE 4**  
**MARKET-WIDE COMMONALITY IN LIQUIDITY (Relative Spread)**  
**PORTFOLIOS BY AGGREGATE LIQUIDITY AND MARKET VALUE**  
**1993-2000**

$$DSP_{jt} = a_j + b_j DSP_{mt} + e_{jt}$$

where  $DSP_{jt}$  is the percentage change from month t-1 to t in liquidity as proxied by the relative spread of portfolio j, and  $DSP_{mt}$  is the concurrent change in a cross-sectional average of the same variable or the market-wide (equally-weighted) relative spread. This table reports results from three differently sorted portfolios according to (i) the sensitivities of returns with changes in the relative bid-ask spread (HLS); (ii) the sensitivities of returns to fluctuations in aggregate liquidity as measured by order flow inducing greater return reversals when liquidity is lower (OFL); market capitalization (MV) from January 1993 to December 2000. HLS1, OFL1 and MV1 represent low sensitivity and small market value respectively.

Portfolios	Slope Coefficient	t-statistic	Adjusted R <sup>2</sup>
HLS1	1.899	(7.30)	0.357
HLS2	1.434	(1.98)	0.090
HLS3	1.004	(4.45)	0.167
HLS4	1.620	(5.11)	0.211
HLS5	1.489	(4.04)	0.140
HLS6	0.839	(2.38)	0.047
HLS7	0.552	(1.21)	0.050
HLS8	0.669	(4.20)	0.150
HLS9	0.328	(1.98)	0.030
HLS10	0.530	(5.59)	0.243
OFL1	0.879	(3.99)	0.137
OFL2	1.034	(5.98)	0.270
OFL3	1.296	(4.71)	0.184
OFL4	1.349	(3.50)	0.107
OFL5	0.660	(2.25)	0.042
OFL6	0.736	(1.98)	0.030
OFL7	0.744	(3.56)	0.120
OFL8	0.434	(2.10)	0.035
OFL9	0.929	(4.65)	0.180
OFL10	0.765	(5.00)	0.203
MV1	1.266	(5.61)	0.245
MV2	1.001	(5.55)	0.241
MV3	0.915	(3.15)	0.087
MV4	1.401	(3.62)	0.114
MV5	1.119	(6.22)	0.286
MV6	0.896	(5.54)	0.240
MV7	1.269	(6.33)	0.294
MV8	0.595	(7.65)	0.379
MV9	0.235	(2.30)	0.044
MV10	0.100	(5.52)	0.239



**TABLE 5**  
**DIFFERENCES BETWEEN ALPHAS OF EXTREME PORTFOLIOS SORTED**  
**ON AGGREGATE LIQUIDITY AND MARKET VALUE**  
**1993-2000**

This table reports the differences in percent per year between estimated alphas based on four asset pricing models; CAPM alphas, Fama-French alphas, and two liquidity-based asset pricing model alphas. We form three differently sorted portfolios according to (i) the sensitivities of returns with changes in the relative bid-ask spread (HLS); (ii) the sensitivities of returns to fluctuations in aggregate liquidity as measured by order flow inducing greater return reversals when liquidity is lower (OFL); market capitalization (MV) from January 1993 to December 2000. HLS1, OFL1 and MV1 represent low sensitivity and small market value respectively.

	alphaHLS10-alphaHLS1			alphaOFL10-alphaOFL1			alphaMV10-alphaMV1		
	Value <sup>1</sup>	$\chi^2$ -test <sup>2</sup>	p-value	Value	$\chi^2$ -test	p-value	Value	$\chi^2$ -test	p-value
CAPM alpha	-1.78	0.048	0.826	-13.86	4.425	0.035	0.30	0.002	0.968
Fama-French alpha	-1.14	0.020	0.889	-13.40	4.086	0.043	0.86	0.015	0.903
CAPM+HLS alpha	-0.02	0.000	0.998	-13.59	4.274	0.039	1.06	0.022	0.883
CAPM+OFL alpha	-1.74	0.045	0.831	-13.37	4.627	0.032	0.45	0.004	0.951

1 In percent per year

2  $\chi^2$ -test of equality between means

**TABLE 6**  
**CROSS-SECTIONAL UNCONDITIONAL AND CONDITIONAL ASSET PRICING MODEL TESTS**  
**1993-2000**

This table contains the time series averages of the monthly coefficients in cross-sectional asset pricing tests using standard Fama-MacBeth methodology. The dependent variable is the monthly return on three differently sorted portfolios according to (i) the sensitivities of returns with changes in the relative bid-ask spread (HLS); (ii) the sensitivities of returns to fluctuations in aggregate liquidity as measured by order flow inducing greater return reversals when liquidity is lower (OFL); market capitalization (MV) from January 1993 to December 2000. The explanatory variables are the betas of the different factors estimated (when possible) with the 35 previous monthly returns to each cross-sectional estimation and the corresponding month itself for a total of 36 observations in each regression. The conditioning variable is the (log) of the aggregate book-to-market ratio, and the models are the standard CAPM and two liquidity-based asset pricing models. The cross-sectional regressions for each month take the following form:

$$R_j = g_0 + g_1 b_{jm} + g_2 b_{jbm} + g_3 b_{jmbm} + g_4 b_{jHLS} + g_5 b_{jHLSbm} + g_6 b_{jOFL} + g_7 b_{jOFLbm} + h_j$$

-----  
**PANEL A: 10 HLS PORTFOLIOS**  
 -----

$g$	$g$	$g$	$g$	$g$	$g$	$g$	$g$	$g$	$R^2$
0.2288 (0.42)	1.0775 (1.40)	-	-	0.0448 (0.12)	-	-	-	-	41.9
0.1607 (0.24)	1.1705 (1.30)	-	-	-	-	-5.258 (-1.40)	-	-	33.4
1.2778 (2.02)	0.1881 (0.22)	-22.160 (-3.44)	-0.3063 (-0.79)	0.5895 (1.11)	0.1904 (0.62)	-	-	-	71.1
1.0058 (1.65)	0.3353 (0.38)	-12.567 (-2.25)	-0.2141 (-0.55)	-	-	-5.5506 (-1.14)	1.0186 (0.78)	-	69.7

-----  
**PANEL B: 10 OFL PORTFOLIOS**  
 -----

$g$	$g$	$g$	$g$	$g$	$g$	$g$	$g$	$g$	$R^2$
-0.8835 (-1.18)	2.2450 (2.21)	-	-	0.2773 (0.38)	-	-	-	-	31.9
-0.2216 (-0.33)	1.5790 (1.91)	-	-	-	-	-0.8743 (-0.40)	-	-	32.5
-0.1617 (-0.16)	1.5408 (1.32)	-18.444 (-3.55)	0.0583 (0.17)	0.6452 (0.71)	0.2385 (0.70)	-	-	-	69.3
0.9389 (1.05)	0.4688 (0.46)	-12.302 (-2.45)	-0.2190 (-0.51)	-	-	-1.1319 (-0.50)	-0.1218 (-0.15)	-	70.3

-----  
**PANEL C: 10 MV PORTFOLIOS**  
 -----

$g$	$g$	$g$	$g$	$g$	$g$	$g$	$g$	$g$	$R^2$
0.5347 (0.87)	0.8529 (1.08)	-	-	0.4998 (0.79)	-	-	-	-	43.7
0.9540 (1.59)	0.4447 (0.59)	-	-	-	-	1.1727 (0.42)	-	-	40.7
1.3824 (2.08)	-0.0674 (-0.07)	-16.334 (-3.13)	-0.324 (-0.99)	0.7414 (0.95)	0.1561 (0.59)	-	-	-	75.8
1.4940 (2.53)	-0.1376 (-0.15)	-20.645 (-3.21)	-0.409 (-1.06)	-	-	3.4146 (0.98)	0.1715 (0.19)	-	72.7

-----