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LIQUIDITY CONSTRAINTS AND CREDIT SUBSIDIES IN AUCTIONS

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Abstract

I consider an auction with participants that differ in valuation and access to liquid assets. Assuming credit is costly (e.g. due to moral hazard considerations) different auction rules establish different ways of screening valuation-liquidity pairs. The paper shows that standard auction forms result in different allocation rules. When the seller can deny access to capital markets or offer credit subsidies, she gains an additional tool to screen agents. The paper derives conditions under which the seller increases profits by way of subsidizing loans. In particular, in a second price auction, the seller always benefits from offering small subsidies. The result extends to a non-auction setting to show that a monopolist may use credit subsidies as a price discrimination device.

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1 Introduction

Liquidity constraints are ubiquitous in economic relationships. When assessing the value of an object, a buyer takes into account the effect of the purchase on her liquidity. Of course, the agent's appraisal of a good's value determines her willingness to pay. But the converse is often also true: the buyer's valuation of the good depends on the amount she is required to pay for it. For instance, an empty-pocketed entrepreneur assigns a different value to a project before and after having a venture capitalist agreeing to invest in it.

The early work on auction theory (Vickrey [41], Myerson [35], Milgrom and Weber [33], Maskin and Riley [24], etc.) abstracts from the effects of liquidity constraints. Recently, however, there has been a growing literature that considers financially constrained bidders in auctions. This paper builds on that literature and advances in two directions: i) it puts forward a framework to analyze auctions with bidders that have price-dependent valuations. And ii) it derives conditions under which a seller profits from providing credit subsidies, both in an auction context and in a monopoly pricing problem. The first contribution generalizes the framework presented by Che & Gale [6]; by allowing for a broader class of utilities, the paper captures several applications like the sale of moral hazard contracts. The question of credit subsidies had only been partially addressed (in Maskin [23], Che & Gale[5] and Zheng[43]) in restrictive settings. This paper shows that subsidizing credits can increase revenue in a large class of environments.

The motivation for studying liquidity constraints is the paramount role they play in economic transactions of significant size. Seldom is a single up-front payment the currency for a costly object because firms are typically reluctant to pay cash for large purchases. Instead, sellers use different tools to facilitate trade. For example: Christie's and Sotheby's, the largest auction houses for art objects, offer buyers pre-arranged favorable financial terms[1]. In privatizing utilities, local governments across the U.S. utilized a system of vouchers designed to alleviate buyers' financial constraints (see Maskin [23]). In developing countries, bidders for infrastructure concessions normally lack the financial strength to be liable in case of a downturn; regulators find partial solutions in sophisticated contracts designed to balance risk-sharing with incentives to control costs (see McAfee and McMillan [26] and [27], Laffont and Tirole [17] and Klein [13]). In some of the FCC auctions for spectrum rights, winning bidders were allowed to spread payments over 10 years at low borrowing rates (see Zheng [43]). In

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[1]See *The New Yorker*, Oct 15, 2001, pp 158-175.

this paper I show that these policies, by reducing the cost of capital can increase competition and raise the seller's expected revenue.

I begin by formulating a general model that captures most situations in which liquidity constraints might impact the demand for a good. The model captures the following four distinct types of problems:

i) *Investments and budget constraints*: Consider the case of bidders with fixed budgets that they have to split into acquiring a venture and investing on it. Paying a high price reduces the capacity for further investments, which results in a less valuable project. This is the case of an entrepreneur that approaches several venture capitalists to finance a project. The final value of the project –in average– will depend on the financing that the entrepreneur can obtain.

ii) *Moral hazard and financial constraints:* A project requires a costly action by the buyer. Outside funding reduces the value of the project by inducing the buyer to choose suboptimal actions. For example, judges in the U.S. sometimes use auctions to assign the role of lead counsel in class-action law suits. Law firms bid the share of the proceeds that they will keep as fees and the judge selects the lowest bidder. As it has turned out, many class-actions end in quick settlements; the winner does not expect additional work to be worthwhile given the size of her stakes.[2]

iii) *Financing costs:* Capital is costly either because capital markets are imperfect or, in the spirit of Townsend [40], because of verification costs in cases of default.

iv) *Decreasing absolute risk aversion:* When the good is a risky venture and buyers have decreasing absolute risk aversion, paying a higher price for the venture results in a higher risk premium.

I then consider the auction of a single good to bidders who have two dimensions of private information. Bidders have private knowledge of both their "ability" and their liquidity. Bidders who are sufficiently liquid act as in the standard independent private values (IPV) model; their ability fully determines their valuations. Constrained bidders, on the other hand, face increasing marginal costs of capital. I present two results with important policy considerations. On one hand, I show that different auction forms result in different winners. Second price and English auction forms accentuate the differences across bidders due to liquidity constraints. Whereas first price and Dutch auctions mitigate asymmetries of liquidity constraints. As a consequence, it turns out that first price auctions dominate second price auctions in terms of revenue. On the other hand, I study the provision of profitable credit subsidies. In a simple setting, I show that a monopolist can sometimes use credit subsidies as

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[2]See *The New Yorker*, Oct 15, 2001, pp 158-175.

an effective means for price discrimination. In auctions the effects of subsidies are stronger, I show that under second price auctions the seller is always better-off by providing a small subsidy.

The intuition for the subsidization result can be seen in a simple example. Consider a car seller who faces two types of customers, each with equal probability. The first type has deep pockets (unlimited cash on hand), while the second is illiquid and has a shadow cost of funds of 25%. Both buyer types value a particular car at 100, while the seller's costs are normalized to zero. Illiquid agents face a cost of capital and therefore are willing to pay up to 80 for the car. A buyer arrives at the dealership, and the seller cannot distinguish his type. If the seller is restricted to make a single take-it-or-leave-it offer, she will price the car at 80. However, she can do better by offer financing. If she pays all the costs of financing,[3] she would be able to price the car at 100 and still capture total demand. Her expected profit increases to 90 because only constrained types require the subsidy.

Offering finance allows the seller to price discriminate; she continues to receive 80 from illiquid types while extracting the full value from other types. In contrast, a single fixed price forces her to internalize the cost of capital of all the population, including buyers that need no finance. Thus, bundling cars with interest rate subsidies acts as an effective means to discriminate and extract more rents.

Note that a key assumption is that borrowing, even with subsidies, is more costly than using internal sources of capital. This is why the liquid type does not try to take advantage of the subsidy. In a future extension it may be worth to endogenize the buyer's decision regarding its sources of capital. In this first approach to the question however, I simply rule out the possibility of outsourcing capital needs before emptying the buyer's own pockets.

Interestingly, liquidity constraints might affect willingness to pay both positively or negatively. Because liquidity can bound an agent's liability, it may diminish downside risk. Board [2] and Zheng [43] show that in auctions where bidders differ in their liabilities, predatory bidding can arise in equilibrium. An illustration might be the case of infrastructure concession bidding in Latin America. Potential concessionaires bid below costs (for procurement contracts). If realized costs are high they can still make profits through ex-post renegotiations. After realizing sunk costs, concessionaires enter a two-sided monopoly relationship with government, and the threat of bankruptcy triggers contract

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[3]Full subsidies turn out to be optimal in this example.

renegotiations.[4] The nature of the problem is that each bidder views the object being auctioned as a different contract, specifically, as a call option with identity-dependent strike price. It would seem like the proper course of action is to modify auction rules to diminish asymmetries in valuations due to asymmetries in liability.

The model builds on, and generalizes the framework of Che & Gale [6]. In their model, a bidder's private signal consist of a valuation and a budget. Bidders face costs of capital that only depend on the size of the loan. I allow for more general price-dependent valuations; i.e., financing costs can depend also on bidders' ability. This extension allows me to capture in a single model the four classes of problems described above, while the Che-Gale framework only embraces the case of financing costs. Section 4 shows that many of their results continue to hold in the generalized setting.

The question of whether it is worth for the seller to provide subsidized financing to increase competition has been raised by Che and Gale [5] and Zheng [43]. Both papers find affirmative answers in a simple environment where bidders only differ in their wealth, as in the car example. The wealthiest bidder always wins; he requires a relatively small credit, thus a small subsidy. On the other hand, the promise of reduced capital costs improves considerably the competitive edge of poorer bidders, raising the resulting price of the auction. Proposition 5 shows that small subsidies always benefit the seller in second price auctions, even when bidders have different valuations. Although the intuition from the simplified setting does not go through, the proof of the proposition uses a similar argument. An important difference of this setting is that subsidies can result in a change of allocation –i.e., in a different winner. Moreover, I show that subsidized winners are always less liquid than winners in non-subsidized auctions. The proposition then shows that the seller can profit from offering subsidies even though, under subsidies, the winner always requires more financing.

A parallel result is obtained for the case of a single buyer. As in the car example, the seller faces a demand that depends on the cost of capital. Even when the seller must absorb the full cost of a subsidy, she may be able to capture more surplus by providing it. The critical condition for this to happen is that the sensitivity of demand to changes in the borrowing rate be large enough –larger than the average fraction of the price that buyers need to borrow. These results compare to the findings in the literature about bundling (see Adams and Yellen [1], McAffee, McMillan and Whinston [28] and Nalebuff [37]).

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[4]Some governments have adopted mechanisms that favor bids that are closer to the mean of accepted bids as a way of avoiding costly renegotiations. Construction contracts in Colombia (for roads and public buildings) usually take this approach.

The paper is organized as follows: Section 2 introduces the model in reduced form. Section 3 extends the model and shows how it fits different applications. Section 4 shows under what circumstances it is profitable for a monopolist to offer credit subsidies. Section 5 develops a method to map the auction problem into an economy where bidders have a single dimensional type. This mapping allows to compare different auctions in terms of revenue. Section 4 analyzes the effect of credit subsidies in first price and second price auctions and other selling mechanisms. Section 5 concludes.

2   The Model

A seller has a single indivisible good that she values at zero. There are $N \geq 1$ potential buyers. In the case of a single buyer, the seller chooses an asking price (take-it-or-leave-it). In the case of multiple buyers, she auctions the good. In most of the applications described in section 3 the good is a business venture; consequently, throughout the paper, it will be called the project or the venture. Buyer $i \in \{1, 2, ..., N\}$ has a two-dimensional type $(\theta_i, w_i) \in \Theta \times W = [0,1] \times [\underline{w}, \bar{w}]$ that determines his preferences.[5] $\theta_i \in \Theta$ will represent buyer i's ability; his potential to exploit the venture. $w_i \in W$ represents i's liquidity –the budget available to buyer i. Types are assumed to be independent and identically distributed across bidders according to the twice continuously differentiable joint probability density function $g : \Theta \times W \to \mathbf{R}^+$. $G : \Theta \times W \to [0,1]$ denotes the corresponding probability distribution function (p.d.f.). Each buyer's type is his private information. All agents, including the seller and lenders, share the same prior beliefs regarding the distribution of buyers' types.

I shall restrict attention to auction mechanisms where only the winner pays. The following reduced form of utilities will be assumed –section 3 develops several applications. Without loss of generality, the utility of not participating or not winning the auction is normalized to zero, while the utility of a type (θ, w) who pays x for the good is

$$u(\theta, w, x) = V(\theta) - x - C(\theta, x - w). \tag{1}$$

Here $V(\cdot)$ represents the potential value of the project to a buyer of a given ability, and $C(\theta, d)$ the internal cost to a θ-ability buyer of obtaining debt d from outside sources. The case $C = 0$ corresponds to the standard IPV model with risk neutral bidders.

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[5]I will adopt the convention that a female is selling the good to N male buyers.

Borrowing costs $C(\cdot, \cdot)$ depend on the available borrowing rates. A buyer who borrows d is asked to repay $R(d)$ in the future. $C(\theta, d)$ denotes the cost to a θ-ability buyer of acquiring such debt. There are two reasons why this cost may depend on the parameter θ. First, the risk of default may vary with ability. Second, the amount of debt can affect the buyer's choice of actions in the future. The first reason stems from adverse selection in financing contracts: a contract with identical terms imposes different costs to different buyers. The second stems from moral hazard: debt commitments affect the behavior of the buyer in a type dependent way.

Credit subsidies induce a debt repayment function $r(\cdot) \leq R(\cdot)$. Debt repayment functions will be called borrowing rates. In light of lower borrowing rates, buyers' cost of borrowing changes. Let $c(\cdot, \cdot)$ denote borrowing costs at subsidized rates r. Borrowing rates affect costs only through the amount of implied debt repayments. In other words, c must satisfy:

$$c(\theta, d) = C\left(\theta, R^{-1}(r(d))\right). \tag{2}$$

Seller's cost of subsidy SC from offering rates r to any buyer that borrows d is given by:[6]

$$SC(d) := d - R^{-1}(r(d)) \tag{3}$$

This means that the seller can take money from her own pocket and repay immediately a fraction of the loan to the lender.[7] The buyer undertakes debt with a third party. He only commits to a given future repayment. The seller pays the difference between the debt that the buyer asks and the debt that corresponds to the future repayment at borrowing rate $R(\cdot)$. Figure 1 illustrates this subsidy technology. The buyer receives d and commits to repay $r(d)$, the seller, by paying $d - R^{-1}(r(d))$, buys her way out of the financing scheme.

An important characteristic of this subsidy technology is that it does not give the seller any information about the buyer conveyed by debt repayments. Since the value of a debt contract depends on the winner's private information, the seller could exploit the positive correlation between expected debt repayments and winner's type as a means to extract more rents. This would be a case of the linkage principle.[8] By ruling out borrowing schemes that leak information, the seller derives benefits from subsidies solely from the effect on price; which is the main concern here.

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[6] An equality preceeded by a colon ":=" will signify that what follows is a definition.

[7] The results follow through if the seller had to pay the difference between borrowing rates. That is, if $SC = R - r$.

[8] This would work with any type of payments that are increasing in the projects realized value (e.g., equity, revenues, etc.). Hansen [9] presents a couple of interesting examples; more on the linkage principle can be found on Milgrom and Weber [33].
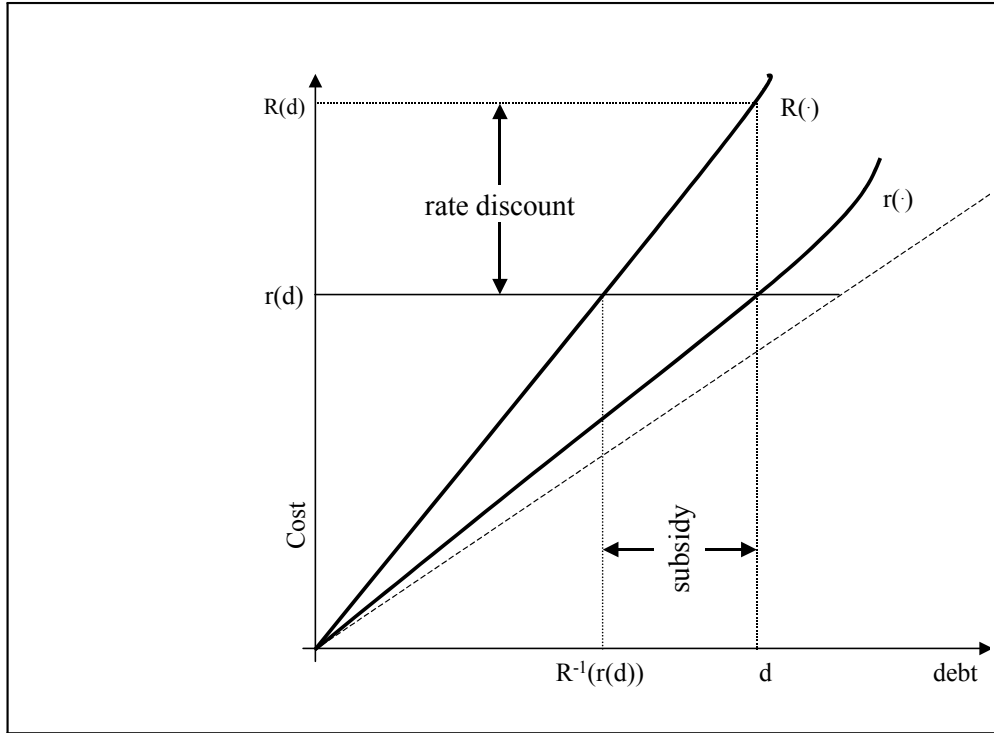
Figure 1

The following assumption imposes some minimum restrictions on the class of borrowing rates and subsidies that will be allowed.

Assumption 1 (A1 - borrowing rates):
$R(\cdot)$, $r(\cdot)$ and $(R - r)$ are all non negative, continuous, non decreasing and weakly convex functions with $R(0) = r(0) = 0$

The assumption does not play a very important role except from providing a bound for subsidy costs.[9] It ensures that the marginal rates of borrowing and the marginal discounts on the rates are non decreasing on the size of loans.

To introduce the definitions below it will be convenient to represent explicitly the dependence of borrowing costs on the underlying borrowing rate. Let $\psi(\theta, d; \rho)$ denote borrowing costs when the debt repayment function is ρ. For example, $\psi(\theta, d; R) = C(\theta, d)$ and $\psi(\theta, d; r) = c(\theta, d)$. Similarly,

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[9] A slightly different bound could be found easily if the assumption was dropped.

let

$$U^{\rho}(\theta, w, x) = V(\theta) - \min(x, w) - \psi(\theta, x - w; \rho).$$

A condition that will be important for most results is that debt is costly.

Definition 1 *Debt is costly with respect to borrowing rates ρ if and only if the inequality*

$$\psi(\theta, d'; \rho) - \psi(\theta, d; \rho) \geq d' - d$$

holds for all θ and all d, d', such that $d' > d \geq 0$.

Costly debt means that buyers only use external sources of capital as a last resort. Assuming debt is costly rules out the possibility that excess liquidity has any negative effect.[10] This is a key assumption because it means that buyers do not borrow more than they need to –no buyer has incentives to lie about its liquidity after being awarded the project. .

Assumption 2 (A2 - costly debt):
Debt is costly with respect to market borrowing rates $R(\cdot)$.

So far little has been said about the form of borrowing costs. In general, the function C covers all liquidity-related costs, and not only those directly linked to borrowing (see applications in section 3). Assumption 3 imposes some mild conditions on the form of borrowing costs. It says that for sufficiently liquid types borrowing costs are zero and, when positive, they are smooth.[11]

Assumption 3 (A3 - borrowing costs): For every θ,
$C(\theta, \cdot)$ is continuous. $\exists d(\theta)$ such that
$C(\theta, d) = 0$ for $d < d(\theta)$, and
C is twice continuously differentiable for $d > d(\theta)$, with $C_2 > 0$.
Moreover, $d(\theta)$ is non decreasing in θ.

The assumption says that borrowing costs are non negative and, when positive, they are increasing in the size of debt. It is worth noting that under the costly debt assumption, $C_2 \geq 1$ for $d > 0$. This

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[10]E.g., if liquidity has the role of liability; more liquidity represents more downside risk in a project. Board (2000) and Zheng (2001) consider this effect.

[11]Subscripts are used to denote the argument of partial derivatives. For example: $C_1 := \frac{\partial C(\theta, d)}{\partial \theta}$ and $C_2 := \frac{\partial C(\theta, d)}{\partial d}$.

guarantees that $u(\theta, w, x) \leq V(\theta) - x$; moreover, the bound is only reached when $C = 0$, that is for non binding liquidity constraints.

Given that a buyer's outside alternative is worth zero, he is willing to pay for the object any price that gives him a positive utility. His willingness to pay is defined in the natural way.

Definition 2 *Given a borrowing rate function ρ, a type (θ, w) has willingness to pay p iff $U^\rho(\theta, w, p) = 0$*

Buyers that can afford to pay the potential value without incurring borrowing costs are said to be *unconstrained*.

Definition 3 *A (θ, w) buyer is unconstrained if his willingness to pay with borrowing rate R is grater or equal to $V(\theta)$.*

Unconstrained buyers have quasi-linear utility functions for prices below their willingness to pay. From the costly debt assumption $w > V(\theta)$ is a necessary condition for a type (θ, w) to be unconstrained. Hence, his utility for $x \leq V(\theta)$ is simply $u(\theta, w, x) = V(\theta) - x$. For any given price, it follows that constrained types derive less utility that an unconstrained type of the same ability.

Note that it has been assumed implicitly that the only external source of capital available to buyers is through debt contracts. Most results hold under other forms of financing. I choose a specific form to avoid discussions about optimal capital structure; there are several reasons to choose then debt. On one hand, it is the most widely used source of capital and, on the other hand, it provides a natural way to introduce subsidies.[12]

Note also that all buyers have access to the same borrowing terms; i.e. $R(d)$ is type independent. This modelling assumption emphasizes the results since it would be less costly to subsidize buyers if some information about types were available.

The next section describes a more detailed model that fits this reduced form and that can be easily interpreted in several applications.

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[12]It is also worth noting, however, that Towsend (1979), Innes (1993), and others show the optimality of debt contracts in similar settings.

3   Interpretations and Applications

This section extends the description of the model to show how the reduced form presented in section 2 captures many cases of interest. In particular, four applications are specifically modelled. They describe the following possible effects of liquidity constraints: a) direct effect of financing costs, b) moral hazard costs due to borrowing needs, c) suboptimal investments due to financing costs, and d) with DARA utility functions, the risk premium of a risky venture depends on the price and the buyer's initial level of wealth.

In all cases the project will be treated as having an uncertain value. The timing of the problem is depicted in Figure 2. At date zero, buyers learn their types and the rules of the auction, and they submit bids. At date one the seller awards the project and determines the price. The winner contracts with lenders and pays the required transfer to the seller. The seller transfers the subsidized part of the loan (if any) to lenders. At date two, the true value of the project, which is observable to all parties, is realized. Buyers decide whether to repay debt or hand the project to lenders, either way, the relationship is terminated.

Figure 2

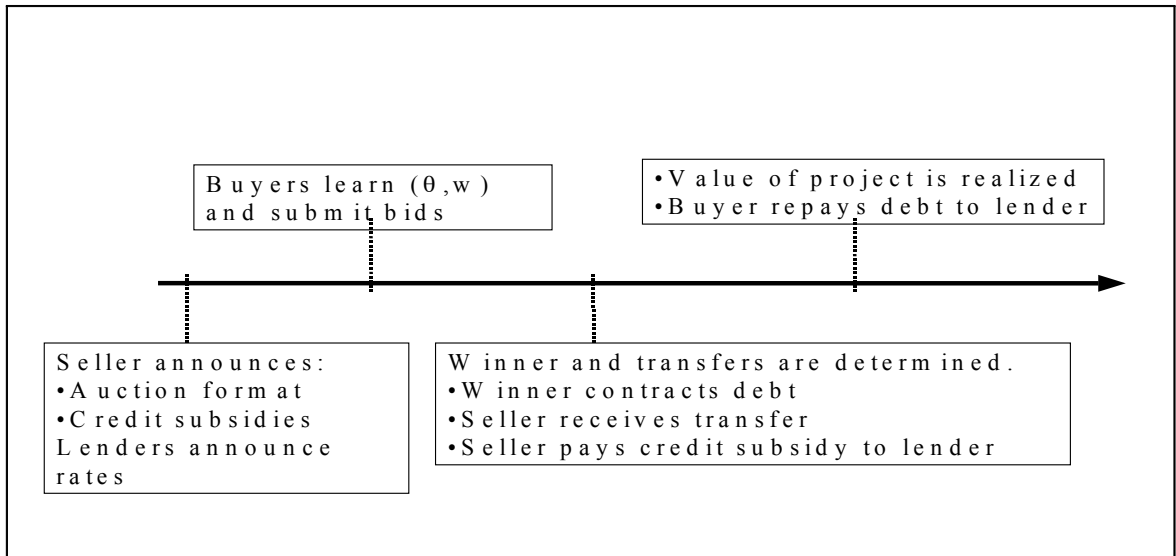At the time of the sale, agents do not know the true value of the project, say output, but they have a distribution over its possible realizations. This distribution depends on the buyer's ability characteristic and his post-purchase level of liquidity. That is, at date one, the time of the auction, a type (θ, w) buyer that faces a price x for the project knows that realized output at date two is

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distributed according to the p.d.f. $\mu(v; \theta, x - w)$. Output is verifiable by all parties and has support in $[0, \bar{v}]$. The discount factor is normalized to zero.

The modelling assumptions that i) higher ability is better and ii) that more liquidity gives more flexibility to a buyer are reflected on the probability distribution of output. The family of probability measures $\{\mu(\cdot; \theta, d)\}_{\theta \in [0,1], d \in \mathbf{R}}$ is ordered according to first order stochastic dominance (FOSD). Formally:

Assumption 4 (A4 - output distribution):

i) For all θ, $d, d' \leq d(\theta)$ implies $\mu(\cdot; \theta, d) = \mu(\cdot; \theta, d') = \bar{\mu}(\cdot, \theta)$.

ii) For all v, $\mu(v; \cdot, \cdot)$ is non-increasing in θ and non-decreasing in d.

The first part of the assumption indicates that all unconstrained types with the same ability have the same distribution over output. The second part orders output distributions according to types. That is, higher ability or higher liquidity yields a better output distribution, in the FOSD sense.

Borrowing is restricted to be in the form of debt contracts[13]. Lenders can not observe buyers' private information, and therefore must offer identical terms to all of them.

Definition 4 *A debt contract is a binding agreement between buyer and lender that commits the former to repay* $\max[R(y), v]$ *at date two, in exchange for a date 1 transfer of y from the latter.*

Suppose that on top of repaying debt, buyers may have an additional cost of debt $K(\theta, d) \geq 0$. K depends on a buyer's ability parameter θ and the size of debt d. This cost is used in modelling a moral hazard application; it will represent the cost of equilibrium levels of effort. Apart from being non negative, it will be assumed that for all θ, $\{E[v|\theta, d] - K(\theta, d)\}$ decreases in d. That is, although the cost of effort K may decrease on d, it does not do so abruptly.

Buyers are risk neutral and therefore the utility of a type (θ, w) that faces a price x can be written as:

$$u(\theta, w, x) = E\left[(v - R(x - w))^+ | \theta, x - w\right] - K(\theta, x - w) - \min(x, w) \tag{4}$$

Where $E[\cdot|\theta, d]$ denotes the expectation operator with respect to the measure $\mu(\cdot|\theta, d)$. Note that the fact that buyers prefer to use internal financing rather than external is already implicit in the utility

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[13]Cf. footnote on page 10.

function (they take as little debt as it is necessary). To guarantee that this is in fact the case, an equivalent assumption to A2-costly debt must hold. The relevant condition is the following:

A2' (costly debt): For all θ and $d' > d \geq 0$,

$$E\left[\min\left(v, R(d)\right)^+ | \theta, d\right] + K(\theta, d) - (d' - d) \geq E\left[\min\left(v, R(d')\right)^+ | \theta, d'\right] + K(\theta, d')$$

In general, the costly debt assumptions could be replaced by the assumption that lenders observe buyers' w and offer only the necessary amount of credit. The relevant property for the analysis is that buyers do not keep money in their pockets when using debt.

The reduced form of the model can be obtained from the following definitions.

$$v(\theta, d) := E[v|\theta, d] - K(\theta, d)$$

$$V(\theta) := v(\theta, d(\theta)) = E[v|\theta, d(\theta)] - K(\theta, d(\theta))$$

$$C(\theta, d) := \{V(\theta) - v(\theta, d)\} + E[\min(v, R(d))|\theta, d]$$

Where the cost function has been written as to emphasize its two components: the bracketed term represents efficiency loss, and the last term represents direct costs of borrowing. From the assumption on output distribution and on the cost K, it should be noted that u is increasing in θ, and that assumption 3 is implied.

This general representation allows for several applications. The following subsections describe a handful of relevant cases. The treatment is informal; I simply point out ways of adapting the model to different classes of problems.

3.1 *Financing Costs*

The first application considers the case of pure financial costs. It reflects the idea that bidders do not have free and unlimited access to capital markets.

Let output distributions depends only on ability; i.e., $\mu(\cdot; \theta, d) = F(\cdot|\theta)$. Assume there are no costs of financing other than debt repayments, that is $(K = 0)$. It is worth noting that even in this case, borrowing costs depend on buyers' ability. This fact corresponds to the prevalence of adverse

selection problems in credit markets. Since lenders offer the same terms to all buyers, the cost of debt is higher for high-ability types because they default less often. If debt was not costly to all types, low-abilities would be the first to find in their advantage to ask for large loans. That is, borrowing costs increase in θ.[14]

The utility of a buyer (θ, w) that pays x for the project can be written as:

$$u(\theta, w, x) = E[v|\theta] - \min(x, w) - E[\min(v, R(x - w))|\theta]$$
$$= V(\theta) - \min(x, w) - C(\theta, x - w).$$

Since $\min(v, R(x - w))$ is a non decreasing function of v, its conditional expected value increases in θ due to FOSD. The complete expression, however, does increase in θ.

3.2   *Moral Hazard and Financing*

Now consider a slightly more elaborated application that is as prevalent as the previous one in the context of contract auctions. Borrowing creates a moral hazard concern by reducing the buyers' incentives to take efficient decisions. This is, for example, the case of auctions for representation in class action law suits discussed in the introduction.[15]

Suppose that the probability distribution over output depends on a costly action from the buyer. Let e, for effort, denote this action and $h(\theta, e)$ be the disutility of exerting effort e for a θ-ability buyer. Let $F(v|e)$ denote the distribution over output conditional on the level of effort. Suppose that for $e' > e$, $F(\cdot|e')$ dominates $F(\cdot|e)$ in the first order stochastic dominance sense.

After acquiring the project at a price p, a (θ, w) buyer solves the problem:

$$\max_e \left\{ E\left[(v - R(x - w))^+ |e \right] - h(\theta, e) - \min[w, x] \right\}.$$

Assume there is a unique solution $e(\theta, x - w)$ to the above problem, and that it increases in ability, its first argument, and decreases in the size of debt, its second argument. Letting, $\mu(\cdot; \theta, d) =$

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[14]In this context, the costly debt assumption could be interpreted as lenders offering high rates to mitigate the cost of adverse selection.

[15]See *The New Yorker*, Oct 15, 2001, pp 158-175.

$F\left(\cdot|e\left(\theta,d\right)\right)$ and $K\left(\theta,d\right) = h\left(\theta, e\left(\theta,d\right)\right)$, it follows that the problem is an instance of the model presented in this section. Note that here, $d\left(\theta\right) = 0$ for all θ.

When the buyer does not take debt, he exerts the efficient level of effort: $e^*\left(\theta\right) = e\left(\theta,0\right)$. The reduced form of utilities (1) can therefore be obtained from defining:

$$V\left(\theta\right) = E\left[v|e^*\left(\theta\right)\right] - h\left(\theta, e^*\left(\theta\right)\right)$$
$$C\left(\theta,d\right) = V\left(\theta\right) - \left\{E\left[\left(v - R\left(d\right)\right)^+|e\left(\theta,d\right)\right] - h\left(\theta, e\left(\theta,d\right)\right)\right\}$$

The cost function incorporates both costs of capital and the efficiency loss due to moral hazard. This setting allows the costly debt assumption to hold even under competitive capital markets. If the efficiency loss: $V\left(\theta\right) - \left\{E\left[v|e\left(\theta,d\right)\right] - h\left(\theta, e\left(\theta,d\right)\right)\right\}$ is large enough, then, debt could be costly to all agents even if lenders charge the rate at which they break even. Put differently, if the incentive effects of debt are sufficiently large, the costly debt assumption will be satisfied automatically.

3.3   Investments and Budget Constraints

Contracts acquired through auctions many times require large investments from the winning party. For example, this is typically the case in contracts for exploiting mineral rights, or telecommunication licenses that require investment in networks. With liquidity constrained buyers, the problems raised by post-auction investment decisions are similar in nature to the moral hazard concerns analyzed in the previous subsection.

Suppose the value of the project depends on the buyer's ability and the amount of investments. That is, the distribution of the project is $F\left(v|\theta, I\right)$ where I denotes investments. Assume that the family of distributions is ordered in the FOSD sense both with respect to θ and I.

The buyer faces the following post-auction maximization problem:

$$\max_I \left\{E\left[\left(v - R\left(I + x - w\right)\right)^+|\theta, I\right] - \min\left(I, \left(w - x\right)^+\right)\right\}.$$

That is, the level of debt depends both on the price of the auction and the buyer's investment decision. Buyers use their budgets for both the auction and investments.

Suppose the above problem is well-behaved and has a unique solution $I(\theta, x - w)$ that is non decreasing in its first argument and non increasing in its second. Again, this turns out to be an instance of the general model; it suffices to define implicitly:

$$\mu(\cdot; \theta, d + I(\theta, d)) = F(\cdot | \theta, I(\theta, d))$$

$$K(\theta, d) = 0$$

3.4 *Decreasing Absolute Risk Aversion*

Finally, another application of the model might be that of analyzing behavior of risk averse buyers; more precisely of buyers with decreasing absolute risk aversion (DARA).

Like in the financing costs application assume output distribution only depends on ability –i.e., $\mu(\cdot; \theta, d) = F(\cdot | \theta)$. In this case assume there are no borrowing costs; either because internal and external costs of funds is the same, or because $\underline{w} \geq V(1)$ and no buyer needs financing.

Buyers' utility is determined by their final level of wealth. All buyers have the same Von Neumann-Morgenstern utility function $U(\cdot)$ which displays DARA. A type (θ, w) buyer that acquires the project at price x cares about

$$E[U(w - x + v) | \theta].$$

Let $R(\theta, x - w)$ denote the *risk premium* for a θ-ability buyer that is left with wealth $(w - x)$. That is, R solves:

$$E[U(w - x + v) | \theta] = U(E[w - x + v | \theta] - R(\theta, x - w)) \tag{5}$$

DARA simply implies that R is increasing in its second argument. By letting $C(\theta, d) = d^+ + R(\theta, d)$, the argument of U in the right hand side above is a case of expression (1).

The shape of U will be relevant for analyzing first price auctions and similar mechanisms. But in the case of second price auctions and the monopoly problem of section 4, U can be dropped off the analysis. The relevant information in these latter cases is a buyer's certainty equivalent (willingness to pay) for the project. That is, buyers behavior can be studied from the argument of U in the right hand side of expression (5)

4 Selling to a single agent

This section studies whether a monopolist should offer credit subsidies. In many industries, soft financing terms are a common way of implicitly reduce prices from time to time; e.g., loans to buy cars or appliances include a period of zero percent rates. Proposition 1 shows that the seller can use credit subsidies as an effective tool for price discrimination when she can not explicitly discriminate.

Consider a monopolist that faces a population of consumers with (θ, w) distributed according to G. When the monopolist is restricted to choose a single price for the good, she will choose the profit maximizing price. With zero costs and no capacity constraints, her profits are just the price times demand at such price. Given a price p, demand is the fraction of population that is willing to buy at price p, that is:

$$Q(p) = \Pr\{(\theta, w) : u(\theta, w, p) \geq 0\}. \tag{6}$$

Let $A(p)$ and $A^s(p)$ denote the set of buyers that are willing to pay p for the good when facing market borrowing rates $R(\cdot)$ and subsidized borrowing rates $r(\cdot)$ respectively. That is:

$$A(p) := \{(\theta, w) : u(\theta, w, p) \geq 0\}$$
$$A^s(p) := \{(\theta, w) : V(\theta) - \min(p, w) - c(\theta, p - w) \geq 0\}$$

Subsidies are profitable if their costs are smaller than the benefits resulting from an increase in demand. The response in demand due to offering subsidized borrowing rates $r(\cdot) \leq R(\cdot)$ is given by:

$$\Delta Q = \Pr\{c(\theta, x - w) \leq V(\theta) - p \leq C(\theta, x - w)\}$$
$$= \Pr(A^s(p)) - \Pr(A(p)).$$

The seller's expected cost of subsidizing loans by offering rates $r(\cdot)$ is:

$$SC = E_{\theta,w}\left\{(p - w)^+ - R^{-1}(r(p - w)) | A^s(p)\right\} \Pr(A^s(p)). \tag{7}$$
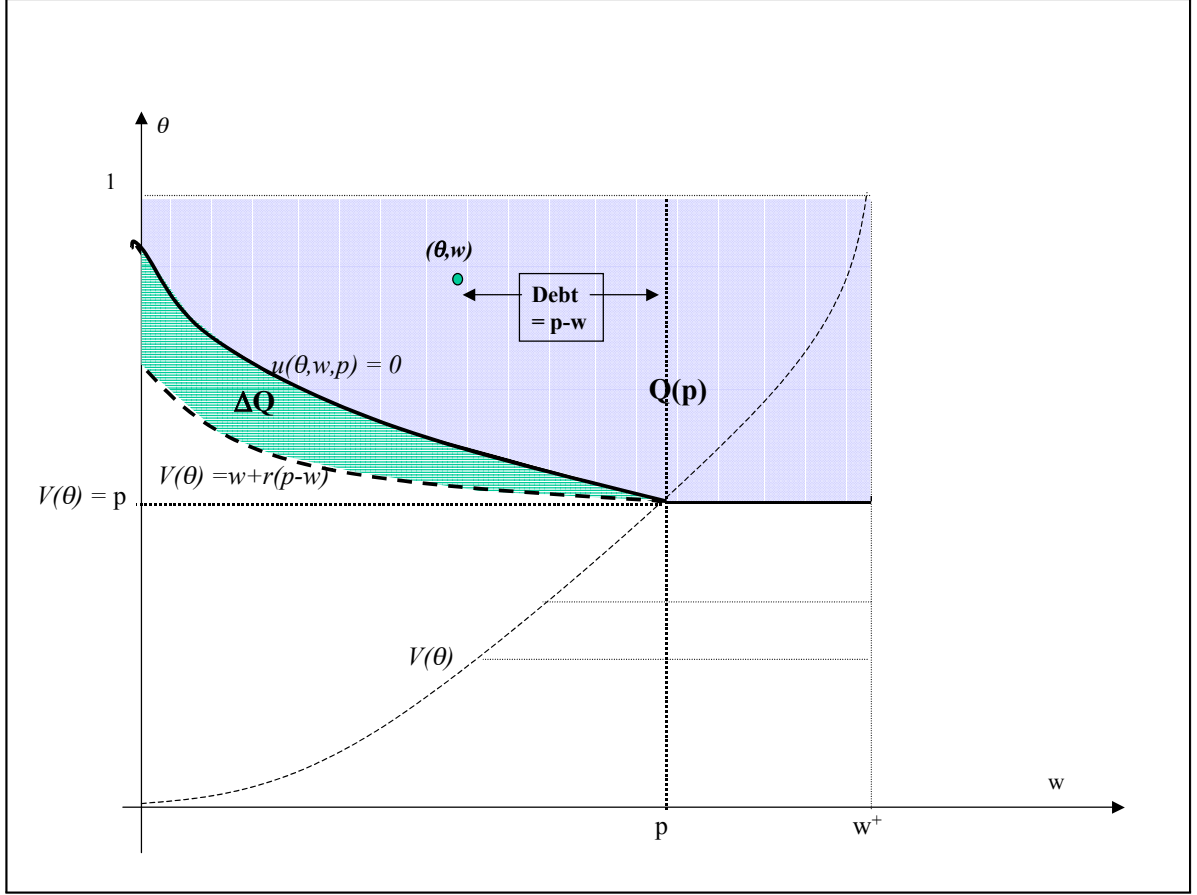
Figure 3

Figure 3 shows the space of types: abilities are in the vertical axis, and the horizontal axis measures liquidity levels. Demand, $Q(p)$, is the probability mass on the light shaded area above the curve $u(\theta, w, p) = 0$. With subsidies, demand increases capturing all types that lie in the dark shaded region, ΔQ. Types in the area ΔQ do not buy at price p unless they are subsidized. As Figure 3 shows, any type (θ, w) that buys at price p contracts debt $(p - w)^{+}$. The subsidy to a type (θ, w) costs the seller $(p - w)^{+} - R^{-1}(r(p - w))$. Expected subsidy costs SC are obtained by taking expectation over all types in the demand region –both light and dark shaded areas. Instead of calculating the exact cost, a convenient bound is used.

Since $R(d) \geq r(d)$ is increasing and convex, it follows that

$$d - R^{-1}(r(d)) \leq R(d) - r(d).$$

Now, given that $R - r$ is also increasing and convex and that $R(0) = r(0) = 0$, then, for $d \in [0, D]$

$$R(d) - r(d) \leq \frac{d}{D}[R(D) - r(D)]$$

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Now, the maximum debt that any agent contracts is $p - \underline{w}$; hence, subsidy costs SC are bounded above by:

$$CS \leq \frac{1}{p - \underline{w}} \left[R(p - \underline{w}) - r(p - \underline{w}) \right] E_{\theta,w} \left\{ (p - w)^+ \, | A^s(p) \right\} \Pr(A^s(p)) \tag{8}$$

Let Δr be the largest possible rate of subsidy; that is, $\Delta r := \frac{R(p-\underline{w}) - r(p-\underline{w})}{p - \underline{w}}$. The expectation on the right hand side of (8) represents the average size of loans. That is, the bound is constructed by assuming the fraction of each loan that is subsidized is maximal.[16] The following is then a sufficient condition for credit subsidies to be profitable:

$$\frac{\Delta Q}{\Delta r} \geq \frac{E_{\theta,w} \left\{ (p - w)^+ \, | A^s(p) \right\} \Pr(A^s(p))}{p} \tag{9}$$

That is, demand's response to decreasing the borrowing rate needs to be greater than the average fraction of price that buyers want to defer. The next proposition summarizes the result.

Proposition 1 *Suppose that subsidized borrowing rates r satisfy A2-costly debt. Then, subsidizing credit at rates r increases seller's profits whenever the ratio average-loan to price is smaller than demand's sensitivity to changes in the borrowing rate–i.e. if condition (9) holds.*

The proposition is quite intuitive. If demand is sensitive to marginal changes in the borrowing rate but, overall, the amount of borrowing is not too large, the benefits of a credit subsidy outweigh its costs.

Note that, although condition (9) depends on the price p, nothing prevents it to hold at the optimal price. That is, at the price that the monopolist would offer if subsidies were not possible. To see this consider figure 3 again. Suppose p is the optimal price –without subsidies. Even though moving to a lower price would decrease seller's revenue, using credit subsidies might increase it. Suppose there is a large mass of consumers on the top right corner of the figure and also a large mass in area ΔQ. Then, a price reduction may not be profitable because consumers on the top right corner pay less. A credit subsidy, on the other hand, does not alter the price for such consumers (they do not require borrowing) but does for the mass of consumers in ΔQ. Likewise, car sellers offering temporary low rates continue to receive up-front payments from some (unconstrained) consumers, while they also manage to attract new customers.

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[16]A tighter bound could be achieved by using a piece-wise linear approximation to the borrowing rate schedule.

5 Auctions

This section considers the sale of the project to one out of N agents via an auction. An auction game with its equilibrium is transformed into an economy of agents with single dimensional types. Whenever the potential valuation, $V(\cdot)$, is sufficiently low for buyers with $\theta = 0$, the induced setting can be constructed as to have agents with quasi-linear utilities. It is shown that different auction forms differ in their allocation properties. For example, second price and first price auctions yield different winners. In the induced setting the difference is translated to the distribution over possible types.

Che & Gale [6] present a framework to study standard auctions with financially constrained bidders. This section capitalizes on their work; in particular, to transform the auction into a new economy I use a similar argument to the one they apply to revenue comparisons. The basic differences are i) allowing for a broader class of utilities. And ii) the explicit use of a path in the type space that captures marginally unconstrained buyers.

Restrict attention to mechanisms where i) bidders submit a single dimensional bid and the highest bid wins the auction, ii) only the winner pays, and iii) there is a symmetric pure strategy Nash equilibrium bidding function that is non decreasing in both components of bidders' types. The restriction permits the most commonly used types of auctions: first and second price auctions (which, in this setting, are equivalent to English and Dutch auctions respectively). It rules out other forms, such as all-pay mechanisms and lotteries that are useful for relaxing liquidity constraints (see, e.g. Laffont and Robert [15]), but that are politically hard to implement.

Definition 5 *An environment is a triple (N, Ω, T) where N is the number of buyers, $\Omega = \Delta^N \times \mathbf{R}^N$ is the outcome space, and $T = \Theta \times W$ is the space of possible types.*

Where $\Delta^N = \left\{ p \in [0,1]^N : \sum p_i = 1 \right\}$ denotes the N dimensional simplex. Outcomes are then determined by a vector of winning probabilities and a vector of payments: the price that each bidder pays if he wins. The auction forms that will be considered are games where bidders submit a single scalar bid. Buyers know their own types and submit a one dimensional bid. A bid then consists on a non negative real number, and a bid profile is a vector in $\mathbf{R}_+^N$. Formally:

Definition 6 *An auction is a pair $(S, \langle p, x \rangle)$ where $S = \mathbf{R}^N$ is the set of bid profiles, and $\langle p, x \rangle$ maps bids into outcomes.*

The function $\langle p, x \rangle : S \rightarrow \Omega$ consists of a probability function $p : S \rightarrow \Delta^N$ and a transfer function $x : S \rightarrow \mathbf{R}^N$. When buyers play $s \in \mathbf{R}^N$, $p_i(s)$ and $x_i(s)$ represent the probability that buyer i is awarded the project, and the transfer he must pay if he's awarded the project, respectively.

An auction game is a Bayesian game that is fully specified by i) an environment, ii) an auction, and iii) a description of buyers' utilities and beliefs. Unless specified differently, the utility of a type (θ, w) is $u(\theta, w, x)$ when he acquires the project at a price x, and zero when he does not get the project. All buyers share the same beliefs about each other's types. Namely, all assume that types are independently drawn from the distribution $G(\cdot, \cdot)$. The following notation will be adopted: $\theta_{-i} = \{\theta_j\}_{j \neq i} \in \Theta_{-i} = [\times_{j \neq i} \Theta] = [0,1]^{N-1}$, $w_{-i} \in W_{-i} = [\underline{w}, \bar{w}]^{N-1}$, $b_{-i}(\theta_{-i}, w_{-i}) = \{b(\theta_j, w_j)\}_{j \neq i}$, etc.

Definition 7 *A symmetric pure strategy Nash equilibrium bidding function (SEBF) of an auction $(S, \langle p, x \rangle)$ is a function $b : \Theta \times W \rightarrow S$ that satisfies*

$$b(\theta_i, w_i) = \arg\max_b E_{\Theta_{-i} \times W_{-i}} \left[p_i(b, b_{-i}(\theta_{-i}, w_{-i})) u(\theta_i, w_i, x_i(b, b_{-i}(\theta_{-i}, w_{-i}))) \right].$$

for every buyer i and every type $(\theta_i, w_i) \in \Theta \times W$.

When a SEBF $b(\cdot, \cdot)$ is continuous and non decreasing in both arguments it will be called a *regular SEBF*.

Definition 8 *An auction is regular if it has a regular SEBF.*

Let $L : \Theta \rightarrow W$ be a non-decreasing function such that $L(0) = \underline{w}$ and $L(1) = \bar{w}$. Suppose $(S, \langle p, x \rangle)$ is a regular auction with symmetric equilibrium bidding function $b(\cdot, \cdot)$. From the regularity of $b(\cdot, \cdot)$, there is a representative from the graph of L for every possible bid. More formally;

Lemma 1 *Suppose $b(\cdot, \cdot)$ is a regular SEBF of the auction $(S, \langle p, x \rangle)$. Then, if $b = b(\theta, w)$ for some $(\theta, w) \in \Theta \times W$, there exists $\left(\tilde{\theta}, \tilde{w}\right) \in \{(\theta, L(\theta)) : \theta \in [0,1]\}$ with $b\left(\tilde{\theta}, \tilde{w}\right) = b$.*

Proof. Immediate. Since b is monotonic in both arguments, $b(0, \underline{w}) \leq b \leq b(1, \bar{w})$. The lemma follows from continuity of b along the graph of L. ∎

The function L is used in mapping the auction game into a setting where types have a single dimension. The idea is to use the graph of L as the type space in the new economy. Although

the method only requires L to be increasing and continuous, its usefulness lies in being able to choose a convenient candidate; e.g., the function that traces out the marginal unconstrained type for each level of ability. This avoids having liquidity constraint considerations in the new economy; hence agents' payoffs are quasi-linear. This analysis follows Che and Gale [6] who implicitly use a similar transformation; they look at bidding behavior among the wealthiest agents –i.e., along $\{(\theta, \bar{w}) : \theta \in [0, 1]\}$. In their case however, liquidity costs only depends on a buyer's budget.

The first step is to associate all types that bid the same amount with a representative in the graph of L. Lemma 1 indicates that all types will have such representative. Then, given a regular SEBF $b(\cdot, \cdot)$, the induced measure over the ability space Θ can be constructed as follows. Let

$$A(\varphi) := \{(\theta, w) : b(\theta, w) \leq b(\varphi, L(\varphi))\}.$$

Then, define the measure induced by the equilibrium b and the function L as:

$$G_b(\varphi; L) := \iint_{A(\varphi)} g(\theta, w) \, d\theta dw.$$

Auctions with equilibriums b and B induce
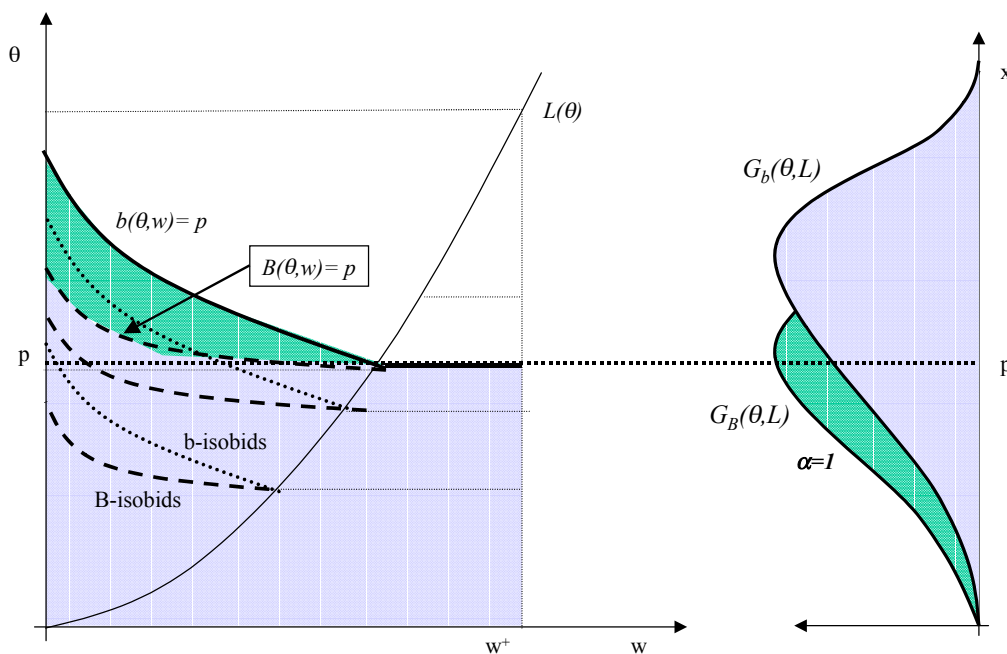different probability measures

Figure 4

Now consider the auction $(S, \langle p, x \rangle)$ in an economy where agents' preferences are parametrized by $\varphi \in \Theta$, and types are distributed according to $G_b(\cdot; L)$. A type φ that pays x for the good has utility $U(\varphi, x) = u(\varphi, L(\varphi), x)$ and utility zero if he does not get the good. Call this the *economy induced by b and L*. It is straightforward to show that $B(\varphi) = b(\varphi, L(\varphi))$ defines a symmetric equilibrium of such game.

Figure 4 illustrates how the new economy is constructed. The curve $L(\cdot)$ becomes the vertical axis on the right hand side picture. The picture on the left describes the space of types (θ, w). Isobid curves are depicted for two auctions with SEBF $b(\theta, w)$ and $B(\theta, w)$. All types in a single isobid curve on the left hand side are associated with a single point on the vertical axis of the picture on the right. Thus, points on this axis represent different sets of bidders for each auction. The picture shows the probability distribution that these two different equilibriums induce.

Lemma 2 *Suppose $b(\cdot, \cdot)$ is a regular SEBF of the auction $(S, \langle p, x \rangle)$ in the environment (N, Ω, T) with independent private types drawn from G. Then $B(\varphi) = b(\varphi, L(\varphi))$ is a symmetric equilibrium when the same auction rules are applied to the environment (N, Ω, Θ) with independent private types drawn from the distribution $G_b(\cdot; L)$.*

Proof. See appendix. ∎

The proof is straightforward, the intuition can be described as follows. Given that the bidding function is an equilibrium for all possible types, it is also an equilibrium for types along the graph of L. The induced measure over the new type space simply guarantees that agents along the graph of L solve the same problem in both settings.

Corollary 1 *A regular auction with equilibrium b yields the same expected price in the original economy as in the economy induced by b and L.*

Proof. See appendix ∎

I use the term "expected price" rather than "expected revenue" to avoid confusion when dealing with questions regarding subsidies. As with the proposition, the proof follows from the construction of the probability measure, and again it is straightforward.

From the assumption on borrowing costs (A3) remember that $C(\theta, x - w) = 0$ whenever $x - d(\theta) \leq w$. In words, unconstrained buyers are not affected by borrowing costs; their utility function is simply:

$u(\theta, w, x) = V(\theta) - x$. Lemma 2 could then be used to construct an induced economy in which all agents are unconstrained. However, this is only possible if the set of unconstrained types contains a path that connects the lowest possible type, $(0, \underline{w})$, with the highest, $(1, \bar{w})$. Note that for each level of ability θ, types (θ, w) are unconstrained if and only if $w \geq d(\theta)$. That is the graph of $d(\cdot)$, $\{(\theta, d(\theta)) : \theta \in [0, 1]\}$, traces out marginally unconstrained types. Thus, such construction is possible whenever $d(0) \leq \underline{w}$ and $d(1) \leq \bar{w}$. This is formalized in the next lemma.

Lemma 3 *Assume A3-borrowing costs and suppose $d(0) \leq \underline{w}$ and $d(1) \leq \bar{w}$. Then there exists a non-decreasing function $L : \Theta \rightarrow W$ with $L(0) = \underline{w}$, $L(1) = \bar{w}$ such that all types in the graph of L are unconstrained. That is, $(\theta, w) \in \{(\theta, L(\theta)) : \theta \in [0, 1]\}$ implies that $C(\theta, V(\theta) - L(\theta)) = 0$.*

Proof. Let

$$L(\theta) = \max\{\underline{w}, \theta\bar{w} + (1 - \theta)d(\theta)\}$$

It is clear that $L(0) = \underline{w}$ and $L(1) = \bar{w}$. Since d is increasing, so is L. Moreover, from construction, $L(\theta) \geq d(\theta)$ and from A3, this implies that

$$C(\theta, V(\theta) - L(\theta)) = 0$$

for all θ. ∎

The assumptions of the lemma are not hard to satisfy. Perhaps the most restrictive consideration is that the lowest type faces no liquidity constraints. But this is satisfied whenever the potential value of the project is sufficiently low for low abilities. For example, this is the case when $V(0) = 0$, a reasonable assumption. In general, the condition will be satisfied if paying $V(0)$ does not represent a threat to any agent's liquidity. Even if this condition does not hold, it would be possible to expand the type space by including lower abilities and obtain an approximation where the condition holds.

The following proposition makes use of the revenue equivalence theorem (see for example Milgrom [31]). If bidders in the induced auction have quasi linear preferences and the lowest type gets a payoff of zero, the revenue equivalence theorem holds in the induced setting. That is, expected price of the auction is the same as that of a Vickrey auction.

Proposition 2 *Let $(S, \langle p, x \rangle)$ be a regular auction with SEBF $b(\cdot, \cdot)$ and assume that the conditions for lemma 3 are satisfied. Suppose that L is constructed as in lemma 3. Then, if type $(0, \underline{w})$ expects zero profits from the auction, the expected price of the auction equals*

$$E_\varphi\left[V\left(\varphi^{(2)}\right)\right]$$

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where $\varphi^{(2)}$ denotes the second order statistic from $\{\varphi_1, ..., \varphi_N\}$, the types of bidders in the economy induced by L and the SEBF b.

Proof. From lemma 3 agents have quasi linear preferences. By assumption, the lowest type gets no rents and from lemma 2 the symmetric equilibrium bidding function is increasing. By the rules of the auction, the allocation rule corresponds to that of a Vickrey auction. These are the conditions for the revenue equivalence theorem to hold (see e.g. Milgrom [31]); hence the result follows. ∎

Figure 4 shows that two auctions with different equilibriums are transformed into the new economy defining different probability distributions over types. In the case depicted, the function $L(\theta)$ traces out marginally unconstrained types. That is, level curves –isobids– for both auctions are flat to the right of L and not necessarily so to the left. If isobid curves that meet at the graph of L follow the depicted pattern –i.e., b-isobids lie above B-isobids, then the distribution induced by B and L first order stochastically dominates the distribution induced by b and L. Note that the choice of L is important to have such dominance. If a path to the left of L was chosen instead, the same could not be said of the induced measures.

5.1 Standard Auctions

This subsection discusses some properties of the most common types of auctions: English, Dutch, first price sealed bid, and second price sealed bid auctions. Within this model, English auctions are strategically equivalent to Vickrey –second price sealed bid– auctions, and Dutch auctions are strategically equivalent to first price auctions. The discussion focuses on first price and second price. In first price and second price auctions, bidders submit bids in closed envelopes. The object is awarded to the highest bidder who then pays his bid, in the former case, or the second highest bid, in the latter.

An English auction is one in which bidders offer prices out loud, each time increasing the last announced bid. When bidding stops, the object is awarded to the last bidder to speak. The agent that values the object the most ends up paying the value of the second highest bidder. It is equivalent to a second price auction in this context because bidders find the information about other bidders behavior irrelevant. Similarly, first price auctions are equivalent to Dutch auctions. In a Dutch auction, an auctioneer names a ridiculously high price and continues by naming prices in descending order until someone agrees to buy the object. This is equivalent to a first price auction because in both cases bidders must simply decide their bid. In one case they submit it in a closed envelope, in the other they stop the auctioneer if and when she reaches the desired price.

Perhaps the most tangible effect of introducing liquidity constraints is that both auctions define different allocation mechanisms. That is, with the same pool of buyers, each auction may determine a different winner in equilibrium. The result is shown in proposition 3. Figure 4 illustrates this result: as long as two auctions do not have exactly the same isobid curves, their ways of determining winners are going to be different. In standard models of auction theory this effect is not present because types have only one dimension and so do bids. Here, bids define an ordering of types, different orders may and do arise from different auction rules.

It is also shown that first price auctions yield more revenue than their second price counterparts. The difference results from the fact that while bidding in a first price auctions bidders know exactly the financial burden they will face, in second price auctions they do not. The effect works against a price-maximizing seller.

Start by considering equilibrium bidding in both auctions. It is readily shown that in second price auctions it is a dominant strategy to bid one's willingness to pay. As definition 2 states, willingness to pay is the price at which a buyer is indifferent between buying or not the project.

Lemma 4 *It is a dominant strategy in a second price auction for buyer i with type (θ, w) to bid his willingness to pay. That is, to bid $b(\theta, w)$ such that*

$$u(\theta, w, b(\theta, w)) = 0$$

Proof. Let B denote the highest bid from the rest of the buyers. If buyer i bids $b > b(\theta, w)$, his situation only changes if $B \in [b(\theta, w), b]$. In such a case he ends with a negative payoff instead of getting zero. Similarly, if he bids $b < b(\theta, w)$, his situation only changes if $B \in [b, b(\theta, w)]$. In which case he gets nothing by deviating whereas he would get a positive profit by bidding $b(\theta, w)$. ∎

Equilibrium strategies in the high bid auction are not as easy to derive and I will not attempt to do so. It will not always be the case that there is a symmetric equilibrium that is increasing in both components of types. The following proposition provides a sufficient condition in order to have a well behaved equilibrium. The proposition is a direct application of Milgrom [30].

Lemma 5 *Suppose $B(\theta, w)$ is a symmetric equilibrium of the first price auction. If $u(\theta, w, x)$ is continuously differentiable, and $u(\theta, \cdot, \cdot)$ and $u(\cdot, w, \cdot)$ are log-supermodular, then $B(\cdot, \cdot)$ is non decreasing in (θ, w)*

A function $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ is log-supermodular if $\log\left(f\left(x,y\right)\right)$ has increasing differences in $\left(x,y\right)$. If f is twice differentiable, log-supermodularity is equivalent to the condition $\frac{\partial^2 f(x,y)}{\partial x \partial y} f\left(x,y\right) \geq \frac{\partial f(x,y)}{\partial x} \frac{\partial f(x,y)}{\partial y}$.

Proof. Let $H\left(x\right)$ denote the probability distribution function of the highest bid among all other agents if they bid according to B. That is:

$$H\left(x\right) := \left(\Pr\left\{\left(\theta,w\right) : B\left(\theta,w\right) \leq x\right\}\right)^{N-1}$$

Any type $\left(\theta,w\right)$ buyer then solves the following problem:

$$\max_x H\left(x\right)\left[u\left(\theta,w,x\right)\right]$$

Fix θ. Theorem 1 in Milgrom [30] states that as long as u is continuously differentiable and $u \geq 0$, if $u\left(\theta,\cdot,\cdot\right)$ is log-supermodular, then $B\left(\theta,\cdot\right)$ is non decreasing in w.
Exactly the same argument holds when interchanging the roles of θ and w. ∎

Assuming the first price auction has a well-behaved equilibrium, there seems to be no reason to assume that both auctions will allocate the project to the same bidder. The problem that a buyer faces in the first price auction is considerably different than under a second price auction. There is no reason to expect that the level curves of both solutions will match. The following proposition shows that in fact the identity of the winner depends on the auction rules.

Proposition 3 *Let $B\left(\theta,w\right)$ denote an equilibrium bid function for the first price auction. Suppose the assumptions of lemma 5 hold. Suppose also that for all θ, borrowing costs are convex, i.e. $C_{22}\left(\theta,\cdot\right) \geq 0$.*
Then , if an unconstrained type has the same willingness to pay than a constrained type, the bid of the latter is greater or equal than that of the former.

Proof. Consider buyer i and let $H\left(x\right)$ denote the equilibrium probability distribution function of the highest bid among all other agents. That is:

$$H\left(x\right) := \left(\Pr\left\{\left(\theta,w\right) : B\left(\theta,w\right) \leq x\right\}\right)^{N-1}$$

Assume all other bidders bid according to B and let z denote $i's$ willingness to pay. Type $\left(\theta,w\right)$ buyer i solves the problem

$$\max_x H\left(x\right) u\left(\theta,w,x\right)$$

If he is unconstrained then $u(\theta, w, x) = V(\theta) - x = z - x$ for any $x \leq z$

Now suppose that i's type is $\left(\tilde{\theta}, \tilde{w}\right)$ and that he is constrained but with the same willingness to pay as type (θ, w). In this case, his utility for any bid $x \leq z$ is given by:

$$u\left(\tilde{\theta}, \tilde{w}, x\right) = V\left(\tilde{\theta}\right) - x - C\left(\tilde{\theta}, x - \tilde{w}\right)$$
$$= (z - x) + \left(C\left(\tilde{\theta}, z - \tilde{w}\right) - C\left(\tilde{\theta}, x - \tilde{w}\right)\right) \geq (z - x)$$

Let $\Delta C(x) = C\left(\tilde{\theta}, z - \tilde{w}\right) - C\left(\tilde{\theta}, x - \tilde{w}\right)$. For $\beta \in [0, 1]$, let $S(\beta, x) := (z - x) + \beta \Delta C(x)$. By showing that $S(\beta, x)$ is log-supermodular, Theorem 1 in Milgrom **??** implies that the solution $x^*(\beta)$ to $\max_x H(x) S(\beta, x)$ is non decreasing in β. The constrained type bids $x^*(0)$, whereas the unconstrained type bids $x^*(1)$. S is log-supermodular because $S(\beta, x) S_{12}(\beta, x) - S_1(\beta, x) S_2(\beta, x)$

$$= -C_2\left(\tilde{\theta}, x - \tilde{w}\right) \{(z - x) + \beta \Delta C(x)\} + \Delta C(x) \left(1 + \beta C_2\left(\tilde{\theta}, x - \tilde{w}\right)\right)$$
$$= \Delta C(x) - (z - x) C_2\left(\tilde{\theta}, x - \tilde{w}\right) \geq 0$$

The last inequality follows from convexity of $C(\theta, \cdot)$. ∎

The intuition for the proof is as follows. Any price below the willingness to pay results in a higher payoff for the constrained type. It follows that the cost of shading more his bid while risking to loose the auction is higher for the constrained type.

The proposition shows that constrained types have better chances of winning under a first price auction that under a second price auction. The result has some policy implications. There are situations where the auctioneer would like the result of the auction to favor unconstrained types. For example, consider the case of bidding for concessions in Latin America, as mentioned in the introduction. Reasonably, the seller prefers to award contracts to buyers that have less credible threats of bankruptcy; that is, to types with sufficient liquidity. Opting for a second price auction would then bias the result in the right direction.

If the auctioneer wants to favor constrained types, a first price auction would be more desirable. The previous result, as next proposition shows, also implies that first price auctions raises more expected revenue. First price auctions enhance the competitiveness of constrained bidders exerting upwards pressure on prices. The next section shows that the seller may even find worthwhile to subsidize constrained bidders as a means to increase revenue.

Proposition 4 *Under the assumptions of lemma 3 and proposition 3, first price auctions yield more expected revenue than second price auctions.*

Proof. From lemma 3 it is possible to transform both auction forms into a setting with quasi linear preferences. From proposition 3 it follows that the distribution over types that the first price auction induces is better in the FOSD sense to the distribution induced by the equilibrium of the second price auction. Finally, from proposition 2 expected revenue is given by $E\left[V\left(\theta^{(2)}\right)\right]$, where the expectation is taken with respect to the induced measure. The result follows. ∎

The result can be understood by comparing it to the case of risk averse bidders. As Maskin and Riley [24] show high bid auctions dominate second price auctions in terms of expected revenue. Risk averse bidders fear more the risk of not winning the auction and are willing to forego some rents. In second price auctions they do not internalize this effect because the dominant strategy consists in capturing all possible rents –regardless of risk preferences. Similarly, constrained types know exactly their financial costs when bidding in a first price auction. In second price auctions, by contrast, they have to estimate the resulting financial burden. In a way, the second price auction equilibrium forces bidders to internalize the full cost of financing in their bids, while only a fraction is consumed in the end.

6   Credit Subsidies in Auctions

Analog to section 3, credit subsidies may serve as a means to increase expected revenue in an auction. When the seller is a monopolist, she weighs the cost of giving subsidies against the benefit of *increasing demand*. A similar effect occurs in auctions; the cost of subsidies is the same, but the benefit results from *increasing competition.* Constrained buyers are able to compete more fiercely due to reduced borrowing costs. Profitability of credit subsidies hinges on whether the increase in expected price outweighs total costs of subsidies.

Consider the sale of an art object through an auction to two bidders, c and u. Suppose both of them have valuations drawn from a common distribution with support in $[V - \varepsilon, V + \varepsilon]$. Bidder u

is unconstrained and can pay up-front; however, bidder c is constrained and has no liquid assets at the time of the auction. If the cost of money is higher than 2ε, the seller will always get less than $V - \varepsilon$ in an English auction. By offering subsidies to the unconstrained type, the seller generates more competition.[17] With full subsidies, the price of the object is the minimum of the two valuations (higher than $V - \varepsilon$), and she will only give the subsidy with probability $\frac{1}{2}$.

Credit subsidies cause all constrained types to increase valuations. Since the effect is different on each bidder, the result of the auction might change from the benchmark of a non-subsidized auction. That is, the winner's identity depends on subsidies. Moreover, it turns out that under subsidies the winner always has a tighter liquidity constrained and a higher ability that in the non-subsidized case.

The approach to credit subsidies in this section will be slightly different from the one taken in section 3. Instead of looking at the total effect of a given subsidy, only marginal effects will be considered. The result is in terms of when it is profitable to give at least a small subsidy. I restrict attention to second price auctions. This allows the model to capture the effects of subsidies omitting the analytical complications of first price auctions and more general mechanisms.

From lemma 4 shows that bids correspond to willingness to pay. Subsidies modify willingness to pay by reducing bidders 'costs of borrowing. A type (θ, w) buyer that is willing to pay b without subsidies, is then willing to undertake $d = b - w$ in debt. From expression 2 it follows that under subsidized borrowing, would be willing to undertake debt equal to $r^{-1}(R(d))$. That is, the level of debt that commits him, with subsidies, to the same repayment that he would commit with no subsidies. Hence, such type increases his bid in a the subsidized auction from b to $b' = b + (r^{-1}(R(d)) - d)$.

By the same token, the cost of subsidy to the seller, when

For a small subsidy the order of bids will hardly be affected. Hence, the increase in price corresponds to the debt increase of the second highest bidder. Let b denote the second highest bid in the benchmark case (no subsidies). Let $w^{(1)}$, $w^{(2)}$ denote the liquidity parameters of the first and second highest bidders respectively. From the analysis above, price increases by $r^{-1}(R(b - w^{(2)})) - (b - w^{(2)})$ –the jump on the *second highest* bid. The subsidy, on the other hand, corresponds to the difference on the size of loan that the *highest bidder* applies for. Other effects being of second order, the key to have

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[17]In fact she offers subsidies to both agents. As long as borrowing is still costly, the unconstrained type will prefer to pay from his pocket.

positive marginal effects from subsidies lies in the fact that higher bidders have better distributions over the liquidity parameter. That is, conditional on one bidding more than the other, it is more likely that $w^{(1)} \geq w^{(2)}$ because bidding is increasing in w. The next proposition formalizes this analysis.

Proposition 5 *Suppose the seller can offer credit subsidies that satisfy the subsidized rate assumption (A3), and suppose that each bidder's θ and w are affiliated[18]. Then, a small enough subsidy increases seller's expected revenue in a second price auction.*

Proof. The proof proceeds in three steps. First it is shown that altering the order of bids is only a second order effect of subsidies. The second step determines the price increase and cost subsidies in terms of the bids and liquidity parameters of the first and second highest bids. Finally, it is shown that in expectation the subsidy is profitable when the order of bidders does not change.

Notation: Consider the benchmark case of the auction with no subsidies and borrowing rates defined by the repayment function R. Let $b_1, b_2, ...$ and $w_1, w_2, ...$ represent bids and liquidity parameters of the highest bidder, second highest bidder and so on. Let b'_n represent the bid that the n-th highest bidder in the benchmark case submits in the auction with subsidies. And let B_n represent the $n-th$ highest bid in the auction with subsidies. That is, $B_1 \geq B_2 \geq ... \geq B_N$ ranks the set of bids $\{b'_1, ..., b'_N\}$ from highest to lowest.

1) (Changes in order of bids is second order)

Let $r(\cdot)$ represent subsidized rates and assume $|R(x) - r(x)| \leq \delta$ for some $\delta > 0$. B_2 is the price of the subsidized auction. As long as the order of the two highest bids does not change, it will be the case that $B_2 = b'_2$. More precisely, B_2 will be higher than b'_2 only if $\max(b'_3, ...b'_N) > b'_2$ and lower only in cases where $b'_1 < b'_2$. Since no bidder increases his price by more than δ, the probability of these events is bounded by $\Pr\{b_1 - b_2 < \delta\}$ and $\Pr\{b_2 - b_3 < \delta\}$ respectively. Moreover, the bid difference will also be bounded so that $|B_2 - b'_2| < \delta$. That is,

$$b'_2 + \delta \Pr\{b_1 - b_2 < \delta\} \leq B_2 \leq b'_2 + \delta \Pr\{b_2 - b_3 < \delta\}$$

Noting that $\Pr\{|b_i - b_2| < \delta\} \to 0$ as $\delta \to 0$ for $i \in \{1, 3\}$ we can write

$$B_2 = b'_2 + o(\delta).$$

Where $o(x)$ is a function that satisfies $\lim_{x \downarrow 0} \frac{o(x)}{x} = 0$. That is, the first order effect on price comes only from the effect on the second highest bidder. Similarly, first order changes on subsidy costs can

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[18]Roughly, (θ, w) are affiliated if they are possitively correlated when conditionning on every possible subset of $\Theta \times W$. See Milgrom and Weber (1982) for a more rigourous definition.

be viewed as coming from not having the order of bidders altered. To see this it suffices to notice that the maximum cost of subsidy is bounded by δ:

$$\left| r^{-1}\left(x\right) - R^{-1}\left(x\right) \right| \leq \left| x - R\left(r^{-1}\left(x\right) \right) \right| \leq \delta.$$

The inequalities hold because $R'\left(\cdot \right) \geq 1$ –this follows from A2. Now, letting $P_\delta = 1 - \Pr\left\{ b_1' = B_1; b_2' = B_2 \right\} \leq \Pr\left\{ \left| b_i - b_2 \right| < \delta \right\}$, the expected cost of subsidy CS can be bound by

$$E\left[CS \right] \leq \left(1 - P_\delta \right) E\left[CS | b_1' = B_1; b_2' = B_2 \right] + P_\delta \delta$$

So that the cost of subsidy can be written as:

$$E\left[CS \right] = E\left[CS | b_1' = B_1; b_2' = B_2 \right] + o\left(\delta \right)$$

2) (Benefits and costs). Each bidder commits to the same debt repayment in both auctions. If the second highest bidder is constrained, he will offer a bid equal to his wealth plus the value of a bond that commits him to repay some fixed amount x after output is realized. The difference in bids for both auctions depends on the value of such bond. Hence, in the subsidized auction, he bids more by $r^{-1}\left(x\right) - R^{-1}\left(x\right)$ –where $x = R\left(b_2 - w_2 \right) = r\left(B_2 - w_2 \right)$. The winner of the subsidized auction will need to borrow $B_2 - w_1$. From 3, the cost to the seller of giving him a subsidy is $\left(B_2 - w_1 \right) - R\left(r^{-1}\left(B_2 - w_1 \right) \right)$. Profitability of subsidies hinges then on whether this last expression is smaller than $r^{-1}\left(x\right) - R^{-1}\left(x\right)$. Since $r^{-1}\left(\cdot \right) - R^{-1}\left(\cdot \right)$ is increasing, profitability depends on $x = r\left(B_2 - w_2 \right) \geq r\left(B_2 - w_1 \right)$.

3) (Profitable subsidies in expectation) It remains to be proved that $r\left(B_2 - w_2 \right) \geq r\left(B_2 - w_1 \right)$ in expectation. Let $b\left(\theta, w \right)$ denote the SEBF of the auction with subsidies.
Milgrom and Weber [33] (theorem 22 P. 1119) show that if $\left(\theta, w \right)$ are affiliated, the set A is increasing,[19] and $f\left(\theta, w \right)$ is a non decreasing function; then

$$E\left[f\left(\theta, w \right) | A \right] \geq E\left[f\left(\theta, w \right) \right] \geq E\left[f\left(\theta, w \right) | A^c \right].$$

Now define the set $A\left(p \right) := \left\{ \left(\theta, w \right) : b\left(\theta, w \right) \geq p \right\}$ and note that for any p, $A\left(p \right)$ is increasing. Fix B_2 and let $f\left(\theta, w \right) := -r\left(B_2 - w \right)$. For any $q < p$, the theorem can be applied to the set $A\left(p \right)$ using the probability distribution conditioning on $A\left(q \right)$. It follows then that:

$$E\left[-r\left(B_2 - w \right) | b\left(\theta, w \right) \geq p \right] = E\left[f\left(\theta, w \right) | A\left(p \right) \right]$$

$$\geq E\left[f\left(\theta, w \right) | A\left(q \right) - A\left(p \right) \right]$$

$$= E\left[-r\left(B_2 - w \right) | p \geq b\left(\theta, w \right) \geq q \right]$$

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[19] Meaning that its indicator function 1_A is non decreasing in each argument.

Letting $q \uparrow p$, by the monotone convergence theorem, we have:

$$E\left[-r\left(B_2 - w\right) | b\left(\theta, w\right) \geq p\right] \geq E\left[-r\left(B_2 - w\right) | b\left(\theta, w\right) = p\right]$$

For $p = B_2$, the left hand side is the expectation of $-r\left(B_2 - w_1\right)$ for a given price and the right hand side is the expectation of $-r\left(B_2 - w_2\right)$ for the same given price. Taking expectations over all possible prices, the result follows.

This completes the proof. Summarizing: for sufficiently small δ the effect of a subsidy is always positive in terms of seller's revenue. ∎

The proof exploits the implicit advantages of types with higher levels of liquidity. Given that these types are more competitive, the winner is less likely to have a large financial burden. The seller benefits from fiercer competition while paying a small cost: the selection mechanism is still biased towards liquid types. Although credit subsidies benefit more illiquid types, the subsidy does not eliminate all costs due to liquidity constraints.

7 Summary and Conclusions

This paper considers the effects of liquidity constraints in selling mechanisms. By introducing bidder's liquidity constraints into an independent private values model it is shown that many basic properties of standard auctions fall apart. Perhaps the most striking feature is that by using auctions with single dimensional bids, different auction rules define different mechanisms to screen bidders. Thus familiar forms of auctions, like 1st and 2nd price yield different winners in equilibrium.

Ceteris paribus, illiquid agents are in disadvantage with respect to liquid types due to costs of financing. English and second price auctions accentuate this disadvantage because buyers cannot anticipate the final burden of financial costs. In equilibrium, they bid their willingness to pay, internalizing the maximum financial burden they would take. Dutch and first price auctions on the other hand, mitigate the effect of asymmetries in liquidity. In these auction forms, bidders can fully account for exact financial costs in their bidding strategies. Not surprisingly, Dutch and first price auction rules have the further effect of raising more revenue to the seller. This last effect parallels the consequences of having risk averse instead of risk neutral bidders, as Maskin and Riley [24] show.

The paper extends the methods developed by Che & Gale [6] to study auctions with bidders that have two dimensions of private information in two ways. First, it introduces general price-dependent valuations to the framework, and shows the importance of this extension in capturing several cases of interest –e.g. the auction of a moral hazard contract. Second, it shows how it is always possible to map the setting into an economy where bidders have a single dimension of private information.

The most pragmatic and interesting results of the paper, however, are concerned with a different question; one that has been given little importance in the literature.[20] By relaxing buyers' financial constraints through credit subsidies, can the seller improve her objective? The answer is positive under weak assumptions. The key condition is that, even at subsidized rates, borrowing must be costly for buyers. This assumption prevails whenever capital markets are imperfect. Moreover, introducing moral hazard costs, the assumption may coexist with perfect competition in capital markets.

Together with costs of capital, differences in liquidity constraints raise the possibility of using capital costs as a price discriminating device. The paper shows that in a broad set of circumstances, a monopolist or an auctioneer can increase their rents by providing credit subsidies. The monopolist captures a larger demand by way of lowering borrowing rates. The auctioneer raises the expected receipts from the auction by way of improving the competitive position of constrained bidders. While the monopolist requires the response of demand to subsidies to be sufficiently large for subsidies to be profitable; the auctioneer always benefits from a small subsidy when using a second price auction. The effect of subsidies in auctions is stronger because all bidders react to the promise of low borrowing rates, and the winner, who is the only one to receive the subsidy, is in average the less needy.

The bottom line: by absorbing a part of the financial burden of its customers, the seller gains an additional dimension in designing pricing schemes. As a monopolist, it allows her to capture a larger demand without explicitly decreasing prices. As an auctioneer, credit subsidies allow her to increase competition.

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[20] As noted in the introduction, the question is partially approached in Che & Gale (1996), Maskin (1992) and Zheng (2000). But none of this paper go beyond an unrealistic example with bidders that only differ in their liquidity.

8   Appendix

Proof. of lemma 2, page 23 .

Consider the bidding decision of buyer i with type φ_i in the new setting. Assume all other bidders bid according to $B\left(\cdot\right)$. Let $\varphi_{-i} = \{\varphi_j\}_{j \neq i}$ and similarly define θ_{-i} and w_{-i}. Denote $B_{-i}\left(\varphi_{-i}\right) := \{B\left(\varphi_j\right)\}_{j \neq i}$, and $b_{-i}\left(\theta_{-i}, w_{-i}\right) := \{b\left(\theta_j, w_j\right)\}_{j \neq i}$. The problem for buyer φ_i is represented by:

$$\max_z E_{\varphi_{-i}}\left[p\left(z, B_{-i}\left(\varphi_{-i}\right)\right) U\left(\varphi_i, x\left(z, B_{-i}\left(\varphi_{-i}\right)\right)\right)\right]$$

$$\Leftrightarrow \max_z E_{\varphi_{-i}}\left[p\left(z, b_{-i}\left(\varphi_{-i}, L\left(\varphi_{-i}\right)\right)\right) U\left(\varphi_i, x\left(z, b_{-i}\left(\varphi_{-i}, L\left(\varphi_{-i}\right)\right)\right)\right)\right]$$

$$\Leftrightarrow \max_z E_{\theta_{-i} \times w_{-i}}\left[p\left(z, b_{-i}\left(\theta_{-i}, w_{-i}\right)\right) u\left(\varphi_i, L\left(\varphi_i\right), x\left(z, b_{-i}\left(\theta_{-i}, w_{-i}\right)\right)\right)\right]$$

In the first two lines, the expectation is taken with respect to the product measure $\left[\times G_b\left(\cdot; L\right)\right]^{N-1}$ and in the last line it is taken with respect to $\left[\times G\left(\cdot, \cdot\right)\right]^{N-1}$. The first equivalence follows from the fact that $b_{-i}\left(\varphi_{-i}, L\left(\varphi_{-i}\right)\right) = B_{-i}\left(\varphi_{-i}\right)$. The last one follows from construction of $G_b\left(\cdot; L\right)$. Since $b\left(\cdot, \cdot\right)$ is an equilibrium in the original setting, it follows that

$$z^* = b\left(\varphi_i, L\left(\varphi_i\right)\right) = B\left(\varphi_i\right)$$

is a solution.

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Proof. of corollary 1, page 23.

Let $\varphi := \{\varphi_i\}_{i \in N}$, $B\left(\varphi\right) := \{B\left(\varphi_i\right)\}_{i \in N}$, and similarly for θ, w and $b\left(\theta, w\right)$. It follows that:

$$E_\varphi\left[\sum_{i=1}^N p_i\left(B\left(\varphi\right)\right) x_i\left(B\left(\varphi\right)\right)\right]$$

$$= E_\varphi\left[\sum_{i=1}^N p_i\left(b\left(\varphi, L\left(\varphi\right)\right)\right) x_i\left(b\left(\varphi, L\left(\varphi\right)\right)\right)\right]$$

$$= E_{\theta \times w}\left[\sum_{i=1}^N p_i\left(b\left(\theta, w\right)\right) x_i\left(b\left(\theta, w\right)\right)\right]$$

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