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Input-Output Economics and Material Flows

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Abstract: This paper argues that resources constitute the fundamental area of overlap between the interests of input-output economists and industrial ecologists. Three misconceptions about input-output economics obscure this fact: the frequent failure to utilize combined quantity and price input-output models, treatment of value-added as a monetary concept only, and the belief that all input-output models assume a linear relationship between output and final deliveries. The paper dispels these misconceptions by describing a quantity input-output model with resources measured in physical units and the corresponding price model with both resource prices and product prices. The model is illustrated with a numerical example of a hypothetical economy and analysis of a scenario where that economy is subsequently obliged to extract a lower grade of ore. Then three other input-output models are presented: a model closed for household consumption, a dynamic model, and a model of the world economy. Unlike the basic model, the last two are non-linear in final deliveries and in factor prices while also retaining the desirable features of the basic model.

Key words: scenario analysis, price and quantity models, factors of production, dynamic model, world model

> Two fields of scientific inquiry can be interconnected effectively only through a clear conceptual overlap. Moreover, the overlapping (that is, the common) concepts must have proven their internal operational effectiveness separately in each one of the adjoining disciplines. Wassily Leontief, 1959

1. INTRODUCTION

The theory of international trade based on comparative advantage is the most ambitious of economic theories as it explains the operation of the entire world economy in terms of consumption, production, and factors of production in each individual region. Factors of production are those inputs that are required for production but can not themselves be produced in business establishments (at least not in a single production period) and are, furthermore, of limited mobility. For this reason, the available quantities, or endowments, of these factors in a region constrain its production capacity. Thus the relatively lowest-cost producer of some product can increase production only until the available amount of some factor is exhausted; then a higher-cost producer must take over. The potentially limiting factors are taken to be labor, capital, and land, where "land" implicitly includes not only the soil but also everything on or under the surface such as fresh water, energy resources, metals, and other mineral ores.

Despite the importance for economists of the theory of international trade, its empirical implementation in models of the world economy has been relatively limited until now, for reasons to which I return in the last section of this paper. Someplace between the abstract theory of international trade and the large body of empirical work analyzing individual economies, the crucial economic roles of land, water, and mineral resources[1] have become obscured. Physical measures of factor endowments and factor requirements for production are absent from economic databases, and land is no longer treated as a factor of production distinct from capital assets. The place of physical measures of the factor inputs to production has been largely taken over by the monetized concept of "value-added," defined for an individual production establishment and, by extension, for an entire sector, as the residual when the payments for inputs produced in other sectors are subtracted from the sector's total revenues. Value-added is sometimes disaggregated into employee compensation, profits, and a residual.

Input-output economics accords a place of privilege to physical quantities measured in physical units. However, this potential has not been fully exploited because input-output models, like other economic models, are typically implemented using databases in money values only. Today, there is significant and growing interest in input-output economics coming from quarters outside of economics. In particular, industrial ecologists, often with a background in engineering, are using input-output models to analyze the flows of materials and energy through an economy in order to quantify the environmental impacts associated with particular products and processes. Industrial ecologists are attracted to the basic input-output model because it can absorb their data about physical flows at a moderate level of detail and capture the fundamental interdependence of all parts of an economy while also revealing physical relationships. These colleagues bring to collaboration with input-output economists not only new areas of expertise but also new questions and concerns, namely about the environment, that can stimulate new thinking on both sides. A deeper collaboration between industrial ecologists and input-output economists requires dispelling three main misconceptions. I attempt to do that in this paper by exhibiting relevant work from the literature in input-output economics and indicating specific areas where new thinking is required. These misconceptions follow:

Misconception 1. The general form of the basic input-output model is $(I - A)\,x = y$, where all variables are measured in money values.

Fact: The basic input-output model includes both a quantity model and a price model. The quantity model tracks flows of products (goods and services) throughout the economy, and the price model determines their unit prices. This fact is widely ignored because the quantity model is typically implemented in money units, a special case of quantity unit, and the special case is unfortunately mistaken for the general case. The product flows represented in the basic model include mineral commodities but not mineral resources, land, or water. However, the latter can, and should, be represented in the basic input-output model, but they are generally not visible because of the second misconception.

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[1] I distinguish 3 types of mineral resources: fuels, metals (primarily used as materials), and non-metals in their natural settings. The *mineral commodities* are obtained from *mineral resources* by processes that include extraction and preliminary cleaning or concentration. For brevity, I sometimes use the word resources to include all mineral resources as well as land and water.

Misconception 2. Value-added, the difference between a sector's revenues and its outlays for the products produced by other sectors, is a money value without a direct physical counterpart.

Fact: There is a direct physical counterpart to value-added, and this is the appropriate representation for the quantity model. Value-added is the sum of payments to all factors of production, generally defined to include labor and capital. However, mineral resources, land, and water are also factors of production, and the time has come to explicitly include them as such in both quantity and price models. Conceptually, resource flows of factor inputs should not be confounded with inter-industry flows of products in a quantity model. The theoretical significance of their role as sources of value-added in the price model should be simultaneously represented.

Misconception 3. The basic input-output model has two shortcomings that limit its usefulness for industrial ecologists. First, input-output models deal only with flows, but industrial ecologists need to analyze stocks also. Second, input-output models assume a linear relationship between final deliveries and outputs, but non-linear models are needed to adequately represent and analyze many phenomena, namely stock-flow relationships.

Fact: The dynamic input-output model represents stock-flow relationships and has long been recognized as an important extension of the basic input-output model for both theory building and empirical analysis. Other extensions of the basic input-output model are equally important for industrial ecologists, notably the input-output model of the world economy, which captures the sectoral interdependencies across all trading partners. These models do not assume a linear relationship between output and final deliveries (or between product prices and factor prices). Additional model extensions will no doubt be developed in the course of the collaboration of input-output economists and industrial ecologists, and they should likewise not be limited by the assumptions, or the simple mathematical form, of the basic input-output model.

The objective of this paper, then, is to describe three fundamental properties of input-output economics. The first property is the relation between the quantity input-output model and the price input-output model, a distinction not made frequently enough even by input-output economists. Sectoral outputs and factors are measured in appropriate units, such as tons, kWh, or dollars' worth, in the quantity model, while the unit prices of individual factors and products figure in the price model. Just as the quantity model follows the "supply chain," for an individual product or an entire bill of goods, the price model makes it possible to track the "value chain" for the same final deliveries.

Second, value-added is disaggregated into the payments of wages, profits, and rents for specific categories of resources measured in appropriate units. While this step is easily achieved in practice, it is conceptually fundamental and provides the vital and explicit link to collaboration of input-output economists with industrial ecologists by way of energy use, material flows, and increasingly the use of land and water. As Wassily Leontief said almost a half century ago in the quote that opens this paper, each partner's separate interest in the area of overlap is necessary to assure effective collaboration across disciplines. The congruence of value-added and factors of production with material flows is precisely such an area of overlap.

Third, input-output economics is a conceptual framework for analyzing applied problems rather than a particular mathematical formula or a specific body of data. While most applications to date use only the basic linear model, this constitutes only the simplest representation of input-output relationships. Three different types of models are described and contrasted in order to demonstrate how the input-output framework provides conceptual guidance for developing new models to analyze new problems. These start with the simplest example, which like the basic input-output model is still based on a matrix inverse, and progresses to representations of other kinds of relationships.

The rest of the paper is divided into 5 sections. Section 2 is devoted to a discussion of scenarios about sustainable economic development. The paper begins with this topic because there is no point in developing new models and techniques until it is clear what questions they are intended to address. Section 3 replaces the most common implementation of a basic input-output model with all variables measured in money units by an equally simple framework that distinguishes a quantity model from a price model and includes an income equation that makes explicit the links between them. Section 4 shows how value-added in the monetized model is replaced by factor quantities including physical measures of all inputs in the quantity model and the corresponding unit prices for resources as well as products in the price model and provides a numerical example for a hypothetical economy to demonstrate the concepts.

The concepts of input-output economics can also be applied to more complex relationships than the models of Sections 3 and 4, and the fifth section provides brief descriptions of three important examples. The first example extends the basic model by adding row-and-column pairs describing incomes and outlays of households to the coefficient matrix and then inverting it. The next example is the dynamic input-output model, where profits (row) and investment (column) for each sector are related to the planned increase in production capacity for that sector as represented by a difference equation. The final example is an input-output model of the world economy, represented by a linear program where each region's output is constrained by physical measures of factor availability.

2. SCENARIOS ABOUT SUSTAINABLE DEVELOPMENT

While growth is a common criterion for gauging economic progress, it is only one of several considerations for sustainable development, which takes multiple economic, environmental and social considerations equally into account. Defining scenarios for sustainable development is a substantial challenge, and it is discussed briefly in this section. It is an equally demanding but distinct challenge to analyze their plausibility and their implications. Analyzing scenarios about sustainable development requires a model that does not build in assumptions about growth and that can handle the representation of resources and environmental pollution. An input-output framework is ideally suited to analyzing scenarios about prospects for sustainable development.

The purpose of a scenario analysis is to evaluate a scenario by identifying bottlenecks, recognizing unexpected opportunities, and quantifying a variety of implications. Scenarios are both economic and physical: economic in that they reflect assumptions about how people make their livelihoods and use their incomes and physical because they involve assumptions about technological choices, resource use, and environmental degradation. Contrary to the assumptions built in to many kinds of economic models, a scenario may assume the adoption of technologies that are more expensive but environmentally more desirable than the ones they replace, or of lifestyles that involve less rather than more consumption of material goods. Input-output models are well suited to analyzing these kinds of scenarios because the models do not incorporate the maximization assumptions about profits and consumption that are common to virtually all other kinds of economic models. Ultimately a model can be used to analyze a scenario only if the scenario assumptions are well defined and quantifiable in terms of the variables and parameters figuring in the model. The level of detail of an input-output model and the nature of most of its parameters (inputs per unit of output) are well-suited to the representation of scenarios about sustainable development.

I offer one scenario as an example. In this scenario consumers in rich countries shift from high-calorie meat-based diets to a lower overall intake and a mix of foods that is mainly plant-based. This switch is important from an environmental point of view because a plant-based diet is generally less resource-intensive than one based on meat. A change in the diet would be represented by changes in the composition of consumption, requiring different patterns of production. The analysis needs to assess changes in domestic production, in imports and exports, and in the relative prices of different foods. All of these will have an impact on the use of land and water, mineral resources and chemical products, and so on.

At the same time that consumers in the rich countries might be motivated to make such changes in their eating habits, the growing populations of the developing countries will surely aim at adding animal products to their diets. In particular, China can be expected to import increasing quantities of feed grain for livestock, other agricultural and food products, and possibly fresh water. With an input-output model of the world economy, one could evaluate the extent to which less resource-intensive diets in the rich countries could offset future improvements in diets in developing economies and assess the land and water requirements of the scenario, the implication for different sectors in different geographic regions, and the impact on relative prices of agricultural products. For further discussion of this scenario, see (Duchin forthcoming 2005). This scenario can also be analyzed in the context of a one-region model, but then changes in the region's imports and exports need to depend on exogenous assumptions.

3. THE QUANTITY MODEL AND ITS PRICE DUAL

3.1 The Quantity and Price Models

The equation $(I - A)x = y$ and the so-called Leontief inverse matrix $(I - A)^{-1}$ are often treated as comprising the entire analytic core of input-output economics. This matrix equation is only the simplest relationship that describes the interdependence of the inputs and outputs of different parts of an economy. Yet this equation alone, coupled with the assumption that all variables are measured in money units, is used in the vast majority of empirical input-output studies, and it is the one into which, increasingly over the past several years, industrial ecologists have incorporated their data on product life cycles and material flows. There is a certain irony in this choice of input-output model for an analysis that accords importance to physical quantities of energy use and material flows, as will become apparent below. The objective of this section is to provide a better alternative.

The familiar equation is only an abbreviated form of the basic input-output model. The full model for an economy described in terms of n sectors requires 3 equations:

$$(I - A)\, x = y \qquad (1)$$

$$(I - A')\,p = v \qquad (2)$$
$$p'y = v'x. \qquad (3)$$

where A is the n x n input-output matrix, and x, y, p, and v are n x 1 vectors: x is the vector of output levels, y is final deliveries, p is unit prices, and v is value-added per unit of output. Each sector's output is quantified in a unit appropriate for measuring the characteristic product of that sector. Thus steel and plastics would be measured in tons,[2] electricity in kWh, and computers and automobiles in numbers of standard units (i.e., number of computers of average capability). Even some service sector output may be measured in a physical unit, such as number of insurance policies. However, some sectors have output mixes that are so heterogeneous as to be more usefully measured in the money value of output, say dollars' worth of business services. An input-output model places no restriction on the choice of units for measuring output, whether physical or monetary units, nor does it require that all quantities be measured in the same unit. The resulting table, and the coefficient matrix derived from it, can be constructed with no conceptual difficulty in a mix of units and. In the coefficient matrix A derived from a mixed-unit flow table, the ijth element is equal to the ijth element of the flow table divided by the jth row total (since it makes no sense to calculate the jth column total in a mixed-unit table). The A matrix may instead be constructed directly as a coefficient matrix using engineering information, such as that developed for the use phase of life-cycle studies.

Equation (1) is called a quantity input-output model. If variables are measured in physical quantities such as tons or computers, the corresponding technical coefficients are ratios of physical units such as tons of plastic per computer. If y is given, the solution vector x represents the quantities of sectoral outputs.

Equation (2) is the input-output price model, and the components of the vector of unit prices are price per ton of plastic, price per computer, etc. For a sector whose output is measured in dollars in Equation (1), for example business services, the corresponding unit price is simply 1.0. With this equation one can compute the impact on unit prices of changes in technical coefficients (A) or in value-added per unit of output (v). Finally, Equation (3), called the income equation, is derived from the first two: this identity assures that the value of final deliveries is equal to total value-added, not only in the actual base-year situation for which the data have been collected but also under scenarios where values of parameters and exogenous variables are changed.

It generally escapes notice that Equation (1) has the attributes of a *quantity* model when, as is most frequently the case, the outputs of all sectors are all measured in money units. One component of the output vector, in money terms, would be the value of the output of plastic or steel, each figure being the implicit product of a quantity and a unit price, but with inadequate information to distinguish the quantity from the price. Under these circumstances, there is no perceived benefit from a separate price model: all elements of the price vector in (2) would be 1.0, and the price model is therefore deemed to be trivial. This is a faulty conclusion, however, since even in this extreme case (i.e., where are quantities are measured in a money unit), the price model provides additional information: it yields the percentage *changes* in unit prices associated with changes in A or v. When, as this paper recommends, some of the variables of the quantity model are measured in non-monetary physical units, the solution prices are in money values per physical unit.

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[2] These would be tons of a standard product, such as a certain quality of steel.

For industrial ecologists no characteristic of an economic model can be more important than the systematic distinction of quantities from prices and the use of compatible quantity and price relationships. Once the data have been collected for a quantity model, very little additional information is needed to also implement the price model (only the unit prices of factors). Some input-output economists have long made use of mixed-unit quantity with and without corresponding price models; examples are (Leontief, Carter and Petri 1977, Duchin 1990, and Duchin and Lange 1998).

3.2 Tracking the Value Chain

The quantity input-output model can track inputs and outputs along the full supply chain by identifying and quantifying both direct and indirect inputs to the final products under analysis. Links in the chain are revealed in the power expansion to the solution for the output vector in the quantity model:

$$x = y + Ay + A^2y + A^3y + \ldots, \qquad (4)$$

where y is the vector of products delivered and each succeeding term on the righthand side represents the direct outputs required to deliver the preceding round of inputs. This equation follows from the easily verified fact that $I = (I - A)(I + A + A^2 + A^3 + \ldots)$, or $(I - A)^{-1} = (I + A + A^2 + A^3 + \ldots)$, and from Equation (1), $x = (I - A)^{-1}y$. The contribution of each succeeding term is smaller than the one before, so a good approximation to total production, x, is achieved if the righthand side is truncated after several rounds.

While this power expansion is well known, it is less appreciated that the price model can also be written in this form:

$$p = v + A'v + A'^{2}v + A'^{3}v + \ldots. \qquad (5)$$

This equation shows that the price of a product is equal to the value-added paid out in the sector producing the product plus the value-added for all direct inputs and all rounds of indirect inputs. Using the price equation one could disaggregate the price of, say, food into value-added received at the farm, the food-processing sectors, and the supermarket. These conceptual linkages are even more useful when value-added is disaggregated, as below, according to its main factor components.

4. FACTOR INPUTS: THE CONCEPTUAL LINK BETWEEN ECONOMIC VALUE-ADDED AND RESOURCE FLOWS

4.1 Value-Added as Payments to Factors of Production

The case was previously made that an input-output table, or matrix, in only money values obscures the underlying physical flows because it fails to distinguish the quantity from the unit price of a product, for example the number of cars from the price per car or the amount of steel from the price per ton. Nowhere is this shortcoming more strikingly problematic, however, than in the case of value-added.

Also called net output, value added in the input-output tables prepared by statistical offices is essentially the amount paid for the use of labor and capital.[3] But quantities of labor and capital can in principle be measured, and the objective of this section is to replace the monetized notion of value-added by its quantity and price components: quantities of factor inputs for the quantity model and unit prices of factors for the price model. Land, mineral resources, and water are included as factor inputs. Since many sources of water are not priced, it is not surprising that they are left out of monetized accounts. But obviously water has as much claim for inclusion in the analysis of scenarios for sustainable development as land and minerals, resources that are converted to priced commodities before being absorbed into the production process.

Mineral resources are generally not included as factors of production because they are considered “free gifts of nature.” In input-output tables the mining sectors purchase goods and services from other sectors as well as the labor and capital for transforming the purchased inputs into resource commodities, such as a ton of processed coal or iron pellets or alumina. The sector’s commodity output is duly recorded as product, but the resource input of raw coal or iron ore or bauxite is simply not represented! The challenge of this section is to make these resources visible.

For classical economists, rent, the income earned by the owners of land and other resources, was kept conceptually distinct from profits and wages. However, rents on land are not explicitly represented in input-output tables as a component of value-added because they are relatively small for most sectors except agriculture and mining, especially in the industrialized economies, and because economists today treat land as a capital asset that earns profits just like built capital.

Three steps are needed for a proper representation of factors of production: to interpret value-added as payments to factors of production; to add land, water and mineral resources to capital and labor as distinct factors of production with unit prices (wages for labor, profits on capital, and rents or royalties on resources), and to represent the quantities of factors of production in the quantity model and their unit prices in the price model. In this way resource inputs can be represented in the input-output model using the same concepts and units employed by industrial ecologists.

4.2 Model with Factor Inputs and Factor Prices

The proposed representation provides direct links from mineral resources and their extraction to the processing and use of mineral commodities in other sectors of the economy. Following these links one can calculate the resource content for any final bill of goods and quantify the use of those resources at all points along the supply chain. In the price model, the rents earned by the owner of the resource, usually the sovereign state where the resource is located, can be distinguished from profits earned in the extracting sector, usually by foreign concessionaires, and at subsequent stages of processing and fabrication. The price model can trace, for a given product, not only the total value-added but also the incomes earned by individual factors in every sector that has contributed to its production. The sum of all these factor incomes, those paid out

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[3] This ignores the relatively small residual consisting of subsidies and certain taxes.

directly in the producing sector and indirectly in those sectors whose outputs it purchases, is the unit price of the product.

The basic input-output model with explicitly identified factor inputs and factor prices is shown in Equations (1′ - 3′) where value-added, v, is disaggregated into k components, each described by a quantity and a price. Thus $v = F'\pi$, where F is the k x n matrix of factor inputs per unit of output, and π is the k-vector of factor prices. Defining f as the k-vector of total factor use in physical units, the equations are as follows:

$$(I - A)x = y \qquad (1'a)$$

and

$$Fx = f \qquad (1'b)$$

$$(I - A')p = F'\pi \qquad (2')$$

$$p'y = \pi'Fx. \qquad (3')$$

4.3 Numerical Example

This section provides a numerical description of a hypothetical economy using a basic input-output quantity model, price model, and income equation with the explicit representation of resources measured in physical units as factors of production (Equations (1′ - 3′)). The example provides a concrete illustration of the concepts described earlier; in particular it demonstrates the relationships between products and factors and between the quantity model and the price model.

The hypothetical economy in question produces wheat, coal, iron pellets, machinery and electricity using labor, capital, land, raw coal, and iron ore as factors of production. Outputs of the first three sectors are measured in tons; machinery is measured in number of units, and electricity in kWh. Land is measured in hectares, raw coal in tons, iron ore in tons of metal content, labor in person-years, and capital in the money unit, dollars. The coal mining sector extracts and cleans the raw coal and sells a coal commodity while the iron mining sector extracts iron ore and concentrates it to pellets. The factor prices are rents on the resource inputs, the wage rate for labor, and the rate of return on capital. The example quantifies the indirect reliance of other sectors’ products on raw coal and iron ore and the portions of the prices of the products that correspond to payments for these factors.

The A and F matrices for the hypothetical economy are shown in Table 1. Table 2 shows values for the exogenous variables (y and π) and the solution values for endogenous variables (x, p, f, and v), where factor use is calculated as $f = Fx$, and value-added is calculated as payments for all factors per unit of output, or $v = F'\pi$.

Table 1. A and F Matrices for a Hypothetical Economy

A Matrix	Wheat	Coal Mining	Iron Mining	Machinery	Electricity
Wheat	0.020	0.000	0.000	0.000	0.000
Coal mining	0.000	0.023	0.214	0.259	0.833
Iron mining	0.000	0.000	0.286	0.556	0.139
Machinery	0.020	0.068	0.143	0.111	0.278
Electricity	0.049	0.045	0.179	0.370	0.056

F Matrix	Wheat	Coal Mining	Iron Mining	Machinery	Electricity
Land	0.245	0.045	0.107	0.000	0.000
Raw Coal	0.000	1.250	0.000	0.000	0.000
Iron Ore	0.000	0.000	1.071	0.000	0.000
Labor	0.196	0.182	0.286	0.444	0.056
Capital	0.980	2.727	5.714	11.111	16.667

Note: See text for units

Table 2. Exogenous and Endogenous Variables

	Exogenous	Endogenous		
	y	x	v	p
Wheat	100	102	6.23	9.28
Coal mining	0	44	9.66	15.23
Iron mining	0	28	8.32	35.99
Machinery	5	27	7.56	51.29
Electricity	12	36	4.00	38.06
	π	f		
Land	15	30		
Coal	5	55		
Iron	2	30		
Labor	12	50		
Capital	0.2	1280		

Note: See text for units.

According to the F matrix (Table 1), raw coal is input only to the coal mining sector, which requires 1.25 tons of resource input for each ton of commodity coal. Likewise, iron ore is input only to the iron mining sector, which requires about 1.07 tons of resource input for each ton of iron pellets it delivers. The rent on land (see the vector of factor prices, π , in Table 2) is assumed to be \$15 per hectare per year, the annual rents (or royalties) on raw coal and iron ore are \$5 and \$2 per ton, respectively, and wages are \$12 per person-year. The capital stock, consisting of buildings and equipment, is measured in dollars' worth, and the rate of return on capital is 20% (Table 2).

To quantify the dependence of all sectors on the individual resource inputs, we calculate the k x n matrix $F(I - A)^{-1}$, where each entry measures the amount of one factor (corresponding to the row) required directly and indirectly to deliver a unit of final deliveries of the product (corresponding to the column). This matrix is shown as Table 3. According to the first and last entries in Table 3, 0.265 hectares of land are required to deliver a ton of wheat and $47 of capital to deliver a kWh of electricity to final users.

Comparing this matrix with F element by element (Tables 1 and 3) shows that, even though not all factors are required directly in each sector (i.e., there are zeroes in F), every sector makes use of all factors at least indirectly (i.e., there are only non-zero entries in $F(I - A)^{-1}$). In particular, delivering100 tons of wheat to final users requires (reading down the first column in Table 3) 27 hectares of land, of which most (25) is used directly to grow the wheat, but also 16 tons of raw coal and 9 tons of iron ore, both of which are entirely attributable to their use in the production of machinery and electricity purchased by establishments in the wheat sector.

Table 3. Factor Requirements to Satisfy Final Deliveries ($F(I - A)^{-1}$)

	Wheat	Coal	Iron	Machinery	Electricity
Land	0.265	0.074	0.272	0.268	0.184
Raw coal	0.160	1.548	1.500	1.829	2.273
Iron ore	0.087	0.175	2.171	2.336	1.011
Labor	0.287	0.356	1.117	1.739	1.049
Capital	4.438	8.821	33.372	55.355	46.619

Note: See text for units.

Following a similar logic, unit prices can be disaggregated, using Equation (2′), into the portion paid, directly and indirectly, to each factor of production. Table 4 shows the matrix $\hat{\pi} F(I - A)^{-1}$, with the elements of the price vector (being the column totals) as the bottom row[4].

Table 4. Product Prices Diasggregated by Individual Factors ($\hat{\pi} F(I - A)^{-1}$)

	Wheat	Coal Mining	Iron Mining	Machinery	Electricity
Land	3.97	1.11	4.07	4.02	2.76
Raw Coal	0.80	7.74	7.50	11.68	11.37
Iron Ore	0.17	0.35	4.34	3.66	2.02
Labor	3.45	4.27	13.40	20.87	12.58
Capital	0.89	1.76	6.67	11.07	9.32
Unit Price	9.28	15.23	35.99	51.29	38.06

Note: Unit prices as total payments to factors. Column headings refer to products and row headings to factors.

Thus the income from a ton of wheat (column 1 of Table 4) is paid out mainly for land ($3.97) and labor ($3.45) for a total of $7.42 out of $9.28, of which most of the labor and almost all the land are used directly in the production of wheat (seen by comparing with the components of $\pi' F$). By contrast, about 30% of the price of a machine ($11.68 plus $3.66, or $15.34, out of $51.29) or a kWh of electricity ($11.37 plus $2.02, or $13.39, out of $38.06) goes to pay rents for resources, even though neither resource is directly exploited by these sectors.

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[4] From Equation (2′) we know that $p' = \pi' F(I - A)^{-1}$; using $\hat{\pi}$ (a diagonal matrix) in place of π in the equation provides a disaggregation of p into individual factors.

4.4 Scenario about Resource Degradation

Now consider a simple scenario where the same economy is forced to extract iron ore with a lower metal content. The objective is to quantify how much this deterioration will cost the economy, in terms of the use of resources, the production of output, and price increases, relative to the baseline. We assume that this resource deterioration requires of the iron-mining sector 20% more machinery and electricity and 20% more labor and capital to convert a larger quantity of lower-grade ore (in order to achieve a given metal content) to a ton of iron pellets.

Redoing the calculations (using Equations 1′, 2′ and 3′ and the new input coefficients) shows higher sectoral output levels, factor use, and unit prices. Total factor payments increase 6% from $1641 to $1755 to deliver the same quantities of output to final users. Most outputs and most factor inputs, except for wheat and land, increase by about 10%. The unit price increase is steepest for the iron commodity (23%) and substantial for machinery (14%), since the latter makes intensive although indirect use of iron ore. It is lowest for wheat and the coal commodity (4% each), which make little use of iron ore either directly or indirectly.

5. CLOSURE OF THE BASIC INPUT-OUTPUT MODEL FOR CONSUMPTION, INVESTMENT, AND TRADE

5.1 "Closure" of the Basic Model

The basic input-output model is an "open" model for a single country or other geographic region and a single time period. The openness refers to the fact that consumption, investment, and exports are all columns of final deliveries whose levels are exogenous -- that is, specified from outside the model -- rather than being endogenously determined by the model. Thus there is no way to assure that, under alternative scenarios, outlays for consumption will be consistent with the endogenous earnings of labor, that investment will be consistent with earnings on capital stock, and that exports and imports will shift in consistent ways.

The basic model can be used to address many kinds of questions about economic interdependency that cannot be approached in other ways. This fact accounts for its continuing popularity. However, analyzing scenarios about sustainable development runs up against limitations of the basic model, notably the fact that consumption, investment, and trade levels are exogenous. The three extensions of the basic input-output model described in this section have been developed to meet this challenge. First, the input-output model is said to be closed for households when consumption and employment are made endogenous by relating them through one or more mathematical equations. In the simplest case, the earnings of labor are distinguished from other components of value-added and directly linked to household consumption, thus "closing" the model for households. Second, a dynamic input-output model is described that relates product flows to capital stocks. The dynamic model disaggregates the return on capital from other components of value-added and provides closure for investment outlays and the return on capital. Investment flows are associated with increases in the capital stock, and the price of the product includes a return on the capital stock required for its production. Finally, a world model results when the one-region model is closed for trade flows with all potential trade partners. An input-output model of the world economy with trade based on comparative advantage provides closure for a region's imports and exports by linking them to production and trade of all other regions. This trade model accords a prominent theoretical role to factor endowments, the total physical supply in each potential trade partner of each factor of production.

There are many ways of achieving closure for a model that contains an input-output matrix. The resulting model remains an input-output model only if the closure is multi-sectoral, that is, involves all sectors simultaneously. Thus in a dynamic input-output model the magnitude and composition of investment are determined at a sectoral level, and capital goods ordered by one sector are produced in the appropriate quantities and with designated time lags by the sectors producing those particular capital goods. This approach to closure is different from a dynamic model with an aggregate investment function for the economy as a whole or one with sectoral production functions that do not specify the sectoral composition of investment. It is also to be distinguished from the simplest kind of closure, described below for household consumption, where rows and columns are added to the coefficient matrix, which is then inverted.

The simplest closure for the basic input-output model retains the assumption of a linear relationship between output and final deliveries in the quantity model and between prices and value-added in the price model. This is the case in the closure for households described below. The second example of a dynamic input-output model makes use of a matrix difference equation with relationships that are no longer linear functions of final deliveries and value added. The final example utilizes a linear program, where production and consumption in each region are subject to constraints on the availability of the factors of production, inducing both trade flows in the quantity model and rents on scarce factors in the price model. In the linear program, both the objective function and the constraints are linear functions of the independent variables (as the name "linear program" implies), but the relationships between output and final deliveries, and between prices and value-added, are no longer linear.

These closures require additional information in the form of new variables and parameters as well as additional assumptions about the logic of the relations among these variables and between them and those of the basic model. While the three individual closures have been achieved and implemented, there is at this time no operational input-output model that is simultaneously closed for consumption, investment, and trade: this would be a dynamic world model closed for households.

5.2 Household Consumption

The closure of the basic input-output model for households provides the best example of a true conceptual extension to the basic model, but one that can still be represented with the same mathematical model, a linear relationship of the key variables involving a matrix inverse. This kind of model will be useful for the study of sustainable consumption. Industrial ecologists are concerned mainly with production and technology, but in the last few years they have turned increasing attention to the industrial ecology of household consumption (Hertwich, forthcoming 2005). This focus reflects the conviction that technological change cannot deliver sustainable development in the absence of changes in household lifestyles, mainly behaviors regarding diet, housing, and mobility. Changes in household consumption patterns have direct and indirect effects on factor use, including resources and employment, as well as on household income, and these in turn feed back on consumption. Closure of the basic input-output model for households captures this feedback loop and assures that household income and consumption outlays are consistent. The idea of extending an input-output table in this way is attributed to Stone, who made major contributions to national accounting (Stone 1975), with other particularly important contributions by Pyatt and Thorbecke (1976) and Keuning (1995) among others.

The simplest closure for households is achieved by starting from the n x n A matrix and adding one column and one row so that the resulting matrix is of dimensions (n + 1) x (n + 1). The new column of consumption coefficients is taken from final deliveries, and the new row of labor coefficients comes from factor inputs. If the output of households in the quantity model is number of workers, the row unit is workers per unit of each sector's output and the column unit is consumption per worker. In the price model, the unit price for the household sector is the wage rate. The matrix is manipulated in exactly the same way as its n x n counterpart, and a matrix inverse is calculated to provide a solution to the linear system. (If household output in the quantity model is measured in money values, the row unit is employee compensation per unit of sectoral output and the column unit is outlays for a given sector's product as a share of the total value of consumption.)

With the closure for household consumption, an important relationship has become endogenous. Now if changes are made in the A matrix, economy wide labor requirements, consumption quantities, the wage rate, and product prices will all adjust consistently. The quantity model using the expanded matrix now assures that enough labor is employed to satisfy consumption requirements, and the price model assures that wages are adequate to cover workers' costs of production, i.e., to purchase the consumption bundle.

The social accounting matrix (or SAM) is the name given to the extension of an input-output table that treats other categories of final deliveries in the way just described for households. (The name social accounting table would have been less confusing.) The SAM is converted to a coefficient matrix and manipulated like an input-output table. In this way it makes explicit the links between different categories of value-added (factors used and income earned) and corresponding categories of final deliveries (deliveries made and income spent). Today SAMs are compiled in many statistical offices, mainly in developing countries, as part of their National Accounts. They may contain several categories of households and different types of workers. Like input-output tables, they are generally compiled and analyzed in money values only. Duchin developed a mixed-unit SAM for Indonesia and constructed quantity and price models to analyze a scenario about technological changes (Duchin 1998). The analysis demonstrated that what may look like an increase in income for a certain category of household when analyzing only a money-value SAM may in fact be an increase in the number of such households coupled with not an increase but actually a decline in the average income per household.

5.3 Dynamic Model

The capital stock consists of infrastructure, buildings, machinery and equipment that are essential for production and consumption. These durable goods require energy, materials and other resources for their production, and after their economic or physical lifetime is exhausted, they are a major source of wastes and a secondary source for materials. The dynamic input-output model represents the demand for capital goods on the part of each producing sector and provides sectoral detail for the input requirements for resources and products to produce the capital goods.

Leontief formulated the dynamic input-output model shown in Equation (6) in terms of a difference equation with dated coefficient matrices, including a new matrix describing capital requirements (the B matrix), that distinguished technological structures at different points in time (Leontief 1970). The exogenous vector of investment, formerly part of final deliveries, was replaced by an expression where a matrix of stock-requirement coefficients is multiplied by the anticipated increase in output between the present time period and the subsequent period. This is written for the quantity model as a difference equation:

$$(I - A_t)\, x_t - B_{t+1}\, (x_{t+1} - x_t) = c_t, \qquad (6)$$

where c_t includes all final deliveries except investment goods. Interestingly, Leontief entitled the article "The Dynamic Inverse," stressing the fact that this was a linear system that could still be represented by a matrix and its inverse.

Unfortunately, this version of the model has features that limit its usefulness for empirical investigation: nonnegative solutions for the output vectors cannot in general be assured. Duchin and Szyld (1983) relaxed some of the unrealistic constraints in Leontief's model by defining two additional variables: each sector's production capacity and additions to capacity during a given time period. The new model introduced a non-linearity by allowing for unused capacity when output is falling: no expansion of capacity takes place if there is unused capacity, so a sector can fail to grow and still function normally (rather than having its capital stock turned back into raw materials when the investment term, $B_{t+1}\, (x_{t+1} - x_t)$, becomes negative). This characteristic made the dynamic input-output model operational for empirical analysis and is particularly well suited for analyzing scenarios that focus on development and not on growth. The model was used in an empirical investigation of the impact of computer-based automation on employment in the US over the period from 1963 to 2000 (Leontief and Duchin 1986), and a related model was used by Edler and Ribakova (1993) in a study of technological change in the German economy. The first empirical study using both dynamic quantity and price models was carried out by Duchin and Lange (1992).

Time distinguishes stocks from flows: it takes time to accumulate capital stocks, and they are durable goods with a longer lifetime than other products. The dynamic input-output model described above deals with the time lags required to put new capacity in place: only a uniform one-year lag is represented in the simple version of Equation (6). Leontief and Duchin (1986) also represented the replacement of existing capacity as durable goods become worn out or obsolete. No attempt has yet been made to use the dynamic input-output model to determine the potential of the depleted capital stock for reuse or for recycling of materials.

5.4 Trade

Challenges to sustainable development involving the extraction, processing, use, disposal, and reuse of resources are of a global nature. Many of the poorest economies are heavily dependent on agricultural production and resource extraction for export earnings. Their economic well-being depends upon the quantities of resources they can export and the prices of these exports relative to the cost of their manufactured imports (i.e., their terms of trade). Important influences are: barriers to trade in potential importers, including escalating tariffs (where the tariff is low on a raw material but progressively higher as the resource is more highly processed), and the amount of rent or royalty received by the owner of a resource relative to the profits earned by the industry that extracts and markets it -- especially when the profits are earned in another country.

The first input-output model of the world economy was conceived by Leontief (1974) and implemented by Leontief, Carter, and Petri (1977) to analyze scenarios about future economic development. It required a massive data collection effort and represented a major computational challenge for that time. It was run as a mixed-unit quantity model with rudimentary elements of a price computation and rudimentary dynamics. There was no attempt to base trade flows on comparative costs. That model was subsequently refined and the database expanded and updated for more specialized empirical studies, including the projection of future mineral and energy use (Leontief, Koo, Nasar and Sohn 1983) and evaluation of a scenario for sustainable development (Duchin and Lange 1994). This and many other models of the world economy can be used only by teams of researchers: they require a far larger number of exogenous assumptions and far more data than one-region models, and their use is cumbersome and labor-intensive. Such models have not been of much interest to theorists, including trade theorists, because they do not incorporate a concept of comparative advantage.

Recently Duchin revisited this framework in ways that should make it easier and more attractive to use. The new framework includes both a quantity model and a price model, and both are based on a fully general and operational conception of comparative advantage with production limited by resource availability (2005 forthcoming). It assigns a crucial role to resource endowments in different geographic regions as physical constraints on production and calculates scarcity rents even on unpriced resources, such as fresh water. These constraints introduce a nonlinearity into the model. Called the World Trade Model, it is a linear programming model of the world economy: the primal and dual correspond to the quantity and price input-output models, respectively, while the equality of the primal and dual objective functions corresponds to the income equation of a one-region model.

The World Trade Model offers several practical advantages. Because it has more theoretical structure about trade than its predecessor, the new model requires many fewer and simpler equations. Data requirements are also lower: for each country or region it requires only the information base of the basic input-output model plus a vector of total factor availability (e.g., coal reserves or the size of the labor force) and a vector of factor prices. The World Trade Model retains many features of the Leontief, Carter and Petri model, but in addition it makes a fully multisectoral determination of endogenous trade flows and product prices. In line with its intended use for analyzing scenarios about sustainable development, it minimizes factor use for given (exogenous) regional consumption rather than maximizing consumption for given factor use.

A country's requirements for resources, goods, and services may be met through domestic extraction and production or else through imports that are purchased in exchange for exports. In principle a country trades when it is cost-effective to do so, and changes in technologies and consumption patterns impact the calculation of cost-effectiveness. This calculation requires a direct comparison of the cost structures in all potential trading partners. Such comparisons cannot be achieved in a one-region framework.

6. CONCLUDING COMMENTS

Fuels, materials, other minerals, land and water are crucial for sustaining life both directly and indirectly through their roles in the production of goods and services. The ways in which we use resources are the single most important consideration for environmental degradation. Resource use is the common concern of input-output economists and industrial ecologists. The two groups have common interests in the availability of many types of data and mathematical models for analyzing the use of resources. But in both cases the inquiries need to be driven by the questions to be addressed rather than by what data have been collected or what techniques are in the toolkit. At the extreme, it is more useful to ask probing questions and address them in a preliminary way with scanty data and simple methods than to analyze trivial questions or carry out only formal exercises with highly massaged databases and elaborate techniques. All the better, of course, to address important questions with ample, high-quality data and relevant models.

Globalization today involves an unprecedented extent of transfer of technologies and emulation of institutions and lifestyles. The fact that resources are unevenly distributed over the globe lends critical importance to the terms on which they are obtained and the division of labor in processing them. The prospects for dramatic changes in building design and material use, the substitution prospects for specific resources in different uses, the magnitude of recycling that is practical to achieve -- all these are questions to be addressed. We need to formulate the big questions to frame our subject and only then determine what data and methods may be needed to address them. I am optimistic that we can make striking progress at this time in these directions through the collaboration of industrial ecologists with input-output economists.

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