"Technological Unemployment: A New View"

by

Peter Hans Matthews

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DEPARTMENT OF ECONOMICS
MIDDLEBURY COLLEGE
MIDDLEBURY, VERMONT 05753

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TECHNOLOGICAL UNEMPLOYMENT: A NEW VIEW

Peter Hans Matthews

Department of Economics
Middlebury College
Middlebury, Vermont 05753

802/443-5591 (tel)
802/443-2084 (fax)

peter.h.matthews@middlebury.edu

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Abstract: This paper extends the now familiar Shapiro-Stiglitz (1984) model of labor market behavior to reconsider the controversial proposition that some forms of innovation have persistent displacement effects. In particular, it finds that when distinctions between random production failures and reduced effort level are difficult to draw, the adoption of new methods of production that compel more effort, break down more often and/or allow for closer supervision will sometimes induce technological joblessness. The possible magnitude of such dislocation, its welfare effects and the possibilities for intervention are then discussed in detail.

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Technological Unemployment: A New View*

> Tess . . . started on her way up the dark and crooked lane or street not made for hasty progress; a street laid out before inches of land had value, and when one-handed clocks sufficiently subdivided the day.
>
> Thomas Hardy, from *Tess of the d'Urbervilles*, as quoted in Thompson (1967: 56)

1. Introduction

In a characteristic flourish, the late historian E. P. Thompson (1967) used the "street laid out ... when one-handed clocks [sufficed]" as a point of departure in his landmark paper on the discipline of work and the measurement of time. The introduction of reliable mechanical clocks into the first proto-factories, an "innovation" that allowed those who supervised production to monitor better the effort of individual workers, altered forever the historical relationship between them, as economic historians since Landes (1983) have underscored. The diffusion of such clocks constitutes an important, but often overlooked, form of technical change: the rise in output per worker was *not* predicated on the acquisition of new tools or different skills per se, but on "improvements" in the *conditions* of production.

Hounshell's (1984) seminal work on the establishment of mass production methods in the United States provides another important example: the rapid spread of

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the "line" in manufacture was in some measure attributable to the attendant increase in minimum effort and/or the pace of production, as well as the enhanced surveillance of workers. In illustrative terms, the "five dollar workers" in Ford's automobile factories expended more of themselves, and found fewer opportunities for "private behavior," than before.

Modern workers also confront innovation of this kind: whatever their other benefits, computers facilitate their closer supervision. In an argument that echoes Marglin (1974) more than, for example, Landes (1986), Perelman (1998) concludes that computerization has contributed less to social welfare than to the perfection of "command and control structures" both in and out of the workplace. Given the absence of well-defined legal parameters (Anton and Ward 1998) on the surveillance of workers in the United States, Greenlaw and Prudeanu (1997) conclude that little but the potential adverse consequences of "demoralization" preclude routine monitoring of computer use, including, of course, electronic mail.[1]

A formal characterization of this phenomenon requires a model in which workers exert some control over effort - the question of supervision does not otherwise arise - and capitalists are able to choose between "techniques" that differ both in terms of required effort and the measurement of performance. The "effort extraction literature" - a literature that includes, but is much broader than, the Shapiro and Stiglitz (1984) variant of the efficiency wage hypothesis[2] or EWH - therefore pro-

[1] For example, it has been possible for some time to count keystrokes, watch operators' screens, monitor phone use and even location, et cetera. In response, Congress considered considered, but did not pass, the Privacy for Consumers and Workers Act (PCWA) in 1997.

[2] Bowles (1986), for example, contrasts the "neo-Hobbesian" treatment of the problem in mainstream models with his own, which is rooted in Marx (1867). Rebitzer (1993) reviews the contributions of radical political economists to this literature in more detail.

vides a natural framework for analysis. Several contributions to this literature merit particular attention in this context. In their empirical work, for example, Bowles, Gordon and Weisskopf (1984, 1989) have underscored the connection between the "social structure(s) of accumulation" and the extraction of labor, and argue that the ascendance of conservative economics in the 1980s produced an increase in the *share* of profits in national income, but not (until the mid-1990s, perhaps) in the adjusted *rate* of profit. This suggests that capitalists found, and perhaps still find, themselves under pressure to "discover" methods of production that undercut the representative worker's bargaining position. It then seems reasonable to suppose that innovations which increase the pace of production and/or the likeihood that "non-performance" is detected will become more attractive than is otherwise desirable.

It also seems plausible that such methods will "fail" or break down more often, and Levine (1989) considers the implications for the Shapiro-Stiglitz model if capitalists who observe low effort levels cannot be sure whether the cause is in fact "shirking"[3] or measurement error. A similar problem exists when capitalists possess reliable measures of aggregate effort/output, but are uncertain when the value of this measure falls which workers are "responsible." In either case, both shirkers and non-shirkers are sometimes dismissed in the resulting equilibrium, a violation of the "just cause principle." In this context, the enforcement of just cause legislation, often advocated on the grounds of fairness, can become Pareto efficient.

In the model articulated here, capitalists are able to increase both the minimum effort level and the likelihood of detection with the introduction of new, perhaps

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[3] Most radical and some mainstream (Bulow and Summers 1986, for example) economists are uncomfortable with this characterization of the "contested exchange" (Bowles and Gintis 1993) between capitalist/firm and worker, but the term has proven a durable one, and is adopted here.

capital intensive, methods of production. To extend Adam Smith's (1776) famous example, the increase in the number of pins each worker produces following the division of labor is in some measure attributable to the strict discipline, increased effort and closer supervision that such specialization induces. With this increase in effort, however, comes an increase in the likelihood of failure: as individual workers are pushed toward their respective limits, breakdowns become more frequent. The worker who "points" drawn wire - one of eighteen (!) distinct operations that Smith (1776: 5) enumerated - will fall short of her target more often than her predecessors, who oversaw the production of pins from start to finish. Furthermore, supervisors will no doubt find it more difficult to distinguish this sort of non-performance from the more deliberate withdrawal of effort, a variant of Levine's (1989) model. This information problem has important social and economic consequences: an otherwise productive worker whose "failure" is a random event differs in most relevant senses from one who is unproductive because she contests the terms of her wage bargain.

In another contribution to the literature, Agénor and Aizenman (1997) invoke the EWH to characterize both the labor market displacement and segmentation effects of skill-biased technical change in a two sector model of the sort described in Bulow and Summers (1986). Their results lead to one of the most important, and controversial, issues considered here: to the extent that the use of computers and other innovations have increased the pace of production and/or enhanced capitalists' surveillance powers, is *permanent* "technological unemployment" a possible result? After the optimization problems of workers and capitalists have been characterized in some detail, the fourth and fifth sections of this paper consider the model's possible equilibria, and evaluate these on the basis of reasonable parameter values. It is concluded that technological displacement is sometimes possible, even probable, a result that in turn calls for some discussion of the model's welfare implications and the possibilities for intervention.

In this context, the paper should also be understood as a contribution to the debate that Ricardo (1817: 388) ignited almost two centuries ago, when he reversed himself on the so-called "machinery question" :

> These were my opinions, and they continue unaltered, as far as regards the landlord and the capitalist; but I am [now] convinced, that the substitution of machinery for human labor, is often very injurious to the interests of the class of laborers.

This said, the rationale for the displacement effect described below is different, in both form and spirit, from either Ricardo's or those of his intellectual heirs (Duchin and Leontief 1986, for example). It also differs from less common Keynesian treatments (Asimakopulos 1988, for example) of the phenomenon.

2. Workers

Following Shapiro and Stiglitz (1984), the existence of H identical and infinite-lived workers is supposed, each of whom cares about real wages and the pace of work in each (discrete) period. If the representative worker is further assumed to be risk neutral, her within period vNM function will have the form:

$$v^i(\omega^i, e^i) = \omega^i - e^i \tag{2.1}$$

after normalization, where ω^i is the real wage and e^i is some measure of effort. The substitution of real labor income for consumption in (2.1) can be rationalized on the grounds that workers do not, or perhaps cannot, save between periods, but this is not essential: as Shapiro and Stiglitz (1984) observe, the same specification obtains if capital markets are assumed to be perfect.[4] The observation that capital markets are not (in this sense) perfect, however, and that workers sometimes find it difficult to borrow at *competitive* rates of interest (Ausubel 1991), implies that $(1+i)^{-1}$ is

[4] This second interepretation invites debate, however. Critics of the EWH have sometimes claimed that if capital markets were indeed perfect, the introduction of performance bonds would eliminate the incentive to shirk (Carmichael, 1990). There is little evidence, however, that the markets for such bonds function well, if at all.

not the relevant discount rate, so the simple rate of time preference θ is used here instead.

If attention is restricted to long run (steady state) equilibria, the notion of a "job offer" is not difficult to formalize. The j^{th} capitalist, $j = 1, \ldots, M$, $M << H$, who chooses the k^{th} method of production offers her workers ω_k^j each period until the latter are either "separated" or dismissed for non-performance. It is further assumed that capitalists must choose between two methods of production - a mixture is excluded - and that this choice determines the minimum required effort level e_k^j where $e_2 > e_1$. Workers must in turn decide whether to expend this amount of effort or none at all - it is impossible, in other words, to "shirk a little bit." The choice of technique also determines the conditional probabilities f_k^j that a worker "fails" in a particular period despite the expenditure of effort, and d_k^j that non-performance, the result of either no effort or a breakdown in production, is detected *ex post*, where, in the spirit of the previous discussion, $d_2 > d_1$ but $f_2 > f_1$. It is also assumed that capitalists cannot distinguish between the two sources of non-performance, but similar results would obtain if the power to do so were imperfect. What matters, in other words, is that non-performance is a mixed signal, which forces each capitalist to decide what percentage s_k^j of detected workers should be dismissed.

The observation that workers will sometimes fail and that capitalists cannot tell the difference between such failure and "non-cooperative behavior" has important social and economic consequences, not least of which are its effects on the composition of the jobless pool. The standard Shapiro-Stiglitz model predicts that all *equilibrium* worker-capitalist separations will be the result of quits or involuntary separations, since there is no incentive to withhold effort if the "no shirking condition" (NSC) is satisfied. It is a weakness of this model, therefore, that the observed behavior of labor markets suggests otherwise: as Okun (1981) and others have reminded economists interested in labor market microfoundations, a sizeable proportion of separations are *not* worker-initiated. Within the framework of the model described here, some workers will quit and others will be dismissed, but those fired for non-performance will not be shirkers: because the incentive condition is met in equilibrium, all capitalist-initiated job loss will be the result of random failure. Whether or not this enhances the realism of the model is unclear: the capitalist or firm now assumes a more active role, but the implication that *all* dismissals are without cause is no less problematic.

It is further supposed that individual capitalists must precommit to a particular method of production at the start of each period, and that this choice is observable, so that all communication between capitalists and prospective hires about the values of e_k^j, f_k^j and d_k^j is credible. The same cannot be assumed about dismissal policies, however. If the search for replacement workers were costly, the choice of an optimal dismissal rate would be a complicated one. On the one hand, each capitalist would have some incentive to announce that dismissal rates are high because such behavior would, if believed, reduce the real wage required to induce effort. On the other hand, if workers do not then shirk, capitalists should be reluctant to dismiss for non-performance because those who are dismissed will be otherwise productive workers whose failure was a random event, in which case a smaller dismissal rate would reduce search costs. To underscore the basic themes of this paper, however, it will be assumed that search costs are zero, that capitalists' previous behaviors are well known, and that the parameters of their profit-maximization problem, including the exogenous likelihood q that a worker who neither shirks nor fails will quit her position at the end of a particular period, are common knowledge.

If the M capitalists are identical and each sets the same terms in equilibrium, the derivation of the incentive conditions is involved, but not difficult. Let $(\omega_1, e_1, f_1, d_1, s_1)$ and $(\omega_2, e_2, f_2, d_2, s_2)$ denote the "contracts" for first and second method workers, and define $V_{1,k}$, $V_{2,k}$ and V_3 to be (resp.) the lifetime utilities of non-shirkers hired to work with the k^{th} method, shirkers hired to work with the k^{th} method, and jobless workers.[5] The precise form of the NSCs follows from two

[5] This assumes that the contracts are unique, an assumption that is sensible for

initial observations:

Proposition 1. *The lifetime utilities of the non-shirkers under contract are:*

$$V_{1,k} = \frac{\omega_k - e_k + \theta(f_k d_k s_k + q(1 - f_k d_k s_k))V_3}{1 - \theta(1-q)(1 - f_k d_k s_k)} \qquad k = 1, 2 \tag{2.2}$$

and

Proposition 2. *The lifetime utilities of the shirkers under contract are:*

$$V_{2,k} = \frac{\omega_k + \theta(d_k s_k + q(1 - d_k s_k))V_3}{1 - \theta(1-q)(1 - d_k s_k)} \qquad k = 1, 2 \tag{2.3}$$

Proof(s): The proofs for these and most of the propositions that follow are collected in an appendix.

A worker hired to use method k will expend the required effort e^k if, and only if, the reward for such "co-operation" is at least as large as that for non-cooperation or, in other words, $V_{1,k} \geq V_{2,k}$. It follows from (2.2) and (2.3) that this condition can be expressed as:

$$\frac{\omega_k - e_k + \theta(f_k d_k s_k + q(1 - f_k d_k s_k))V_3}{1 - \theta(1-q)(1 - f_k d_k s_k)} \geq \frac{\omega_k + \theta(d_k s_k + q(1 - d_k s_k))V_3}{1 - \theta(1-q)(1 - d_k s_k)} \tag{2.4}$$

which, after simplification and elimination of the ">" , can be expressed as:

Proposition 3. *The incentive conditions (NSCs) are:*

$$\omega_k = \left(\frac{1 - \theta(1-q)(1 - d_k s_k)}{\theta(1-q) d_k s_k (1 - f_k)}\right) e_k + (1-\theta)V_3 \qquad k = 1, 2 \tag{2.5}$$

Most of the properties of (2.5) are simple extensions of standard results: the real wage rates ω_1 and ω_2 needed to induce effort levels e_1 and e_2 are each decreasing functions of the rate of time preference θ and the (respective) probabilities of detection d_1 and d_2, and increasing functions of the separation rate q, the effort effort

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the techniques considered below.

levels, and the welfare of jobless workers V_3. In addition, however, ω_k is also a decreasing function of the failure rates f_k and the "dismissal policies" s_k. Neither of these properties comes as a surprise: as the likelihood that method k workers fail rises, for example, the likelihood that non-shirkers will lose their jobs at the end of each period also rises, which reduces the "punishment value" of dismissal, and forces capitalists to increase ω_k. Likewise, as the dismissal rate s_k increases, the likelihood that non-performance, if detected, is punished, and this reduces the value of the incentives capitalists must provide.

The conditions (2.5) are not reduced forms, of course. The welfare of jobless workers V_3 cannot be one of the model's primitives - in this one sector model, it is a function of the exogenous value of jobless benefits $\bar{\omega}$ and the probabilities of rehire, denoted a_1 and a_2. (The model *does* admit a two sector interpretation, however, one that should be recalled in the calibration exercises that follow: if it is assumed that all the workers not hired to use one of the two methods are then "absorbed" into a "dual" sector without the same information/control problems, the value of $\bar{\omega}$ can be understood as a constant second sector wage, a variant of the Bulow and Summers (1986) model. Viewed from this perspective, the existence of a constant, or even falling, jobless rate is *not* inconsistent with technological displacement, an important feature of this model in the current macroeconomic climate.) Inasmuch as the comparative statics of variations in jobless benefits/dual sector wages are well known, the value of $\bar{\omega}$ is set equal to zero here.[6] Some additional assumptions

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[6] The premise needs some qualification. Until Albrecht and Vroman's (1999) recent contribution, attention was limited to the Shapiro-Stiglitz model in which each capitalist compensated the workers *she* dismissed at some predetermined rate. It would be more realistic to suppose, however, that the state both determines and distributes such benefits, with some reliance on dedicated tax revenues. On the basis of their more elaborate model, Albrecht and Vroman (1999) compare the properties of a proportional payroll tax and one that reflects firms' "experience ratings," and find that the latter is associated with more jobs, less shirking and more output.

on the "matching mechanism" are needed, and it is supposed that each job seeker receives at most one offer at the start of each period. The probabilities a_k that a particular worker receives an offer from a method k capitalist are exogenous from the perspectives of each worker and capitalist but not, of course, in the aggregate, where a_1 and a_2 move to equilibrate the markets for labor power in the long run.[7] With this in mind, their influence on the welfare of job seekers V_3 is not difficult to formalize:

Proposition 4. *If the incentive conditions (2.5) are satisfied, then:*

$$V_3 = \frac{a_1 V_{1,1} + a_2 V_{1,2}}{1 - \theta(1 - a_1 - a_2)} \tag{2.6}$$

This result has the simple, but important, implication that $\lim_{a_1+a_2 \to 1} V_3 = a_1 V_{1,1} + a_2 V_{1,2}$, which confirms the critical intuition behind all such models: as labor markets become less slack, and the combined likelihood of rehire approaches one, the expected punishment value of dismissal, the difference between $a_1 V_{1,1} + a_2 V_{1,2}$ and V_3, approaches zero.

Combined, the first and fourth propositions constitute three conditions in three ($V_{1,1}$, $V_{1,2}$ and V_3) unknowns, which allows the reduced forms for each to be calculated, and for the NSCs to be written in terms of the model's primitives:

Proposition 5. *If the incentive conditions (2.5) are satisfied, then:*

$$V_3 = \frac{1}{1-\theta}\left(\frac{a_1(\omega_1 - e_1)}{1 - \theta(1 - a_1)(1 - q)(1 - f_1 d_1 s_1)} \quad + \right.$$

[7] The reference to the "long run" - the term "potential" would be preferable, perhaps - is in deference to those who argue, with cause, that there is no room for effective demand, and therefore traditional macroeconomic policies, in such models. Equilibrium in an effort elicitation model is perhaps best understood in terms of the *maximum*, rather than the actual, number of workers under contract each period, notwithstanding the recent work of Kimball (1994) and others on the disequilibrium properties of the Shapiro-Stiglitz model.

$$\frac{a_2(\omega_2 - e_2)}{1 - \theta(1 - a_2)(1 - q)(1 - f_2 d_2 s_2)} \tag{2.7}$$

Proposition 6. *The reduced form incentive conditions (NSCs) are:*

$$\begin{aligned}\omega_k = &\left(\frac{1 - \theta(1-q)(1 - d_k s_k)}{\theta(1-q) d_k s_k (1 - f_k)}\right) e_k \\ &+ \frac{1}{\Delta}\left(\frac{1}{1 - \theta(1 - a_1)(1 - q)(1 - f_1 d_1 s_1)}\right)\left(\frac{1 - \theta(1-q)(1 - f_1 d_1 s_1)}{\theta(1-q) d_1 s_1 (1 - f_1)}\right) a_1 e_1 \\ &+ \frac{1}{\Delta}\left(\frac{1}{1 - \theta(1 - a_2)(1 - q)(1 - f_2 d_2 s_2)}\right)\left(\frac{1 - \theta(1-q)(1 - f_2 d_2 s_2)}{\theta(1-q) d_2 s_2 (1 - f_2)}\right) a_2 e_2\end{aligned} \tag{2.8}$$

for $k = 1, 2$, *where*

$$\Delta = \frac{(1 - a_1)(1 - a_2)(1 - \theta(1-q)(1 - f_1 d_1 s_1))(1 - \theta(1-q)(1 - f_2 d_2 s_2)) - a_1 a_2}{(1 - \theta(1 - a_1)(1 - q)(1 - f_1 d_1 s_1))(1 - \theta(1 - a_2)(1 - q)(1 - f_2 d_2 s_2))} \tag{2.9}$$

It is an important, and perhaps surprising, feature of (2.8) that the difference between first and second method wages $\omega_1 - \omega_2$ is a function of the institutional parameters θ and q and the *conditions* of production e_k, f_k and d_k, but *not* the probabilities of rehire a_k or the jobless rate, which means that the *technical* conditions of production do not exert an influence on either. Furthermore, this premium is reminiscent of Adam Smith's (1776) modern treatment of the compensating differential.

3. Capitalists

The representative capitalist's choice of dismissal rates can be inferred before the profit maximization problem is described in detail. Observe first that given the all other capitalists' demands for first and second method workers and therefore the levels of compensation ω_1 and ω_2, as well as the value of their "fallback position" V_3, it is not difficult to show, using an argument similar to that in the proof of the third proposition, that the j^{th} capitalist must offer:

$$\omega_k^j = \left(\frac{1-\theta(1-q)(1-d_k s_k^j)}{\theta(1-q)d_k s_k^j(1-f_k)}\right)e_k + (1-\theta)V_3 \qquad k = 1,2 \tag{3.1}$$

With V_3 fixed from the perspective of the individual capitalist, it becomes clear that the right hand side of (3.1) is decreasing in s_k^j, and that each capitalist will minimize her own labor costs with $s_1^j = s_2^j = 1$, a decision that creates (negative, of course) externalities for her rivals. That is,

Proposition 7. *All capitalists will choose the "harshest" possible dismissal policies:* $s_k^j = 1$ *for all* j *and* k.

Within this framework, then, competition for labor power maximizes the number of unjust dismissals: because the NSCs are satisfied in equilibrium, the enforcement of harsh dismissal policies means that *all* random failure results in job loss, and that absent the exogenous quit/separation rate, *all* job loss is the consequence of such failure. Whatever the robustness of this feature, it is, to repeat an earlier claim, at least consistent with the observation that dismissal for non-performance is an important source of additions to the jobless pool in the "real world."

Given the choice of dismissal policies, the expressions for ω_k^j and ω_k can be consolidated:

$$\omega_k^j = \omega_k = \psi_k e_k + (1-\theta)V_3 \qquad k = 1,2 \tag{3.2}$$

where:

$$\psi_k = \frac{1-\theta(1-q)(1-d_k)}{\theta(1-q)d_k(1-f_k)} \qquad k = 1,2 \tag{3.3}$$

which implies that:

$$\omega_1^j - \omega_2^j = \omega_1 - \omega_2 = \psi_1 e_1 - \psi_2 e_2 \tag{3.4}$$

where V_3 is understood to be exogenous with respect to ω_k^j but not ω_k. As alluded to in the previous section, (3.4) implies that the method-based wage differential is *not* a function of the likelihoods of rehire a_1 and a_2 or therefore the "thickness" of labor markets.

The differences in the *conditions* of production - effort levels, detection rates and failure rates - complicate the characterization of capitalist behavior; for purposes of exposition, a streamlined, but suggestive, version of the profit maximization problem is considered here. Suppose first that while capitalists must precommit to one of the two methods at the beginning of each period, the capital or fixed costs of each are, for the moment, equal. It is further assumed that the two methods exhibit constant returns to *effective* labor below some predetermined limit[8] *and that this return is constant across methods*, a restriction that limits attention to differences in the conditions of production alone:

$$Q_k^j = \begin{cases} \alpha L_k^j & \text{if} \quad 0 \leq N_k^j \leq \bar{N} \\ \alpha \bar{L}_k & \text{if} \quad N_k^j \geq \bar{N} \end{cases} \tag{3.5}$$

where L_k^j is the number of units of effective labor hired and N_k^j is the number of non-shirkers under contract, and where $M\bar{N} > H$. It follows that within this

[8] This is a restriction with some basis in the empirical literature. Even Bils (1987), who finds that the short run marginal costs of production are procyclical, attributes this to the use of more expensive overtime labor, not the existence of diminishing returns *per se*. Furthermore, he also finds that this behavior is *not* mirrored in prices, in which case reliance on a fixed mark-up over constant average variable costs is not unreasonable. See also Blinder *et al* (1998). Inasmuch as macroeconomic considerations have influenced the specification of firm behavior, and not *vice versa*, this is perhaps an example of the "macrofoundations of micro" (Colander 1993).

context, the existence of an equilibrium in which all (or some) capitalists choose the second method will be the result of increased effort levels and/or detection rates, and nothing else.[9]

The need to choose between otherwise similar methods of production is a consequence of the fact that it is labor power, not effective labor, that is exchanged in the market. The capitalist who hires N_k^j non-shirkers commands $e_k(1-f_k)N_k^j$ units of effective labor - each of the $(1-f_k)N_k^j$ workers who does not fail exerts effort e_k. The production function(s) can therefore be expressed:

$$Q_k^j = \begin{cases} \alpha e_k(1-f_k)N_k^j & \text{if} \quad 0 \le N_k^j \le \bar{N} \\ \alpha e_k(1-f_k)\bar{N} & \text{if} \quad N_k^j \ge \bar{N} \end{cases} \tag{3.6}$$

Since there is no reason to suppose that $e_1(1-f_1)$ is equal to $e_2(1-f_2)$, the marginal product of labor power, as opposed to effective labor, is *not* uniform. From another perspective, given the behavior of her rivals, the j^{th} capitalist will have real annual per unit labor costs of $\omega_k^j/\alpha e_k(1-f_k)$ when she chooses method k.

Last, it will be assumed that competition in product markets is imperfect, and that each capitalist exerts some power over prices.[10] In particular, suppose that no matter what the mixture of first and second method capitalists, their combined sales revenues will be a proportion $(1+\tau)$ of total labor costs, where τ, a mark-up of sorts, is sufficient to more than cover fixed/capital costs. Thus, if all capitalists choose the first method, for example, the aggregate demand for labor power schedule will be horizontal (until $M\bar{N}$, of course, at which point it is vertical) with $\omega_1 = \alpha e_1(1-f_1)/(1+\tau)$. Since the profits of no single capitalists are large enough to

[9] Other things being equal, the assumed difference in failure rates *reduces* the attractiveness of the second method.

[10] There is no need to commit to one "imperfection" over another in this context. The model is more or less consistent with several of those outlined in, for example, Startz (1989) or Silvestre (1992).

influence τ, a first method capitalist would therefore have an incentive to switch to the second if her new per unit labor costs $\hat{\omega}_2^j/\alpha e_2(1-f_2)$, where $\hat{\omega}_2^j$ is understood to be the wage she must offer second method workers when all other capitalists have committed to the first method, are less than $\omega_1/\alpha e_1(1-f_1)$ or, substituting for ω_1, if $\hat{\omega}_2^j/\alpha e_2(1-f_2) < 1+\tau$.

A final technical restriction is then needed to ensure that both methods are feasible. For reasons that will become clear in the next section, it will be sufficient to suppose that $\theta(1-q)[d_k(1-f_k)^2\alpha^* + (1-d_2)] > 1$ for both k, where $\alpha^* = \alpha/(1+\tau)$ is a scaled value of output per unit of effective labor. This serves, in effect, as an intuitive lower bound on α.

4. Equilibrium

4.1 The General Approach

The conditions under which labor markets clear are not difficult to state in abstract terms. Suppose that M_1 and $M_2 = M - M_1$ capitalists choose the first and second methods and that their combined demands for first and second method workers are $N_1(\omega_1, \omega_2)$ and $N_2(\omega_1, \omega_2)$. For each pair (ω_1, ω_2), these determine the likelihoods of rehire a_1 and a_2 and, on the basis of the Implicit Function Theorem, the inverse demand functions $\omega_1 = \omega_1(a_1, a_2)$ and $\omega_2 = \omega_2(a_1, a_2)$. Combined with the NSCs (2.8), a second pair of expressions for ω_1 and ω_2 that depend on a_1 and a_2, there are four independent relations in four unknowns, solutions of which constitute clearance of the labor market(s). An equilibrium is then defined to be a quadruple $(\omega_1, \omega_2, a_1, a_2)$ such that labor markets clear *and no capitalist regrets her choice of technique.*

The second step in this construction, the connection between the numbers of first and second method workers N_1 and N_2 and the probabilities of rehire a_1 and a_2, remains to be elaborated, however. The equalization of flows into and out of various "states" in the labor market requires that:

Proposition 8. *In equilibrium, the probabilities of rehire a_1 and a_2 are:*

$$a_k = \frac{(f_k d_k + q(1 - f_k d_k))N_k}{H - (1-q)((1 - f_1 d_1)N_1 + (1 - f_2 d_2)N_2)} \qquad k = 1, 2 \tag{4.1}$$

As one would expect, there is a positive relationship between a_k and N_k: the greater the number of workers hired to use method k, the greater the likelihood of rehire a_k.

The conditions under which a second method equilibrium exists, and the labor market consequences of the establishment of such an equilibrium, are the principal concern here, however, and the previous restrictions on production and price formation facilitate the characterization of these. In particular, absent a measure zero coincidence, "mixed equilibria" in which both methods are used are impossible, in which case attention can be turned to a simple(r) question: If all capitalists have committed to one method or the other, will one or more have an incentive to deviate from this outcome?

4.2 Existence of a (Unique) Second Method Equilibrium

Consider the market for second method workers when all capitalists use this method, as pictured in Figure 1. The intersection E_0 of the demand for labor power and the incentive condition or NSC (2.8), plotted here as a function of N_2 rather than a_2, determines the values of ω_2^* and N_2^*. As the diagram suggests, workers under contract will receive $\alpha e_2(1-f_2)/(1+\tau)$ each period, and each capitalist will receive a share $1/M$ of total profits, equal to $(\tau/1+\tau)\alpha e_2(1-f_2)N_2^*$. The j^{th} capitalist will have no incentive to switch if per unit labor costs after doing so, $\hat{\omega}_1^j/\alpha e_1(1-f_1)$, where $\hat{\omega}_1^j$ is the compensation she must offer her new first method workers, conditional on other capitalists' commitment to the second method, exceed current per unit labor costs, equal to $1/(1+\tau)$.

How does the j^{th} capitalist know what wage $\hat{\omega}_1^j$ she must offer if no first method contracts exist? The answer is contained in the constant differential (3.4): in terms of the current notation, $\omega_2^* - \hat{\omega}_1^j$ is equal to $\psi_1 e_1 - \psi_2 e_2$, where ψ_1 and ψ_2 are as defined in (3.3). Given the initial value of ω_2^*, it follows that per unit costs with the first method will be:

$$\begin{aligned}\frac{\hat{\omega}_1^j}{\alpha e_1(1-f_1)} &= \frac{1}{\alpha e_1(1-f_1)}(\omega_2^* + \psi_2 e_2 - \psi_1 e_1) \\ &= \frac{1}{\alpha e_1(1-f_1)}\left(\frac{\alpha e_2(1-f_2)}{1+\tau} + \psi_2 e_2 - \psi_1 e_1\right) \qquad (4.2)\end{aligned}$$

No capitalist will switch from the candidate second method equilibrium[11] if this value is more that $(1+\tau)$ or, after some simplification, if:

$$\alpha^*(e_2(1-f_2) - e_1(1-f_1)) > (\psi_2 e_2 - \psi_1 e_1) \qquad (4.3)$$

[11] It is assumed here that if per unit labor costs are equal, capitalists will use the "older" method.

Figure 1. The Labor Market When All Capitalists Choose the Second Method

where, once more, $\alpha^* = (1+\tau)^{-1}\alpha$ is an adjusted or "marked down" output per worker. The intuition for (4.3) is straightforward: the left hand side is the scaled difference, positive or negative, in the marginal products of labor power for the two methods, and the right hand side is the difference in the wages needed to secure workers' co-operation with each.

Because the second method is assumed to increase both effort levels and failure rates, the sign of $e_2(1-f_2)-e_1(1-f_1)$ is uncertain. If the more "modern" method features a higher marginal product of labor power - that is, if $e_2(1-f_2) > e_1(1-f_1)$ - the "no switch condition" (4.3) can be written:

$$\alpha^* > \frac{\psi_2 e_2 - \psi_1 e_1}{e_2(1-f_2) - e_1(1-f_1)} \tag{4.4}$$

Two sub-cases can then be distinguished: (a) if $\psi_1 e_1 \geq \psi_2 e_2$ - that is, the numerator in (4.4) is less than or equal to zero - then no capitalist will switch to the first method, no matter what the value of α^*, since the second method is more productive and workers hired to use it are "cheaper," but (b) if $\psi_1 e_1 < \psi_2 e_2$, the value of α^* must exceed a critical bound $\bar{\alpha}$ for defection to be unprofitable. The lower bound is needed in this case because the switch to the first method reduces the value of the incentive the capitalist must provide - that is, the difference $\psi_2 e_2 - \psi_1 e_1$ is negative - which means that the assumed difference in output per unit of effective labor, an increasing function of α^*, must more than offset this.

If, on the other hand, $e_2(1-f_2)$ is less than (or equal to) $e_1(1-f_1)$ - in other words, if the marginal product of labor power with the "older," more "primitive," method is at least as high - the condition under which no capitalist will switch from the second method becomes:

$$\alpha^* < \frac{\psi_1 e_1 - \psi_2 e_2}{e_1(1-f_1) - e_2(1-f_2)} \tag{4.5}$$

if $e_1(1-f_1) \neq e_2(1-f_2)$, and $\psi_1 e_1 > \psi_2 e_2$ otherwise. Once more, there are two sub-cases to consider: (a) if $\psi_1 e_1 > \psi_2 e_2$, (4.5) establishes $\bar{\alpha}$ as an *upper* bound on α^*, but (b) if $\psi_1 e_1 \leq \psi_2 e_2$, capitalists will switch *en masse* to the first method, no matter what the value of α^*. In the first sub-case (a), an upper bound is needed because under the relevant conditions, the second method is less productive but also less expensive than the first, and if the difference in productivities, which increases in α^*, is not too large, capitalists will still not have an incentive to switch to the first method. In the second, no second method equilibrium exists because it is both less productive and more expensive. The inference that neither sub-case is plausible from an historical standpoint - it seems reasonable to suppose that for each of the innovations described earlier, the effort effect was dominant - is tempting, but premature: there are no doubt hundreds, even thousands, of smaller innovations for which this is not the case.

There is another, perhaps more intuitive, interpretation of these conditions, based on three simple ratios:

$$\mu_e = \frac{e_2}{e_1} \qquad \mu_f = \frac{1-f_1}{1-f_2} \qquad \mu_d = \frac{d_2(1-\theta(1-q)(1-d_1))}{d_1(1-\theta(1-q)(1-d_2))}$$

each of which ranges from 1 to ∞.[12] The first of these, μ_e, the ratio of the second to first method effort levels, is a natural measure of the size of the *effort effect* associated with the former, while the second, μ_f, the ratio of first to second method success rates, can be considered an index of the *failure effect*. The third ratio, μ_d, is a measure of the *detection effect* associated with the second method: given the rate of time preference θ and separation rate q, it rises with d_2 and falls with d_1. Recognizing that $\psi_1/\psi_2 = \mu_d/\mu_f$, the preceding discussion can then be summarized as:

Proposition 9. *A second method equilibrium will exist if:*

$$\mu_e > (=)\mu_f \qquad \text{and} \qquad \mu_f\mu_e \leq (<)\mu_d \tag{4.6}$$

or

$$\mu_e > \mu_f \qquad \text{and} \qquad \mu_f\mu_e > \mu_d \qquad \text{and} \qquad \alpha^* > \bar{\alpha} \tag{4.7}$$

[12] Given the restrictions on e and f, this is trivial to show for μ_e and μ_f. It is not difficult, but tedious, to show this for μ_d.

or

$$\mu_e < \mu_f \qquad \text{and} \qquad \mu_f \mu_e < \mu_d \qquad \text{and} \qquad \alpha^* < \bar{\alpha} \tag{4.8}$$

where μ_e, μ_d and μ_f are the effort, detection and failure effects associated with that method.

The first pair of inequalities (4.6) claims, in other words, that a second method equilibrium exists if both the effort and detection effects are, in a precise sense, "large enough." This should come as no surprise, of course: because the two methods of production are otherwise identical, capitalists will find the second method attractive for two reasons - more effort and closer supervision - and unattractive for one - more frequent breakdowns - and if the effort and detection effects are both substantial, there will be no reason to switch to the first. The second set of inequalities (4.7) asserts, on the other hand, that if the detection effect is not substantial, a second method equilibrium will still exist if the effort effect dominates the failure effect *and* the marginal product of labor power, and therefore the difference in the marginal products of effective labor, is also, in the sense described above, substantial. The third and final set (4.8) implies that a second method equilibrium will sometimes exist even if the effort effect is smaller than the failure effect, provided the detection effect is substantial and the marginal product of labor power is not. Combined, (4.7) and (4.8) suggest that either the effort or detection effect must be dominant for a second method equilibrium to exist.

Last, it is not difficult to show that under the restrictions imposed here, the conditions under which a second method equilibrium exists are also those under which individual capitalists have an incentive to deviate when all choose the first method. That is, if one of these conditions are satisfied, the second method equilibrium is unique, but if none are, there will exist a unique first method equilibrium.

4.3 Technological Unemployment

Suppose that a unique second method equilibrium exists. Will it exhibit technological unemployment in the precise sense that N_2^* is smaller than N_1^*, the number of workers hired when capitalists *must* choose the first method? (I have also used "structural displacement" to describe this phenomenon, but this comes with a *caveat*: because the effects are permanent, not persistent, this is *not* the medium term disequilibrium phenomenon most macroeconomists have in mind when such terms are used.) The answer is no, not under all conditions: the power to extract more effort from each non-shirking worker does not *always* mean that fewer workers are hired. The surprise, perhaps, is that within this framework, the displacement of workers does *not* turn on differences in effort levels, but rather on the differences in detection and failure rates.

To see this, note first that when all capitalists have chosen the second method, in which case $a_1 = 0$, the relevant NSC (2.8) simplifies to:

$$\begin{aligned}\omega_2^* &= \left(\frac{1-\theta(1-q)(1-d_2)}{\theta(1-q)d_2(1-f_2)}\right)e_2 + \frac{a_2^*}{1-a_2^*}\left(\frac{1}{\theta(1-q)d_2(1-f_2)}\right)e_2 \\ &= \left(\frac{1-\theta(1-a_2^*)(1-q)(1-d_2)}{\theta(1-a_2^*)(1-q)d_2(1-f_2)}\right)e_2 \qquad (4.9)\end{aligned}$$

where:

$$\begin{aligned}1-a_2^* &= 1-\frac{(f_2d_2+q(1-f_2d_2))N_2^*}{H-(1-q)(1-f_2d_2)N_2^*} \\ &= \frac{H-N_2^*}{H-(1-q)(1-f_2d_2)N_2^*} \qquad (4.10)\end{aligned}$$

Substitution of (4.10) into (4.9) and some simplification produces:

$$\omega_2^* = \left(\frac{(1-\theta(1-q)(1-d_2))H-((1-f_2d_2)-\theta(1-d_2))(1-q)N_2^*}{\theta(1-q)d_2(1-f_2)(H-N_2^*)}\right)e_2 \qquad (4.11)$$

Because $\omega_2^* = \alpha^* e_2(1-f_2)$, (4.11) can then be solved for N_2^*:

$$N_2^* = \left(\frac{\theta(1-q)(d_2(1-f_2)^2\alpha^*+(1-d_2))-1}{\theta(1-q)(d_2(1-f_2)^2\alpha^*+(1-d_2))-(1-q)(1-f_2d_2)}\right)H \qquad (4.12)$$

or, in more compact notation:

$$N_2^* = \left(\frac{\psi_6}{\psi_6 + \psi_8}\right)H \tag{4.12}$$

where

$$\psi_6 = \theta(1-q)(d_2(1-f_2)^2\alpha^* + (1-d_2)) - 1 \quad \text{and} \quad \psi_8 = f_2 d_2 + q(1 - f_2 d_2) \tag{4.13}$$

and the jobless rate $u_2^* = 1 - (N_2^*/H)$ in a second method equilibrium will therefore be $\psi_8/(\psi_6 + \psi_8)$.

Even in this streamlined model, the solution for u_2^* is perhaps "messier" than one would like, but its properties are consistent with intuition. An increase in the value of α^*, for example, the result of either a broad increase in output per unit of labor power α or a decrease in capitalists' mark-up τ, causes the value of ψ_6 to rise and the jobless rate u_2^* to fall. In graphical terms, the demand for labor power schedule shifts upward against a fixed NSC curve, so that more workers are hired, and each is paid more. An increase in the value of θ, on the other hand, is also associated with an increase in ψ_6 and thus a decrease in u_2^*, but for different reasons: because workers care more about future income, the NSC curve shifts *downward* - that is, the wage required to induce effort falls for each N_2 - while the demand schedule is unaffected, in which case more workers will be hired at a constant real wage. For similar reasons, a decrease in the separation/quit rate will also be associated with reduced joblessness.

The comparative statics of variations in d_2 and f_2 are more complicated, of course, but differentiation of u_2^* with respect to the failure rate, for example, confirms that an increase in f_2 is associated with a rise in joblessness: the demand for labor power schedule shifts downward because the representative non-shirker is now less productive, while the incentive condition or NSC shifts upward, because the likelihood of dismissal without just cause has risen, both of which tend to drive the number of hires and the wage rate downward. In a similar vein, differentiation of (4.13) with respect to d_2 demonstrates that better supervision is associated with increased joblessness, too: output per unit of effective, *non-shirking*, labor is unchanged, but the wage required to induce effort e_2 rises.

The same expression (4.13) also rationalizes the observation that the displacement question does not involve the comparison of effort levels, a much less intuitive result: neither ψ_6 nor ψ_8 are functions of e_2, so neither is u_2^*. As a matter of arithmetic, the effort variable e_2 is "lost" because both sides of (4.9) are proportional to it. The left hand side is proportional to e_2 because the supposition that the marginal product of labor power is constant implies that $\omega_2^* = \alpha^* e_2(1 - f_2)$, while the right hand side exhibits this feature because $v(\cdot, \cdot)$ is linear - that is, workers are assumed to be risk neutral. If either of these is not the case - if, for example, workers are instead assumed to be risk averse (see Chatterji and Sparks' 1991 effort elicitation model, for example) - then N_2^* will depend on e_2, as well as f_2 and d_2. From a graphical standpoint, the structure of the model ensures that if the pace of production rises, both the NSC and the demand for labor power schedules will shift upward the same amount, measured from N_2^*.

For identical reasons, the number of workers N_1^* under contract and the associated jobless rate u_1^* when all capitalists use the first method of production are:

$$N_1^* = \left(\frac{\psi_5}{\psi_5 + \psi_7}\right) H \qquad \text{and} \qquad u_1^* = \frac{\psi_7}{\psi_5 + \psi_7} \tag{4.14}$$

where, of course:

$$\psi_5 = \theta(1-q)(d_1(1-f_1)^2\alpha^* + (1-d_1)) - 1 \qquad \text{and} \qquad \psi_7 = f_1 d_1 + q(1 - f_1 d_1) \tag{4.15}$$

It follows, therefore, that when a second method equilibrium exists, it will involve technological unemployment if:

$$\frac{\psi_8}{\psi_6 + \psi_8} > \frac{\psi_7}{\psi_5 + \psi_7} \qquad \text{or} \qquad \frac{\psi_8}{\psi_6} > \frac{\psi_7}{\psi_5} \tag{4.16}$$

The two sides of (4.16) differ with respect f and d alone and, on the basis of the previous comparative statics exercises, the sign of the "displacement effect" is ambiguous: the more substantial the detection effect, the greater the difference between the right and left hand sides of (4.16), but the more substantial the failure effect, the smaller this difference becomes. Furthermore, it is not difficult to show that for admissable, if not reasonable, parameter values, the latter sometimes dominates, in which case the number of workers under contract will *rise* in a second method equilibrium.

The comparison of ω_1^* and ω_2^* is much simpler. Since workers receive $\omega_1^* = \alpha^* e_1(1-f_1)$ if all capitalists commit to the first method, it follows that ω_2^* will be more or less than ω_1^* as $e_2(1-f_2)$ is more or less than $e_1(1-f_1)$ or, in terms of the effects identified earlier, as the effort effect μ_e is more or less than the failure effect μ_f. If the previous conjecture that μ_e will in practice exceed μ_f is correct, it follows that fewer workers are hired in second method equilibria but those under contract are paid more, in which case there is a trade-off to be considered.

5. Numbers and Policies

5.1 Model Realism

If the pursuit and eventual introduction of technologies that increase effort levels, failure rates and the extent of supervision are in fact an important historical phenomenon, the existence of a second method equilibria should be more than "possible." It is important, then, that the results of the previous section be re-evaluated on the basis of reasonable parameter values. The approach adopted here will be to choose values for both the common and first method parameters, and then calculate the size of the effort effect, and its labor market consequences, associated with second method equilibria for various combinations of f_2 and d_2. Given the results in Summers and Poterba (1986) or Kletzer (1998), for example, a combined separation/quit rate of fifteen percent per annum ($q = 0.15$) is a sensible, if conservative, first choice. The representative worker's rate of time preference θ will be set equal to 0.95, a value that corresponds, in the case where workers are able to borrow against future income, to a real interest rate of five percent per annum. On the other hand, there is little aggregate data on rates of industrial breakdown, and even less on the "all or nothing" sort of failure assumed here: to err on the side of caution, f_1 will be set equal to 0.10. In the context of Gordon's (1990) work on the ratio of managers and supervisors to production workers, it will also be assumed that the conditional likelihood of detection with even the first method is substantial: $d_1 = 0.50$[13]

The normalization of the representative worker's vNM function and the independence of the jobless rate u and the effort level e then force the values of α^* and e_1. *Under the single sector interpretation of the model*, for example, the choice $\alpha^* = 15$, which implies that $u_1^* = 0.0428$ or 4.28 percent when all capitalists select the first method, is consistent with recent US experience. Given this value of α^*, the choice $e_1 = 3$ then implies that each worker would receive $\omega_1^* = 40.50$ thousand per annum, and if $\tau = 0.30$, would produce 52.65 thousand output, both of which are also more or less consistent with recent data.[14] On the basis of (4.1), the implied likelihood of rehire a_1^* is 0.812 - that is, 81.2 percent of those in the jobless pool are (re)hired each period.

The terms of the ninth proposition then allow "critical effort effects," the values of μ_e required to induce capitalists to switch to the second method, to be calculated with ease, and Table 1 reports these for various second method failure $f_2 \geq f_1 = 0.10$ and detection $d_2 \geq d_1 = 0.50$ rates. Consider, for example, the case where $f_2 = 0.15$ and $d_2 = 0.60$. The size of the failure effect $\mu_f = (1 - f_1)/(1 - f_2)$ is therefore

[13] The second method can of course be interpreted as one that requires some, or perhaps more, workers to supervise others.

[14] To the extent that the value of e_1 is smaller than expected, the model is either less robust than desirable or, consistent with Juster (1986) or Bewley (1999), workers do not dislike effort as much as once believed. It should be noted, however, that the former does not mean that the broader framework should be abandoned: the introduction of a second, or dual, sector, for example, makes the model more flexible.

1.059, while the size of detection effect μ_d is almost identical, 1.057. Recall from the first condition (4.6) that a unique second method equilibrium exists when the effort effect μ_e exceeds the failure effect $\mu_f = 1.059$ *and* the product of the two, $\mu_e \mu_f = 1.059\mu_e$, is less than or equal to the detection effect $\mu_d = 1.057$ or, in other words, $\mu_e \leq 0.998$, a contradiction. The second condition (4.7), on the other hand, claims that such an equilibrium will exist if $\mu_e > \mu_f$ (that is, $\mu_e > 1.059$) *and* $\mu_e \mu_f > \mu_d$ (that is, $\mu_e > 0.998$) *and* output per unit of labor power α^* exceeds some critical bound $\bar{\alpha}$. Given the assumed values of d_2 and f_2, the bound is equal to $(1.64e_2 - 4.92)/(0.85e_2 - 2.70)$, and this will be less than $\alpha^* = 15$ when $e_2 > 3.20$ or $\mu_e > 1.07$. All three inequalities will be satisfied, therefore, when $\mu_e > 1.07$, the value reported in Table 1. The third condition (4.8), which requires that $\mu_e < 1.059$ *and* $\mu_e < 1.057$ *and* $\mu_e > 1,07$, involves another contradiction. In fact, all but one of the "not all" entries in Table 1 are based on the second condition.[15]

One is tempted to conclude, then, that second method equilibria are possible, even probable, for modest increases in the effort/pace of production, despite the increase in the breakdown rate. It seems almost certain, for example, that each of the possible historical examples mentioned earlier - clocks, computers and production lines - would meet such a test.

The patterns in Table 1 are consistent with intuition, of course: other things being equal, for example, the higher the failure rate f_2 or the lower the detection rate d_2, the greater the increase in the effort level and/or the pace of production needed to sustain a second method equilibrium. Furthermore, the threshold effort effect seems more sensitive to variations in f_2 and than d_2: when $f_2 = 0.20$, for

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[15] The exception is the case $(f_2 = 0.15, d_2 = 0.80)$: the *first* condition will be satisfied if μ_e lies between 1.059 and 1.074, the second is satisfied if it exceeds 1.074, while the terms of the third condition are met if it lies between 1.057 and 1.059. It follows, then, that a second method equilibrium exists if, to two decimal places, the size of the effort effect is 1.06 or more.

example, an increase in d_2 from 50 to 80 percent, the critical value of μ_e falls from 1.17 ($e_2 = 3.50$) to 3.42 ($e_2 = 3.42$), but when $d_2 = 0.60$, a rise in the failure rate from 15 to 25 percent causes the critical value of μ_e to rise from 1.07 ($e_2 = 3.20$) to 1.26 ($e_2 = 3.79$). This hints, but perhaps no more than that, that capitalists will have some incentive to hunt for methods that increase effort or pace on the basis of closer supervision - an increase in the number of forepersons, for example - rather than those which do so through simple speed up.

Table 2 reports the jobless rates in second method equilibria and the change - that is, the displacement effect - relative to the first method outcome (4.28 percent) for the critical effort effects.[16] In the particular case ($f_2 = 0.15, d_2 = 0.60$) considered above, for example, it is not difficult to determine that if a second method equilibrium exists, the value of u_2^* will be 4.72 percent, which implies that the "rate of technological unemployment" or RTE will be 0.44 percent. The data suggest that the RTE will be quite sensitive to variations in both f_2 and d_2. At one extreme, a substantial increase in the failure rate, from 10 to 25 percent and a sufficient (28 percent) rise in effort levels, from 3.00 to 3.83, is associated with an increase in the overall jobless rate from 4.28 to 8.36 percent, or 4.08 percent technological displacement if the detection rate remains constant. At the other, if the failure rate remains constant and the detection rate rises from 50 to 80 percent, the jobless rate *falls*, to 3.02 percent, in which case the RTE is *negative*.

Some readers will be concerned that the data in Table 3, which reports the annual real income of workers in a second method equilibrium, seem inconsistent with the previous comparative statics exercises, where it was observed that higher failure rates are associated with *lower* wages, other things being equal, or that the detection rate exerts no *direct* influence on compensation. There is no contradiction,

[16] The use of critical e_2 values does not matter much in this context - if a second method equilibrium exists, the value of u_2^* is not a function of e_2, for reasons discussed earlier - but it is relevant for the other data reported here.

		d_2			
		0.50	0.60	0.70	0.80
	0.10	NA	All(1.00)	All(1.00)	All(1.00)
	0.15	3.23(1.07)	3.20(1.07)	3.18(1.06)	3.17(1.06)
f_2	0.20	3.50(1.17)	3.47(1.15)	3.46(1.15)	3.42(1.14)
	0.25	3.83(1.28)	3.79(1.26)	3.76(1.25)	3.74(1.25)

Table 1. Critical Effort Levels (and Effort Effects) For A Second Method Equilibrium

		d_2			
		0.50	0.60	0.70	0.80
	0.10	NA	3.71(-0.56)	3.32(-0.96)	3.02(-1.26)
	0.15	5.35(+1.08)	4.72(+0.44)	4.27(-0.01)	3.93(-0.34)
f_2	0.20	6.69(+2.41)	5.96(+1.69)	5.45(+1.17)	5.06(+0.79)
	0.25	8.36(+4.08)	7.52(+3.25)	6.93(+2.66)	6.49(+2.21)

Table 2. Jobless Rates (and Changes) in Second Method Equilibrium
For Critical Effort Levels

		d_2			
		0.50	0.60	0.70	0.80
	0.10	NA	40.5(0.00)	40.5(0.00)	40.5(0.00)
	0.15	41.2(+0.68)	40.8(+0.30)	40.5(+0.05)	39.0(-1.48)
f_2	0.20	42.0(+1.50)	41.6(+1.14)	41.5(+1.02)	41.0(+0.54)
	0.25	43.1(+2.59)	42.6(+2.14)	42.3(+1.80)	42.1(+1.58)

Table 3. Real Wages ($th) (and Changes) in Second Method Equilibrium
For Critical Effort Levels

		d_2			
		0.50	0.60	0.70	0.80
	0.10	NA	83.9(+2.73)	85.9(+4.77)	87.5(+6.35)
	0.15	79.1(-2.09)	82.1(+0.89)	84.3(+3.13)	86.0(+4.88)
f_2	0.20	76.6(-4.53)	79.9(-1.27)	82.4(+1.19)	84.3(+3.12)
	0.25	72.8(-7.41)	77.3(-3.84)	80.0(-1.12)	82.2(+1.02)

Table 4. Likelihood of Rehire (and Change) in Second Method Equilibrium
For Critical Effort Levels

in fact, because other things are *not* equal here - the compensation of workers is calculated for the smallest second method effort levels consistent with equilibrium (that is, for the critical effort effect) and, as Table 1 evinces, these are not constant across (f_2, d_2) combinations. In particular, because an increase in the failure rate is also associated with an increase in the threshold effort effect, the consequences for compensation are uncertain *a priori*. This said, one could infer that more often than not, real wages rise - in the extreme case, almost \$3000 per annum - and that there is often a trade-off of sorts in second method equilibria, between better compensation and increased labor market slack. If such a trade-off exists, it has important implications for social welfare, and these are considered in more detail in the next section.

Last, the likelihoods of rehire in a second method equilibrium are listed in Table 4, and these data underscore the subtle relationship between the unemployment rate and the rate of rehire or its continuous time analogue, the mean jobless spell. Consider once more the case where $f_2 = 0.15$ and $d_2 = 0.60$: relative to the first method equilibrium, the jobless rate rises, from 4.28 to 4.72 percent, but so does the likelihood of rehire, from 81.2 to 82.1 percent. That is, the jobless rate and the mean jobless spell move in *opposite* directions, a result that in the context of recent labor market behavior - a substantial but gradual fall in the jobless rate and, for some time, little or no movement in jobless spells - merits attention. At the least, it prompts a simple, but often overlooked, question: Which is the better measure of labor market slack?[17] The divergence of these two measures owes to the fact that, in this particular case, the number of job seekers at the start of each period is also larger: although the proportion of these who receive an offer is greater than it was in the first method equilibrium, so, too, is the absolute number of workers who do

[17] Matthews and Kandilov (2000) consider this question from the perspective of the empirical Phillips curve, and find that *in this context*, the "cost of job loss," which is a function of a not u, is a more reliable measure.

not. From another perspective, while the jobless pool is larger, so is the rate at which it "turns over."

5.2 Social Welfare

The use of identical workers facilitates the construction of social welfare functions, more so if one is further prepared to assume that workers alone are also the shareholders in firms. Even without this restriction, however, the sum of workers' satisfactions each period, $Nv(\omega, e) + (H - N)v(0, 0)$, is an obvious candidate measure. Because workers are further assumed to be risk averse, this particular measure "collapses" to $N(\omega - e)$ or, scaled, $(1 - u)(\omega - e)$, where u is the jobless rate.

It then follows that within the framework described here, second method equilibria will cause social welfare to rise if $(1-u_2^*)(\omega_2^*-e_2)$ is more than $(1-u_1^*)(\omega_1^*-e_1)$ or, after substitution for ω_1^* and ω_2^*, if:

$$\frac{e_2}{e_1} = \mu_e > \frac{(1 - u_1^*)(\alpha^*(1 - f_1) - 1)}{(1 - u_2^*)(\alpha^*(1 - f_2) - 1} \tag{5.1}$$

Since neither u_1^* nor u_2^* are functions of e_1 and/or e_2, this condition establishes a *lower* bound for the effort effect. Some will wonder whether this result constitutes a paradox of sorts: shouldn't the likelihood that social welfare rises *decrease* as second method effort levels rise? The answer is no, not for this model, because once a second method equilibrium is established, the resulting rise in real wages more than offsets the increase in required effort.

With this in mind, the values of the right hand side of (5.1) are reported in Table 5 for the same combinations of f_2 and d_2 used in the previous section, under the assumption that the effort effects are sufficient to sustain a second method equilibrium.

To several decimal places, the numbers are almost identical to those in Table 1 - that is, if the effort effect is "large enough" to induce capitalists to switch from the first method, it will also be associated with an increase in social welfare. Given the displacement and compensation data in Tables 2 and 3, this means that from a collective standpoint, workers will often be better off in a second method

		d_2			
		0.50	0.60	0.70	0.80
	0.10	NA	All	All	All
	0.15	1.08	1.07	1.06	1.05
f_2	0.20	1.17	1.16	1.15	1.15
	0.25	1.27	1.26	1.25	1.25

Table 5. Threshold Effort Effects for Improvements in Social Welfare

		d_2			
		0.50	0.60	0.70	0.80
	0.10	NA	50.7(0.30)	50.9(0.50)	51.7(0.66)
	0.15	50.6(0.27)	50.5(0.13)	50.5(0.06)	50.5(0.08)
f_2	0.20	50.9(0.55)	50.9(0.50)	51.0(0.64)	50.7(0.25)
	0.25	51.3(0.93)	51.3(0.86)	51.2(0.78)	51.1(0.75)

Table 6. Output per Person (and Change) in Second Method Equilibria
For Critical Effort Levels, in Thousands

equilibrium with higher wages but more labor market slack. Whether or not these results explain, or even contribute to an explanation of, the infrequent nature of "Luddite-like" resistance to technological change, even when it is the conditions of production that are affected, is a matter for future research.

In cases where redistribution is possible, income per capita, $Q/H = \alpha e(1 - f)(N/H) = \alpha e(1 - f)(1 - u)$, is another reasonable measure of social welfare.[18] Given the same parameter values, income per capita is 50.4 thousand constant dollars per annum in a first method equilibrium, and Table 6 lists the income per capita and absolute increase/decrease in second method equilibria *for the threshold effort effect*. In each cases, output per capita rises, even if the difference is sometimes small.

5.2 Policy Implications (?)

It is reasonable to wonder whether there is a rationale, let alone scope, for intervention within the narrow confines of this model - for reasonable parametrizations, after all, second method equilibria are associated with an increase in the sum of workers' utilities. There are nevertheless at least two sorts of state initiatives that merit attention here. First, even if social welfare increases, it should be recalled that the same parametrization also suggests that workers are displaced in second method equilibria. This is not the case, however, with "pure" technical change - that is, an increase in the value of α rather than the effort level e or the likelihood of detection d - which, within this framework, causes the real wage ω to rise *and* labor market(s) to be less slack. (In graphical terms, "improvements" in the methods, as opposed to the conditions, of production cause the horizontal labor demand schedule to shift upward, but leave the incentive constraint or NSC unchanged.) To the extent that firms or capitalists must invest resources to pursue new methods

[18] This is not income per capita in the strict sense, however, because total output Q is measured relative to the number of *workers*, not workers *and* capitalists.

of either sort, and that their decisions are sensitive to financial and tax incentives, there emerges a possible role for public policies. While the model is *not* detailed enough to provide rigorous support for such a claim, it is at least consistent with a public preference for "basic" over "organizational" research.

Second, the state can mitigate whatever displacement does occur if it can either reduce the separation/quit rate q or somehow reduce the cost of effort to individual workers, inasmuch as both of these cause the NSC to shift downward and, within the context of the model, induce firms to hire more workers without a decrease in compensation. What sorts of policies might achieve this? Whatever their other consequences, those which reduce stress in the workplace - anxieties about health or child care, discrimination and harassment, for example - would perhaps have such an effect.

6. Conclusion

Recent work on the "effort extraction problem" has rekindled interest in the classical distinction between labor power and effective labor: because the abilities of capitalists or their designates to monitor effort are limited, individual workers exercise some control over their contributions to production. The failure to connect the resultant "workplace discipline" issues with capitalists' choice of technique is a weakness of the EWH literature, however, one that this paper attempts to redress. Although the model described here exhibits few hard and fast properties, it does suggest that methods of production will sometimes be chosen for unusual reasons - that is, increased effort or pace of production, or closer supervision. The observation that such methods are also often associated with increased failure rates moderates, but does not eliminate, their attractiveness. Furthermore, for reasonable parameter values, the widespread adoption of such methods can produce substantial technological dispacement.

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Appendix 1. Proofs of Propostions.

Proof of Proposition 1: There are four scenarios under which the method k non-shirker could nevertheless lose her position at the end of a particular period. With likelihood $f_k d_k s_k$, she will fail, be detected and then dismissed; with likelihood $f_k d_k(1-s_k)q$, she will fail, be detected, not dismissed but quit for other reasons; with likelihood $f_k(1-d_k)q$, she will fail, not be detected, but quit; and with likelihood $(1-f_k)q$, she will not fail but quit. The likelihood that she will find herself without work at the start of the next period, then, despite the fact that she is not a shirker, is equal to the sum of these or, collecting terms, $f_k d_k s_k + q(1 - f_k d_k s_k)$, and the likelihood that she retains her position is therefore $1 - (f_k d_k s_k + q(1 - f_k d_k s_k)) = (1-q)(1 - f_k d_k s_k)$. Given the form of the objective function (2.1), Bellman's Principle implies that:

$$V_{1,k} = \omega_k - e_k + \theta((1-q)(1 - f_k d_k s_k)V_{1,k} + (f_k d_k s_k + q(1 - f_k d_k s_k))V_3)$$

which, after collection of terms, is the desired result.

Proof of Proposition 2: There are three "roads to joblessness" for the method k shirker. The likelihood that she will be detected and then dismissed in a particular period is $d_k s_k$; the likeihood that she will be detected, not dismissed but quit for other reasons is $d_k(1-s_k)q$; and the likelihood that she will not be detected but nevertheless quit is $(1-d_k)q$. The likelihood that she will be without work at the end of each period is therefore, after some simplification, $d_k s_k + q(1 - d_k s_k)$, which implies that with likelihood $1 - (d_k s_k + q(1 - d_k s_k)) = (1-q)(1 - d_k s_k)$, she will retain her position from one period to the next. Under the assumption that $e = 0$ when she shirks, Bellman's Principle implies that:

$$V_{2,k} = \omega_2 + \theta((d_k s_k + q(1 - d_k s_k))V_3 + (1-q)(1 - d_k s_k)V_{2,k})$$

which, after collection of terms, is the desired result.

Proof of Proposition 4: *If* the conditions (2.5) are satisfied, a jobless worker will receive, and accept, an offer of method k work with likelihood a_k, but will not be offered either first or second method work with likelihood $1 - a_1 - a_2$. It follows from the definitions of $V_{1,k}$ and V_3 that:

$$V_3 = a_1 V_{1,1} + a_2 V_{1,2} + (1 - a_1 - a_2)\theta V_3$$

which, after simplification, is (2.6).

Proof of Proposition 5: Substituting for $V_{1,1}$ and $V_{1,2}$ in (2.6):

$$\begin{aligned}(1 - \theta(1 - a_1 - a_2))V_3 = a_1 &\left(\frac{\omega_1 - e_1 + \theta(f_1 d_1 s_1 + q(1 - f_1 d_1 s_1))V_3}{1 - \theta(1-q)(1 - f_1 d_1 s_1)}\right)\\ &+ a_2\left(\frac{\omega_2 - e_2 + \theta(f_2 d_2 s_2 + q(1 - f_2 d_2 s_2))V_3}{1 - \theta(1-q)(1 - f_2 d_2 s_2)}\right)\end{aligned}$$

Multiplying both sides by $(1-\theta(1-q)(1-f_1d_1s_1))(1-\theta(1-q)(1-f_2d_2s_2))$ yields, after some simplification,

$$\psi_0 = a_1(1-\theta(1-q)(1-f_1d_1s_1))(\omega_1-e_1) + a_2(1-\theta(1-q)(1-f_2d_2s_2))(\omega_2-e_2)$$

where

$$\begin{aligned}\psi_0 = {} & (1-\theta(1-a_1-a_2))(1-\theta(1-q)(1-f_1d_1s_1))(1-\theta(1-q)(1-f_2d_2s_2)) \\ & - a_1\theta(1-\theta(1-q)(1-f_2d_2s_2))(f_1d_1s_1+q(1-f_1d_1s_1)) \\ & - a_2\theta(1-\theta(1-q)(1-f_1d_1s_1))(f_2d_2s_2+q(1-f_2d_2s_2))\end{aligned}$$

It is then tedious, but not difficult, to show that ψ_0 can be rewritten as:

$$\psi_0 = (1-\theta)(1-\theta(1-a_1)(1-q)(1-f_1d_1s_1))(1-\theta(1-a_2)(1-q)(1-f_2d_2s_2))$$

which can be substituted into the previous expression to obtain the desired result.

Proof of Proposition 6: Rewrite (2.5) and (2.6) as:

$$\omega_1 = \psi_1e_1 + (1-\theta)V_3 \qquad \omega_2 = \psi_2e_2 + (1-\theta)V_3$$

$$(1-\theta)V_3 = a_1\psi_3(\omega_1-e_1) + a_2\psi_4(\omega_2-e_2)$$

where

$$\psi_1 = \frac{1-\theta(1-q)(1-d_1s_1)}{\theta(1-q)d_1s_1(1-f_1)} \qquad \psi_2 = \frac{1-\theta(1-q)(1-d_2s_2)}{\theta(1-q)d_2s_2(1-f_2)}$$

and

$$\psi_3 = \frac{1}{1-\theta(1-a_1)(1-q)(1-f_1d_1s_1)} \qquad \psi_4 = \frac{1}{1-\theta(1-a_2)(1-q)(1-f_2d_2s_2)}$$

Substitution for $(1-\theta)V_3$ in the expressions for ω_1 and ω_2 and collection of terms leads to, in matrix terms:

$$\begin{pmatrix} 1-a_1\psi_3 & -a_2\psi_4 \\ -a_1\psi_3 & 1-a_2\psi_4 \end{pmatrix}\begin{pmatrix}\omega_1 \\ \omega_2\end{pmatrix} = \begin{pmatrix}\psi_1e_1 - a_1\psi_3e_1 - a_2\psi_4e_2 \\ \psi_2e_2 - a_1\psi_3e_1 - a_2\psi_4e_2\end{pmatrix}$$

The solution is:

$$\begin{pmatrix}\omega_1 \\ \omega_2\end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} 1-a_2\psi_4 & a_2\psi_4 \\ a_1\psi_3 & 1-a_1\psi_3 \end{pmatrix}\begin{pmatrix}\psi_1e_1 - a_1\psi_3e_1 - a_2\psi_4e_2 \\ \psi_2e_2 - a_1\psi_3e_1 - a_2\psi_4e_2\end{pmatrix}$$

where

$$\Delta = (1-a_1\psi_3)(1-a_2\psi_4) - a_1a_2\psi_3\psi_4 = 1 - a_1\psi_3 - a_2\psi_4$$

To obtain the NSCs, observe that:

$$\omega_1 = \frac{(1-a_2\psi_4)(\psi_1 e_1 - a_1\psi_3 e_1 - a_2\psi_4 e_2) + a_2\psi_4(\psi_2 e_2 - a_1\psi_3 e_1 - a_2\psi_4 e_2)}{\Delta}$$
$$= \frac{(1-a_2\psi_4)\psi_1 e_1 + a_2\psi_2\psi_4 e_2 - a_1\psi_3 e_1 - a_2\psi_4 e_2}{\Delta}$$
$$= \frac{(1-a_1\psi_3 - a_2\psi_4)\psi_1 e_1 + a_1\psi_3(\psi_1 - 1)e_1 + a_2\psi_4(\psi_2 - 1)e_2}{\Delta}$$
$$= \psi_1 e_1 + \frac{a_1\psi_3(\psi_1 - 1)e_1 + a_2\psi_4(\psi_2 - 1)e_2}{\Delta}$$

and, likewise:

$$\omega_2 = \psi_2 e_2 + \frac{a_1\psi_3(\psi_1 - 1)e_1 + a_2\psi_4(\psi_2 - 1)e_2}{\Delta}$$

To establish the equivalence of these and (2.8), note that:

$$\psi_3(\psi_1 - 1) = \left(\frac{1}{1-\theta(1-a_1)(1-q)(1-f_1 d_1 s_1)}\right)\left(\frac{1-\theta(1-q)(1-f_1 d_1 s_1)}{\theta(1-q)d_1 s_1(1-f_1)}\right)$$

and

$$\psi_4(\psi_2 - 1) = \left(\frac{1}{1-\theta(1-a_2)(1-q)(1-f_2 d_2 s_2)}\right)\left(\frac{1-\theta(1-q)(1-f_2 d_2 s_2)}{\theta(1-q)d_2 s_2(1-f_2)}\right)$$

Proof of Proposition 8: Since the conditional likelihood of dismissal is one, a proportion $f_1 d_1$ of all first method workers will fail, be detected and then dismissed each period, and an additional $q(1-f_1 d_1)$ percent will not fail but quit for other reasons in equilibrium, so that $(f_1 d_1 + q(1-f_1 d_1))N_1$ first method workers will lose their positions at the end of each period. In flow equilibrium, the same number will be hired from the pool of job seekers at the start of the next period. Likewise, $(f_2 d_2 + q(1-f_2 d_2))N_2$ second method workers will be hired and fired each period. It also follows that $N_1 - (f_1 d_1 + q(1-f_1 d_1))N_1 = (1-q)(1-f_1 d_1)N_1$ first method workers and $(1-q)(1-f_2 d_2)N_2$ second method worrkers will retain their positions from one period to the next, so that the total number of job seekers must be $H - (1-q)((1-f_1 d_1)N_1 + (1-f_2 d_2)N_2)$. The likelihood of rehire a_k is the ratio of new method k hires to the total number of jobless workers, the stated result.