Using Optimal Control to Characterize the Economic and Ecological Implications of Spatial Externalities

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Abstract

This study uses optimal control theory to examine the dynamic features of agricultural landscapes characterized by spatial externalites. A stylized system of agricultural production and groundwater flow is used to illustrate how spatial externalities affect land use decisions over time. Policy alternatives are also considered.

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1. INTRODUCTION

For decades, environmental economists have discussed the implications of externalities as a cause of market failure. According to Baumol and Oates (1988,[1]) an externality exists "whenever some individual's (say A's) utility or production relationships include real (that is, nonmonetary variables) whose values are chosen by others (persons, corporations, governments) without particular attention to the effects on A's welfare." Many issues of concern with regard to natural resources, such as the contamination of air or water, fall under the category of negative environmental externalities; the contamination is produced as a side effect of some production process, and it then affects the utility (or production value) of individuals who are not involved in those production decisions. Environmental economists have illustrated that if the disutility to the affected individuals is not taken into account in the original production decision, then an excessive level of production will be engaged in, and a socially excessive amount of contamination will be produced.

Economists have also explored a range of possible solutions to externality problems. These solutions range from command and control approaches that specify maximum production or contaminant levels to incentive- (or market-) based approaches that induce the original producer to take into account the cost that their production externalities are imposing on others when they are making their production decisions. This accountability may be enforced through taxes on production or contaminated emissions, for instance, or through the implementation of marketable permits schemes that force to producers to pay for their contamination through the purchase of permits. These approaches, and the concepts of externalities in general, have been described in detail throughout the literature.

The proliferation of techniques enabling dynamic social optimization, with its emphasis on the future impacts of current production decisions, brought a new dimension to the analysis of environmental externalities. Some environmental externalities form stock pollutants, for instance, whose production and utility effects are not limited to the time period in which they are produced. Designing a policy to compel polluters to take into account the costs of emitting a ton of carbon dioxide must take into account not only the current costs of that emission but also the future stream of costs that arises from having that extra ton is the atmosphere (Falk and Mendelsohn, 1993[3]).

There is a final dimension to the analysis of environmental externalities that has received less attention in the literature, but that is equally relevant in the analysis and design of policy solutions. Just as the effects of externalities extend, and vary, temporally, they also vary spatially. In their discussion of policies to deal with externalities, Baumol and Oates acknowledge the complexity introduced when geographical distinctions among polluters must be taken into account; permit and tax systems that might otherwise appear, at least theoretically, quite simple to implement become cumbersome to administer when spatial distinctions are involved (see Braden et al. 1989[2], Henderson, 1977[5], Morgan, 1999[6]).

Unfortunately for the simplicity of environmental policies, spatial distinctions with respect to external environmental effects probably represent the rule rather than the exception. An examination of the spatial aspect of environmental externalities suggests that the effects of externalities, and the extent to which they enter into individuals' production or utility functions, operate at a number of different levels. Certain types of contamination, such as upper atmosphere ozone pollution, have global effects, so that individuals located hundreds of miles away from the source of contamination are affected as substantially as those immediately adjacent to the source of contamination. At this global level, the effects of ozone pollution can be considered nonspatial, for there is little geographical distinction in how the externality enters distinct production and utility functions. However, most types of contamination, including ozone pollution, have local effects as well; the magnitude of the effects at the local level may far exceed the magnitude of the global effects, and these effects are often spatially differentiated. This spatial differentiation can either arise from distance effects, in which individuals farther from the source of the externality are less strongly affected by it, or from directional effects, where the effects of an externality are not felt uniformly at a given distance from the source point. Directional differentiation arises from a combination of natural processes and channels of dispersion, such as wind patterns or ground- or surface- water flow patterns, and institutional factors such as the positioning of wells or monitoring stations. Although a watershed's groundwater flow is physically fixed, for instance, the effects of each individual producers' contribution to groundwater contamination, and the producers' "responsibility" for those effects in the design of policy, will depend on both the producer's location within the watershed and where within the watershed a drinking water well is placed.

The presence of such geographical distinctions considerably complicates the implementation of policies designed to force the internalization of external costs in producer's production decisions (see, e.g., Goetz and Zilberman, 2000[4] and Morgan, 1999[6]). Recent studies have examined, for instance, the design of permit schemes in watersheds where contamination is measured at a single well. Such schemes require the use of large integrated models to calculate a transfer coefficient that describes how production activity at the source translates into contamination at the measurement point; this calculation must be made separately for each individual producer in order to design an efficient structure of permit trading ratios. An individual distant from a measurement well, for instance, may require fewer emissions permits for the same amount of production activity as an individual immediately adjacent to the well; the ratio of permits required for activity arises from a comparison of the producers' transfer coefficients.

While acknowledging the presence and importance of institutionally imposed complications such as those mentioned in the case of placement decisions for drinking water wells, this study focuses on the complications introduced into policy design by purely physical processes in the creation of directional externalities. A highly stylized system of agricultural production and groundwater flow is used to illustrate how directional externalities can appear in an optimal control analysis of land use decisions, what their environmental and ecological effects would be on a steady state solution, and what implications they have for possible policy alternatives.

2. METHODOLOGY: OPTIMAL CONTROL

Dynamic optimization problems seek a solution to the question of what is the optimal path of resource use over time for the management interval of interest. A solution to a dynamic optimization problem therefore provides an optimal magnitude for every choice variable at any given point in the management interval. Optimal control is a solution method for dynamic optimization problems that establishes and then solves necessary conditions for optimal resource use over time. Optimal control divides a system's endogenous variables into control variables, which are the decision variables available to the system's manager, and state variables, which describe the state of the system's components. The state variables evolve within the system according to a series of first order differential equations called the equations of motion.

One set of necessary conditions for an optimal control solution applies to a solution for a steady state, which is an equilibrium solution in which the magnitude of the state and choice variables remains constant over time, with an infinite time horizon. Steady state solutions conform to the popular notion of sustainability in that they describe a decision solution that maximizes a given objective function subject to the condition that the state of the system is unchanging in perpetuity. Such an analysis permits a type of comparative statics that is analogous to that traditionally applied in static analyses; this approach might explore, for instance, how sensitive the steady-state, or sustainable, levels of choice and control variables are to changes in exogenous variables. This research will use a steady-state analysis to explore how the sustainability of a highly stylized production system is affected by the exogenous variables that form that system's physical framework.

3. MODEL DESCRIPTION

Consider the case of multiple agricultural producers, each of whom produce using the same production function. Production on each parcel is a function of effort applied (x_i), generic hydrological contamination (h_i), and level of contaminant abatement selected (a_i).

$$P_i=f(x_i, h_i, a_i)$$

where i indexes agricultural production units. The control, or decision, variables in this scenario are x_i and a_i. The state variables are h_i, and the level of contamination changes according to the equations of motion, which describe what physical processes operate on the contamination level in each period and how it responds to changes in agricultural effort, abatement levels, or the hydrological contamination present:

$$\dot{h}_i = g_i(x_i, x_{\neg i}, h_i, h_{\neg i}, a_i, a_{\neg i})$$

Note that the state equation $g_i(\cdot)$ is indexed by i; this allows for differences in how the level of contamination changes in each production unit. Also, the equation of motion for cell i can depend on activity and decisions outside of cell i ($\neg i$). In this framework, therefore, it is the state equation that describes the structure of the externality, which in this case will be illustrated by the movement of hydrological contamination among the cells.

In this model, the underlying physical process of contaminant flow creates an economic externality because production is sensitive to level of contamination; the production decisions in the contaminant source cell therefore impact the production function in the contaminant receptor cell. A social optimizer who optimizes aggregate production value of the landscape would take into account this contaminant flow, and the effect that upstream production has on downstream production (both in the current and future time periods), but an individual optimizer would not. The two optimization processes therefore produce different steady state solutions, with different levels of both the choice variables (effort and abatement) and the state variable (contamination level in each cell). Under the individual optimization, with no policy intervention, the downstream producer making optimization decisions must take the incoming contaminant flow (and its effect on current and future productivity) as given and optimize accordingly. The difference between the aggregate landscape value under the socially optimal solution and the individually optimal solution represents the cost of having no policy intervention to correct for the externality.

The specific analytic form of the state equation can vary depending on how the physical processes underlying the externality operate; the form of the state equation and the resulting steady state solution will vary, for instance, depending on whether contaminant movement from one cell to the next is a function of existing hydrological levels, differences in contaminant concentration, effort levels, etc. The cost of a non-intervention policy will also vary, therefore, depending on the physical structure underlying the externality and how it appears in the state equation. It is important to emphasize, of course, that a truly realistic representation of the underlying physical process would require more than a single ordinary differential equation. The simplification of the model, which allows for analytical tractability, is useful as an illustration of the sensitivity of both costs and policy recommendations to the underlying physical process.

To illustrate the effect of different forms of externality on the steady state solutions, the social cost of non-intervention, and the structure of an appropriate policy, consider the following example. Consider a landscape consisting of two cells, each of which produces according to the following function:

$$f(x_i, h_i, a_i) = x_i - b \cdot x_i^2 - h_i^2 - c \cdot a_i^2$$

Suppose in addition that cell 1 is upstream of cell 2, and that hydrological contamination flows downstream. This flow enters into the state equation for each cell as follows:

$$\dot{h}_1 = x_1 - a_1 - \text{flow1}$$

$$\dot{h}_2 = x_2 - a_2 + flow1 - flow2$$

Where flow1 represents the amount that flows from cell 1 to cell 2 in each time step, and flow2 represents the amount that flows out of cell 2, and from there out of the production system, in each time step. It is the specific form of flow1 and flow2 that captures the exact nature of the physical flow and the spatial externality it generates. This study will examine the case of three possible scenarios:

1. flow1 and flow2 are both constants determined by some exogenous physical structure

   flow1= $\mathbf{f_1}$

   flow2= $\mathbf{f_2}$

2. flow is a simple percentage of existing hydrological contamination level:

   $$flow1 = f \cdot h_1$$

   $$flow2 = f \cdot h_2$$

3. flow is a simple percentage of current agricultural effort:

   $$flow1 = f \cdot x_1$$

   $$flow2 = f \cdot x_2$$

Each of these scenarios produces different socially and individually optimal solutions. Section Four will compare the equilibrium values of state and control variables under the three different externality types to their respective socially optimal equilibrium values. Section 5 will discuss policy solutions that can be designed to correct spatially explicit externalities in a dynamic context.

4. RESULTS

The first scenario, in which flow of pollutants is a constant value, is an interesting illustration of a subtle distinction that exists within the definition of an externality. Under this scenario, the socially and privately optimal equilibrium values appear as shown in Table 1. Clearly, there are no differences between the socially and privately optimal production activities and stock contamination levels. This is because the economic externality arises not directly from the contaminant flow itself, but from the impacts of upstream activities on the downstream activity decisions. In this case, where flow is a constant, the upstream production choices themselves do not impact in any way downstream production choices, therefore no externality exists, no wedge is driven between the socially and privately optimal decisions, and there is no difference between the solutions.[1]

This observation highlights the fact that there may not be social cost associated with the flow itself; a cost only exists if this flow acts as a conduit through which upstream activity can impact downstream productivity. The purpose of intervening policy in the case of a downstream pollutant flow is therefore not to correct for the presence of the flow; the presence of the flow is a natural physical process that is taken into account in determining the socially optimal levels of the state and control variables. The fact that the downstream cell has to abate an additional amount to compensate for the f_1 arriving from upstream is not inherently inefficient. The inefficiency arises from the externality, or the

[1]The only aspect of the upstream production decision that affects the downstream cell is the decision to produce at all; no wedge is driven as long as the socially optimal solution is not a corner solution. If it were socially optimal for the upstream cell *not* to engage in agriculture at all, then the social optimal could diverge from the private optimal as a result of the presence of a fixed contaminant flow. That scenario is not addressed in this paper.

Variable	Socially Optimal Eq. Level	Privately Optimal Eq. Level
x_1	$\frac{1+2cf_1}{2(b+c)}$	$\frac{1+2cf_1}{2(b+c)}$
h_1	$\frac{c(1-2bf_1)r}{2(b+c)}$	$\frac{c(1-2bf_1)r}{2(b+c)} = cra_1$
a_1	$\frac{1-2bf_1}{2(b+c)}$	$\frac{1-2bf_1}{2(b+c)} = x_1 - f_1$
x_2	$\frac{1-2c(f_1 - f_2)}{2(b+c)}$	$\frac{1-2c(f_1-f_2)}{2(b+c)} = x_1 - 2c(2f_1 - f_2)$
h_2	$\frac{c(1+2b(f_1-f_2))r}{2(b+c)}$	$\frac{c(1+2b(f_1-f_2))r}{2(b+c)} = cra_2$
a_2	$\frac{1+2b(f_1-f_2)}{2(b+c)}$	$\frac{1+2b(f_1-f_2)}{2(b+c)} = x_2 + f_1 - f_2$

Table 1: Socially and privately optimal levels when flow is a constant

impact of upstream decisions on downstream productivity. In this case, beyond the decision to produce at a non-zero level, the upstream decisions do not directly affect the flow, and therefore do not affect downstream choices. There is therefore no economic inefficiency in this system that would require policy intervention.

Consider in contrast the case where the magnitude of flow is a function of the accumulated contamination level in a cell. In this case, upstream production decisions, and the level of upstream choice variables, will impact on downstream productivity through their effect on h_1; differences in h_1 are transported to the downstream cell through the flow parameter. The results under this scenario are shown in Table 2[2].

	Socially optimal eq. levels	Privately optimal eq. levels
x_1	$\frac{c(1+cfr)+b(1+c^2f^2(f+r)^2+cf(3f+2r))}{2(c^2+bc(2+cf(3f+2r))+b^2(1+c^2f^2(f+r)^2+cf(3f+2r)))}$	$\frac{1+cf(f+r)}{2(c+b(1+cf^2+cfr))}$
h_1	$\frac{c(cr+b(r+cf(f+r)^2))}{2(c^2+bc(2+cf(3f+2r))+b^2(1+c^2f^2(f+r)^2+cf(3f+2r)))}$	$\frac{c(f+r)}{2(c+b(1+cf^2+cfr))}=c(f+r)a_1$
a_1	$\frac{c+b(1+cf(3f+r))}{2(c^2+bc(2+cf(3f+2r))+b^2(1+c^2f^2(f+r)^2+cf(3f+2r)))}$	$\frac{1}{2(c+b(1+cf^2+cfr))}=x_1-fh_1$
x_2	$\frac{c(1+cf^2)+b(1+c^2f^2(f+r)^2+cf(3f+2r))}{2(c^2+bc(2+cf(3f+2r))+b^2(1+c^2f^2(f+r)^2+cf(3f+2r)))}$	$\frac{c+b(1+cf(f+r))^2}{2(c+b(1+cf^2+cfr))}$ $=b(1+cf(f+r))x_1+ca_1$
h_2	$\frac{c(f+r)(c+b(1+2cf(f+r)))}{2(c^2+bc(2+cf(3f+2r))+b^2(1+c^2f^2(f+r)^2+cf(3f+2r)))}$	$\frac{cf(c+b(1+2cf(f+r)))}{2(c+b(1+cf^2+cfr))}=cfa_2$
a_2	$\frac{c+b(1+2cf(f+r))}{2(c^2+bc(2+cf(3f+2r))+b^2(1+c^2f^2(f+r)^2+cf(3f+2r)))}$	$\frac{c+b(1+2cf(f+r))}{2(c+b(1+cf^2+cfr))}$ $=x_2+f(h_1-h_2)$

Table 2: Socially and privately optimal levels when flow is a function of the accumulated contamination level in a cell

[2]For simplicity, the results presented in the following graph assume that r=0.

Under this scenario, an externality does exist. In the upstream cell, a private optimization produces a higher equilibrium level of x_1 and h_1, and a lower equilibrium abatement level.[3] The downstream cell, whose private optimization process is sensitive to the upstream contaminant level, also carries a higher contaminant level (h_2), and in addition suffers from higher abatement costs, and a lower optimal agricultural effort level (x_2). Under a social optimum, the upstream decisions contributing to contamination should take into account not only the discounted future costs that will accrue to the upstream cell itself, but the discounted future costs that accrue to the downstream cell as well. Under a private optimum, the upstream cell only considers the discounted future costs to itself in the optimization procedure, and therefore carries a higher contamination level. The result of the externality is therefore not only a loss in overall landscape productivity, but an increase in landscape contamination. It is this increase in landscape contamination that produces the decline in productivity; other possible impacts of increased contaminant runoff from cell 2, unrelated to productivity, are not taken into consideration in this approach. Such impacts would create an even wider wedge between the social and privately optimal steady states, for when taken into account in the social optimization they would result in a decline of equilibrium contamination levels, though the private optimization results would remain unchanged.

Different results are obtained under the third scenario, in which flow is a function of current effort rather than current contamination levels. The results are reported in Table 3. Again, an xternality exists that is driving the difference between socially and privately optimal equilibrium solutions. As in the

[3]Assuming nonnegativity constraints on b, c, and f

	Socially optimal equilibrium values	Privately optimal equilibrium values
x_1	$\frac{b+c-3cf+2cf^2}{2(b^2+c^2(f-1)^4+bc(2-4f+3f^2))}$	$\frac{1}{2(b+c(f-1)^2)}$
h_1	$\frac{cr(1-f)(b+c-3cf+2cf^2)}{2(b^2+c^2(f-1)^4+bc(2-4f+3f^2))}$	$\frac{cr(1-f)}{2(b+c(f-1)^2)} = cra_1$
a_1	$\frac{(1-f)(b+c-3cf+2cf^2)}{2(b^2+c^2(f-1)^4+bc(2-4f+3f^2))}$	$\frac{(1-f)}{2(b+c(f-1)^2)} = (1-f)x_1$
x_2	$\frac{b+c-3cf+3cf^2}{2(b^2+c^2(f-1)^4+bc(2-4f+3f^2))}$	$\frac{b+c-3cf+2cf^2}{2(b+c(f-1)^2)^2}$
h_2	$\frac{cr(b-c(f-1)^3)}{2(b^2+c^2(f-1)^4+bc(2-4f+3f^2))}$	$\frac{cr(b-c(f-1)^3)}{2(b+c(f-1)^2)^2} = cra_2$
a_2	$\frac{b-c(f-1)^3}{2(b^2+c^2(f-1)^4+bc(2-4f+3f^2))}$	$\frac{b-c(f-1)^3}{2(b+c(f-1)^2)^2} = x_2 - f(x_2 - x_1)$

Table 3: Socially and privately optimal levels when flow is a function of current agricultural effort

scenario above, the private optimization produces a higher h_1 and x_1, and lower a_1, and a higher h_2 and a_2 and lower x_2, than the social optimization. There are a couple of interesting things to note about both the social and private equilibria in this case. Most noticeably, only the optimal state variable levels h_1^* and h_2^* are sensitive to the discount rate. This is because when the externality arises from flow related to the control variable rather than to the state variable, then its downstream effect can be compensated for in a single time period. The downstream cell prevents future repercussions of this period's increased upstream activity simply by abating more in the current time period. Therefore optimal levels of the control variables do not depend on the discount rate, which would indicate how heavily future costs should be taken into account now, because no future costs are generated.

The levels of the state variables remain sensitive to the discount rate, however, because the accumulating contamination levels always have future costs through the reduction of future productivity. Note that if r=0, so that future impacts count as much as current impacts, then the optimal contamination level at any point on the landscape is 0. The optimum strategy is always to simply maintain minimum contamination and maximum productivity. As r increases, all of the decision variables remain the same, but the level of the state variable contamination increases. Increasing r means that it may be optimal to maintain a certain level of contamination if the costs of abating it in the present time period are not justified by the discounted stream of future avoided costs.

This is not the case when flow is a function of the state variable (scenario 2). In this case, increased upstream production effort x_t will continue to be felt downstream in periods t+1, t+2, etc., due to the effect on h_t, which flows downstream over time. The levels of all of the choice variables are therefore sensitive to how heavily those future costs "count" in today's decisions, which is reflected in the discount rate r. Also in contrast to scenario3, even with r=0, the optimal level of hydrological contamination is always nonzero. This difference arises because when flow is a function of contamination levels, the only way for each cell to take advantage of the natural cleansing value derived from the flow is to maintain a positive contamination level. Maintaining a zero level for the upstream cell, for instance, would require abating everything entering the system, $a_1=x_1 - fh_1= x_1$ if $h_1=0$. At this abatement level, the landowners would lose the abatement cost benefits of the natural flow. Therefore it is both privately and socially optimal to maintain some level of background contamination when the flow mechanism takes this form.

5. POLICY SOLUTIONS

Taxes or subsidies that alter production costs are commonly suggested as a means of changing the level at which production, along with its external side effects, is engaged in. Taxes or subsidies may be placed on any of the variables in a production optimization problem. In the scenario described here, they could change the farm's production function as shown:

$$f(x_i, h_i, a_i) = x_i - b \cdot x_i^2 - h_i^2 - c \cdot a_i^2 - tax1 \cdot x_i$$

or

$$f(x_i, h_i, a_i) = x_i - b \cdot x_i^2 - h_i^2 - c \cdot a_i^2 - tax2 \cdot h_i$$

or

$$f(x_i, h_i, a_i) = x_i - b \cdot x_i^2 - h_i^2 - (c - subs1) \cdot a_i^2$$

where tax1 is a tax on a cell's agricultural effort, tax2 is a tax on a cell's contamination level, and subs1 is a subsidy on the cost of abatement. Is it possible to use taxes or subsidies such as these to correct the externalities described in scenarios 1,2, and 3 above?

As explained above, scenario 1 does not represent a true externality, and therefore requires no policy intervention to correct. For the remaining scenarios, Table 4 shows the taxes or subsidies that would need to be applied in order to induce the entire system to operate at socially optimal levels.

	scenario 2	scenario 3
tax1	None	$\frac{cf(b-c(f-1)^3)}{b^2+c^2(f-1)^4+bc(2-4f+3f^2)}$
tax2	$\frac{cf(c+b(1+2cf(f+r))}{c^2+bc(2+cf(3f+2r))+b^2(1+c^2f^2(f+r)^2+cf(3f+2r))}$ $= - f \lambda_2$	None
subs1	None	None

Table 4: Taxes and subsidies to correct three types of externalities

Note that when the flow is a function of effort, the only effective tax is a tax on effort, and when flow is a function of contaminant level, the only effective tax is a tax on contaminant level. Under scenario 3, when flow is a function of effort, taxing upstream hydrology does not affect the upstream landowner's decision about effort level at all; effort and abatement levels remain constant regardless of contaminant tax, only the pooled stock of contamination changes. The downstream landowner therefore continues to receive an excessive contaminant flow from the higher-than-optimal upstream effort level. Under both scenarios, a subsidy on abatement provides a production incentive in the wrong direction; a non-negative subsidy is in fact a lowering of production costs that encourages production effort to rise rather than fall. The only way to lower effort levels to the socially optimal level is to *tax* abatement; this returns effort to socially optimal levels, but skews the other incentives, so that abatement is lower and hydrological contamination is higher.

Note that it is the upstream cell's behavior that the policy is intended to alter; the downstream cell's private optimization process produces the social optimum if the upstream cell behaves optimally. This result arises because there are no external effects considered for the downstream cell's activities; the presence of downstream costs for flow2 would change that result. In the present model, an efficient policy requires the asymmetric application of a tax or subsidy– i.e. the tax or subsidy would apply only to the upstream cell.

From a local perspective, however, there are problems involved with the application of a tax to the upstream cell. In particular, if the amount collected for the tax leaves the system altogether (i.e. is transferred to neither the upstream or the downstream cell), then the presence of the tax itself changes the landscape's social optimum. If upstream agricultural effort results in a loss to the system through the tax mechanism, for instance, then a local social planner would prefer to redistribute agricultural effort, with a lower amount upstream and larger amount downstream. The redistribution of effort and contamination across the entire system would moderate the total loss generated by the tax mechanism. This tension between scales of optimization may result in complex and counter-intuitive policy measures at different levels of authority.

If, on the other hand, the tax revenue stays within the closed landscape system, then the policy can be used to achieve a social optimum that remains undistorted. Suppose, for instance, that the tax proceeds were transferred to the downstream cell in a lump sum payment. The tax and the payment would cancel one another out in the social planner's objective function, and the social optimum would remain unchanged. Such a system, however, still requires the governing authority to establish in advance the price that must be charged as a tax in order for the upstream cell to arrive at the socially optimal activity level.

Suppose instead that the policy were set up as a permit system rather than a tax. The structure of an appropriate permit system would have to depend on the structure of the physical flow. As in the tax system described above, the permit system would be asymmetrically applied in that only the upstream cell would be required to have permits for their production activity. When flow is a function of effort, then permits would apply to effort, and when flow is a function of contamination, then permits would apply to contamination. The upstream cell would be required to purchase from the downstream cell the right to either engage in upstream effort or to accumulate contamination, depending on the structure of the physical flow. The downstream cell maintains the right to either sell or not sell the permits, depending on the price that is agreed upon.

The advantage to this system, from a policy-maker's perspective, is that the price of permits, and the quantity transferred, are jointly determined by the parties themselves, and therefore do no need to be established in advance. The governing authority is responsible only for determining the appropriate number of permits to issue. Surprisingly, the downstream cell can be issued an unlimited number of permits, and the number transferred will still settle to the socially optimal level x_1^* or h_1^*, depending on which level requires permits. Through the negotiation process, the price of the permits will also arrive at equilibrium levels equivalent to the tax levels shown above. Intuitively, this result makes sense; the taxes are imposed to represent the interests of the lower cell in terms of the marginal cost imposed by upstream activity. When the lower cell represents its own interest through the negotiating process, the same result will be arrived at.

As with other permit schemes, it turns out that the initial distribution of permits does not affect the efficiency of the permitting policy if the proper number of permits (i.e. h_1^* or x_1^*) are allocated and as long as a market for those permits exists. The upstream cell would simply purchase the remaining permits from the downstream cell. But this permit scheme, like others, is vulnerable to the critique that if the policymaker knew h_1^* , and therefore knew the optimal number of permits to issue, then why not simply impose a pollution flow standard on the upstream cell and be done with it? How could a permitting scheme be used to accommodate the fact that the permitting agency may not have enough information on the production systems in each cell to determine h_1^* or x_1^*?

As seen above, if a greater-than-necessary number of permits is issued to the downstream cell (i.e., $\bar{h}_2 > h_1^*$)[4], there is no efficiency loss because the remaining permits would simply remain unsold; the upstream cell is not willing to pay enough for each permit to induce the downstream cell to sell them. Does the same result hold if a greater-than-necessary number of permits are allocated to the upstream cell? When the permits are issued upstream, then the downstream cell has the option of purchasing any permits to prevent h_1 contamination, however the marginal analysis breaks down if the number of

[4]A barred variable represents the number of permits issued, while the subscript on barred variables denotes the cell to which the permits are issued.

permits exceeds $\hat{h}_1$ or $\hat{x}_1$, the privately optimal upstream hydrology or effort levels. Suppose $\overline{h}_1 > \hat{h}_1$ permits are issued. Even in the absence of a market, the upstream cell would not use more than $\hat{h}_1$ permits, but in order to alter the upstream cell's production decisions, the downstream cell would have to purchase all $\overline{h}_1 - \hat{h}_1$ excess permits. If the additional cost incurred to begin abatement, which totals $\text{price} * (\overline{h}_1 - \hat{h}_1)$, is greater than the potential net benefits of permit trading that accrue to the downstream cell, then the downstream cell will opt not to participate in the market and will instead settle for the results of the upstream cell's private optimization process. Such a result would represent an inefficient policy outcome. This problem could be addressed by simply issuing the upstream cell enough permits to cover its current contaminant level, $\hat{h}_1$. Permit trading from this point should result in an efficient level of h_1, with the downstream cell finding it worthwhile, in terms of avoided costs, to purchase $(\hat{h}_1 - h_1^*)$ permits.

What would happen if too few permits were offered ($\overline{h}_1 < h_1^*$)? Regardless of initial distribution, the same outcome would be achieved– the upstream cell would purchase, or retain, all of the permits available, and would operate at $\overline{h}_1$.

6. CONCLUSIONS

Designing appropriate policies for pollution control in a spatial context requires an understanding of the biophysical processes underlying the externality, both to determine the socially desired level of pollution flow and pollution control as well as to determine which types of policies will be effective at generating them. The size and shape of the wedge between social and private optimal production levels generated by an agricultural externality will depend, for instance, on whether the external effect is due to the magnitude of current upstream agricultural effort or the magnitude of the contaminant accumulation that results from that effort. The structure of the external flow will also determine the extent to which current activity translates into future costs downstream.

When the external flow increases with increasing upstream hydrological contamination level, the appropriate policy alternative is a tax applied to upstream hydrology. Such a policy would induce the upstream landowner to take into account the effect that the accumulating contamination is having downstream. The objective of the tax is not necessarily to drive the upstream contamination to zero. There is a certain social value generated by the natural flow of the contaminant out of the system without expenditure on abatement; although it is unfortunate that the contaminant must first flow through the downstream cell, even a social planner would choose to take advantage of this free cleaning service, which can only be done if contaminant levels are non-zero.

When the contaminant flow between cells increases with upstream agricultural effort, then the appropriate policy response to correct the externality is a tax applied to upstream effort levels. In this case, the upstream landowner can maximize the private benefit derived from the natural cleansing flow of the landscape by increasing effort levels, but he has the appropriate incentives to keep contaminant levels low. This is accomplished by increasing both effort and abatement levels, at the expense of downstream cells. The imposed tax on effort corrects this incentive.

It is possible to design permit schemes to correct for the market failure as well. These schemes are site-specific in that only parties whose activities generate externalities (in this case the upstream cells) are required to have permits, and process-specific in that the permits, like the taxes, must be applied to the appropriate flow process– either the effort level or the contaminant level. At the extreme, in a two-party model, issuing either $\bar{h}_1 = \infty$ or $\bar{h}_2 = \hat{h}_2$ mimics a Coasian scenario, and negotiation (or trading) results in a socially efficient outcome. When permits are split between the parties, whether the outcome of the negotiation process is socially efficient depends on whether an efficient number of permits ($\bar{h}_1 + \bar{h}_2 = h_1^*$) are issued.

An understanding of the nature of spatial flow relationships is therefore critical in the search for ways to correct for market failure as well as in characterizing what the socially desirable objectives are. This model presents highly simplified examples of such flow processes, and the diverging results produced in terms of landscape hydrology, producer incentives, and policy measures.

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