Weather Derivatives: Managing Risk with Market-Based Instruments

by

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Practitioners' Abstract:

Accurate pricing of weather derivatives is critically dependent upon correct specification of the underlying weather process. We test among six likely alternative processes using maximum likelihood methods and data from the Fresno, CA weather station. Using these data, we find that the best process is a mean-reverting geometric Brownian process with discrete jumps and ARCH errors. We describe a pricing model for weather derivatives based on such a process.

keywords: derivative, jump-diffusion process, mean-reversion, stochastic volatility, weather.

JEL Classification:

Introduction

Despite extreme price volatility, sensitivity of yields to fluctuations in temperature and precipitation, and expressed demand for some form of risk management tool (Blank and McDonald), there are few risk management alternatives available to California fruit growers. Indeed, because most fruits are highly perishable and are inelastic in demand, there are few in the agricultural economy who experience returns volatility greater than fruit growers. Such volatility imposes significant costs on both the industry and society as it forces growers to adopt enterprise diversification, crop rotation, capital reserve, off-farm employment or downstream integration strategies that they would otherwise not attempt if their primary activities were more stable. Recent modifications to the federally subsidized crop insurance program through the Crop Insurance Reform Act of 1994 and the Agricultural Risk Protection Act of 2000 provide incentives for insurance companies to develop a wider range of insurance products for specialty crop producers, but these have not met with the level of success that was originally hoped. Even with these reforms, many believe that traditional crop insurance simply will not work for fruit growers due to the heterogeneity of risks they face and their inherently entrepreneurial nature. Moreover, these initiatives promise levels of government expenditure that are not likely to be sustainable given current budget projections and, perhaps most importantly, represent an extent of government intervention in an industry in which government regulation of any type, even if financially beneficial for all, is an anathema. Consequently, the only long-term solution to the demand for an effective risk management tool in the California fruit industry must be a market-based one.

In this regard, weather derivatives represent a potentially promising solution. Although "over the counter" weather products (those that are not traded on a formal exchange) exist for rainfall, snowfall, humidity and temperature, the latter are the most common. Consequently, we believe that they are the most likely to of use to agricultural risk managers. Broadly defined, weather derivatives provide firms the ability to manage volumetric risk that derives from unusual weather events or seasonal deviations from longer term climatic norms. Coupled with conventional price-hedging or forward contracting, weather derivatives provide a revenue risk management capability that has proven to be attractive to many different types of firms. In fact, Turvey reports that some 4000 weather derivative transactions took place in the year 2000 worth approximately $8.0 billion (Weather Risk Advisory Ltd.) Given the broad range of firms that may be able to use weather derivatives to great advantage, this value should only increase. Achieving this liquidity, however, requires a more general understanding of how these tools work.

There are five essential elements to every weather contract: (1) the underlying weather index, (2) the period over which the index accumulates, typically a season or month, (3) the weather station that reports daily maximum and minimum temperatures, (4) the dollar value attached to each move of the index value, and (5) the reference or "strike" value of the underlying index (Cao and Wei). Essentially, weather derivatives are contingent securities that acquire value when the temperature is either greater than or less than some benchmark value, typically 65 degrees fahrenheit, at some reference location.[2] Each day the temperature averages greater than this benchmark contributes one cooling degree day (CDD) to the value of the cumulative CDD index, whereas each day below adds to a heating degree day (HDD) index. At the agreed expiry date, the holder of a put (call) will receive a

[2] Although precipitation derivatives also exist, temperature-based derivatives are more common and serve to illustrate the concept equally well.

payment if the cumulative amount of the underlying index falls below (rises above) the strike level. The amount of the payment is equal to the number of CDD or HDD above the strike level multiplied by some notional dollar value per unit of the index. Ideally, the buyer of the derivative is thus compensated by the writer for an amount that offsets the real business losses that have been incurred as a result of the weather pattern that emerges. For example, an amusement park owner would buy a CDD put that pays out if there is a string of unusually cold days. The value accumulated with the long put position will help offset the lost revenue from customers who have stayed away during the adverse weather period. If, on the other hand, the intervening period was unusually hot so that the CDD index rises well above the strike level, then the put will expire worthless and the amusement park owner is out the premium he paid at the initiation of the contract to the writer of the put, but is happy to do so because his business revenue likely more than compensates for the price of this "insurance policy." Clearly, the application to farming is directly analogous to the amusement park owner. A fruit grower, for example, would likely buy a CDD call so that he or she is compensated if a string of unusually hot weather causes a reduction in either yield or fruit quality compared to that expected had the CDD index reached the strike value over the growing period. Despite the apparent attractiveness of this ability to pass revenue risk to another, there are some problems that have, until now, limited the usefulness of weather derivatives as agricultural risk management tools.

Although weather derivative researchers and analysts commonly cite agricultural producers as likely users of weather derivatives, there has been little interest to date (Dischel). Several factors contribute to this including the lack of a forward market in a relevant weather index, potential basis risk, problems defining meaningful weather data, and a lack of accurate pricing models (Dischel; Turvey). First, although the Chicago Mercantile Exchange (CME) began trading degree-day futures and options for a number of major U.S. cities in the Fall of 1999, the fact that weather is a local phenomenon and micro-climates often differ radically within small geographic areas mean that the CME products are of little use to most agricultural producers. Second, without a traded instrument to from part of a riskless hedge, conventional preference-free Black-Scholes pricing models cannot be used to price weather derivatives. However, alternative approaches based on the weather-state variable are available that take into account the market price of risk, albeit they are not as simple to apply as standard option pricing models. Third, basis risk is likely to be a significant problem for assets based on weather indices. Basis risk, in this case, refers to the difference between a weather futures index value based in a particular city and the true value of the same weather index defined for a specific firm. Basis risk also arises from the fact that revenue, particularly in agriculture, derives from both rain and heat fluctuation. Precipitation and temperature are not perfectly correlated, nor do they have linear relationships to yields and market prices. Related to this problem is the fact that temperature varies continuously from region to region, whereas precipitation risk is discrete, often occurring in some fields, but leaving others only yards away dry. If weather is specific to very small geographic areas as we expect, then collecting useable data and defining a relevant index are both vitally important and potentially difficult.

Of all these potential problems, it is the absence of realistic pricing models developed specifically for the idiosyncracies of weather derivatives that is largely responsible for the large bid-ask spreads typical of quoted derivatives (Cao and Wei). While improved liquidity, institutional and regulatory changes, and better data collection methods can help solve the first three problems, academic research may help solve the final one. Admittedly, the complexities in both modeling and estimating processes underlying any weather index create challenges for any attempt to price weather derivatives. However, in this paper, we use well understood methods to find the "best" index model from among several viable alternatives. With this process, we then may

be able to create a potentially useful pricing model for California fruit growers. Specifically, the objectives of this paper are to: (1) determine the nature of the process underlying a temperature index for Central California, and (2) to sketch the requirements for an appropriate pricing model based on our preferred weather process for CDD weather derivatives. We begin the paper by describing the unique features of weather processes and defining several possible stochastic processes that may be used to describe the path of a CDD index through a growing season. Next, describe the data we use to estimate each of the CDD process models. Fourth, we provide a discussion of how we estimate the parameters of each of these processes and how we test among several competing empirical models in order to find the best CDD respresentation. A fourth section provides a discussion of the results obtained by comparing the fit of each index model while the final section concludes our findings and suggests several avenues for future research. In this final section we also draw several implications for the likely value of weather derivatives to growers who face problems in accessing effective ways to manage revenue risk.

Empirical Model of California Weather

Overview of Method

As is clear from our objective statement, we focus our efforts in this paper on the specific issue of determining the correct form of the stochastic processes underlying weather derivative pricing models. Consequently, our research method consists of two stages: first, we develop and estimate alternative models for stochastic processes underlying a weather index (CDD) for California fruit growers and, second, we describe a pricing model that is consistent with the preferred underlying weather process. Because we focus on temperature measures from the Fresno, CA weather station, we leave open the question of how geographic basis risk and local micro-climates impact the effectiveness of a weather derivative-based hedging program. We begin, however, by describing these data in more detail.

Data Sources and Sample Description

The weather data for this study are from the U.S. National Climatic Data Center for a weather station located in Fresno, CA Based on prior analyses of the optimal length of a data series required to estimate a weather process (Dischel), we estimate each of several candidate models with 30 years of daily temperature data for each weather station. The data consist of daily maximum temperature, minimum temperature, and average temperature. With these measures, we construct an index of cooling degree days (CDD) for a growing season that is assumed to run from May through July – the critical phase of final fruit development for the soft fruit (peaches, plums, and nectarines) and table grapes grown in the Fresno area. Although the temperature series is not directly applicable to any particular grower, primarily because it is gathered at the Fresno Air Terminal, the proximity of many growers to Fresno and the relative topographical homogeneity of the surrounding area should minimize the basis risk that would likely exist for growers located farther away from the weather station. Table 1 provides summary statistics for the CDD index for a series of 5-year intervals from 1970 to 2000. Contrary to what many believe, these data do not suggest that a "heat island" effect has been responsible for a general rise in temperatures in the Fresno area over this time period. With these data, we consider a variety of alternative forms for the underlying weather process.

Alternative Stochastic Processes

When pricing any derivative security, the accuracy of the pricing model depends critically upon the nature of the process for the underlying security or, in our case, the weather process from which the derivative derives its value. In this analysis, we consider four alternatives of increasing complexity and, correspondingly, decreasing parsimony: (1) geometric Brownian motion, (2) a jump-diffusion, or Poisson-normal mixture model, (3) mean-reverting jump-diffusion and (4) a geometric, mean-reverting jump-diffusion model. In each of the latter cases, there is also considerable empirical evidence that the volatility of weather processes may be stochastic, so we account for this possibility by using a simple autoregressive, conditional heteroskedastic (ARCH) error structure (Jorion; Hull and White). To determine which process provides a better fit to the weather data, we conduct pairwise comparisons of related models using likelihood ratio tests. In the simplest case, we assume the weather state-variable follows a geometric Brownian motion (GBM) akin to that used in a typical Black-Scholes model:

$$dW/W = \alpha\, dt + \sigma\, dz, \tag{1}$$

where W is the CDD index, α is its instantaneous rate of change, σ is the standard deviation of the process, and dz defines the Wiener process with properties: $E(dz) = 0$ and $E(dz^2) = dt$. Next, we define $w = \log(W_t/W_{t-1})$ as the daily percentage change in the weather index so that $\mu = \alpha - \sigma^2/2$ is the mean growth rate of w. Intuitively, a GBM process is a continuous-time version of a discrete random walk and is typical of many price processes, particularly those of stock prices traded on public equity markets.

However, many authors recognize the weakness of this assumption for not only weather processes, but for many other real-world processes as well. These authors identify biases that arise in attempting to apply this model to generate accurate price predictions (Merton; Ball and Torous; Jarrow and Rosenfeld; Jorion; Naik and Lee; Bates; Hilliard and Reis). Perhaps not surprisingly, the simplifying assumption of GBM is even less likely to hold for the evolution of temperatures as for stock prices, exchange rates, or interest rates. Specifically, weather indices are not likely to follow GBM processes because changes in the index from day to day (ie. the daily temperatures) are: (1) seasonal, (2) highly non-linear, (3) mean reverting, and (4) likely to experience significant periodic jumps. In the context of stock prices, Merton argues that jumps are likely to occur because news arrives to financial markets in discrete, often unpredictable intervals. Similarly, Jorion argues that foreign exchange markets are likely to experience jumps due to exchange rate regime realignments and periodic currency devaluations. Like market news, extreme weather events tend to occur infrequently and contain unique sources of peril for crops. Consequently, we represent a more plausible weather index process in terms of a jump-diffusion model of the form:

$$dW/W = (\alpha - \lambda\phi)dt + \sigma\, dz + \phi\, dq, \tag{2}$$

where now σ is the variance of the weather process conditional on no discontinuities, q is the Poisson counter with mean arrival rate λ, and ϕ is random percentage jump in the weather index conditional on a Poisson event. Moreover, in the base model we assume the random variable $(1 + \phi)$ is log-normally distributed: $\ln(1 + \phi) \sim N(\gamma - 0.5\delta^2, \delta^2)$ and the distribution of q is given by:

$$dq = \begin{bmatrix} 0 \text{ with probability } 1 - \lambda\, dt \\ 1 \text{ with probability } \lambda\, dt \end{bmatrix}. \tag{3}$$

Following Johnson and Barz, we generalize this model in the empirical example below, but we assume that the jump magnitude is log-normal, rather than exponentially, distributed. Recent research in this area finds that an assumption that jumps are distributed double-exponential (Kou) represents a preferable alternative for options on futures and interest rate derivatives, while Lewis provides a more general option valuation method for a broad class of jump-diffusion processes. Either way, a jump-diffusion process represents not only infrequent jumps in temperature, but non-normal skewness and, with slight modification, mean reversion as well – both likely attributes of any well defined weather process. In fact, Bates (1991, 1996) shows that accounting for discrete jumps explains such pricing anomalies as volatility smiles in both stock price and foreign exchange data.

The specification in (2) is, however, a simplification in that each of the parameters is conditional on a particular variance value. Hull and White and Jorion, on the other hand, recognize that other types of processes can explain the same excess kurtosis that is typical of weather distributions, namely a mixture of normals or one with stochastic volatility. Stochastic volatility, in the sense of Engle (1982) means that the second moment of the distribution varies over time. Incorporating stochastic volatility into (2) is relatively straightforward as we can write an expression for the conditional volatility as a first-order ARCH process:

$$h_t = E_{t-1}(\sigma_t^2) = \gamma_0 + \gamma_1(w_{t-1} - \alpha)^2, \qquad \textbf{(4)}$$

and estimate each of the parameters simultaneously by substituting h in for σ^2 in the likelihood function defined below. Some researchers, however, believe that weather indices tend to follow processes that exhibit even more complex behavior than that described by the stochastic-volatility, jump-diffusion specification described up to this point.

In particular, Pirrong argues that the process represented in (2) is insufficient for energy or weather derivatives because jumps tend to be one-sided, with prices rising upon the arrival of a new weather system, but declining slowly over time. Standard jump-diffusion models also implicitly assume that the jump is permanent, whereas jumps in a weather index are likely to revert quickly to the mean. An additional complication in estimating jump parameters is the fact that jump probabilities are likely to be seasonal. While the latter point is obviated by considering only season-specific weather indices, Cao and Wei, using CDD and HDD indices for several major U.S. metropolitan areas, find that daily temperatures are indeed strongly mean-reverting. To accommodate these characteristics, Johnson and Barz propose a deseasonalized, geometric mean-reverting jump-diffusion model. In fact, they show that such a process outperforms standard Brownian motion, geometric Brownian motion and basic Orstein-Uhlenbeck processes in explaining California electricity prices, each with or without similar jump processes. Incorporating each of these features, the geometric mean-reverting model is as follows:

$$dW/W = \kappa(\alpha - \lambda\phi - \ln W)dt + \sigma dz + \phi dq, \qquad \textbf{(5)}$$

where κ is the rate of mean-reversion. Again, we estimate this model using maximum likelihood methods as a generalization of (2) above, under alternative assumptions regarding the jump-distribution, and compare goodness of fit using a series of likelihood ratio tests. However, estimating these models using maximum likelihood is not the only approach.

Estimation Method

In fact, there are three alternative methods for estimating the parameters of the jump-diffusion weather process given by (5): (1) direct maximum likelihood estimation as in Ball and Torous (1983, 1985), Jorion or Jarrow and Rosenfeld, (2) implied estimation of derivative moments using an existing price series (Hilliard and Reiss) and (3) a least-squares estimator (Bates 1991, 1996). There are advantages and disadvantages of each approach. One problem with maximum likelihood estimation is the amount of data that are required. Because jumps are, by definition, infrequent, identifying jumps requires a long data series. However, over longer time series, the earlier periods may not be relevant to the current period in terms of either the volatility or the amplitude of the Poisson process (Hilliard and Reis). Second, the number of events is somewhat arbitrary, depending upon the researcher's definition of what constitutes "unusual" in a statistical sense. Third, the estimated parameters are those of the true process, so the researcher must make assumptions as to the form of preferences and technology underlying the data generating process. However, implied estimation with weather derivatives faces an even more fundamental problem -- the lack of derivative pricing data on which to base the estimates. There are existing weather derivative price data series, but there is some question as to how efficiently these are priced, whether the information on which they are based are accurate, and how relevant they are for growers in the Fresno area. Consequently, we adopt the direct maximum likelihood approach of Ball and Torous (1983, 1985); Jarrow and Rosenfeld, and Jorion.

With this approach, we test each of the specifications defined above against each other using a series of likelihood-ratio tests. These specifications include: (1) geometric Brownian motion (GBM), (2) mean-reverting geometric Brownian motion (MRGBM), (3) geometric Brownian motion with a log-normal jump (GBM-J), (4) a mean-reverting version of (3) (MRGBM-J), (5) an auto-regressive conditional heteroscedastic version of (3) (GBM-J-ARCH), and (6) an auto-regressive conditional heteroscedastic version of (4) (MRGBM-J-ARCH). The most general of these specifications, including mean reversion, log-normal jumps and ARCH errors is:

$$L(w \mid \beta) = -T\lambda - \frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\sum_{n=0}^{N} \frac{\lambda^n}{n!} \frac{1}{\sqrt{h_t + \delta^2 n}} \exp\left(\frac{-(w_t - (\alpha + h_t/2 + n\delta^2/2 - n\phi - \ln w)(1 - e^{-\kappa}))^2}{2(h_t + \delta^2 n)}\right)\right], \quad \textbf{(6)}$$

for *M* observations of log-relatives of the weather index: $(w_t = \ln(W_{i,t}/W_{i,t-1}))$ where λ is the Poisson intensity parameter, σ^2 is the volatility of the continuous part, δ^2 the volatility of the discrete part, and N is the mean jump size.[3] Following Ball and Torous (1985), we define *n* as the random realization of a shock to revenue, and fix *N* at a value likely to include all possible occurrences of a shock (for most data sets, three has proven to be sufficient), and maximize (6) with respect to the remaining parameters. We define each of the other likelihood functions in a similar way, but with appropriate substitutions for the maintained form of the stochastic process for w_t. With the appropriate numerical optimization algorithm, the likelihood function in each case converges rapidly and is robust over a range of starting values. Because each of the first five models is nested within the sixth, we select the best from among competing specifications with likelihood ratio tests. If these tests that one of the latter five models is preferred, then the weather derivative pricing model that follows will be of non-standard form.

[3] This assumption is more general than Ball and Torous (1985), who assume a jump size of mean zero.

Determination of Appropriate Pricing Model

For purposes of this paper, we discuss the likely alternative pricing models, but leave implementation for future work. To serve as the basis for an effective weather derivative trading program, we define an appropriate pricing model as one that: (1) is consistent with the underlying weather index process, (2) reflects the fact that a CDD index is cumulative, unlike stock prices, exchange rates, or other asset prices, and (3) is understandable by users in the field, whether extension agents, bankers, suppliers, or growers themselves. Originally, practitioners used simple "burn rate" models or modified existing Black-Scholes based pricing models to the weather problem by defining the underlying security in terms of the CME cooling degree day (CDD) or heating degree day (HDD) contracts defined for a specific urban location. Burn rate models have one key advantage that explains their popularity among practitioners -- ease of use. However, several problems exist with this approach (Dischel; Turvey; Pirrong). Most importantly, derivatives that are priced according to a burn rate model will trade infrequently, if at all, because there is no way to update the probabilities of adverse weather events. Without such volatility, there is no opportunity for the parties to arbitrage risk, so there will be no liquidity in the market (Turvey). Introducing a more complex process for the underlying index means that the simple averaging process implied by a burn rate model will not work, so a contingent claim (or dynamic programming) approach similar is necessary.

Yet, we cannot apply a standard analytical solution method without some modification. In fact, there are a host of well-documented problems associated with applying a simple Black-Scholes approach to a problem with a fundamentally different underlying payoff structure (Nelken; Dischel 1999). Most importantly, the underlying weather index is not a traded asset, but rather a state variable.[4] Therefore, the risks associated with weather are non-hedgeable so any derivative pricing model based on a weather index must include the market price of risk in an equilibrium pricing framework. The lack of a forward market for weather indices also means that there will be no way for traders to price derivatives and, hence, trade them on a continuous basis. Further, Pirrong believes that highly non-linear and seasonal weather indices mean that there is no stochastic representation that can easily fit into a contingent claim framework. Nonetheless, Turvey argues that the absence of a hedgeable asset can be addressed by applying the risk-neutral pricing model of Cox and Ross under the simplifying assumptions that the mean drift rate of the weather index and the "weather beta" are both zero.[5] Hilliard and Reis use a similar approach to value commodity options where jump-risks are assumed to be systematic and, therefore, undiversifiable. Using this assumption, we recognize that a risk-free hedge is unavailable, so we can potentially estimate a "risk neutralized" weather process that incorporates agents' marginal utility of wealth and the jump-diffusion characteristic of a CDD weather index (Bates 1991):

$$dW/W = -\lambda^*\phi^*dt + \sigma dz^* + \phi^* dq^*, \qquad (7)$$

[4] Although there are CDD and HDD futures contracts on the CME, these apply to only a handful of major metropolitan areas, so cannot be used to hedge weather risks for growers in Central California, or many other agricultural areas for that matter.

[5] This latter assumption is justified on the grounds that it is unlikely that the market portfolio can have any impact on the number of degree days, but it is not necessarily true that weather events do not impact the market portfolio.

where $\lambda^* = \lambda[1 + E(\Delta J_w / J_w)]$, $\phi^* = \phi + cov(\phi, \Delta J_w / J_w)/[1 + E(\Delta J_w / J_w)]$, and J_w is the marginal utility of wealth. With this process so defined, we then form the fundamental partial differential equation in the usual way and solve for the derivative price analytically. For example, in the most general MRGBM-J-ARCH process case, a CDD call option assumed to be exercisable only at expiry will have a price equal to:

$$V^e = S\sum_{n=0}^{\infty}\left(\frac{e^{-\hat{\lambda}t}(\hat{\lambda}t)^n}{n!}\right)e^{-r^*t}[z\Phi(-d_2) - w\Phi(-d_1)] \qquad (8)$$

where:

$$d_1 = \left(\frac{\ln(w/z) + 1/2v^2t}{v\sqrt{t}}\right), \quad d_2 = \left(\frac{\ln(w/z) - 1/2v^2t}{v\sqrt{t}}\right), \qquad (9)$$

$v^2 = h_t + n\delta^2/t$, $r^* = r - \lambda\phi^* - \kappa\ln w + n\gamma/t$, δ^2 is the variance of the log of N, $\gamma = \log(1+\phi^*)$, and $\hat{\lambda} = \lambda(1+\phi^*)$. Therefore, the value of a weather-based call option on a weather index that follows a composite jump-diffusion process is simply a weighted average of its value under each possible realization of the random number of discrete events, given adjustments to the variance of revenue and the discount rate. While this model should provide accurate weather derivative prices, others suggest that an equilibrium-pricing approach is a preferable way of pricing such non-standard claims, particularly when the jump risks are systematic and the payoff path-dependent (Cao and Wei; Pirrong). Comparing derivative prices calculated using each of these approaches may be a fruitful avenue for future research, but here we focus on the structure of the underlying weather process.

Results and Discussion

In this section, we first provide pairwise comparisons of each alternative CDD process, and then briefly discuss the implied structure of the preferred model. Although others apply this method to estimate the parameters of stock price processes (Ball and Torous 1983, 1985; Jarrow and Rosenfeld) and foreign exchange rates (Jorion), this represents the first attempt to parameterize a non-financial series. As Ball and Torous (1985) suggest, a Bernoulli jump-diffusion model provides useful starting values for the maximum likelihood estimation procedure. With these starting values, convergence of each model occurs within 40 iterations using a Newton-Raphson non-linear solution algorithm. Table 2 provides the likelihood ratio statistics we use to compare the various CDD models.

First, we compare the base geometric Brownian motion (GBM) model to a jump-diffusion alternative with Poisson arrival times and log-normal jump-magnitude (GBM-J). At a 5% level of significance, the likelihood ratio test clearly favors the unrestricted model. Second, we compare a mean-reverting geometric Brownian motion (MRGBM) process to the base GBM model. With one degree of freedom and a 5% level of significance, the likelihood ratio test statistic again suggests rejection of the null hypothesis that the reversion parameter (6) is zero. Given the apparent superiority of both mean-reversion and jump-diffusion elements

relative to the base model, we next compare a mean-reverting process with jumps (MRGBM-J) to one that does not include discrete shocks (MRGBM). At a 5% level, we reject the null hypothesis that each of the jump parameters (arrival rate, λ, jump variance, σ, and jump magnitude, ν) is equal to zero, so conclude that the preferred specification likely includes discrete jumps. To test whether the preferred jump-diffusion process is mean-reverting as well, we next compare the mean-reverting jump-diffusion process (MRGBM-J) to the base model with jumps (GBM-J). With one degree of freedom and a 5% significance level, the chi-square statistic of 192.72 provides clear support for the mean-reverting model.

Due to previous evidence of the importance of stochastic volatility in financial series (Bates 1991), we conduct a fifth test that compares the preferred jump-diffusion model to a first-order autoregressive conditional heteroscedastic (ARCH) variant of the base GBM-J model (GBM-J-ARCH). In this case, we only marginally reject the null hypothesis at a 5% level that the additional ARCH parameter is equal to zero. Given this preference for an ARCH specification, our final test compares a mean-reverting version (MRGBM-J-ARCH) to a non-reverting specification. Again at a 5% level and one degree of freedom, the results in table 2 suggest rejecting the non-reverting model. Therefore, we find that our preferred model is a mean-reverting geometric Brownian motion with log-normal jumps and first-order autoregressive conditional heteroscedastic errors.

The results in table 3 not only provide evidence as to the preferred model, but illustrate sharp qualitative differences in the implications of each process as well. For example, comparing the preferred model to GBM finds that the simpler process overstates the mean drift rate of the series by a factor of three. Critically, however, by ignoring mean-reversion, the GBM specification leaves open the possibility that the CDD index can wander away from its long-term average indefinitely. This is not likely to happen in reality, so should not be reflected in weather derivative prices. Second, if we rely on forecasts of a CDD index generated by a GBM process, we would also miss the fact that much of the cumulative index value comes not from prolonged, small deviations from the reference value (65o F), but often in short, discrete jumps in temperature. Statistically, the results in table 2 show the importance of jumps in the weather process through the large incremental improvements in fit by each jump-diffusion model relative to its continuous analog. Although the CDD process is physical, rather than financial, these results are consistent with the implications of ignoring "fat tails" in financial asset returns processes found by Bates (1991, 1996); Naik and Lee; and Jorion. Third, although we find that allowing for stochastic volatility improves the fit of each model, including and ARCH error component is apparently not as important as in other contexts. Specifically, in their comparison of alternative option pricing models, Bakshi, Cao and Chen provide evidence from S&P 500 index options that allowing for stochastic volatility provides the largest incremental gain in option pricing "fit" from among each of the Black-Scholes extensions that they consider. By using stocks traded on organized exchanges, however, they are able to use indirect market evidence to demonstrate the superiority of their stochastic volatility models, whereas we necessarily rely on direct estimation.

Conclusions

This study determines the appropriate form for the stochastic process underlying a weather derivative pricing model for risks specific to the Central Valley of California (Fresno). The creation of such a model is important to the industry as they lack many of the risk management tools that growers of traditional crops take for granted. Previous research into similar types of processes (electricity, stock prices, exchange rates) have found the usual geometric Brownian motion assumption to be inadequate. To investigate whether this is also the case for a CDD index, we consider six alternative specifications and test among them using a series of likelihood ratio tests. In this

way, we find that the preferred model is instead a mean-reverting, geometric Brownian motion process with first-order autoregressive errors and a log-normally distributed jump term. While we do not formally estimate derivative prices consistent with this index, we derive a pricing model that is appropriate for a CDD index that follows such a process.

As this study concerns only the nature of the underlying weather process for a single region, opportunities for future research that build on this foundation are clear. First, it would be of considerable practical value to calculate a series of weather derivative prices consistent with the preferred underlying process and use these in simulated revenue-risk management programs for California tree fruit growers to determine their potential effectiveness. Second, issues of basis risk and micro-climate variation can be considered by estimating similar processes for adjacent weather stations and determining their correlation with the Fresno series. Third, research is also needed into derivatives for other key weather variables – precipitation, heating degree days during the winter season, or derivatives specifically for catastrophic frost risks such as that which hit the California orange industry in the winter of 1998. Fourth, it is always the case that other researchers could extend our model by considering more complex error structures, such as GARCH, or higher order ARCH processes. Finally, as we suggest above, comparing weather derivative prices under risk-neutral and equilibrium pricing solution methods would be an important contribution to this emerging field of study.

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Table 1. Summary of Fresno Weather Data by Half-Decade

Decade	N	Mean CDD	Std. Deviation	Minimum CDD Index	Maximum CDD Index
1970 - 1974	460	387.14	314.41	0.00	1145.00
1975 - 1979	460	369.73	302.33	0.00	1145.00
1980 - 1984	460	423.86	366.48	0.00	1343.50
1985 - 1989	460	443.95	355.07	0.00	1302.50
1990 - 1994	460	382.41	319.09	0.00	1193.50
1995 - 1999	460	354.42	317.76	0.00	1157.00

Table 2. Likelihood Ratio Specification Tests for CDD Index

Models[1]	Restrictions	Critical χ^2	Estimated χ^2
(1) GBM vs. GBM-J	3	7.815	7,021.46
(2) MRGBM vs. GBM	1	3.840	9.94
(3) MRGBM-J vs. MRGBM	3	7.815	7,204.24
(4) MRGBM-J vs. GBM-J	1	3.840	192.72
(5) GBM-J-ARCH vs. MRGBM-J	1	3.840	4.40
(6) MRGBM-J-ARCH vs. MRGBM-J	1	3.840	50.42

[1] A 5% level of significance is used for all critical values. The likelihood ratio chi-square statistic is calculated as: $LR = 2 (LLF_U - LLF_R)$ where LLF is the log-likelihood function value.

Table 3. CDD Index Stochastic Process Estimation Results

Model[1]	Parameter Estimates							LLF
	8	"	*	6	N	(0	(1	
(1) GBM	N.A.	0.064 (22.381)	N.A.	N.A.	N.A.	0.022 (37.147)	N.A	1326.86
(2) GBM-J	0.219 (19.419)	0.025 (53.649)	0.025 (10.452)	N.A.	0.175 (20.828)	0.035 (21.582)	N.A	4837.59
(3) MRGBM	N.A.	0.022 (11.169)	N.A.	0.002 (10.966)	N.A.	0.022 (56.853)	N.A	1331.83
(4) MRGBM-J	0.266 (19.093)	0.032 (7.357)	0.020 (11.171)	0.004 (15.947)	0.409 (12.207)	0.024 (18.586)	N.A	4933.95
(5) GBM-J-ARCH	0.054 (8.433)	0.036 (57.366)	0.113 (5.373)	N.A.	0.394 (10.214)	0.036 (18.374)	0.251 (16.082)	4936.16
(6) MRGBM-J-ARCH	0.265 (17.564)	0.011 (10.866)	0.019 (11.088)	0.003 (41.854)	0.419 (16.274)	0.023 (17.527)	0.006 (1.752)	4961.36

[1] In this table, N.A. = not applicable, GBM = Geometric Brownian Motion, MRGBM = Mean Reverting Geometric Brownian Motion, GBM-J = Geometric Brownian Motion with Log-Normal Jump, MRGBM-J = Mean Reverting GBM with Log-Normal Jump, GBM-J-ARCH = GBM with Log-Normal Jump and Autoregressive Conditional Heteroskedastic error term, MRGBM-J-ARCH = Mean Reverting GBM with Log-Normal Jump and Autregressive Conditional Heteroskedastic error term.