Inventory and Transformation Risks in Soybean Processing

Roger A. Dahlgran[*]

*Paper presented at the NCR-134 Conference on Applied Commodity Price
Analysis, Forecasting, and Market Risk Management
St. Louis, Missouri, April 22-23, 2002.*

* Associate Professor, Department of Agricultural and Resource Economics, Economics,
Bldg#23, University of Arizona, Tucson, Arizona 85721-0023. Phone: (520)-621-6254, FAX
(520)-621-6250, Email: dahlgran@u.arizona.edu.

Inventory and Transformation Risks in Soybean Processing

Practitioner's Abstract

This study examines strategies for hedging processing operations generally and uses soybean processing as a specific example. The approach assumes a mean-variance utility function but because of the focus on hedging, the analysis concentrates on risk minimization with risk defined as the variance of the processing margin from its currently expected level. We find that risk so defined contains three components. These are (1) the risk of input/output cash price misalignment at the time of transactions, (2) the risk resulting from the firm's inability to utilize inputs and produce outputs in proportion to the mix that minimizes risk in cash market transactions, and (3) the risk of price change during the time between the purchase of inputs and the sale of outputs. The first two risk components are transformation risk while the third is inventory risk. The relationships between inventory and transformation risks were examined using daily price data from January 1, 1990 through March 23, 2000. Our analysis indicates that inventory risk is the largest of the three components, it increases in a roughly linear relationship with the temporal separation between pricing of inputs and outputs, it is the risk that is hedged with usual hedging models, and that hedging reduces this risk by a proportion of its amount. Consequently, even when hedged, processors face risks that increase with the time that separates the pricing of inputs and outputs and this risk is far larger than the risk of product transformation. In soybean processing, the proportion of risk eliminated through hedging reaches a peak for process lengths of one week with gradual declines thereafter. We also find that the risk-minimizing hedge ratios for soybean meal and soybean oil depend on the length of the anticipated hedging period.

Keywords: risk management, process hedging, soybean crushing.

Introduction

Processing alters the form of agricultural commodities but is usually not instantaneous so that ultimately both the time and the form dimensions of a commodity are simultaneously altered. To hedge processing operations, the price risks of both product transformation and inventory holding require attention as different hedging strategies have the potential to reduce risk from each source. The soybean-processing sector provides an ideal sector in which to study these issues because of the abundance of cash and futures prices for soybeans and soybean products. Cash prices for soybeans, soybean oil and soybean meal are reported daily, as are prices for futures contracts traded on the Chicago Board of Trade and the Mid American Exchange. This sector is also attractive for study because futures contracts for the three commodities are actively traded. This arrangement gives input-output price linkages in both the cash and the futures markets and cash-futures linkages for inputs as well as outputs.

Soybean processing consists of crushing and flaking the soybean then removing the oil with hexane (Chicago Board of Trade, 1985). The hexane is evaporated from the oil then reused. This process yields eleven pounds of oil per sixty-pound bushel of soybeans. After extracting

the oil and solvent, the material remaining is toasted and ground into 47 pounds of soybean meal (44 percent protein if hulls are not removed prior to processing, 49 percent if the hulls are removed). Gross processing margin is defined in the trade as the difference between the revenue from the soybean meal and oil and the cost of the soybeans. Typically, hedging seeks to reduce price-induced variation in the gross processing margin.

Several strategies for hedging soybean-processing risk are described in the literature. In a *one-to-one hedge*, each unit of cash market commitment is matched with a corresponding unit of futures market commitment. In the more general risk minimizing *direct hedge*, each unit of cash market commitment is hedged with a futures commitment in the same commodity where the futures commitment is chosen to minimize the risk of a cash commodity price change. In a *multi-contract hedge*, each unit of cash market commitment is hedged with commitments in several futures contracts where these futures commitments are chosen to minimize the risk of a cash commodity price change.[1] The futures contracts used may differ by maturity, may specify different commodities (i.e., a cross-hedge), or may specify other non-commodity financial instruments (currencies, securities, indices, or weather).

The soybean futures crush and reverse crush speculative spreads can also play a role in specifying hedging strategies. The crush spread involves a long soybean futures position, and short soybean meal and soybean oil futures positions in the ratios of 47 pounds of meal and 11 pounds of oil for each bushel of soybeans. With the *one-to-one crush hedge*, the processor is long one bushel in a soybean crush spread for each anticipated bushel to be processed. A generalization of one-to-one crush hedge is the *proportional crush hedge*. Here, the soybean processor uses the crush spread in proportion to the cash market soybean position that minimizes the risk of cash commodity price changes.

Literature Review

Hedging theory, initially presented by Johnson (1960) and Stein (1961), treats a commodity market position as part of a portfolio that may also contain a futures market position. Hedging corresponds to the futures position that maximizes utility, assumed to be a linear function of the mean and variance of returns. Ederington (1979) reports that in terms of risk management (as measured by the variance) this approach is superior to the traditional one-unit futures to one-unit cash approach. Anderson and Danthine (1980, 1981) extended the linear mean-variance approach by including multiple futures contracts in the portfolio. This extension allows risk management through multi-commodity hedging and cross hedging. Our study relies heavily on the Anderson and Danthine hedging formulation. Myers and Thompson (1989) examine the issues of whether hedge ratios are most appropriately estimated from price levels, changes, or returns. From this examination, they derive a generalized approach to hedge ratio estimation. Hedge ratio estimation under this generalized approach utilizes regression estimates based on deviations from the conditional mean at the time the hedge is implemented. Our study utilizes conditional regression estimates.

---

[1] Fackler and McNew (1993) refer to this as a multi-commodity hedge. Because the processor has a multi-commodity cash market position without hedging, we define this as a multi-contract hedge so that the multiple position is explicitly in futures markets. An additional advantage of this definition is that it allows consideration of multiple maturities in the same futures contract.

Production and process hedges have long been of interest in agricultural economics. Some examples of production hedging include the cattle feeding hedge with corn, feeder cattle, and live cattle futures (Leuthold and Mokler, 1979; Shafer, Griffin and Johnson, 1978), and the hog feeding hedge with live hog, soybean meal and corn futures (Kenyon and Clay, 1987). The soybean processing hedge, involving soybean, soybean oil and soybean meal futures, is a short-term application of production hedges. Tzang and Leuthold (1990) present the timing of soybean processing hedge transactions. At the beginning of the planning horizon, the processor hedges by buying soybean futures and selling soybean meal and soybean oil futures. At the beginning of the production phase, the processor buys soybeans, sells the soybean futures contracts, and begins processing soybeans. Finally, at the end of the production run, the processor sells the soybean oil and soybean meal, and buys soybean oil and soybean meal futures to close the hedge.

Tzang and Leuthold (1990) and Fackler and NcNew (1993) investigated various soybean-processing hedge strategies. Tzang and Leuthold use weekly cash and futures prices from January 1983 through June 1988 to examine multi- and single-commodity hedges over 1-, 2-, 6-, 9-, and 15-week hedging horizons. Fackler and McNew use monthly average cash and futures prices to examine three hedging strategies: multi-commodity hedges, single-commodity hedges, and proportional crush-spread hedges. The multi-commodity approach has recently been extended to cross-hedging applications in the cottonseed-processing sector where cottonseed and its products are hedged with futures contracts for soybeans, soybean products, and various feed grains (Dahlgran, 2000; Rahman, Turner, and Costa, 2001).

Missing from production and processing hedging studies is a clear distinction between transformation risk and inventory risk. In making this distinction, we obtain a better understanding of the types of risks faced, the magnitude of these risks, and the potential for hedging each of these risks. This distinction leads to the hypothesis that the primary and controllable source of risk is due to the inventories that surround the processing operation. By comparison, the risk of physical product transformation (i.e., variation in the instantaneous processing margin) is small over the short run as the correlation among the cash prices is nearly as high as among the futures prices. Also none of the previous studies use daily price data which more closely reflects soybean processors' decision-implementation time domain. In this regard, we utilize data with a shorter periodicity than analyzed heretofore. Our overall objective in this study is to disaggregate soybean-processing risks into inventory holding and transformation components, to conceptualize the behavior of each component, to assess the relative magnitudes of each, and to formulate strategies that can be used to hedge each of these risks. We use daily data to accomplish these objectives. Along the way, we compute optimal hedge ratios.

Theoretical Model

Our model assumes a production process where a primary input (x_t) is used in period t to produce a bundle of outputs ($\mathbf{y}_{t+L}$) in period t+L. The assumptions about the timing of this process are illustrated in figure 1. Gap A in figure 1 illustrates that the temporal separation (L) of input utilization and output receipt might in part reflect non-instantaneous production. Alternatively, this time lag could represent time as an input required for product transformation, as in the
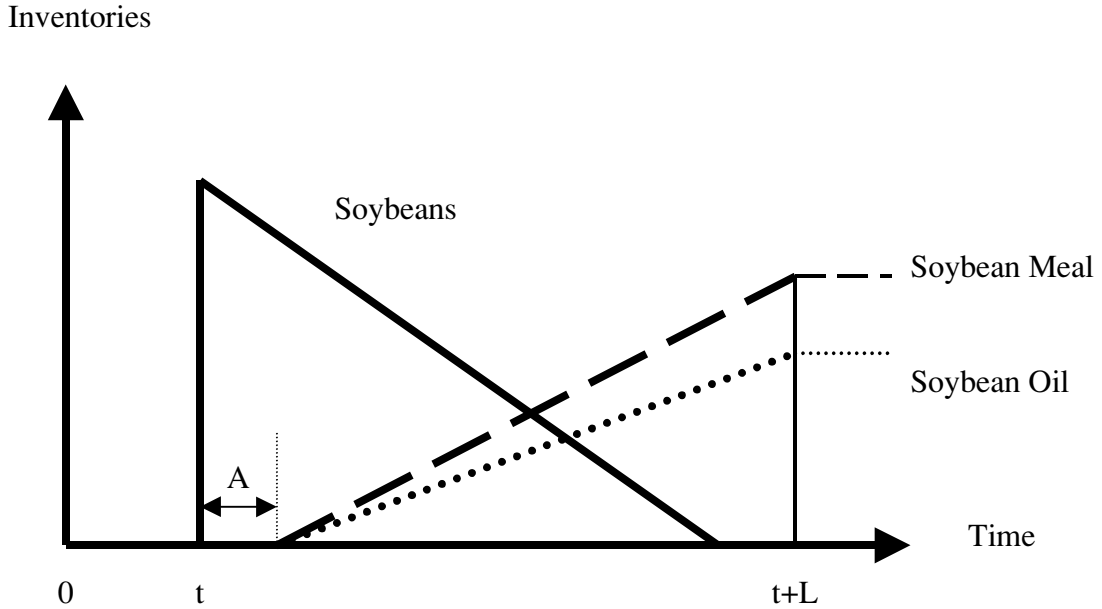
4

Inventories

Figure 1. Transaction timing and sources of risk in soybean crushing.

required aging of wine or in livestock growth. A third possibility is that the lag (L) is due to transactions costs associated with either input purchases or output sales. To minimize these costs, a firm will accumulate inventories from which to service a limited number of transactions (Ravindran, Phillips and Solberg, 1987). In this case, L represents the length of time between the purchase of the input and the sale of the output. More generally, L represents temporal separation of input and output pricing.

Our model also assumes fixed production coefficients, a characteristic of many commodity-processing industries. This production technology is represented as $\mathbf{y_{t+L}} = \alpha\, x_t$ where $\mathbf{y_{t+L}}$ and α are length-m row vectors representing outputs and transformation coefficients, respectively. The output and input quantities are more succinctly represented by the row vector $\mathbf{q_t} = [\ \mathbf{y_{t+L}} : -x_t\ ] = x_t\,[\alpha : -1\ ] = x_t\,\mathbf{\eta}$ with inputs represented as negative quantities. For soybean processing, where a 60-pound bushel of raw soybeans yields 47 pounds of meal and 11 pounds of oil, $\alpha = [\ 47, 11\ ]$ and $\mathbf{\eta} = [\ 47, 11, -1]$.

Spot or cash prices corresponding to $\mathbf{q_t}$ are contained in the vector $\mathbf{s_t} = [\ \mathbf{p_{t+L}} : r_t\ ]$ where $\mathbf{p_{t+L}}$ represents prices for the m outputs ($\mathbf{y_{t+L}}$) and r_t represents the price of the input (x_t). Using this notation, profit (gross crushing margin) for production initiated at time t is

$$\pi_t = \mathbf{q_t}\,\mathbf{s_t}' = x_t\,\mathbf{\eta}\,\mathbf{s_t}'. \tag{1}$$

The presence of $\mathbf{p_{t+L}}$ in $\mathbf{s_t}$ implies that π_t is a random variable with

5

$$E(\pi_t \mid \Omega_t) = x_t \, \boldsymbol{\eta} \, \bar{\mathbf{s}}_t\,', \text{ and} \tag{2a}$$

$$V(\pi_t \mid \Omega_t) = \mathbf{q}_t \, \mathrm{Cov}(\,\mathbf{s}_t, \mathbf{s}_t)\, \mathbf{q}_t' = x_t^2 \, \boldsymbol{\eta} \, \mathrm{Cov}(\,\mathbf{s}_t, \mathbf{s}_t)\, \boldsymbol{\eta}' \tag{2b}$$

where $\bar{\mathbf{s}}_t = E_t(\,\mathbf{s}_t \mid \Omega_t)$, Ω_t represents the information available at time t, and

$$\mathrm{Cov}(\mathbf{s}_t, \mathbf{s}_t) = \begin{bmatrix} \mathrm{Cov}(\mathbf{p}_{t+L}, \mathbf{p}_{t+L}) & \mathrm{Cov}(\mathbf{p}_{t+L}, \mathbf{r}_t) \\ \mathrm{Cov}(\mathbf{r}_t, \mathbf{p}_{t+L}) & \mathrm{Cov}(\mathbf{r}_t, \mathbf{r}_t) \end{bmatrix}.$$

Expressing $\mathbf{p}_{t+L} = \mathbf{p}_t + (\mathbf{p}_{t+L} - \mathbf{p}_t) = \mathbf{p}_t + \Delta^L \mathbf{p}_t$, gives

$$\mathrm{Cov}(\mathbf{s}_t, \mathbf{s}_t) = \begin{bmatrix} \mathrm{Cov}(\mathbf{p}_t, \mathbf{p}_t) + \mathrm{Cov}(\Delta^L\mathbf{p}_t, \Delta^L\mathbf{p}_t) + 2\,\mathrm{Cov}(\,\mathbf{p}_t, \Delta^L\mathbf{p}_t) & \mathrm{Cov}(\mathbf{p}_t, \mathbf{r}_t) + \mathrm{Cov}(\Delta^L\mathbf{p}_t, \mathbf{r}_t) \\ \mathrm{Cov}(\mathbf{r}_t, \mathbf{p}_t) + \mathrm{Cov}(\mathbf{r}_t, \Delta^L\mathbf{p}_t) & \mathrm{Cov}(\mathbf{r}_t, \mathbf{r}_t) \end{bmatrix}.$$

If price changes and levels are uncorrelated, then $\mathrm{Cov}(\mathbf{p}_t, \Delta^L\mathbf{p}_t) = 0$, and $\mathrm{Cov}(\Delta^L\mathbf{p}_t, \mathbf{r}_t) = 0$ so

$$\mathrm{Cov}(\mathbf{s}_t, \mathbf{s}_t) = \begin{bmatrix} \mathrm{Cov}(\mathbf{p}_t, \mathbf{p}_t) & \mathrm{Cov}(\mathbf{p}_t, \mathbf{r}_t) \\ \mathrm{Cov}(\mathbf{r}_t, \mathbf{p}_t) & \mathrm{Cov}(\mathbf{r}_t, \mathbf{r}_t) \end{bmatrix} + \begin{bmatrix} \mathrm{Cov}(\Delta^L\mathbf{p}_t, \Delta^L\mathbf{p}_t) & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \Sigma_{ss} + \Sigma_{\Delta^L p, \Delta^L p}.^{[2,3]}$$

Hence, $V(\pi_t \mid \Omega_t) = x_t^2 \{\, \boldsymbol{\eta}\,\Sigma_{ss}\,\boldsymbol{\eta}' + \boldsymbol{\eta}\,\Sigma_{\Delta^L p, \Delta^L p}\,\boldsymbol{\eta}'\,\} \tag{2c}$

The processing margin risk component, $x_t^2 \{\, \boldsymbol{\eta}\,\Sigma_{\Delta^L p, \Delta^L p}\,\boldsymbol{\eta}'\,\}$, designates the risk of output price change after pricing the inputs. The instantaneous processing margin risk, $x_t^2 \{\boldsymbol{\eta}\,\Sigma_{ss}\,\boldsymbol{\eta}'\}$ can be further disaggregated. One component results from a divergence between technical production coefficients and the market-price cointegrating coefficients. To represent soybean crushing, let the subscripts m, o, and b indicate soybean meal, soybean oil, and soybeans, respectively. By definition, the market-price cointegrating vector is the vector $\boldsymbol{\eta}^* = [\,\alpha^*_m, \ \alpha^*_o, \ -1]$ which

---

[2]   The existence of correlation between price levels and price changes would imply that price changes could be predicted based on price levels. This condition implies consistently profitable arbitrage and violates the notion of informationally efficient markets. We will test for and reject the notion that price levels can be used to predict daily price changes later in this paper.

[3]   This formulation assumes that t=0 in figure 1. In other words, the hedge is placed when soybeans are acquired. Alternatively, we could assume that $t > 0$ allowing anticipatory hedging of the soybean purchases and subsequent crushing. Such an assumption would not fundamentally alter our analysis but it would require the investigation of two different sequential time lags, t and L. In this case, both p_t and r_t are random variables so the covariance matrix $\Sigma_{\Delta^L s, \Delta^L s}$ becomes

$$\begin{bmatrix} \mathrm{Cov}(\Delta^{t+L} p_0, \Delta^{t+L} p_0) & \mathrm{Cov}(\Delta^{t+L} p_0, \Delta^t r_0) \\ \mathrm{Cov}(\Delta^t r_0, \Delta^{t+L} p_0) & \mathrm{Cov}(\Delta^t r_0, \Delta^t r_0) \end{bmatrix}.$$

multiplies the spot price vectors, [$\mathbf{S_m}$, $\mathbf{S_o}$, $\mathbf{S_b}$], and results in a stationary series (ε_t). This definition is expressed in regression form as

$$S_{b,t} = \alpha_0 + \alpha^*_m\, S_{m,t} + \alpha^*_o\, S_{o,t} + \varepsilon_t. \tag{3a}$$

Factor demand theory dictates that on the margin, the value of an input should equal the sum of the value of its marginal products. The technical production relationship suggests a soybean valuation relationship of

$$S_{b,t} = -\,C + 47\, S_{m,t} + 11\, S_{o,t} + \varepsilon_t \tag{3b}$$

with C representing the gross crushing margin so that the price cointegrating vector is $\eta = [\,47,\, 11,\, -1\,]$. While this relationship applies for a firm, η^* may differ from η because of market behavior. More specifically, if in a localized market, assembly costs increase with increased processing, then a crushing firm faces an upward-sloping input supply function. On the output side, soybean meal and soybean oil have differing distribution costs and differing geographic markets with differing demand responses. Both the input supply and the output demands are subject to random shocks. The simultaneous interaction of supply and demand can lead to price responses such that the more general (3a) holds instead of (3b).[4]

---

[4]   Represent a geographic market for soybeans and soybean products as

$P_b = a_0 + a_1\, Q_b + \varepsilon_b$,	price-dependent soybean supply ($a_1 \geq 0$).	(3.1)
$P_m = b_0 + b_1\, Q_m + \varepsilon_m$,	price-dependent soybean meal demand ($b_1 \leq 0$).	(3.2)
$Q_m = 47\, Q_b$,	implied soybean meal supply.	(3.3)
$P_o = c_0 + c_1\, Q_o + \varepsilon_o$,	price-dependent soybean oil demand ($c_1 \leq 0$).	(3.4)
$Q_o = 11\, Q_b$,	implied soybean oil supply.	(3.5)
$47\, P_m + 11\, P_o - P_b = d_0 + \varepsilon_c$	crushing margin.	(3.6)

The comparative static multipliers are

$$
\begin{bmatrix} \Delta P_b \\ \Delta P_m \\ \Delta P_o \\ \Delta Q_b \end{bmatrix}
= \frac{1}{D}
\begin{bmatrix}
48^2 b_1 + 11^2 c_1 & -48\, a_1 & -11\, a_1 & a_1 \\
48\, b_1 & -a_1 + 11^2 c_1 & -48 \times 11 b_1 & 48\, b_1 \\
11\, c_1 & -48 \times 11 c_1 & -a_1 + 48^2 b_1 & 11\, c_1 \\
1 & -48 & -11 & 1
\end{bmatrix}
\begin{bmatrix} \Delta \varepsilon_b \\ \Delta \varepsilon_m \\ \Delta \varepsilon_o \\ \Delta \varepsilon_c \end{bmatrix}
\tag{3.7}
$$

where $D = 47^2\, b_1 + 11^2\, c_1 - a_1$.

The effect of various shocks (ε_b, ε_m, and $\varepsilon_o \neq 0$), the relative price changes ($\Delta P_b / \Delta P_m$ and $\Delta P_b / \Delta P_o$), and the requirements for the expected price effects ($\Delta P_b / \Delta P_m = 47$ and $\Delta P_b / \Delta P_o = 11$) are shown below. Note that no market configuration consistently gives the expected effects over all sources of shock.

The processing-margin risk impact of a difference between the price cointegrating vector and the technical production coefficients vector is expressed as

$$(\eta - \eta^*)\, \Sigma_{ss}\, (\eta - \eta^*)' = \eta\, \Sigma_{ss}\, \eta' + \eta^*\, \Sigma_{ss}\, \eta^{*'} - 2\, \eta\, \Sigma_{ss}\, \eta^{*'}. \tag{4a}$$

In estimating the last two terms

$$
\hat{\Sigma}_{ss}\hat{\eta}^{*'} =
\begin{bmatrix}
s_{mm} & s_{mo} & s_{mb} \\
s_{om} & s_{oo} & s_{ob} \\
s_{bm} & s_{bo} & s_{bb}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix} s_{mm} & s_{mo} \\ s_{om} & s_{oo} \end{bmatrix}^{-1}
\begin{bmatrix} s_{mb} \\ s_{ob} \end{bmatrix} \\
-1
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
0 \\
0 \\
\begin{bmatrix} s_{bm} & s_{bo} \end{bmatrix}
\begin{bmatrix} s_{mm} & s_{mo} \\ s_{om} & s_{oo} \end{bmatrix}^{-1}
\begin{bmatrix} s_{mb} \\ s_{mo} \end{bmatrix} - s_{bb}
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
-s_{bb}(1-R^2)
\end{bmatrix}
\tag{4b}
$$

where s_{ij} represents the mean commodity i and j cross product of the L-period price differences

and $\begin{bmatrix} \hat{\alpha}_m^* \\ \hat{\alpha}_o^* \end{bmatrix} = \begin{bmatrix} s_{mm} & s_{mo} \\ s_{om} & s_{oo} \end{bmatrix}^{-1} \begin{bmatrix} s_{mb} \\ s_{ob} \end{bmatrix}$. Therefore,

$$(\eta - \hat{\eta}^*)\, \hat{\Sigma}_{ss}\, (\eta - \hat{\eta}^*)' = \eta\, \hat{\Sigma}_{ss}\, \eta' + s_{bb}(1-R^2) \tag{4c}$$

where R^2 is the coefficient of determination from estimating (3a). Rearranging (4c) and substituting for $\eta\, \Sigma_{ss}\, \eta'$ in (2c) gives

$$\hat{V}(\pi_t \mid \Omega_t) = x_t^2 \{ s_{bb}(1-R^2) + (\eta - \hat{\eta}^*)\, \hat{\Sigma}_{ss}\, (\eta - \hat{\eta}^*)' + \eta\, \hat{\Sigma}_{\Delta^L p, \Delta^L p}\, \eta' \}. \tag{4d}$$

The terms in this expression represent three distinct processing-margin risk sources. The first represents the risk of input/output price misalignment. If input and output prices align contemporaneously without error ($R^2 = 1$), then this term disappears. The second term represents

Shock	$\Delta P_b/\Delta P_m$	$\Delta P_b/\Delta P_o$	$\Delta P_b/\Delta P_m = 47$ requires	$\Delta P_b/\Delta P_o = 11$ requires
$\Delta a_0 \neq 0$	$47 + (11^2 c_1/47 b_1) \geq 47$	$11 + (47^2 b_1/11 c_1) \geq 11$	$c_1=0$ and $b_1 \neq 0$, or $b_1 = -\infty$	$b_1=0$ and $c_1 \neq 0$, or $c_1 = -\infty$
$\Delta b_0 \neq 0$	$47 / [1 - 11^2 (c_1/a_1)] \leq 47$	$a_1 / 11 c_1 \leq 0$	$c_1 = 0$ and $a_1 \neq 0$, or $a_1 = \infty$	$a_1=11^2 c_1$ and $a_1 \neq 0$ and $\text{sign}(a_1) = \text{sign}(c_1)$
$\Delta c_0 \neq 0$	$a_1 / 47 b_1 \leq 0$	$11 / [1 - 47^2 (b_1/a_1)] \leq 11$	$a_1=47^2 b_1$ and $a_1 \neq 0$ and $\text{sign}(a_1) = \text{sign}(b_1)$	$b_1=0$ and $a_1 \neq 0$, or $a_1 = \infty$

production misalignment and identifies risk due to a divergence between the local input and output markets' price cointegrating coefficients and the firm's technical production coefficients. If the price cointegrating vector between soybean crushing outputs and inputs is [47, 11, -1], or if the technical production coefficients are the same as the price cointegrating coefficients, then this term vanishes. Also note that a sufficiently strong negative correlation (covariance) between output prices (σ_{mo} = - [$(\alpha_m-\alpha^*_m)^2 \sigma_{mm} + (\alpha_o-\alpha^*_o)^2 \sigma_{oo}$]/ [2 $(\alpha_m-\alpha^*_m)(\alpha_o-\alpha^*_o)$]) also causes this term to vanish.[5] The third term in (4d) represents the inventory (or time) risk of product transformation as it indicates the risk of output price change over the L periods after pricing inputs but before pricing the output.

Anderson and Danthine's multi-commodity hedging theory is used to determine the potential for hedging these risks. Let $\mathbf{h}_t$ represent period t hedging transactions in the n futures contracts that constitute potential hedge vehicles, and let $\mathbf{f}_t$ represent the prices of these futures contracts. Both $\mathbf{h}_t$ and $\mathbf{f}_t$ are n-length row vectors where $h_{it} > 0$ ($h_{it} < 0$) establishes a long (short) position. Closure of positions with reversing transactions in period t+L is assumed. With the addition of hedging, the profit function (1) is augmented to

$$\pi^*_t = \mathbf{q}_t \, \mathbf{s}_t' + \mathbf{h}_t \, (\mathbf{f}_{t+L} - \mathbf{f}_t)'. \tag{5}$$

The producer's decision variables, $\mathbf{q}_t$ and $\mathbf{h}_t$, are selected in the presence of uncertainty about $\mathbf{p}_{t+L}$ (contained in $\mathbf{s}_t$) and $\mathbf{f}_{t+L}$ where $E_t(\,\mathbf{f}_{t+L} \mid \Omega_t) = \bar{\mathbf{f}}_{t+L}$, Cov($\mathbf{f}_{t+L}- \mathbf{f}_t, \mathbf{f}_{t+L}- \mathbf{f}_t \mid \Omega_t) = \Sigma_{\Delta^L f,\Delta^L f}$, and Cov($\mathbf{s}_t, \mathbf{f}_{t+L}- \mathbf{f}_t \mid \Omega_t) = \Sigma_{\Delta^L p,\Delta^L f}$.[6] As a result,

$$E(\pi^*_t \mid \Omega_t) = \mathbf{q}_t \, \bar{\mathbf{s}}_t{}' + \mathbf{h}_t \, (\bar{\mathbf{f}}_{t+L} - \mathbf{f}_t)', \text{ and} \tag{6a}$$

$$V(\pi^*_t \mid \Omega_t) = \mathbf{q}_t \, (\Sigma_{ss} + \Sigma_{\Delta^L p,\Delta^L p}) \, \mathbf{q}_t' + \mathbf{h}_t \, \Sigma_{\Delta^L f,\Delta^L f} \, \mathbf{h}_t' + 2 \, \mathbf{q}_t \, \Sigma_{\Delta^L p,\Delta^L f} \, \mathbf{h}_t'. \tag{6b}$$

The Anderson and Danthine theory assumes that the firm's objective is

$$\max U(\pi^*_t) = E(\pi^*_t) - (\lambda/2) \, V(\pi^*_t) \,. \tag{7}$$
$$\text{w.r.t. } x_t, \mathbf{h}_t$$

Accordingly, the first order conditions are

---

[5] σ_{mo} = - [$(\alpha_m-\alpha^*_m)^2 \sigma_{mm} + (\alpha_o-\alpha^*_o)^2 \sigma_{oo}$]/ [2 $(\alpha_m-\alpha^*_m)(\alpha_o-\alpha^*_o)$] can be rearranged to obtain $0 = A^2 + 2 \, \rho_{mo} \, A + 1$ where A =[$\sigma_{mm}^{0.5} \, (\alpha_m-\alpha^*_m)$] / [$\sigma_{oo}^{0.5} \, (\alpha_o-\alpha^*_o)$] and ρ_{mo} is the correlation between soybean meal and soybean oil prices. The solution, ρ_{mo} = +/- 1, implies that perfect positive or negative correlation will also cause this risk source to disappear.

[6] Continuing the notion that price levels and changes are uncorrelated,

$$\text{Cov}(\mathbf{s}'_t, \Delta^L \mathbf{f}_t) = \text{Cov}\left(\begin{bmatrix} \mathbf{p}'_t + \Delta^L \mathbf{p}'_t \\ \mathbf{r}_t \end{bmatrix} \Delta^L \mathbf{f}_t \right) = \text{Cov}\left(\begin{array}{c} \mathbf{p}'_t \Delta^L \mathbf{f}_t + \Delta^L \mathbf{p}'_t \Delta^L \mathbf{f}_t \\ \mathbf{r}_t \Delta^L \mathbf{f}_t \end{array} \right) = \text{Cov}\left(\begin{array}{c} \mathbf{0} + \Delta^L \mathbf{p}'_t \Delta^L \mathbf{f}_t \\ \mathbf{0} \end{array} \right) = \Sigma_{\Delta^L p,\Delta^L f} \,.$$

$$\frac{\partial U(\pi^{*}_{t})}{\partial x_{t}} = \boldsymbol{\eta} \cdot [\bar{\mathbf{s}}_{t}' - \lambda\,(\boldsymbol{\Sigma}_{ss} + \boldsymbol{\Sigma}_{\Delta^{L}p,\Delta^{L}p})\,\mathbf{q}_{t}' - \lambda\,\boldsymbol{\Sigma}_{\Delta^{L}p,\Delta^{L}f}\,\mathbf{h}_{t}'] = 0 \text{, and} \qquad (8a)$$

$$\frac{\partial U(\pi_{t})}{\partial \mathbf{h}_{t}} = \bar{\mathbf{f}}'_{t+L} - \bar{\mathbf{f}}_{t}' - \lambda\,\boldsymbol{\Sigma}_{\Delta^{L}f,\Delta^{L}f}\,\mathbf{h}_{t}' - \lambda\,\boldsymbol{\Sigma}_{\Delta^{L}f,\Delta^{L}p}\,\mathbf{q}_{t}' = 0 . \qquad (8b)$$

Optimal hedging is found by solving (8b) for $\mathbf{h}_{t}$ and using $\mathbf{q}_{t} = \boldsymbol{\eta}\,x_{t}$ to get

$$\mathbf{h}^{*}_{t}{}' = \lambda^{-1}\,(\boldsymbol{\Sigma}_{\Delta^{L}f,\Delta^{L}f})^{-1}(\bar{\mathbf{f}}'_{t+L} - \bar{\mathbf{f}}_{t}') - (\boldsymbol{\Sigma}_{\Delta^{L}f,\Delta^{L}f})^{-1}\,\boldsymbol{\Sigma}_{\Delta^{L}f,\Delta^{L}p}\,\boldsymbol{\eta}' \cdot x_{t}.{}^{7} \qquad (9a)$$

This result can be used to demonstrate that the optimal hedge is not necessarily a crush spread corresponding to the production coefficients, [47, 11, -1]. First, we distinguish hedging from speculation and rid ourselves of speculative behavior by assuming that either $\lambda = \infty$ (extreme risk aversion), or $\bar{\mathbf{f}}_{t+L} = \bar{\mathbf{f}}_{t}$ (no expected speculative returns) so that the first term of (9a) vanishes.[8] The hedge ratios for meal, oil, and beans per unit of input (x_{t}) are $(\boldsymbol{\Sigma}_{\Delta^{L}f,\Delta^{L}f})^{-1}\,\boldsymbol{\Sigma}_{\Delta^{L}f,\Delta^{L}p}\,\boldsymbol{\eta}'$ where $(\boldsymbol{\Sigma}_{\Delta^{L}f,\Delta^{L}f})^{-1}\,\boldsymbol{\Sigma}_{\Delta^{L}f,\Delta^{L}p}$ represents estimates of [$\boldsymbol{\beta_{m}} : \boldsymbol{\beta_{o}} : \mathbf{0}$], in the regressions[9]

$$\Delta^{L}S_{i,t} = \beta_{ib}\,\Delta^{L}F_{b,t} + \beta_{im}\,\Delta^{L}F_{m,t}\,\beta_{im} + \Delta^{L}F_{o,t}\,\beta_{im} + \varepsilon_{i,t},\ i = m,\ o \text{ and } t = 1, 2, ...T. \qquad (9b)$$

The notion that the optimal soybean crushing hedging strategy is a crush spread (i.e., that $[\hat{\boldsymbol{\beta}}_{m} : \hat{\boldsymbol{\beta}}_{o} : \mathbf{0}]\,\boldsymbol{\eta}' = \boldsymbol{\eta}'$) requires $47\,\beta_{mm} + 11\,\beta_{om} = 0$, $47\,\beta_{mo} + 11\,\beta_{oo} = 0$, and $47\,\beta_{mb} + 11\,\beta_{ob} = 1$. This hypothesis will be tested.

Hedging effectiveness is assessed by comparing the variance of profits without hedging, (4d), to the variance of profits with hedging

$$\hat{V}(\pi_{t}\,|\,\mathbf{h}_{t} = \mathbf{h}^{*}_{t}) = x^{2}_{t}\{s_{bb}(1 - R^{2}) + (\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}^{*})\hat{\boldsymbol{\Sigma}}_{ss}(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}^{*})' +$$
$$\boldsymbol{\eta}(\hat{\boldsymbol{\Sigma}}_{\Delta^{L}p,\Delta^{L}p} - \hat{\boldsymbol{\Sigma}}_{\Delta^{L}p,\Delta^{L}f}(\hat{\boldsymbol{\Sigma}}_{\Delta^{L}f,\Delta^{L}f})^{-1}\hat{\boldsymbol{\Sigma}}_{\Delta^{L}f,\Delta^{L}p})\boldsymbol{\eta}'\} . \qquad (10)$$
$$+ \lambda^{-2}(\bar{\mathbf{f}}_{t+L} - \bar{\mathbf{f}}_{t})\,(\hat{\boldsymbol{\Sigma}}_{\Delta^{L}f,\Delta^{L}f})^{-1}\,(\bar{\mathbf{f}}_{t+L} - \bar{\mathbf{f}}_{t})'$$

Again ignoring speculative behavior ($\lambda = \infty$ for extreme risk aversion, or $\bar{\mathbf{f}}_{t+L} = \bar{\mathbf{f}}_{t}$ for no expected speculative returns), then making the comparison isolates the hedgeable components of

---

[7]   Though not of particular interest in this study, the optimal input level, x_{t}^{*}, is found by substituting $\mathbf{h}_{t}^{*}$ for $\mathbf{h}_{t}$ in (8a). As discussed by Anderson and Danthine (pp. 1188-9), x_{t}^{*} is the value of x_{t} that equates expected marginal revenue with the risk-premium adjusted marginal cost.

[8]   Speculative behavior is not the focus of this study.

[9]   The last column of $\boldsymbol{\Sigma}_{\Delta^{L}f,\Delta^{L}p}$ is a null vector when t=0. When t > 0, the null vector becomes β_{b} and contains hedge ratios for the anticipatory hedging of soybean purchases.

processing risk. The risk reduction from hedging, $\boldsymbol{\eta}(\hat{\Sigma}_{\Delta^L p, \Delta^L f}(\hat{\Sigma}_{\Delta^L f, \Delta^L f})^{-1}\hat{\Sigma}_{\Delta^L f, \Delta^L p})\boldsymbol{\eta}'$, indicates that only the risk of output-price change during the processing period is hedged.

Empirical Analysis

Our analysis proceeds along three lines. First, the data are examined for nonstationarity as its existence determines appropriate data treatment methods (differences versus levels) for subsequent analysis. Next, hedging effectiveness is compared among various strategies that utilize various futures contract maturities over various hedge intervals. This step identifies the best contract to use for hedging and the penalties associated with alternative contracts. Finally, risk minimizing hedge ratios are examined for various hedging intervals. Of particular interest is the risk penalty associated with using single-commodity instead of multi-commodity hedges.

The data used in this analysis are summarized in table 1. Daily cash prices from January 1, 1990 through March 23, 2000 (2,583 observations) for soybeans (#1 yellow, central Illinois), soybean oil (crude, Decatur Illinois), soybean meal (48% protein, Decatur, Illinois) were obtained from the Bridge/Commodity Research Bureau InfoTech data source. Daily futures prices (open, high, low, and settlement) for each soybean, soybean oil, and soybean meal futures contract traded on the Chicago Board of Trade during this time period were also obtained from this source.[10] Roughly 25,000 futures-market observations on each commodity were available.

The data were tested for covariance nonstationarity, which is caused by unit roots in the data-generating process. Enders (1995; 221-232) describes the Dickey-Fuller method of testing for unit roots by fitting the model $\Delta y_t = a_0 + \gamma y_{t-1} + \varepsilon_t$. If γ is zero, then no relationship between

Table 1. Data summary.[a]

Commodity	Obs	Avg	Std Dev	Units	Futures Maturities
Cash Prices					
Soybeans (S_s)	2,583	606.23	94.57	cts/bu	
Soybean meal (S_m)	2,583	188.69	39.71	$/tn	
Soybean oil (S_o)	2,583	22.86	3.68	cts/lb	
Futures Settlement Prices					
Soybeans (F_m)	25,593	623.17	72.68	cts/bu	Jan, Mar, May, Jul, Aug, Sep, Nov
Soybean meal (F_m)	24,330	187.03	28.51	$/tn	Jan, Mar, May, Jul, Aug, Sep, Oct, Dec
Soybean oil (F_o)	26,049	23.16	2.99	cts/lb	Jan, Mar, May, Jul, Aug, Sep, Oct, Dec

a/ Data source: CRB/Bridge Infotech, on CD. Daily data from 1/2/90 through 3/23/00.

---

[10] The futures contract delivery locations correspond to the cash price locations so concerns about spatial price relationships and spatial price risk are removed from the focus of this analysis.

price levels and changes exists so the data appear to follow a random walk. If on the other hand, γ is not zero, then the data follow a mean-reverting process. When the data are generated by an ARIMA process, $\Delta y_t = \gamma\, y_{t-1} + \sum_{i=1}^{n} b_i\, \Delta y_{t-i} + \varepsilon_t$ is used to test for unit roots. γ not equal to zero still indicates stationarity. The selection of the maximum lag length n relies on Said and Dickey's (1984) finding that an ARIMA(p, 1, q) process can be well approximated by an ARIMA(n, 1, 0) where $n = T^{1/3}$.

These two models were fit to the cash price data where y_t respectively represents soybean, soybean meal, soybean oil, cash prices, and the gross crushing margin ($S_c = -S_s + 47\,S_m + 11\,S_o$). For the ARIMA model, $n \approx 14$ as our data set contains 2,583 observations (T). n is increased to 15 so that three full weeks of lagged values are included as regressors. The results are summarized in table 2. The respective ten-, five-, and one-percent critical values for the Dickey-Fuller test are approximately -2.57, -2.86, -3.43.[11] Hence, the unit root hypothesis cannot be rejected at the five-percent significance level for soybeans, soybean oil, and soybean meal prices, but the hypothesis is rejected for the crushing margin. The series were also tested for higher-order integration with the model $\Delta^2 y_t = a_0 + \beta\, \Delta y_{t-1} + \varepsilon_t$. The respective Dickey-Fuller test statistics of –51.14, -51.01 and –49.13 for soybeans, soybean meal, and soybean oil, respectively, lead to the rejection of the second-order integration of the cash prices. These results lead to two conclusions. First, the daily cash price series for soybeans, soybean oil, and soybean meal all

Table 2. Dickey-Fuller test statistics for unit roots in daily cash and futures prices.

Model:	$\Delta y_t = \gamma\, y_{t-1} + \sum_{i=1}^{p} b_i\, \Delta y_{t-i} + \varepsilon_t$				$\Delta y_t = a_0 + \gamma\, y_{t-1} + \varepsilon_t$			
Cash Prices:	$\hat{\gamma}$	Std Err $\hat{\gamma}$	t-value		$\hat{\gamma}$	Std Err $\hat{\gamma}$	t-value	
Soybeans	-0.00164	0.00144	-1.14		-0.00183	0.00141	-1.29	
Soybean meal	-0.00211	0.00130	-1.63		-0.00215	0.00129	-1.67	
Soybean oil	-0.00196	0.00147	-1.34		-0.00204	0.00144	-1.42	
Soybean crush margin	-0.01083	0.00296	-3.66		-0.01230	0.00292	-4.21	
	Pr > DF				Pr > DF			
Futures Prices:	>10%	5-10%	1-5%	<1%	>10%	5-10%	1-5%	<1%
Soybeans	63	3	1	0	60	6	0	1
Soybean meal	63	2	2	0	61	5	1	0
Soybean oil	64	2	1	0	60	6	1	0
Soybean crush margin	64	1	2	3	42	7	8	10

---

[11]   The Dickey-Fuller test accounts for the asymptotic non-normality of the OLS estimate of γ.

have unit roots so analysis of these data should utilize first differences. Second, the results indicate that the three series are cointegrated with the crushing margin representing a long-run equilibrium relationship among them.

While we could not reject the hypothesis that the technical coefficients constituted the cointegrating vector among the soybean and soybean-product prices, it is not necessarily the best vector. The cointegrating vector is estimated with[12]

$$\Delta S_{s,t} = \ 0.0121 + 29.5775\ \Delta S_{m,t} + 10.3004\ \Delta S_{o,t} \qquad SSE = 44,477 \qquad dfe = 2580 \qquad (11a)$$
$$\quad\ (0.0817)\ \ (0.6596) \qquad\quad (0.3193) \qquad\qquad R^2 = 0.6238$$
$$\quad\ [0.15]\ \ \ [44.84] \qquad\qquad [32.26] \qquad\qquad F = 2,138. \qquad Prob(> F) < 0.0001$$

From (4d), a difference between the cointegrating vector, [29.6, 10.3, -1] and the technical production coefficients, [47, 11, -1], represents a source of processing risk. Testing the hypothesis that these two vectors are equal gives an F statistic with 2 and 2580 degrees of freedom of 3,301. This is significant at well beyond a probability of 0.0001, leading to the conclusion that the difference between the price cointegrating vector and the production coefficients is a significant source of risk for the soybean crusher.

The soybean, soybean meal, and soybean oil futures contract prices were also examined for nonstationarity. A futures-market crushing margin requires common maturity dates among the soybean, soybean oil, and soybean meal futures contracts. These three commodities have simultaneous maturities in the Jan, Mar, May, Jul, Aug, and Sep contracts. Sixty-seven of these contracts for each commodity were traded during the ten-year study period.[13] Each contract generates a time series of price data beginning with the inception of trading in the contract and ending at contract maturity. Each series was examined for nonstationarity using the Dickey-Fuller test. Rather than report individual value of $\hat{\gamma}$ for each regression, the significance levels of the Dickey-Fuller statistics for the 67 contracts are summarized in table 2. For 4 of the 67 contracts, the unit root hypothesis was rejected at the 10% level for soybeans and soybean meal and the hypothesis was rejected once at the 5% level for soybeans and soybean oil (table 2). These results support the notion that the daily futures prices appear to follow a random walk so analysis using price differences is appropriate. Contrary to the cash-market crushing margin, the futures-market crushing margin appears to be covariance nonstationary.[14]

---

[12] Standard errors are in parentheses, t-statistics are in brackets. Various alternative specifications of this model were estimated. These specifications include first-order autoregressive and nonautoregressive models in and differenced and nondifferenced prices. All are valid under the finding that the three series are cointegrated. The estimated serial correlation of 0.998 in the undifferenced specification lends credibility to the specification reported in (11a). The conclusion that the cointegrating coefficients are less than and significantly different from the technical coefficients held across all specifications.

[13] More precisely, sixty-nine contracts, but the January 1990 and May 2001 contracts had too few observations, 15 and 9 respectively, to be included in this analysis.

[14] This result is consistent with weak form efficiency of these markets. Futures-market participants speculate on the crush spread. If the crush spread displayed mean reversion, or serial correlation, then this would imply crush spread predictability or consistently profitable crush spread speculation, i.e., informational inefficiency. Johnson et al. (1991) reach a similar conclusion in a study that utilizes trading rules.

A regression similar to (11a), applied to futures prices, gives

$$\Delta F_{s,t} = -0.0128 + 43.6\ \Delta F_{m,t} + 9.88\ \Delta F_{o,t} \qquad SSE = 105{,}460 \qquad dfe = 17{,}302$$
$$\phantom{\Delta F_{s,t} = }(0.0188) \quad (0.180) \qquad (0.0766) \qquad R^2 = 0.887 \tag{11b}$$
$$\phantom{\Delta F_{s,t} = }[0.68] \qquad [242] \qquad [129] \qquad\ F = 67{,}825 \qquad Pr\ (> F) < 0.0001$$

The estimated futures-price integration vector is tested against H_0: $\alpha = [47, 11, -1]$ and results in an F statistic with 2 and 17,302 degrees of freedom of 545.8 so that we again conclude that the price integration vector is different from the production coefficients. In a variation of (12b), the coefficients were allowed to vary by contract. The coefficient on the soybean meal price change ranged from 47.213 (August 1997) to 34.311 (September 1996) while the coefficients on the soybean oil price change ranged from 13.936 (September, 1996) to 8.498 (January, 1996). These differences were statistically significant at beyond 0.0001 probability. These results indicate that in both the cash and the futures markets, the value of the marginal product from soybean crushing is less than the value imputed by an individual processor.

Cash and futures price nonstationarity has implications for the variance of hedged and unhedged processing margins, hedge ratios, and hedging effectiveness. Consider a random variable (X_t) that displays serial correlation (ρ) and potentially follows a random walk $(\rho=1)$, $X_t - \rho X_{t-1} = e_t$, where $E(e_t) = 0$ and $V(e_t) = \sigma^2_e$. By extension, $X_t - \rho X_{t-L} = \sum_{i=1}^{L}\rho^{i-1}e_{t-i+1}$ where $E(\sum_{i=1}^{L}e_{t-i+1}) = 0$ and $V(\sum_{i=1}^{L}e_{t-i+1}) = (\sum_{i=1}^{L}\rho^{2(i-1)})\ \sigma^2_e$. For our data $\rho=1$, so $\Sigma_{\Delta^L f, \Delta^L f} = L \cdot \Sigma_{\Delta^l f, \Delta^l f}$, $\Sigma_{\Delta^L p, \Delta^L p} = L \cdot \Sigma_{\Delta^l p, \Delta^l p}$, and $\Sigma_{\Delta^L f \Delta^L p} = L \cdot \Sigma_{\Delta^l f, \Delta^l p}$. Thus, from (2c) the variance of the unhedged processing margin is

$$V(\pi_t \mid \Omega_t) = x_t^2\ \{\ \eta\ \Sigma_{ss}\ \eta' + \eta\ \Sigma_{\Delta^L p, \Delta^L p}\ \eta'\ \} = a + b\ L \tag{12a}$$

where $a = x_t^2\ \eta\ \Sigma_{ss}\ \eta'$, and $b = x_t^2\ \eta\ \Sigma_{\Delta^l p, \Delta^l p}\eta'$.

The variance of the hedged processing margin in (10) is

$$V(\pi_t^* \mid \Omega_t) = x_t^2\ \{\ \eta\ \Sigma_{ss}\ \eta' + \eta\ [\Sigma_{\Delta^L p, \Delta^L p} - \Sigma_{\Delta^L p, \Delta^L f}(\Sigma_{\Delta^L f, \Delta^L f})^{-1}\Sigma_{\Delta^L f, \Delta^L p}]\ \eta'\}$$
$$= a + (b\text{-}c)\ L \tag{12b}$$

where $c = x_t^2\ \{\eta\ [\ \Sigma_{\Delta^l p, \Delta^l f}(\Sigma_{\Delta^l f, \Delta^l f})^{-1}\Sigma_{\Delta^l f, \Delta^l p}]\ \eta'\}$. Comparing the variance of the hedged and unhedged and processing margins gives

$$\text{Effectiveness} = (V_{unhedged} - V_{hedged}) / V_{unhedged} = c\ L / (a + b\ L). \tag{12c}$$

Inspection of (9a) reveals that hedge ratios are invariant to the hedge interval if prices are nonstationary. These expressions indicate how hedging metrics should behave over various hedge intervals. The empirical behavior of these metrics will now be assessed.

Figure 2 shows the unhedged and hedged crushing margin risk for various processing intervals, where hedging is done with successively more distant contract maturities. The hedge intervals from one to 130 business days include most actual hedging horizons. These intervals - 1, 2, 3, 5, 7, 10, 15, 20, 30, 50, 60, 80, 100, and 125, business days - correspond to calendar weeks (5 business days), months and quarters. Four different futures maturities are examined to assess the effect of contract selection. The maturities all relate to the date the hedge is closed so that the nearby contract represents the first maturity after hedge closure. As a result, hedge rollovers are not required. Figure 2a reveals that crushing-margin risk increases in a near-linear fashion with the temporal separation between pricing of inputs and output. This is consistent with (12a), indicating that inventory risk rather than transformation risk is hedgeable. The linear unhedged risk function suggested by figure 2a has a relatively small intercept suggesting that transformation risk is small relative to inventory risk.

In a manner consistent with (12b), the variance of hedged crushing margins increases with the temporal separation between input and output pricing regardless of the contract used for hedging (figure2a). However, hedging with the contract that matures the soonest after hedge closure has the least crushing margin risk.

Figure 2b shows the hedging effectiveness of the various contracts and indicates the crushing-margin risk penalty (as a percentage of unhedged crushing margin risk) associated with hedging in more distant contracts. This figure reveals (a) that effectiveness is greatest when hedging in nearby contracts, (b) that very short-term (less than a week) hedging is less effective than longer-term hedging, and (c) that hedging in the nearby contract reduces risk by roughly seventy percent for hedges of a month or more. The notion that short-term hedges appear generally less effective than longer-term hedges is consistent with (12c).

Transformation risk is small, relative to inventory risk, so it is examined separately to assess the relative magnitude of its components. The instantaneous crushing margin variance was 27.5 $(\text{cts/bu})^2$. About eighty percent [21.6 $(\text{cts/bu})^2$] of this variance is attributable to misalignments among input and output prices and the remainder is due to the misalignment between the production coefficients and the price cointegrating coefficients.

The soybean-crushing hedge ratios for the nearby maturity contracts are shown in table 3. These hedge ratios are estimated with

$$\Delta^L S_{i,t} = \beta_{im} \Delta^L F_{m,t} + \beta_{io} \Delta^L F_{o,t} + \beta_{ib} \Delta^L F_{b,t} + \varepsilon_{i,t} \tag{13}$$

for i = m (soymeal), o (soyoil), and t=1, 2, … T, and $E(\varepsilon_{i,t}) = 0$, $E(\varepsilon_{i,t} \varepsilon_{j,t}) = \sigma_{ij}$, $E(\varepsilon_{i,t} \varepsilon_{j,t'}) = 0$ if t ≠ t'. Like Fackler and McNew, the seemingly unrelated regressions estimation (SURE) method is deemed appropriate because of contemporaneous covariances. But unlike Fackler and McNew, we examine the optimal hedge ratios for successively increasing hedge intervals L = 1, 2, 3, 5, 7, 10, 15, 20, 30, 50, 60, 80, 100, and 125. Because the regressions have the identical regressors, the SURE results are identical to the ordinary least squares estimation (OLSE) single-equation results. However, the SURE method allows us to test $[\hat{\beta}_m : \hat{\beta}_o : 0] \eta' = \eta'$, which involves parameter estimates from different equations. Table 3 reports estimated hedge ratios and test results for hedge intervals of one, five, ten, twenty, forty, and sixty days.
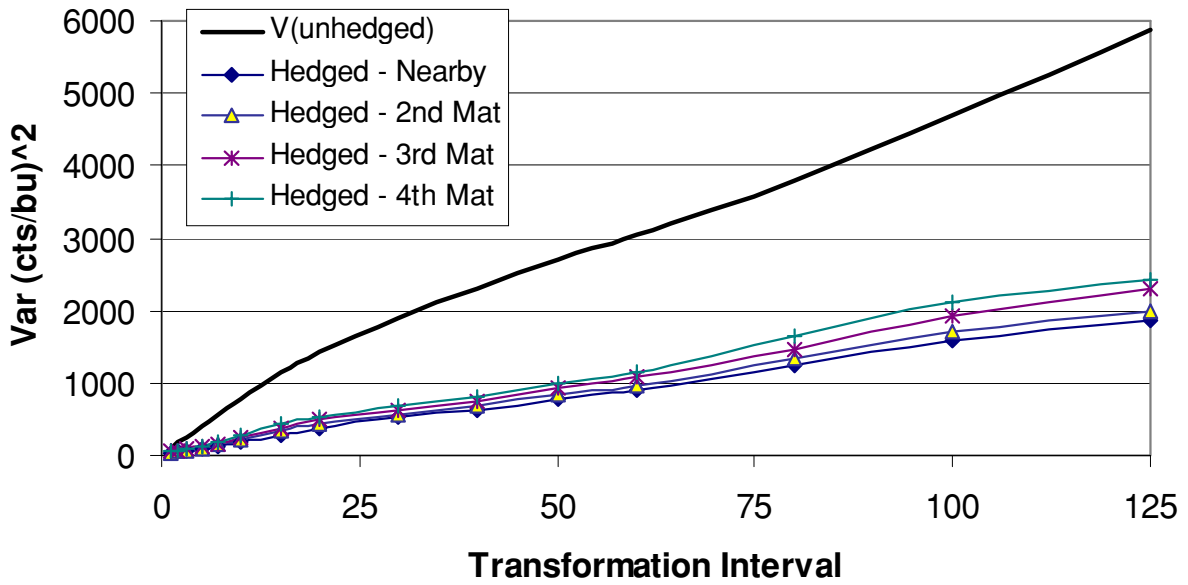
Fig 2a. Variances Hedged and Unhedged
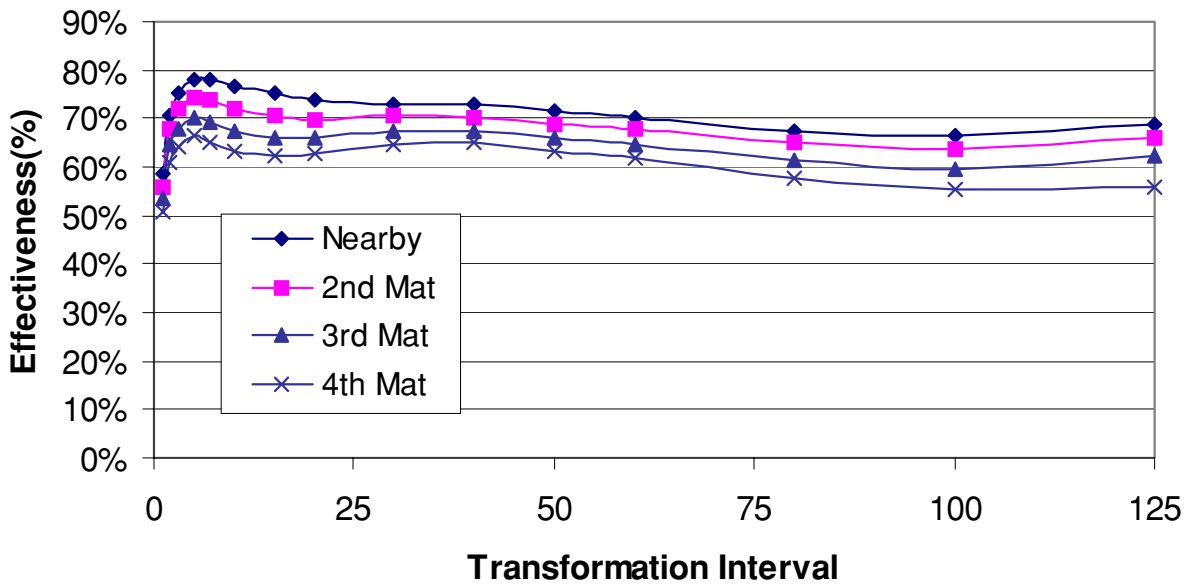

Fig 2b. Hedging Effectiveness by Maturity

Figure 2. Hedging effectiveness over various transformation intervals.

Table 3. Hedge ratio estimation results for various hedge intervals.[a]

Hedge Interval:[b]		1	5	10	20	40	60
Observations:		2580	2576	2571	2561	2541	2521

Multi-commodity Crushing-Hedge Ratios:

		1	5	10	20	40	60
Meal	**Meal [1]**	**0.857******	**1.06****	**1.05**	**0.987**	**0.947**	**0.972**
		(0.0263)	**(0.0240)**	**(0.0298)**	**(0.0377)**	**(0.0416)**	**(0.0430)**
	Oil [0]	-0.0303****	-0.00108	-0.00951	-0.0139	-0.0354***	-0.0394****
		(0.00820)	(0.00779)	(0.00938)	(0.0111)	(0.0111)	(0.0110)
	Beans [0]	0.00272****	-0.000936*	-0.000619	-0.000522	-0.000633	-0.00163*
		(0.000529)	(0.000519)	(0.000664)	(0.000850)	(0.000925)	(0.000932)
	R-sq	0.716	0.779	0.723	0.653	0.645	0.622
	MSE	0.00618	0.0260	0.0665	0.155	0.269	0.393
	F [1,0,0]	10.7****	3.01**	3.09**	7.51****	38.3****	60.9****
Oil	Meal [0]	0.0120	-0.0384	-0.0800***	-0.0935**	-0.252****	-0.289****
		(0.0406)	(0.0294)	(0.0287)	(0.0363)	(0.0501)	(0.0575)
	Oil [1]	**0.984**	**0.989**	**0.963******	**0.978****	**0.960*****	**0.951******
		(0.0127)	**(0.00949)**	**(0.00905)**	**(0.0107)**	**(0.0133)**	**(0.0148)**
	Beans [0]	0.000344	0.000552	0.00195***	0.00191**	0.00502****	0.00578****
		(0.000816)	(0.000633)	(0.000641)	(0.000818)	(0.00112)	(0.00125)
	R-sq	0.8461	0.916	0.928	0.917	0.893	0.872
	MSE	0.0147	0.0386	0.0620	0.144	0.391	0.703
	F [0,1,0]	0.84	1.14	5.79****	2.37*	8.87****	8.63****

Composite Crushing-Hedge Ratios:

	1	5	10	20	40	60
Meal [47]	40.4****	49.2	48.4	45.3	41.7***	42.5**
	(1.32)	(1.17)	(1.42)	(1.79)	(1.97)	(2.04)
Oil [11]	9.40****	10.8	10.2*	10.1*	8.90****	8.61****
	(0.410)	(0.379)	(0.448)	(0.529)	(0.524)	(0.523)
Beans [0]	0.132****	-0.0379	-0.00768	-0.00354	0.0255	-0.0129
	(0.0264)	(0.0253)	(0.0317)	(0.0404)	(0.0438)	(0.0441)
F [47,11,0]	9.73****	2.66**	3.79***	8.05****	38.7****	60.3****

Multi-commodity Anticipatory Crushing-Hedge Ratios:

		1	5	10	20	40	60
Beans	**Beans [1]**	**0.630******	**0.643******	**0.682******	**0.860******	**1.12*****	**1.11*****
		(0.0227)	**(0.0220)**	**(0.0254)**	**(0.0326)**	**(0.0401)**	**(0.0424)**
	Meal [0]	13.6****	12.9****	9.96****	0.146	-10.7****	-9.90****
		(1.13)	(1.02)	(1.14)	(1.44)	(1.80)	(1.96)
	Oil [0]	3.33****	3.19****	3.19****	1.93****	-0.913*	-1.22**
		(0.352)	(0.330)	(0.359)	(0.426)	(0.479)	(0.503)
	R-sq	0.819	0.845	0.823	0.790	0.761	0.737
	MSE	11.4	46.6	97.3	227.6	505.2	814.6
	F [1,0,0]	98.9****	101****	67.2****	40.1****	34.2****	24.4****

a/ Standard errors in parentheses. *, **, ***, and **** denote significantly different from hypothesized value in brackets [ ] at the 0.05 to 0.10, 0.01 to 0.05, 0.001 to 0.01, and the less than 0.001 probability levels.

b/ Lag lengths are in business days.

The multi-commodity, crushing hedge ratios are interpreted as follows. For each pound of meal implicit in current soybean inventories, the crusher should be short .857 pounds of soybean meal futures, long 0.0303 pounds of soybean oil futures, and short 0.00272 bushels of soybean futures. The soybean oil hedge ratios are interpreted similarly. Many of the cross-commodity price effects are significantly different from zero, indicating that the multi-commodity hedge reduces risk significantly beyond that attainable with a single-commodity hedge. The direct hedge ratios are frequently significantly different from unity. The naive one-unit futures per unit spot hedging strategy corresponds to the joint hypothesis that direct-price effects are unity and cross-price effects are zero. For soybean meal, a formal statement of this hypothesis is $H_0: \beta_m = [1, 0, 0]$. The F statistics corresponding to this hypothesis for soybean meal and soybean oil and an indication of significance levels are reported in table 3. The hypothesis is rejected at usual significance levels for at least one of the products for each lag length.

The multi-commodity hedge ratios indicate significant risk reduction associated with hedging soybean meal inventories with soybean meal, soybean oil, and soybean futures. The same conclusion holds for hedging soybean oil. Soybean crushing yields inventories of both meal and oil so the risk minimizing hedge strategies can be converted to per unit of soybeans crushed. These results are shown as composite hedge ratios in table 3. The crushing-spread strategy processing dictates hedge ratios of 47 pounds of meal and 11 pounds of oil per bushel of soybeans crushed. The composite hedge ratios roughly correspond to these values for one- to two-week hedges, but are significantly less for shorter- and longer-term hedges. The joint hypothesis that the technical coefficients are the risk minimizing hedge ratios is rejected at the five-percent significance level for all lag lengths. The degree to which this strategy fails to minimize risk is most extreme for short-term and long-term hedging.

Table 3 also reports multi-commodity hedge ratios for anticipatory crushing hedges. According to these results, a crusher anticipating a soybean purchase in one day should buy 0.63 bushels of soybean futures, 13.6 pounds of soybean meal futures and 3.33 pounds of soybean oil futures per bushel of soybeans to be purchased. The cross-price effects are significantly different from zero while the direct-price effect is significantly different from one. The joint F-test indicates that the naive one-unit futures per unit spot hedging strategy is significantly sub-optimal. This conclusion applies for all hedge intervals reported. We also note that the longer the hedge interval, the greater the direct hedge ratio becomes (1.11 for a 60-day hedge) while the meal and oil cross-hedges move from long to short positions.

Summary and Conclusions

The objective of this paper was to determine the magnitude and behavior of inventory versus transformation risk to soybean processing margins. In the course of evaluating the magnitude of these two sources in a hedged environment, we presented a general framework for modeling processing hedges in both an anticipatory and in-process environment. The in-process case was selected as an application of the model.

Our results indicate that the time or inventory risk due to the temporal mismatch of pricing inputs and outputs greatly exceeds the transformation risk to soybean processing margins. The magnitude of inventory risk increases at a near-constant rate with increasing separation between

input and output pricing. Fortunately, the inventory risk is hedgeable. However, the hedged crushing risk also increases with the temporal separation between the pricing of input and output but its increase is only a fraction of the increase in the unhedged crushing risk.

Hedge ratios for soybean meal and soybean oil were estimated for various production intervals. We found that for very short processing intervals these hedge ratios were below those suggested by the technical coefficients, that for processing intervals of roughly one week, these hedge ratios were roughly equal to (but significantly different from) the technical coefficients, and that for longer processing intervals these hedge ratios decreased. Our results indicate that the contract that matures first after the end of the hedge interval gives the most effective hedge. As predicted by our model, the effectiveness of very short term processing hedges is less than for intermediate term hedges.

Further research remains to be done. Most obvious is the need to investigate the hedging effectiveness of crushing hedges that are implemented in anticipation of crushing. In this application, the purchase of soybeans can be hedged in conjunction with the anticipated production and sale of the resulting soybean oil and meal. Various combinations of production leads and processing intervals need to be simulated. Another useful extension is to compare the inventory and transformation risks of hedged and unhedged processing of other commodities, such as cottonseed.

References

Anderson, R.W. and J. Danthine. "Hedging and Joint Production: Theory and Illustrations." *Journal of Finance* 35(1980):487-501.

Anderson, R.W. and J. Danthine. "Cross Hedging." *Journal of Political Economy* 89(1981):1182-96.

Chicago Board of Trade. *Commodity Trading Manual*. Chicago Board of Trade, Chicago, IL, 1985.

Commodity Research Bureau. CRB InfoTech (data source). http://www.crbtrader.com.

Dahlgran, R.A. "Cross-Hedging the Cottonseed Crush: A Case Study." *Agribusiness* 16(2000):141-158.

Ederington, L.H. "The Hedging Performance of the New Futures Markets." *Journal of Finance* 34(1979):157-170.

Enders, W. *Applied Econometric Time Series*, New York, NY: John Wiley and Sons, Inc., 1995.

Fackler, P.L. and K.P. McNew. "Multiproduct Hedging: Theory, Estimation, and an Application." *Review of Agricultural Economics* 15(1993):521-535.

Johnson, L.L. "The Theory of Hedging and Speculation in Commodity Futures." *Review of Economic Studies* 27(1960):131-59.

Johnson, R.L., C.R. Zulaf, S.H. Irwin, and M.E. Gerlow. "The Soybean Complex Spread: An Examination of Market Efficiency from the Viewpoint of a Production Process." *Journal of Futures Markets* 11(1991):25-37.

Kenyon, D. and J. Clay. "Analysis of Profit Margin Hedging Strategies for Hog Producers." *Journal of Futures Markets* 7(1987):183-202.

Leuthold, R.M. and R.S. Mokler. "Feeding Margin Hedging in the Cattle Industry." *International Futures Trading Seminar Proceedings,* Chicago Board of Trade 6(1979):56-67.

Myers, R.J, and S.R. Thompson. "Generalized Optimal Hedge Ratio Estimation." *American Journal of Agricultural Economics* 71(1989):858-868.

Rahman, S.M., S.C. Turner, and E.F. Costa. "Cross-Hedging Cottonseed Meal." *Journal of Agribusiness* 19(2001):163-171.

Ravindran, A., D.T. Phillips, and J.J. Solberg. *Operations Research Principles and Practice*, 2nd ed. John Wiley and Sons, New York, N.Y. 1987.

Stein, J.J. "The Simultaneous Determination of Spot and Futures Prices." *American Economic Review* 51(1961):1012-1025.

Shafer, C.E., W.L Griffin, and L.D. Johnson "Integrated Cattle Feeding Hedging Strategies, 1972-1976." *Southern Journal of Agricultural Economics* 10(1978):35-42.

Said, S. and D. Dickey. "Testing for Unit Roots in Autoregressive-Moving Average Models of Unknown Order." *Biometrica* 71(1984):599-607.

Tzang, D. and R.M. Leuthold. "Hedge Ratios under Inherent Risk Reduction in a Commodity Complex." *Journal of Futures Markets* 10(1990):497-504.