Incentive-compatible Fiscal Constitutions

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1 Introduction

It is a fact that in most of the countries (both currently and historically) the wealthier a person is, the higher the amount that he or she has to pay in taxes. This feature of a tax system was actually considered by Adam Smith as the first maxim of a good tax regime. In the Wealth of Nations (book 5, chapter 2, part 2), Smith stated this *equality* principle as:

> "The subjects of every state ought to contribute towards the support of the government... in proportion to the revenue which they respectively enjoy under the protection of the state."

On the other hand, the literature on the economy of the state proposes that:

- One important reason for the existence of states is that the society needs an agent who can protect it from foreign predation.
- A good characterization of the behavior of the state must treat it as a private enterprise, interested in maximizing the rents of the ruling elite.
- The focus of the research of constitutional political economists should be the design of Constitutions that provide the state with the incentives to do the job that it is supposed to.

In this paper, we attempt to associate all the previous ideas. We analyze the problem in a very simple Principal-Agent framework. Under imperfect information, we assume that the problem of the society is precisely to design a fiscal constitution such that the state has the incentives to protect the wealth of the society. The interesting result is that, under mild assumptions, such incentive compatible constitution implies a strictly increasing dependance of taxes on the

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wealth of the tax payer[1]. As a by-product, we also find that such constitution cannot be Pareto efficient.

The paper is organized as follows. In the next section, some related literature is reviewed. Then, the next two sections set up and solve the problem. As a comparison benchmark, a Pareto efficient constitution is derived. Those are the two most important parts of the paper. The model has assumptions that range from purely technical to strong economic simplifications. For that reason, before stating the concluding remarks we discuss the assumptions and main weaknesses of the paper. An appendix includes a related problem and the proof of one of the results.

2 Some related literature

When explaining the reasons for the existence of the state, Grossman (1999) quotes First Samuel 8:4-22, when the people of Israel request that the prophet

> "make us a king...[who] may judge us, and fight our battles..."

And Grossman further argues that

> "...assuming that the citizenry has good reason to dread being subjugated by another society more than it fears being exploited by its own king, it is easy to understand why the citizenry perceives itself to be better off with a state to "fight our battles" than without a state."

This view of the problem is consistent with one of the two conflicting ways to think about the relation between state and society – see Grossman (2000). In this context, the state is basically conceived as an agent of the citizenry, the latter being defined as the group of people who are politically decisive. As Rutten (1999) and Grossman (2000) point out, this may be problematic in the sense that, in order for the state to be able to deter predation, it must be endowed, by the society, with a high enough amount of power. Hence, nothing guarantees that that same power will not be used against the citizenry. That is the alternative view of the problem: the state becomes an instrument of the politicians (considered as a class, the ruling elite).

We adopt the view that the state is an agent of the society, who is *hired* to protect the wealth of the society from foreign predation. Our model is simple enough to abstract from any other considerations, such as internal predation (Grossman, 1999) or redistributional policies, which are likely to give results in the same direction as ours. The view that we adopt is, thus, the same as in Downs and Rocke (1994), where the state is the agent of the society that decides whether or not to engage in war with other countries. In that paper, the authors are concerned about how the society can get the state to engage in the same

[1] We will refer to this feature of a tax regime as increasing taxation.

wars the they would engage, were they endowed with the same information as its agent.

Here, we share the view in Down and Rocke (1994) that

> "...the citizens of every state, democratic or not, have an interest in trying to ensure that their chief executive makes decisions that reflect their interests. This problem is particularly difficult in the areas of intervention and interstate conflict in which the chief executive is likely to have access to substantial amounts of information not available to his or her constituency..."

However, we do not concentrate on the relative willingness of the state and the citizenry to engage in wars. We take for granted that some predation by a third agent (whom we will identify as foreigner) will occur, and recognize the fact that the state maximizes an objective function which is increasing in the difference between how much it can collect in taxes and how much it needs to spend in defense. This is equivalent to the view in section 4 of Grossman (1999) or Mendoza (1999).

In our economy, that is the type of abuse to which the state can submit its citizenry. And that is the type of abuse that the citizenry will try to minimize, when designing the constitution. Following Rutten (1999), in its interpretation of the Hobbesian political ideas, political economists should study the mechanisms through which the society would give the state the right incentives to do its job. Our main result is that increasing taxation may be seen as one such mechanism.

In that sense, this work can be seen as what Buchanan (1991) defines as Constitutional Political Economy, in the sense that what we do here is to study the possibility that agents have to impose constraints, rather than the usual analysis in economics, where the object of study is how the agents behave, given the constraints that they face. Moreover, our view of constitutional politics is "contractarian", in the sense that it focuses on interests rather than on theories. That is, we adhere to the von Neumann-Morgenstern paradigm, and deal with the problem of what agents predict will happen in the future by the usual treatment of mathematical expectations. Our main concern is, then, what the objectives of the agents are and, given those, how one of the players (the society, our principal) can give the correct incentives to the other one (the state, our agent).

3 The State as Agent of the Society

The Economy consists of a representative agent and a government. The only public good is the defense of the property of the citizenry from predation by an agent whom we identify as a foreigner.

3.1 The society

The society is represented by a Bernoulli utility function $U : R_{+} \longrightarrow R$, which depends on the wealth that the society keeps after paying taxes and after being preyed on by the foreign agent. Let p denote the proportion of the wealth of the society that is left after predation by the foreigner. Let τ denote the taxes paid by the society. Let W_0 denote the initial wealth of the society. We will normalize $W_0 = 1$, but, nonetheless, denote by pW_0 the gross final wealth of the society, whenever the presence of W_0 makes the equations more intuitive.

Thus, we will denote

$$U = U\left(pW_0 - \tau\right) \tag{1}$$

Assumption U1: *For all $X \in R_{+}$, $U'(X) > 0$ and $U''(X) < 0$.*

The following boundedness assumption is made for technical convenience.

Assumption B1: *There exists $\overline{U} \in R$ such that, $\forall X \in R_{+}$, $|U(X)| < \overline{U}$.*

3.2 Predation Technology:

The proportion of its wealth that the society keeps depends on how much the government spends in defense, S, and how much the foreign predator spends in predation, θ. We suppose that $S \in R$, and that θ is a random variable with support $\Omega \subseteq R$ and distribution $G : \Omega \longrightarrow [0,1]$. We assume that G is absolutely continuous, and denote by g its density function.

Then, $p : R \times \Omega \longrightarrow [0,1]$ is defined by $p = p(S,\theta)$.

Assumption T1: *$p(S,\theta)$ is continuous with $\frac{\partial p(S,\theta)}{\partial S} > 0$, and $\frac{\partial p(S,\theta)}{\partial \theta} < 0$*

In assumption T1, continuity is assumed for technical convenience, whereas the derivatives have the intuition we want: the more the state spends in defense and the less the foreign predator spends in predation, the larger the proportion of its wealth that the society keeps.

3.3 Institutions:

We assume that it is the society who designs the constitution, and the government who decides the expenditure in defense S. We also assume that both of them obey the constitution and that the society cannot directly observe the amount S nor the amount θ, but only the amount p, from which none of the former is discernible[2].

Definition 1 *A constitution is a function $\tau : [0,1] \longrightarrow R$, that defines, for each proportion of the wealth of the society that is left after predation by the foreigner, p, an amount $\tau(p)$ of taxes that the society must pay to the government.*

[2] Usually, the society observes some statistics about S that are produced by the state and controlled by an agency that is likely to be under the influence of politicians. In reality, this statistics may, nonetheless, have some credibility. Here we assume that they have none or, futhermore, that they do not exist.

Thus, all the institutional-legal regime of our economy is given by the constitution $\tau(p)$. τ does not directly depend on S or θ, since none of them is observable. However, the following concept will be useful in some of the analyses:

Definition 2 *Given a constitution τ, a pseudo-constitution is a function $\iota_{\tau,\varphi} : \Phi \longrightarrow R$, for some set Φ, such that there exists a mapping $\varphi : \Phi \longrightarrow [0,1]$ and $\iota_{\tau,\varphi} = \tau \circ \varphi$.*

Remark 3 *In particular the two following classes of pseudo-constitutions are of interest:*

- *Let $\Phi = R \times \Omega$, and $\varphi = p(S,\theta)$, so that $\widetilde{\tau}(S,\theta) = \tau \circ p$ is a pseudo-constitution. This pseudo-constitution is interesting whenever both S and θ are variable but observable.*
- *Let $\Phi = \Omega$, and for any fixed $\widehat{S} \in R$ let $\varphi = p(\widehat{S},\theta)$, so that $\widehat{\tau}_{\widehat{S}}(\theta) = \tau \circ p(\widehat{S},\theta)$ defines a pseudo-constitution. This one is interesting when S is fixed and θ is observable.*

3.4 The State

The state is the agent of the society in determining S. We assume, however, that the objective function of the politicians is to maximize the utility they derive from their wealth, which, we assume, is not subject to predation. Let W_1 be their initial wealth, and suppose that the Bernoulli function $V : R_+ \longrightarrow R$ represents their preferences[3]. Thus, we denote

$$V = V(W_1 + \tau - S) \tag{2}$$

Assumption V1: *For all $X \in R$, $V'(X) > 0$ and $V''(X) < 0$.*
For technical reasons, we assume the following:
Assumption B2: *There exists $\overline{V} \in R$ such that, $\forall X \in R$, $|V(X)| < \overline{V}$.*

3.5 The Constraints:

3.5.1 Technological and Institutional Constraints:

Both the state and the society take as given the fact that p is determined by the function $p(S,\theta)$. Also, since we assume that both of them are constrained by the constitution, they take as given the fact that τ is given by $\tau(p)$. Then, the basic problem for the state is to maximize its von Neumann-Morgenstern *function*:

$$\max_S E_\theta(V(W_1 + \tau(p) - S)) \quad \text{s.t.} \quad p = p(S,\theta) \tag{3}$$

[3] According to van Winden (1983), when describing a State we need to define four features of it: the interests of the politicians, the distribution of power among them, the instruments available to them and the constraints they face. Here, the first is given by equation (2), the second is irrelevant as we assumed a unified ruling elite, the third is simply S, and the fourth is the object of the next subsection.

While, since it is the society who designs the constitution, its basic problem is:

$$\max_{\langle\tau(p)\rangle_{p\in[0,1]}} E_\theta\left(U\left(pW_0-\tau\left(p\right)\right)\right) \quad \text{s.t.} \quad p=p\left(S,\theta\right) \tag{4}$$

where we use the notation $\langle\tau\left(p\right)\rangle_{p\in[0,1]}$ to remark the fact that the problem of the society is to choose the whole *function* $\tau\left(p\right)$, so that $E_\theta\left(U\left(pW_0-\tau\left(p\right)\right)\right)$ is a von Neumann-Morgenstern *functional* of $\tau\left(p\right)$.

3.5.2 Economic Constraints: the Constraints to Constitutional Policy.

When designing the constitution, the society faces two constraints:

- Individual rationality constraint: We assume that if politicians were not in office, they would have an expected utility of $\underline{V}$. Hence, any constitution must guarantee them at least this level of expected utility.
- Incentive compatibility constraint: Since the society does not observe S, but only p, it will want to design a constitution which is free of moral hazard, whenever this problem may be an issue.

4 The Design of the Constitution;

4.1 Pareto Efficient Constitutions:

In order to have a Paretian benchmark, we assume for the moment that there are no informational problems, so that the society itself determines both the amount of taxes paid and the expenditure in defense. Since S is observable, incentives are not an issue, and the constitutional design problem reduces to determining S as well as the pseudo-constitution $\widehat{\tau}:\Omega\longrightarrow R$. The problem is

$$\max_{\left\{\langle\widehat{\tau}(\theta)\rangle_{\theta\in\Omega},S\right\}} E_\theta\left(U\left(p\left(S,\theta\right)W_0-\widehat{\tau}\left(\theta\right)\right)\right) \tag{5}$$

s.t.

$$E_\theta\left(V\left(W_1+\widehat{\tau}\left(\theta\right)-S\right)\right)\geq\underline{V} \tag{6}$$

and, letting $\lambda>0$ denote the shadow price of the Individual Rationality constraint, the first order conditions are given by the continuum of equations[4]

$$\forall\theta\in\Omega,\ \left(-U'\left(p\left(S,\theta\right)W_0-\widehat{\tau}\left(\theta\right)\right)+\lambda V'\left(W_1+\widehat{\tau}\left(\theta\right)-S\right)\right)g\left(\theta\right)=0 \tag{7}$$

and the equation

$$\int_\Omega\left(U'\left(\cdot\right)\frac{\partial p\left(\cdot\right)}{\partial S}W_0-\lambda V'\left(\cdot\right)\right)g\left(\theta\right)d\theta=0 \tag{8}$$

[4] By Lebesgue's dominated convergence theorem, Assumptions B1 and B2 allow us to take the derivative of the integral as the integral of the derivative. This one of the reasons why those conditions are imposed.

Solving from (7) into (8)

$$E_\theta\left(U'(\cdot)\frac{\partial p(\cdot)}{\partial S}W_0\right)=\lambda E_\theta\left(V'(\cdot)\right) \tag{9}$$

Solving for λ, using the normalization $W_0 = 1$ and substituting, we have the Pareto Efficiency condition

$$E_\theta\left(V'(\cdot)\frac{\partial p(\cdot)}{\partial S}\right)=E_\theta\left(V'(\cdot)\right)$$

Here, the left hand side is the expected marginal social utility of expenditure in defense[5], whereas the right hand side is the expected marginal cost for the state. Pareto efficiency requires, then, that the state "internalizes" all the marginal effects of the defense expenditures on the society.

4.2 The Moral Hazard problem:

Obviously, the crux of the problem is that the amount of defense expenditure is neither decided nor observed by the society. The state takes the constitution as given and determines S by the solution to the problem

$$\max_S E_\theta\left(V\left(W_1+\tau\left(p\left(S,\theta\right)\right)-S\right)\right) \tag{10}$$

which has as first order condition

$$E_\theta\left(V'(\cdot)\,\tau'(\cdot)\frac{\partial p}{\partial S}\right)=E_\theta\left(V'(\cdot)\right) \tag{11}$$

in which the state equates its expected marginal utility and cost. The loss in Pareto efficiency comes from the presence of $\tau'(\cdot)$ on the left hand side of the equation. One could be tempted to say that the problem is trivially solved by the society by setting a linear constitution. If the state is risk averse, that is incorrect since it would share risk inefficiently, as we observe in appendix 7.1[6].

4.3 Sound Constitutional Economics (or "An Incentive Compatible Constitution"):

Given the moral hazard problem, the society has to incorporate incentive compatibility constraints in the design of the second-best constitution. This only

[5] By social we do not mean, as is usual in welfare economics, the aggregate across agents of marginal utilities. What we mean here is the marginal utility of the society, as principal of the problem.

[6] In terms of what follows, for the Society to meet the individual rationality constraint of the State with a linear constitution would in general require taxes that are suboptimal from its point of view (too high). In other words, a linear constitution is too costly an incentive mechanism.

means that the society recognizes the fact that politicians are rational agents maximizing their own expected utility functions. When doing sound constitutional politics, people need to incorporate this as a constraint and design the best possible constitution that is both individually rational and incentive compatible.

Before proceeding, it turns out to be convenient[7] to redefine the stochastic process of the problem: given $p = p(S, \theta)$, we can invert the function as $\theta = (S, p)$ and, by the inverse function theorem[8]

$$\begin{aligned} \frac{\partial \theta(\cdot)}{\partial p} &= \left(\frac{\partial p(\cdot)}{\partial \theta}\right)^{-1} < 0 \\ \frac{\partial \theta(\cdot)}{\partial S} &= -\frac{\partial p(\cdot)/\partial S}{\partial p(\cdot)/\partial \theta} > 0 \end{aligned}$$

and we can construct the function $H : [0,1] \times R \longrightarrow [0,1]$, defined by

$$H(p, S) = 1 - G(\theta(S, p)) \tag{12}$$

In this context, p becomes the random variable and S is a control. Thus, we denote by $H(p|S) \equiv H(p, S)$ the distribution function of p conditional on S[9]. For the conditional density, we define

$$h(p|S) = \frac{\partial H(p|S)}{\partial p}$$

so that, by construction

$$h(p|S) = -g(\theta(S,p)) \frac{\partial \theta(S,p)}{\partial p} \geq 0 \tag{13}$$

while

$$\frac{\partial H(p|S)}{\partial S} = -g(\theta(S,p)) \frac{\partial \theta(S,p)}{\partial S} < 0 \tag{14}$$

Definition 4 *For $S_0, S_1 \in R$, $H(p|S_0)$ is said to dominate $H(p|S_1)$ in the fist degree stochastic dominance sense if $\forall p \in [0,1]$, $H(p|S_1) \geq H(p|S_0)$, and $\exists p \in [0,1]$ for which $H(p|S_1) > H(p|S_0)$.*

Theorem 5 *If $S_0 > S_1$, then $H(p|S_0)$ dominates $H(p|S_1)$ in the first degree stochastic dominance sense.*

Proof. It follows directly from definition 4 and equation (14). ■

[7] Holstrom (1979).

[8] Assumption T1 allows us to use it.

[9] To check that H(·) is a bona fide distribution function, notice that $\forall S$, $p \to \infty$ implies $\theta \to \inf \Omega$ so that $G(\theta(\cdot)) \to 0$ and $H(\cdot) \to 1$. On the other hand, $p \to -\infty$ implies $\theta \to \sup \Omega$ so that $G(\theta(\cdot)) \to 1$ and $H(\cdot) \to 0$. Right-continuity and monotonicity of $H(\cdot)$ follow from those of $G(\cdot)$ and $p(\cdot)$

Lemma 6 *If $S_0 > S_1$, then for any constitution, $\tau(p)$, with $\tau'(p) < 1$, we have*

$$E_p\left(U\left(pW_0 - \tau(p)\right)|S_0\right) > E_p\left(U\left(pW_0 - \tau(p)\right)|S_1\right)$$

Proof. If $\tau'(p) < 1$, then $\frac{dU(\cdot)}{dp} = U'(.)\left(1 - \tau'(p)\right) > 0$ and the result follows directly from theorem 1 in Hadar and Rusell (1971), given assumption B1 and theorem 5. ■

The intuition of the theorem and the lemma are simple. The more the state spends in defense, the higher is the likelihood of better results for the society, in the sense of keeping a higher proportion of its initial wealth. If the society has to pay taxes that increase in a less than one-to-one basis with the wealth they keep, then they benefit from the redistribution of the probability mass toward higher values of p.

Assumption MLR: *We assume that the induced distribution function $H(p|S)$ satisfies the monotone likelihood ratio property:*

$$\frac{\partial}{\partial p}\left(\frac{\frac{\partial h(p|S)}{\partial S}}{h(p|S)}\right) > 0$$

Assumption MLR says[10] that if the society has a prior distribution of beliefs about the level of S, which is updated (in a Bayesian manner) after observing p, then the higher the observed p, the "better" the conditional posterior in the sense of first degree stochastic dominance. In other words, after observing high p, the society believes that it is more "likely" that the government spent a high S, than after observing a low p.

In this context, the problem of the society is:

$$\max_{\left\{\langle\tau(p)\rangle_{p\in[0,1]}, S\right\}} E_p\left(U\left(pW_0 - \tau(p)\right)\right) \tag{15}$$

s.t.

$$E_p\left(V\left(W_1 + \tau(p) - S\right)\right) \geq \underline{V} \tag{16}$$

$$S \in \arg\max_{\widehat{S}} E_p\left(V\left(W_1 + \tau(p) - \widehat{S}\right)\right) \tag{17}$$

Assumption B3: *There exists $\overline{V}' \in R$ such that $|V'| < \overline{V}'$.*

Let $A_V(\cdot)$ denote the degree of absolute risk aversion of the politicians:

$$A_V\left(W_1 + \tau(p) - S\right) = -\frac{V''\left(W_1 + \tau(p) - S\right)}{V'\left(W_1 + \tau(p) - S\right)}$$

Assumption DAR: *For all $w \in R_+$, we have that $0 \geq A_V'(\cdot)$.*

Assumption V2: *Given $W_1 \in R_+$ and $\tau(p)$, for all $S \in \left[0, W_1 + \underset{p\in[0,1]}{Inf}\left(\tau(p)\right)\right]$*

$$-\infty < \frac{\partial^2}{\partial \widehat{S}^2} \int_0^1 V\left(W_1 + \tau(p) - \widehat{S}\right) h\left(p\left|\widehat{S}\right.\right) dp\Bigg|_{\widehat{S}=S} < 0$$

[10] See Milgrom (1991).

Assumption B3 is purely technical[11]. Assumption DAR says that the politicians have nonincreasing absolute risk aversion; that is that the richer they are, the lower (or constant) has to be the compensation they would ask in order to engage in a given gamble. Assumptions V1 and V2 give us sufficient conditions to establish that

$$S \in \arg\max_{\widehat{S}} E_p \left(V \left(W_1 + \tau(p) - \widehat{S} \right) \right) \Leftrightarrow \frac{\partial}{\partial \widehat{S}} \int_0^1 V(\cdot)\, h\left(p \,\middle|\, \widehat{S}\right) dp \Bigg|_{\widehat{S}=S} = 0$$

so that we can substitute (17) by the much more easily tractable

$$\frac{\partial}{\partial \widehat{S}} \int_0^1 V\left(W_1 + \tau(p) - \widehat{S}\right) h\left(p \,\middle|\, \widehat{S}\right) dp \Bigg|_{\widehat{S}=S} = 0 \tag{18}$$

and we know that S really is a maximizer and not only a critical point. This is the usually called "*first-order approach.*"

The first order conditions of the problem are equations (16), (18), the continuum $\forall p \in [0,1]$

$$\left(-U'(\cdot) + \lambda V'(\cdot)\right) h(p\,|S) + \mu \left(V'(\cdot) \frac{\partial}{\partial S} h(p\,|S) - V''(\cdot)\, h(p\,|S) \right) = 0 \tag{19}$$

as well as the equation

$$\frac{\partial}{\partial \widehat{S}} \int_0^1 U(\cdot)\, h\left(p \,\middle|\, \widehat{S}\right) dp \Bigg|_{\widehat{S}=S} + \mu \frac{\partial^2}{\partial \widehat{S}^2} \left(\int_0^1 V(\cdot)\, h\left(p \,\middle|\, \widehat{S}\right) dp \right)_{\widehat{S}=S} = 0 \tag{20}$$

where (18) has already been introduced. Here, $\lambda > 0$ is the shadow price of constraint (16) and μ is the one of constraint (18). The sign of μ still has to be determined. Reexpressing (19), we get

$$\frac{U'(pW_0 - \tau(p))}{V'(W_1 + \tau(p) - S)} = \lambda + \mu \left[\frac{\frac{\partial}{\partial S} h(p\,|S)}{h(p\,|S)} + A(W_1 + \tau(p) - S) \right] \tag{21}$$

[11] Together with assumption B2, it allows us to take the second derivative of the integral as the integral of the second derivative (in this case, the conditions need to be stronger than before, since $h(p\,|S)$ is also subject to derivatives). To see why, notice that

$$\int_0^1 \overline{V}' h(p\,|S)\, dp = \overline{V}' < \infty$$

and

$$\int_0^1 \overline{V} \frac{\partial h(p\,|S)}{\partial S} dp = 0 < \infty$$

and that $\forall p \in [0,1]$,

$$\left| \overline{V}' h(p\,|S) \right| \geq \left| V'(\cdot)\, h(p\,|S) \right|$$

and

$$\left| \overline{V} \frac{\partial h(p\,|S)}{\partial S} \right| \geq \left| V(\cdot) \frac{\partial h(p\,|S)}{\partial S} \right|$$

and finally apply Lebesgue's Dominated Convergence theorem.

Theorem 7 *Under all our assumptions, $\mu > 0$.*

Proof. See appendix 7.2. ■

Theorem 8 *Under all our assumptions, if $\tau(p)$ is the optimal individually rational incentive compatible constitution, then $\tau'(p) > 0$.*

Proof. Consider equation (21). Differentiating with respect to p, and reorganizing gives

$$\tau'(p) = \frac{\frac{V'(\cdot)U''(\cdot)}{(V'(\cdot))^2} - \mu\frac{\partial}{\partial p}\left(\frac{\frac{\partial}{\partial S}h(p|S)}{h(p|S)}\right)}{\frac{V'(\cdot)U''(\cdot)+U'(\cdot)V''(\cdot)}{(V'(\cdot))^2} + A'(\cdot)}$$

The numerator of the expression is negative, given $U''(\cdot) < 0$, theorem 7 and assumption MLR. The denominator is negative, given $U''(\cdot) < 0$, $V''(\cdot) < 0$, and assumption DAR. ■

Theorem 8 is the main result of the paper. What it shows is that if the state is risk averse, under weak assumptions a strictly increasing fiscal constitution is designed by the society, in order to give the state incentives to protect the wealth of its people. In view of this result, the pray of the people of Israel to the Prophet Samuel should have been: "make us a king who might fight our battles, but let that King know that heavy taxation will be paid only when He has fought those battles successfully. Thus, we will not have to cry out too much."

5 Discussion:

5.1 The Assumptions:

Several assumptions have been made in this paper. We now discuss them.

5.1.1 Technical assumptions:

First and second order boundedness assumptions (B1, B2 and B3), as well as the assumption of continuity of the predation technology (in assumption T1), and the strict concavity assumption V2, are purely technical. Some of them may indeed be unattractive, but one must notice that they are only sufficient conditions. In specific cases, one could dispose of them, provided that the mathematical procedures that they facilitated are still permissible.

The monotone likelihood ratio and decreasing absolute risk aversion assumptions (MLR and DAR) have technical content, since they are sufficient to determine the sign with which theorem 8 is concerned. However, they have economic meaning, as we tried to argue when we introduced them. In our opinion, those two assumptions are plausible[12].

[12] In fact, standard functional forms, as the exponential density function or the logarithmic Bernoulli utility function satisfy those conditions.

Finally, the risk aversion assumptions of both the citizenry and the politicians (U1 and V1) are more than only technical. If we do not assume it for the former, the optimal constitution may be flat. If we do not assume it for the latter, the state acts as full insurer of the society. Both results are unattractive. Whether the assumptions are plausible or not is left to the reader.

5.1.2 Political Economy Assumptions:

We have assumed that both the citizenry and the politicians take the constitution as a binding constraint. This may be naive on both sides of the problem. One observes tax evasion and government misappropriation much more often than one would like. Also, we have assumed that there is a representative consumer and that the executive cannot be put out of office. We recognize those as limitations of the paper, but try to justify them.

Trying to study increasing taxation in a representative agent model, in which redistributional policy makes little sense, is, at least, an oversimplification. However, the fact that we obtain a increasing taxation result, even without further elements of the issue, should tell us that this isolated effect can only add to the traditional arguments. And in this context, in which there is only one tax payer, tax evasion is less important than in a realistic multi-agent case.

On the other hand, the usual criticism – Rutten (1999), Grossman (2000) – that there is no reason why the state would not abuse its power to overthrow the constitution can also be somewhat avoided. In Grossman and Noh (1990) and Hess and Orphanides (1999) the probability that the state will survive depends on the quality of the exercise of the executive while in office. Our simplifying assumption may be understood as an extreme case of that argument, where the probability of survival is one if the constitution is obeyed, and zero otherwise.

Finally, it is obvious that the scope of this paper is, and has to be, limited. The range of arguments for the existence of the state is broad, and so are the functions that derive from those arguments. In that sense, the following self-criticism of Downs and Rocke (1994) applies here:

> "... to the extent that other issues are more important..., the model will be incomplete."

6 Conclusions:

The goal of the paper was to link the first maxim of taxation of Adam Smith, according to which the richer the tax payer the more taxes he or she must pay, with the existence of the state, and the possibility that the latter may abuse its power. The main result that we obtain is theorem (8). In our extremely simple model, in which the state is created with the sole goal of protecting the wealth of the society from predation by a third agent, but there is imperfect information about the actions of that agent, taxes that are strictly increasing in the wealth of the tax payer arise as the optimal incentive compatible constitution that the society designs. Hence, the first maxim need not only be about fairness: it is also a mechanism by which the citizenry gives the executive the incentives to protect its wealth. The assumptions that are made are, in our opinion, weak.

Moreover, one could broaden the applicability if the type of result that we have obtained here. First, there is no reason why the predator has to be a foreign agent. It could very well be the case of domestic thieves who neither participate on the political processes of the society, nor pay taxes. In this case, S would be understood as expenditure in security and our result would hold unaltered. Secondly, we could abstract altogether from the predation problem and understand p, or some function of it, as an indicator of the qualitative performance of the economy. As long as doing good economic policy may be costly for the state (S), all our conclusions would apply upon reinterpretation[13]. In general, increasing taxation should appear as the second best individually rational-incentive compatible constitution, whenever some action of the state, which is costly to it, positively influences the wealth or income of its society.

As a by-product, the imperfections on the availability of information, which lie in the heart of our argument, imply that such constitution is not Pareto Efficient. But the Pareto Efficient Constitutions would not be give the right incentives to the politicians and are, therefore, inferior from the point of view of the citizenry.

7 Appendix:

7.1 The Risk-Sharing problem

Consider the setting where θ is the random primitive variable. Since τ depends on p, which, for a given S, depends on θ, the problem of designing the Constitutions embeds one of optimal risk-sharing. To isolate this aspect, we take S as fixed at a level $\overline{S}$ and consider the design of the pseudo-constitution $\widetilde{\tau}\left(\overline{S},\theta\right)$:

$$\max_{\left\langle \widetilde{\tau}\left(\overline{S},\theta\right)\right\rangle_{\theta\in\Omega}} E_\theta\left(U\left(p\left(\overline{S},\theta\right)W_0-\widetilde{\tau}\left(\overline{S},\theta\right)\right)\right)$$

s.t.

$$E_\theta\left(V\left(\overline{W}_1+\widetilde{\tau}\left(\overline{S},\theta\right)\right)\right)\geq \underline{V}$$

where $\overline{W}_1=W_1-\overline{S}$.

Let $\lambda\in R_{++}$ denote the shadow price of the individual rationality constraint. The first order conditions of the problem are the continuum defined by[14]

$$\forall\theta\in\Omega,\ -U'\left(p\left(\overline{S},\theta\right)W_0-\widetilde{\tau}\left(\overline{S},\theta\right)\right)+\lambda V'\left(\overline{W}_1+\widetilde{\tau}\left(\overline{S},\theta\right)\right)=0$$

Proposition 9 *Let $\widetilde{\tau}\left(\overline{S},\theta\right)$ be the optimal risk-sharing constitution. Then, $\forall\theta\in\Omega$,*

$$0<\frac{\partial\widetilde{\tau}\left(\overline{S},\theta\right)}{\partial\theta}<\frac{\partial p\left(\cdot\right)}{\partial\theta}$$

[13] In this case, some simplification could be done. If V can be assumed as additively separable in $W_1+\tau\left(p\right)$ and S, one does not need to assume decreasing abosolute risk aversion in order to prove theorem 8 and the proof of theorem 7 comes directly from Holstrom (1979).

[14] See footnote 5.

Proof. Deriving the first order condition with respect to θ, one gets

$$\frac{\partial \widetilde{\tau}\left(\overline{S},\theta\right)}{\partial \theta}=\frac{U''\left(\cdot\right)\frac{\partial p\left(\cdot\right)}{\partial \theta}}{U''\left(\cdot\right)+\lambda V''\left(\cdot\right)}$$

and the result follows from assumptions U1, V1, T1. ■

7.2 Proof of Theorem 7

Here, we proof theorem 7 in the text. The proof is based on Holstrom (1979) although it is somewhat different.

Proof. Recall the first order condition (21)

$$\frac{U'\left(pW_0-\tau\left(p\right)\right)}{V'\left(W_1+\tau\left(p\right)-S\right)}=\lambda+\mu Z\left(p,S\right)$$

where

$$Z\left(p\right)=\left[\frac{\frac{\partial}{\partial S}h\left(p\,|S\right)}{h\left(p\,|S\right)}-\frac{V''\left(W_1+\tau\left(p\right)-S\right)}{V'\left(W_1+\tau\left(p\right)-S\right)}\right]$$

(remember that this is part of the first order conditions, so that S can be taken as fixed) and define the sets

$$\begin{aligned}\Delta_+ &= \{p\in[0,1]\mid Z\left(p\right)\geq 0\}\\ \Delta_- &= \{p\in[0,1]\mid Z\left(p\right)<0\}\end{aligned}$$

since λ is a constant, consider the problem[15]

$$\max_{\langle\tau(p)\rangle_{p\in[0,1]}} E_p\left(U\left(pW_0-\tau\left(p\right)\right)+\lambda V\left(W_1+\tau\left(p\right)-S\right)\right)$$

which has as first order condition, $\forall p\in[0,1]$

$$\frac{U'\left(pW_0-\tau_\lambda\left(p\right)\right)}{V'\left(W_1+\tau_\lambda\left(p\right)-S\right)}=\lambda$$

where we use λ as a subscript to distinguish $\tau_\lambda\left(p\right)$ of $\tau\left(p\right)$. Differentiation of the continuum with respect to p gives

$$\tau'_\lambda\left(p\right)=\frac{U''\left(\cdot\right)V'\left(\cdot\right)}{U''\left(\cdot\right)V'\left(\cdot\right)+V''\left(\cdot\right)U'\left(\cdot\right)}\in[0,1)$$

Now, in order to argue by contradiction of our hypothesis, suppose $0\geq\mu$. Then, $\forall p\in\Delta_+$ we have, since $W_0=1$,

$$\frac{U'\left(p-\tau_\lambda\left(p\right)\right)}{V'\left(W_1+\tau_\lambda\left(p\right)-S\right)}=\lambda\geq\frac{U'\left(p-\tau\left(p\right)\right)}{V'\left(W_1+\tau\left(p\right)-S\right)}$$

[15] Arrow (1971, Appendix to Essay 8), shows that the solutions to this problem are Pareto efficient. Since we are not imposing individual rationality, this solution need not be the same as the optimal risk-sharing Constitution.

and for fixed p we have that

$$\frac{U'(pW_0 - \tau(p))}{V'(W_1 + \tau(p) - S)}$$

is increasing in $\tau(p)$. So that $\forall p \in \Delta_+$, $\tau_\lambda(p) \geq \tau(p)$.

A similar analysis gives us that $\forall p \in \Delta_-$, $\tau(p) \geq \tau_\lambda(p)$.

Now, notice further that $p \in \Delta_- \implies \frac{\partial h(p|S)}{\partial S} < 0$, by assumption C1. This implies that

$$\int_0^1 U(pW_0 - \tau(p)) \frac{\partial h(p|S)}{\partial S} dp \geq \int_0^1 U(pW_0 - \tau_\lambda(p)) \frac{\partial h(p|S)}{\partial S} dp$$

but, by Lemma 6, since $\tau'_\lambda(p) < 1$, we also have

$$\int_0^1 U(pW_0 - \tau_\lambda(p)) \frac{\partial h(p|S)}{\partial S} dp > 0$$

so that

$$\int_0^1 U(pW_0 - \tau(p)) \frac{\partial h(p|S)}{\partial S} dp > 0$$

Now, consider equation (20). We have just shown that the first term in the left hand side is strictly positive. By assumption C2, we must then have $\mu > 0$. This is inconsistent with the assumption $0 \geq \mu$. Then, we conclude $\mu > 0$. ■

8 References

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