ENSAYOS
sobre política económica

Assessing Fiscal Sustainability with Alternative Methodologies

Humberto Mora

Revista ESPE, No. 46-I Edición Especial 2004
Páginas 82-145

Evaluando la sostenibilidad fiscal con metodologías alternativas

Humberto Mora*

El autor es director adjunto de Estudios Económicos del Fondo Latinoamericano de Reservas (FLAR). Agradezco a Álvaro Pachón y a Juan Manuel Julio por facilitarme referencias útiles sobre la aplicación de cadenas de Harkov, así como los comentarios recibidos en los seminarios del Banco Mundial, Fedesarrollo y el Banco de la República, donde se presentó una primera versión de este trabajo. Estoy particularmente agradecido con David Gould, Franz Hamman, Santiago Herrera, Javier Gómez, Roberto Junguito, Enrique Mendoza, Martha Misas, Guillermo Perry, Hernán Rincón y Gabriel Rosas, por los útiles comentarios y sugerencias expresados en esos seminarios, así como con el evaluador anónimo que revisó este trabajo para su publicación en *ESPE*. Así mismo agradezco a Julio Velarde, presidente del FLAR, por permitirme trabajar en este proyecto. Las opiniones incluidas en este trabajo son de la exclusiva responsabilidad del autor y no representan la posición oficial del FLAR.

* Ensayo presentado al Banco Mundial. Correo electrónico: hmora@flar.net

Documento recibido el 19 de mayo de 2004; versión final aceptada el 1 de octubre de 2004.

Resumen

En este trabajo se aplican y analizan tres metodologías alternativas para evaluar la sostenibilidad fiscal de la deuda del Gobierno Central, usando como ejemplos ilustrativos los casos de Colombia y de Costa Rica: la metodología del FMI (2003 a); la metodología de Mendoza & Oviedo (2003); y una nueva metodología que incorpora los efectos de los componentes cíclico y estructural del producto, los gastos y los ingresos del Gobierno sobre la evolución de la deuda, proponiendo un indicador particular sobre la sostenibilidad de la deuda que está definido por los componentes estructurales de estas variables, toda vez que su componente cíclico se anula a lo largo del ciclo completo. Adicionalmente, se desarrolla y aplica una metodología para incorporar explícitamente en el análisis de sostenibilidad la probabilidad de defraudación de la deuda, permitiendo a la vez que dicha probabilidad afecte el nivel de la tasa de interés de la deuda. La característica común de las tres metodologías es la incorporación explícita de la incertidumbre propia del comportamiento de las variables que determinan la evolución de la deuda y la formulación de indicadores específicos que capturan el efecto de la volatilidad sobre la economía, así como su incidencia sobre variables fiscales y financieras. Adicionalmente, las tres metodologías aplicadas usan procedimientos específicos para estimar el valor esperado de las variables relevantes. Los resultados, en especial aquellos que se expresan en términos de probabilidades de defraudación de la deuda, constituyen herramientas analíticas útiles para las autoridades económicas; en particular, para estimar el intervalo de acción de que disponen para adoptar políticas fiscales y medidas correctivas que se caracterizan explícitamente a lo largo del trabajo.

Clasificación JEL: H6: Presupuesto nacional, déficit y deuda.

Keywords: *Sostenibilidad fiscal; volatilidad macroeconómica; valores esperados; cadenas de Markov.*

Comentarios: Álvaro Concha.

Assessing Fiscal Sustainability with Alternative Methodologies

Humberto Mora *

This paper applies and analyzes three alternative methodologies to assess fiscal sustainability of the central government's debt, using as illustrative examples the cases of Colombia and Costa Rica: The IMF's (2003 a); the Mendoza & Oviedo's (2003); and a new methodology that incorporates the effects of the structural and cyclical components of output, government expenditures, and revenues, on the evolution of debt, with a particular indicator of debt sustainability based on the structural components, as long as the cyclical component of debt balances out along

---

Paper presented for the World Bank, Latin America and Caribbean Regional Studies, Project on Debt Sustainability. The author is the Deputy Director of Economic Studies of the Latin American Reserve Fund (FLAR). I thank Álvaro Pachón and Juan Manuel Julio for providing me very useful references for the application of Markov´s chains, as well as the comments received in the seminars at the World Bank, Fedesarrollo, and Banco de la República, where a first draft of this paper was presented. I am particularly grateful to David Gould, Franz Hamman, Santiago Herrera, Javier Gómez, Roberto Junguito, Enrique Mendoza, Martha Misas, Guillermo Perry, Hernán Rincón, and Gabriel Rosas, for the observations and suggestions expressed in those seminars, as well as to the anonymous referee that evaluated this paper for publication in ESPE. I also thank Julio Velarde, President of the FLAR, for allowing me to devote the time requested by this project. The opinions included in this paper are the sole responsibility of the author and do not represent the official position of the FLAR.

* Latin American Reserve Fund (FLAR). Email: hmora@flar.net

Document received May 19th 2004; final version accepted october 1st of 2004.

the full cycle. In addition, a methodology to estimate the probability of default of debt and to treat the interest rate as a function of that probability is developed and applied. The common characteristic of the three approaches is the explicit incorporation of uncertainty about the behavior of the variables determining the evolution of the government's debt, and the formulation of specific indicators that capture the effect of volatility in the economy, as well as their incidence on fiscal and financial variables. In addition, the three methodologies applied use specific procedures to estimate the expected value of the relevant variables. The results, particularly those expressed in terms of the probabilities of default, constitute useful analytical tools for policy makers to foresee the wideness of the period available to undertake corrective fiscal policies, which are explicitly characterized along the paper.

JEL Classification: H6: National budget, deficit and debt.

Keywords: Debt sustainability; macroeconomic volatility; expected values; markov´s chains.

I. INTRODUCCIÓN

The purpose of this paper is to apply and analyze three alternative methodologies to assess fiscal sustainability of the central government's debt, using as illustrative examples the cases of Colombia and Costa Rica.

In Colombia, central government's debt, as a proportion of GDP, has increased from 17.8% in 1997, to 51.3% in 2003; while in Costa Rica, from 32.7% to 40.1%, in the same period. The rapid growth of debt in the Colombian case is the result of a long sequence of primary deficits, that reached the level of 3.1% in 1999 and has been reducing since then, but on average, over the five year's period has been high (1.95%); combined with a low rate of output growth (1.36%) relative to the implicit real interest rate paid on outstanding debt (3.24%), on average. On the contrary, Costa Rica has maintained primary surpluses (1.2%), and a rate of output growth (4.8%) larger than the implicit interest rate (2.6%), on average. Both countries have experienced relatively large central government deficits (5.6% and 3.5%, respectively, on average) and have adopted important fiscal reforms and expenditure measures in the last years, aimed to restructure central government's finances to make them compatible with macroeconomic stability and sustainable growth in the future; but the already mentioned differences in the past, with respect to the variables determining the evolution of debt, make them interesting cases of study for the application of alternative methodologies to assess debt sustainability.

Analysis of debt sustainability are extremely sensitive to the assumptions on the behavior of the main variables determining the evolution of debt in the forecasting horizon, mainly the primary fiscal balance, the real rate of growth of output and the real interest rate paid on outstanding debt. Small changes in the forecasted value of these variables produce completely different conclusions on weather a fiscal program that affects, for example, the primary balance in the future is consistent or not with debt sustainability. In emerging markets these variables exhibit large fluctuations which makes even more difficult to anticipate its future behavior. Thus, for example, in Colombia, in the last five years, the rate of growth of output has fluctuated between -4.2% and 3.6% (in Costa Rica, between 1% and 8.2%); the implicit real interest rate paid on central government's debt, from 2.6% to 6.3% (in Costa Rica from -0.5% to 6.1%); and the primary balanced has been continuously reduced, year after year, from -3.12% of GDP to -0.18%, but its negative sign in all these years fails to meet the basic condition for the sustainability of debt[1] , unless the rate of growth of output exceeds the real interest rate which has only been the case in one of the five years (in Costa Rica it has fluctuated between 0.4% and 1.8%).

The future value of these variable can be seen as consistent of two components. The first one corresponds to its expected value, usually estimated on the basis of historical information, by means of econometric or statistical procedures. The second component corresponds to specific shocks that may affect these variables in the future, independently of its past behavior and which is entirely unpredictable.

The common characteristic of the three approaches applied in this paper is the explicit incorporation of uncertainty about the behavior of the variables determining the evolution of the government's debt, and the formulation of specific indicators that capture the effect of volatility in the economy, as well as their incidence on fiscal and financial variables. In addition, the three methodologies applied use specific procedures to estimate the expected value of the relevant variables.

---

1 For debt to be sustainable, in a general sense, the basic condition that must be met is that the present discounted value of the sequence of future primary balances be larger than or equal to outstanding debt.

The first methodology has been developed and applied by the International Monetary Fund since mid 2002 to assess the sustainability of countries' public debt, in order to broaden the analysis of this subject and evaluate their sensitivity to alternative macroeconomic scenarios (see IMF (2003)). The approach consists of three main elements, the first of which includes the main macroeconomic scenario discussed between the authorities of the country and the IMF and, in this sense constitutes the basis of understandings on the agreed program. The second component develops specific models for projection of the main variables affecting the government's debt, and provides probabilistic distributions for the debt ratio that are used to test the sensitivity of the baseline scenario. The third component interprets the results of debt sustainability in relation to the country's vulnerability to a crisis. In this paper, the second component will be applied to the two countries and their results compared to the other two approaches. As will be seen, one aspect of this approach that results specially useful is the measurement of the degree in which the debt ratio may vary as a result of unpredicted events.

The second framework has been proposed in Mendoza & Oviedo (2003) and consists of two main components. The first one deals with uncertainty concerning government revenues, which are assumed to follow a stochastic Markov's process, and affects debt dynamics. Whenever it reaches a "debt limit", corresponding to the worst possible realization of revenues, the government adjusts expenditures to the minimum. The sensitivity of the "debt limit" to the volatility of revenues, as well as the possible paths of debt under alternative initial conditions are evaluated. The second component develops a stochastic general equilibrium model for a small open economy with tradable and non-tradable goods, whose equilibrium path is affected by exogenous shocks to income and the world interest rate, given the fiscal policy on taxes and expenditure. This model explicitly incorporates the effects on the economy implied by liability dollarization in imperfect international capital markets subject to potential sudden stops. In this paper, the first component will be applied to the two economies.

The estimation of the expected value of revenues in this approach turns out to be particularly relevant for the case of economies that due to institutional factors and external, as well as internal shocks, tend to replicate certain paths already seen in the past. Thus, for example, in the two countries it has been observed that after one or two years of a fiscal reform aimed to increase revenues, the implicit tax rate returns to the previous levels, after agents adjust to the reform. In addition, as in the case of the IMF's methodology, this approach is useful for the measurement of the effect of unforeseen events on the debt ratio.

The third approach incorporates into the analysis of debt sustainability the effects of the structural and transitory components of output, government expenditures, and revenues, on the evolution of debt. In particular, debt dynamics is decomposed into two components, one determined by the structural component of output, interest rates and the primary balance, and the other by their cyclical, or transitory components. Over the whole cycle, the transitory component always balances out and, therefore, has no implications for the sustainability of debt; while the structural component does. An indicator to assess debt sustainability and that incorporates the structural components of the forcing variables in the debt equation is proposed. This indicator might be of particular relevance for economies subject to large cyclical fluctuations coming from external and internal sources, specially in the case of output and the fiscal variables. As will be seen the two economies analyzed in this paper exhibit that type of behavior in the case of output. Sharp differences of interpretation with respect to the sustainability of debt are found between this proposed indicator and the traditional ones. In addition, as in the case of the IMF's methodology, the estimation of the expected value of the main variables uses econometric procedures, and similarly as in the two other approaches, uncertainty is incorporated into the analysis to evaluate its effect on fiscal sustainability.

In addition, the third approach is enriched with the treatment of the interest rate as an endogenous variable, determined in part by the probability of default of the debt, which is estimated for the projection period 2004-2010, in the two countries[2] .

After this brief introduction, the second part of this paper applies the IMF's methodology; the third applies Mendoza & Oviedo's approach; the fourth, presents the structural indicator of debt sustainability; the fifth estimates the probability of default of the debt; and the sixth offers a series of conclusions.

II. SIMULATION OF THE PROBABILITY DISTRIBUTION OF THE DEBT RATIO

As explained, the methodology of the IMF's (2003) paper, has two components. The first one refers to the estimation of the expected value of the variables affecting

2 I thank Guillermo Perry, for having suggested the explicit treatment of the interest rate as an endogenous variable, associated with the probability of default of the debt.

the evolution of debt. The second one incorporates those expected values, as well as a series of random shocks in the forecasting of those variables. In particular, the probability distribution of the debt ratio is simulated for each projection period from a random sample of shocks to the forcing variables of the following debt equation:

(1) $$d_t = \lambda_t d_{t-1} - cb_t \text{ , where } \lambda_t = (1 + i_t) / (1 + \rho_t)(1 + g_t) = R_t / \gamma_t$$

where d_t is the ratio of debt to GDP at the end of period t; cb_t is the primary fiscal balance as a proportion of GDP; i_t is the nominal interest rate; ρ_t is the rate of inflation of the GDP's deflactor; the ratio $(1 + i_t) / (1 + \rho_t)$ corresponds to the gross real interest rate (R_t); and $(1 + g_t)$ is the gross real rate of output growth (γ_t). Therefore, the column vector of forcing variables at time t corresponds to $z_t = \{i_t, \rho_t, g_t, cb_t\}'$, which can be modeled as:

(2) $$z_t = \beta_0 + \sum_{k=1}^{k} \beta_k z_{t-k} + \sum_{j=1}^{J} \delta_j x_{t-j} + \varepsilon_t$$

where x is a vector of exogenous variables, and $\varepsilon_t \sim N(0,\Sigma)$ is a vector of random shocks serially uncorrelated, such that $E(\varepsilon_q, \varepsilon_s) = 0$ for $q \neq s$[3].

Once the parameters in β, δ, and Σ are estimated from historical data, a simulated sample of z is obtained from:

(3) $$z_t = z_t \hat{\beta} + \varepsilon_t = \hat{\beta}_0 + \sum_{k=1}^{K} \hat{\beta}_k z_{t-k} + \sum_{j=1}^{J} \hat{\delta}_j x_{t-j} + \eta_t \text{ , where } \eta_t = W\upsilon_t$$

Where W is the Cholesky's decomposition matrix of Σ, such that $\Sigma = WW'$, and v_t is a vector of identically and independently distributed random shocks drawn from a standard normal distribution, $v_t \sim N(0, I)$.

In the case of Colombia, quarterly data[4] was used from 1995.1 to 2003.4 to estimate (2). All the forcing variables have a unit root, and also, according to the Johansen's

3 ε is a $4XN$ matrix, where N is the number of observations.

4 In a first draft of this paper annual data was used for Colombia, with very poor results since only the inflation rate was related with the other forcing variables.

test, they are cointegrated. Therefore, (2) was estimated in differences of first order of the series including the lagged error terms of the cointegrating equations as explanatory variables. The results of the estimation of the corresponding VAR indicate that only for the equation of the primary balance there are not statistically significant lags of the variables of the system. From these results of the VAR, the non significant lags were excluded and a system of three equations was estimated. The results of this estimation, as well as the Cholesky´s decomposition of the covariance matrix obtained from the VAR are included in Table 1. In the Costa Rican case only annual data was available. Almost all the variables are stationary, except for the primary balance. For this reason, the VAR for the forcing variables of the debt equation was estimated in differences of first order of the series. From the results of estimating the VAR a system of three equations was defined, taking out the non-significant (lags of the) variables. The results of estimating this system are also reported in Table 1, together with the covariance matrix of the residuals and the corresponding Cholesky's decomposition matrix.

On the other hand, a sample of 20.000 random shocks, v, was generated from a standard normal distribution of four variables, one for each of the forcing variables, and for each of the years in the projection period 2004-2008. After multiplying this sample for the Cholesky's decomposition matrix, a sample of size 20.000 for the disturbances of the forcing variables, for each one of these years, was generated according to (3). Also, the results of estimating the systems were used to obtain the forecasted value of the forcing variable in each year. In the case of the primary balances in both countries, for which the results of the VAR system were not satisfactory, the random walk equation was used for forecasting purposes, plugging into it the disturbance generated as explained before. After forecasting the forcing variables, the debt ratio was projected according to (1), for each year. Therefore, in each year, a sample of 20.000 debt ratios were generated.

Graph 1 shows the distribution function of the debt ratio for the projection period 2004-2008, and Table 2 includes the summary statistics of the distribution for each year. The variance of the forecasted ratios is much larger for Costa Rica than for Colombia. This result is due to the relatively lower statistical quality of the estimates for Costa Rica with annual, instead of quarterly data. The mean projected values of the forcing variables for Colombia determine a falling path of the debt ratio, while the opposite occurs for Costa Rica. However, the much larger variance of

Table 1

Results of the Estimation of the VAR for the Forcing Variables
Colombia

Estimation Method: Seemingly Unrelated Regression
Sample: 1996:3 2004:1
Included observations: 32
Total system (balanced) observations: 93
Iterate coefficients after one-step weighting matrix
Convergence achieved after: 1 weight matrix, 51 total coef iterations

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-1.213758	0.182540	-6.649275	0.000000
C(2)	-0.998757	0.161383	-6.188728	0.000000
C(3)	-0.833544	0.161582	-5.158625	0.000000
C(4)	-0.407477	0.156442	-2.604653	0.011300
C(5)	-0.320107	0.121922	-2.625496	0.010700
C(6)	-1.195405	0.464956	-2.571004	0.012300
C(7)	-0.906876	0.399181	-2.271841	0.026300
C(8)	-0.488286	0.214076	-2.280900	0.025700
C(9)	-0.202971	0.055397	-3.663965	0.000500
C(10)	1.981050	0.525287	3.771366	0.000300
C(20)	-0.277822	0.144978	-1.916309	0.059500
C(21)	-0.267358	0.116530	-2.294334	0.024900
C(22)	-0.301372	0.129589	-2.325595	0.023000
C(23)	-0.505640	0.093361	-5.415986	0.000000
C(24)	-0.048941	0.025159	-1.945264	0.055900
C(25)	0.544934	0.205569	2.650862	0.010000
C(26)	-0.358719	0.167868	-2.136919	0.036200
C(30)	0.303316	0.172217	1.761248	0.082700
C(31)	0.718943	0.093225	7.711920	0.000000
C(32)	0.627443	0.147546	4.252541	0.000100
C(33)	1.163137	0.305946	3.801767	0.000300
C(34)	0.528858	0.209753	2.521339	0.014000
C(35)	0.216721	0.035705	6.069743	0.000000
C(36)	-2.157478	0.330273	-6.532400	0.000000
C(37)	-0.418701	0.159937	-2.617910	0.010900
Determinant residual covariance			0.133499	

Table 1 (continuation)

Results of the Estimation of the VAR for the Forcing Variables
Colombia

Equation: D(R_CO) = C(1) * D(R_CO(-1)) + C(2) * D(R_CO(-2)) + C(3) * D(R_CO(-3)) + C(4) * D(R_CO(-4)) + C(5) * D(PI_CO(-1)) + C(6) * D(BP_CO(-1)) + C(7) * D(BP_CO(-2)) + C(8) * D(BP_CO(-3)) + C(9) * ER1(-1) + C(10) * ER2(-1)

Observations:	31	S.D. dependent var	1.830477
R-squared	0.838878	S.E. of regression	0.878200
Adjusted R-squared	0.769825	Durbin-Watson stat	1.993655
Mean dependent var	0.045880	Sum squared resid	16.19593

Equation: D(Y_CO) = C(20) * D(R_CO(-1)) + C(21) * D(R_CO(-2)) + C(22) * D(R_CO(-3)) + C(23) * D(PI_CO(-4)) + C(24) * ER1(-1) + C(25) * ER2(-1) + [AR(1)=C(26)]

Observations:	31	S.D. dependent var	2.394546
R-squared	0.8833	S.E. of regression	0.914563
Adjusted R-squared	0.854125	Mean dependent var	-0.011173
Durbin-Watson stat	2.083439	Sum squared resid	20.074220

Equation: D(PI_CO) = C(30) * D(R_CO(-1)) + C(31) * D(R_CO(-2)) + C(32) * D(R_CO(-3)) + C(33) * D(BP_CO(-1)) + C(34) * D(BP_CO(-2)) + C(35) * ER1(-1) + C(36) * ER2(-1) + [AR(1)=C(37)]

Observations:	31	S.D. dependent var	2.446016
R-squared	0.866566	S.E. of regression	1.020444
Adjusted R-squared	0.825956	Durbin-Watson stat	2.025427
Mean dependent var	-0.052872	Sum squared resid	23.950030

Cholesky's decomposition matrix

	d(r_co)	**d(y_co)**	**d(pi_co)**	**d(bp_co)**
d(r_co)	0.793820	0.000000	0.000000	0.000000
d(y_co)	0.381444	0.736677	0.000000	0.000000
d(pi_co)	-0.522676	-0.860665	0.490194	0.000000
d(bp_co)	0.264908	-0.084138	0.015265	0.604243

PI_CO: annual inflation rate of the GDP's deflactor (%) for Colombia.
R_CO: annual real interest rate (%) for Colombia.
Y_CO: annual real rate of output growth (%) for Colombia.
BP_CO: Ratio of the primary balance to GDP (%) for Colombia.
ER1: Error term of the first cointegrating equation.
ER2: Error term of the second cointegrating equation.
Source: author´s estimates.

Table 1 (continuation)

Results of the Estimation of the VAR for the Forcing Variables
Costa Rica

Estimation Method: Seemingly Unrelated Regression
Sample: 1973 2003
Included observations: 31
Total system (unbalanced) observations: 88
Simultaneous weighting matrix & coefficient iteration
Convergence achieved after: 6 weight matrices, 7 total coef iterations

	Coefficient	**Std. Error**	**t-Statistic**	**Prob.**
C(2)	-0.490111	0.151331	-3.238661	0.001700
C(3)	-0.353944	0.150103	-2.358016	0.020800
C(4)	-9.489820	3.350153	-2.832653	0.005800
C(9)	1.403757	0.327759	4.282899	0.000000
C(11)	0.422448	0.069704	6.060628	0.000000
C(12)	-0.620743	0.117903	-5.264874	0.000000
Determinant residual covariance			1323654	

Equation: D(R_CR) = C(2) * D(R_CR(-1)) + C(3) * D(R_CR(-2)) + C(4) * D(BPRCR(-1))

Observations:	29	S.D. dependent var	39.002490
R-squared	0.391265	S.E. of regression	31.579040
Adjusted R-squared	0.344439	Durbin-Watson stat	2.501730
Mean dependent var	0.341202	Sum squared resid	25,928.130000

Equation: D(Y_CR) = C(9) * D(BPRCR(-2))

Observations:	28	S.D. dependent var	4.095553
R-squared	0.404675	S.E. of regression	3.160018
Adjusted R-squared	0.404675	Durbin-Watson stat	2.101130
Mean dependent var	0.125003	Sum squared resid	269.614300

Table 1 (continuation)

Results of the Estimation of the VAR for the Forcing Variables
Costa Rica

Equation: D(PI_CR) = C(11) * D(R_CR(-2)) + C(12) * D(PI_CR(-1))

Observations:	31	S.D. dependent var	21.430360
R-squared	0.576525	S.E. of regression	14.184200
Adjusted R-squared	0.561923	Durbin-Watson stat	2.413106
Mean dependent var	0.002805	Sum squared resid	5,834.557000

Cholesky's decomposition matrix

	R_CR	Y_CR	PI_CR
R_CR	29.901060	0.000000	0.000000
Y_CR	-0.743861	3.012599	0.000000
PI_CR	-4.972994	0.597107	12.772012

PI_CR: annual inflation rate of the GDP's deflactor (%) for Costa Rica
R_CR: annual real interest rate (%) for Costa Rica
Y_CR: annual real rate of output growth (%) for Costa Rica
BPRCR: Ratio of the primary balance to GDP (%) for Costa Rica
D(X(-i)): indicates the difference of order *i* of the series *X*
Source: author´s estimates.

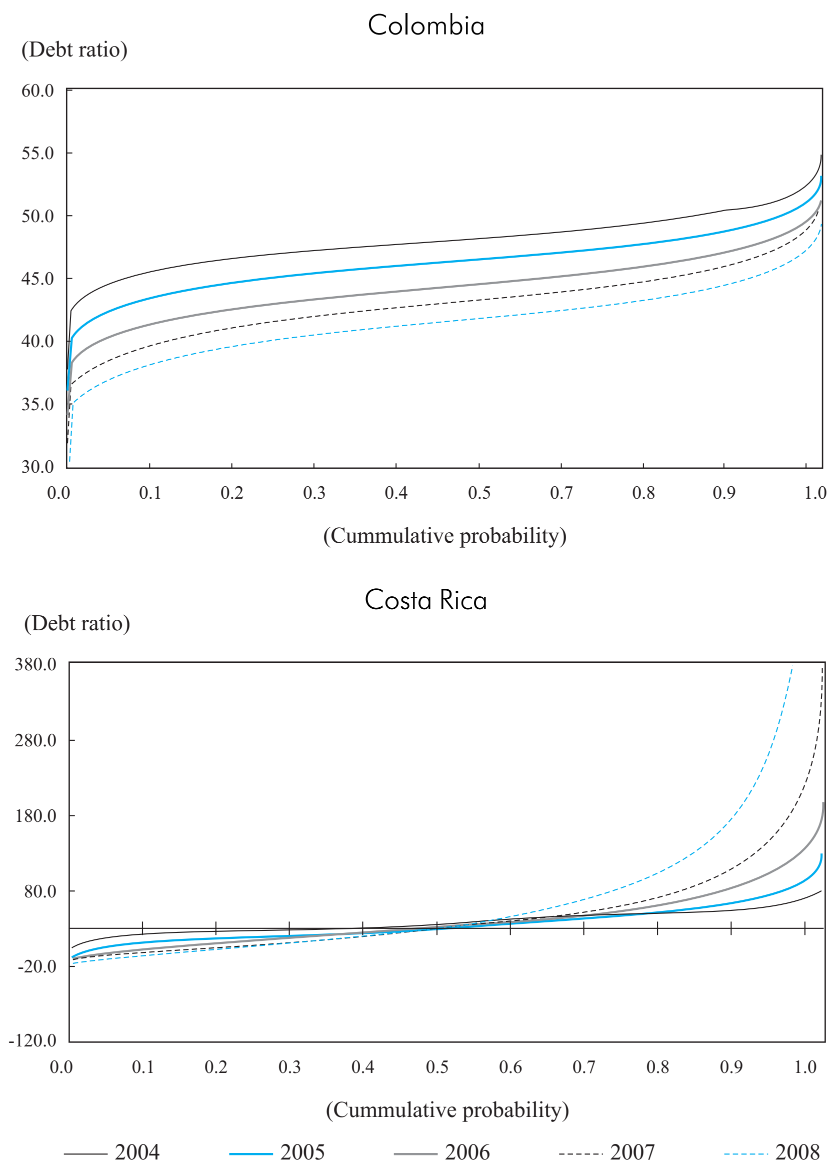
the estimates for Costa Rica imply that some of the projected ratios adopt negative values, which is not the case of Colombia. The 95% and 97.5% limit values[5] obtained from the distribution function increase very rapidly for Costa Rica and their difference to the mean value is very large, while that is not the case for Colombia.

It is important to emphasize that these results are related to the different quality of the estimated models in the two countries, which depends on the frequency of the information used for that purpose. When annual data was used for Colombia, a

5 The limit values obtained from the distribution of the debt ratio, $X_{0.95}$ and $X_{0.975}$, are such that the probabilities Pr.$[X \leq X_{0.95}] = 0.95$, and Pr.$[X \leq X_{0.975}] = 0.975$, respectively.

Graph 1

Distribution Function of the Debt Ratio



Source: author´s estimates.

Table 2

Forescast, 2004-2008

(Percentage)

Forescated Debt Ratio

	2003	2004	2005	2006	2007	2008
Colombia						
Mean value	51.3	47.9	46.2	44.3	42.7	41.3
95.0% limit		51.3	49.9	48.3	47.0	45.8
97.5% limit		52.0	50.6	49.0	47.8	46.7
Maximum value		56.7	55.7	54.6	53.6	53.3
Minimum value		33.9	33.2	32.2	30.6	28.7
Standard deviation		2.1	2.3	2.5	2.6	2.8
Costa Rica						
Mean value	40.1	37.0	35.2	39.9	45.6	73.2
95.0% limit		62.1	79.3	112.8	167.9	319.8
97.5% limit		67.4	92.2	137.0	217.3	440.7
Maximum value		105.9	233.5	321.2	835.3	2,378.5
Minimum value		-9.9	-9.9	-9.9	-9.9	-9.9
Standard deviation		14.5	23.6	37.3	62.2	134.2

Forescated Forcing Variables of the Debt Equation

	2003	2004	2005	2006	2007	2008
Colombia						
Mean real interest rate 1/	2.6	0.1	3.5	3.3	3.1	2.8
Standard deviation of the real interest rate		0.8	0.8	0.8	0.8	0.8
Mean output growth 1/	3.6	5.2	6.2	6.6	5.7	5.3
Standar deviaton of output growth		0.8	0.8	0.8	0.8	0.8
Mean primary balance 2/	-0.2	0.5	0.5	0.5	0.5	0.5
Standard deviation of the primary balance		1.1	1.1	1.1	1.1	1.1
Mean inflation rate 1/	3.6	9.4	8.5	8.7	8.8	8.8
Standar deviation of the inflation rate		0.7	0.7	0.7	0.7	0.7
Costa Rica						
Mean real interest rate 1/	5.3	-6.4	-1.3	0.2	-2.3	-1.6
Standard deviation of the real interest rate		30.0	34.9	38.5	44.2	58.3
Mean output growth 1/	5.6	-1.7	1.7	0.0	0.0	0.0
Standar deviaton of output growth		3.1	3.1	3.4	3.4	3.4
Mean primary balance 2/	1.6	1.6	1.6	1.6	1.6	1.6
Standard deviation of the primary balance		1.0	2.0	3.0	4.0	5.0
Mean inflation rate 1/	6.5	1.0	4.3	-6.1	8.6	-8.4
Standar deviation of the inflation rate		5.0	5.9	9.7	13.9	15.0

1/ Percentage.
2/ Percentage of GDP.
Source: author´s estimates.

large variance of the estimates and significant differences of the 97.5% limit values with respect to the mean value was also obtained[6].

Therefore it may be concluded that even though the IMF's methodology is very useful to illustrate the errors in which econometric forecasting procedures may incur in trying to foresee the debt ratio, these errors are magnified if the information available does not allow to obtain statistically robust estimates of the projected values for the forcing variables. As will be seen in the next section, other methodologies, like the use of Markov chains for estimating the expected value of the relevant variables might be less vulnerable to these extremely large variances of the forecasted ratios, when the transition matrix characterizing the Markov chain is estimated using historical data. Of course, in the basis of this limitation lies the difficulty of capturing through econometric models the (probably changing) relation among the macroeconomic variables determining debt evolution, specially when low frequency data is used. But in contrast with the Markov chains, the econometric methods do not constrain the values adopted by the forecasted variables to those observed in the past.

III. THE USE OF MARKOV'S CHAINS FOR SIMULATING THE BEHAVIOR OF THE DEBT RATIO

The methodology proposed by Mendoza & Oviedo (2003) to evaluate whether or not the stock of public debt is consistent with fiscal solvency has two components. On the one hand, they incorporate uncertainty into the behavior of the ratio of public revenues to GDP and analyze the effect of different stochastic paths of this variable on debt dynamics. On the other hand, they deal with the problem of liability dollarization and fiscal policies, on the expenditure and revenue side, in a dynamic stochastic general equilibrium model that quantifies the impact on debt dynamics of the competitive equilibrium of a two sector (traded and non-traded) small open economy. In this section the first component of their methodology is applied to the cases of Colombia and Costa Rica.

The authors define a sustainable public debt policy under uncertainty as "one for which the government can credibly commit to repay in all states of nature", which

[6] I thank the participants in the seminar at Banco de la República and the anonymous referee of ESPE for having made this observation.

requires the self-imposition on the part of the government of a "debt limit" beyond which it will not borrow. This "debt limit" corresponds to the amount of debt it could service after experiencing an infinite sequence of the worst possible realizations of public revenues. In this state of crisis, revenues, as a proportion of GDP, reduce to the level t and the government is forced to reduce expenditure to the minimum possible ratio, g, since under those circumstances access to debt markets is closed. Therefore, the imposition of the debt limit means compliance with the following condition:

$$(4) \qquad \beta_t \leq (\gamma / R - \gamma)\,(t - g)$$

Where the right hand side corresponds to the steady state level of debt, according to equation (1), but under a perpetual situation of crisis. In this expression, γ and R represent the gross rates of output growth and interest, respectively.

On the contrary, in normal times, the debt evolves according to (1) with revenues and expenditures set at their normal ratios, t and g.

In order to facilitate comparisons of the results with those obtained by the authors, based on the data for Mexico, the ratios of these variables will be defined in a similar way, for the case of the central government. Table 3 contains the resulting estimates of the debt limit. The ratio of revenues to GDP in normal times corresponds to the average of this ratio over the sample period, while the minimum ratio of revenues is equal to that value for normal times, minus two standard deviations[7]. The sample taken to make the calculations corresponds to the period 1990-2003, for both countries[8]. The ratio of expenditures in normal times is set to equate the difference between the average revenue and the average debt in the sample period[9]. The minimum ratio of expenditures to GDP was set at the level consistent with a debt limit of 50% for both countries. It is important to mention that had the minimum ratio

7 Obviously, the resulting estimate of the minimum revenue is, *ceteris paribus*, smaller for countries having a larger revenue volatility.

8 We did not take the longer period available, 1971-2003 in the case of Colombia, and 1972-2003, in the case of Costa Rica, due to the structural changes that have experienced these economies and, particularly, the fiscal variables, along the longer period. Thus, for example, over the whole available sample the average ratio of central government revenues to GDP was 12.6%, for Colombia, and 14.6% for the case of Costa Rica; while in the shorter period 1990-2003, those average were 13.7% and 15.4%, respectively.

9 The average ratio of central government debt to GDP was 25.9% for Colombia, and 32.4% for Costa Rica, over the sample 1990-2003.

Table 3

Variables Used to Estimate the Debt Limit

(Sample 1990-2003)

	Colombia	Costa Rica
Variables		
Gross annual rate of interest (R)	1.0650	1.0650
Gross annual rate of output growth (Gamma)	1.0273	1.0468
Ratio of central government revenue to GDP in normal times (*t*)	0.1366	0.1535
Ratio of central government revenue to GDP in times of crisis (*t*)	0.1031	0.1344
Ratio of central government expenditures to GDP in normal times (*g*)	0.1271	0.1479
Ratio of central government expenditures to GDP in times of crisis (*g*)	0.0848	0.1257
Average ratio of central government debt to GDP	0.2592	0.3238
Debt limit	0.5000	0.5000
Other variables (1990-2003)		
Minimun ratio of central government expenditure to GDP	0.1030	0.1247
Average ratio of central government expenditure to GDP	0.1420	0.1460
Coefficient if variation of central government revenues 1/	12.2522	6.2155
Coefficient if variation of central government expenditures 1/	16.5337	10.0327
Coefficient if variation of the ratio of central government debt to GDP 1/	54.0845	16.5991

1/ Percentage.
Source: author´s estimates.

of expenditures been set at the minimum level effectively observed along the sample period 1990-2003 (10.3% for Colombia, and 12.5% for Costa Rica), the debt limit would had been 0.2% for Colombia, and 56.0% for Costa Rica, instead of 50%, which makes a substantial difference, specially for Colombia. Finally, the real annual interest rate is set at 6.5%, and the rate of output growth to its average over the same sample period.

It is worthy to emphasize that the debt limit of 50% adopted by the authors is completely ad hoc and do not correspond to critical thresholds. Even though this level is in the neighborhood of observable current ratios for Mexico, and also for Colombia, there are countries with current ratios much larger than the 50% where debt is sustainable, and countries with lower level where debt is not sustainable.

Table 3 shows important differences between the two countries in terms of the level of revenues, as well as their volatility. Costa Rica has a higher revenue level and a much lower volatility, for which the minimum level is also higher in this country. However, the same occurs with expenditures. As a result, the coefficient of variation for Colombia´s debt-output ratio is more than three times as large as Costa Rica's. Despite the higher volatility in Colombia, it is interesting to note that the average ratio of expenditures over the sample period is very similar for both countries (14.2% in Colombia versus 14.6% in Costa Rica), while the minimum of that ratio observed between 1990-2003 is significantly smaller in Colombia (10.3% versus 12.5%). Therefore, for a supposed similar initial debt ratio in both countries, the variables that could induce a more rapid increase of debt for Colombia, than for Costa Rica, under this approach, are the lower revenue ratio in times of crisis (10.3% versus 13.4%) and the smaller output rate of growth for the last country (2.7% versus 4.7%).

Table 4 illustrates the effect on the level of the debt limit of alternative scenarios of volatility in revenues, and then, on the minimum value of revenues, as well as the effect on the debt limit of alternative adjustments in expenditures. The upper left position corresponds to the ratio of debt to GDP observed in 2003 in both countries. The adjustment required in outlays, in relation to the normal value of that variable, in order to equate the debt limit to that observed ratio is 4.28 percentage points (p.p.), in the case of Colombia, and 2.04, in the case of Costa Rica[10]. Increases in the volatility of revenues reduce the debt limit and require additional adjustments in expenditures, in order to make the current debt ratio sustainable. Negative debt limits have been set equal to zero in the table. For coefficients of variation of revenues larger than 34.22, in Colombia, and 25.76, in Costa Rica, there is no borrowing. Notwithstanding the previous remarks, it is interesting to note that the minimum level of revenues for which there is no borrowing in Costa Rica (5.44% of GDP) is larger than in Colombia (2.31%). This is due to the higher level of normal outlays in the last country.

Using the variable and parameter estimates included in Table 3, and in order to evaluate debt dynamics, it is assumed that central government revenues follow a Markov process[11].

---

10 The "normal" level, built in the form explained earlier, is 3.6 percentage points lower than the one observed in 2003, for the case of Colombia; but very similar to that value, in the case of Costa Rica.

11 A Markov process is an stochastic process "with the property that, given the value of Xt, the values of Xs, $s > t$, do not depend on the values of Xu, $u < t$; that is, the probability of any particular future behavior of the process, when its present state is known exactly, is not altered by additional knowledge concerning its past behavior" (see Karlin & Taylor (1975, p. 29).

In order to characterize the Markov process for each country, the observed ratio of revenues to GDP in each year, for Colombia, and for Costa Rica, can be allocated to one of seven integer intervals, for the case of Colombia, and six for Costa Rica,

Table 4

Debt Limits Under Alternative Scenarios of the Volatility of Revenues and Adjustment in Expenditures 1/

(Percentage)

Variables and parameter estimates for Colombia

Standard deviation of *t*	Coefficient of variation of *t*	Minimum value of *t* = E(*t*) - 2*s*	Adjustment in *g* 2/			
			0.0428	0.0628	0.0828	0.1028
			gamma 1.0273	R 1.0650	E(*t*) 0.1366	*g* 0.1271
0.0167	12.25	0.1031	51.30	105.85	160.41	214.96
0.0267	19.57	0.0831	0.00	51.30	105.85	160.41
0.0367	26.90	0.0631	0.00	0.00	51.30	105.85
0.0467	34.22	0.0431	0.00	0.00	0.00	51.30
0.0567	41.54	0.0231	0.00	0.00	0.00	0.00
0.0667	48.86	0.0031	0.00	0.00	0.00	0.00

Variables and parameter estimates for Costa Rica

Standard deviation of *t*	Coefficient of variation of *t*	Minimun value of *t* = E(*t*) - 2*s*	Adjustment in *g* 2/			
			0.0204	0.0404	0.0604	0.0804
			gamma 1.0468	R 1.0650	E(*t*) 0.1535	*g* 0.1479
0.0095	6.22	0.1344	40.09	155.05	270.01	384.97
0.0195	12.73	0.1144	0.00	40.09	155.05	270.01
0.0295	19.24	0.0944	0.00	0.00	40.09	155.05
0.0395	25.76	0.0744	0.00	0.00	0.00	40.09
0.0495	32.27	0.0544	0.00	0.00	0.00	0.00
0.0595	38.79	0.0344	0.00	0.00	0.00	0.00

1/ The volatility of revenues affects the minimun values of t ($t = E(t) - 2s$).
2/ Expenditures are adjusted in the following way: $g = g$ - adjustment in g.
Source: author´s estimates.

ranging between less than 11% to 17%, for the first country; and less than 13% to 18%, for the second country. Each interval, like for example, (0,11], or (11,12] corresponds to one state of the Markov's chain. Once theses states are defined, the transition matrix indicates what is probability of shifting from one state, in time t, to other possible states, in time $t + 1$, and is estimated by simply counting the frequencies with which those shifts have occurred. For this reason, the characterization of the Markov process using historical information, constrains the variable of interest, in this case the ratio of revenues, to adopt only one of the states observed in the past, in any future time; as different from the forecasted values on the basis of the parameter estimates of econometric models, like the ones used in the IMF's methodology of the previous section.

Therefore, the use of all available historical information on the ratio of revenues to GDP allows us to define the states of the Markov's chain, as well as the transition matrices for both countries, T_{col} and T_{cr}, shown in Table 5. The table shows the probability estimates of shifting from state *t*, in time t - 1 (row), to state *j*, in time *t* (column), under the assumption that the true probabilities are stationary, and then, independent of time. As mentioned, on the basis of the observed data (1971-2003, for Colombia; and 1972-2003, for Costa Rica) seven states have been defined for Colombia, and six for Costa Rica, with the limits of each state corresponding to integer ratios of revenues to GDP between the smallest and largest observed ratios. None of the matrices presents absorbing states[12]. Table 5 also shows the mean value in each state.

Starting at time zero, the expected value of the ratio of revenues to GDP, in each projection period, $n = 1,\dots, N$, for an initial state *i*, is given by (see Kemeny and Snell (1960)):

$$(5) \qquad \hat{t}_n = \pi_i T^n t$$

Where π_i is an *S* row vector having a 1 in the *i*-th position and zeros elsewhere; *T* is the *SXS* transition matrix, *t* is the *S* column vector of state mean values, and *S* denotes the number of states.

[12] In the case of Colombia, the highest observed ratio of revenues to GDP occurred in 2003 (16.08%). Since we did not have previous observations to estimate the probability of shifting from that state to any other state, we assumed that similarly as in 1990, when it reached the maximum level observed up to that moment (15.84%), in the following period it will go back to the previous state.

Table 5

Transition Matrices for the Ratio of Revenues to GDP: Probability Estimates of Shifting From State *i* (Row), in *t* - 1, to State *j* (Column), in Time *t*

Colombia							
Time *t* - 1	Time *t*						
	11	12	13	14	15	16	17
11	0.6666667	0.0000000	0.3333333	0.0000000	0.0000000	0.0000000	0.0000000
12	0.0909091	0.5454545	0.3636364	0.0000000	0.0000000	0.0000000	0.0000000
13	0.0000000	0.3333333	0.3333333	0.2222222	0.0000000	0.1111111	0.0000000
14	0.0000000	0.0000000	0.2500000	0.2500000	0.2500000	0.2500000	0.0000000
15	0.0000000	0.5000000	0.0000000	0.5000000	0.0000000	0.0000000	0.0000000
16	0.0000000	0.0000000	0.0000000	0.0000000	0.3333333	0.3333333	0.3333333
17	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	1.0000000	0.0000000
Mean value (% of GDP)	10.5242840	11.5063776	12.4523622	13.3233544	14.4461651	15.5821537	16.0844932

Costa Rica						
Time *t* - 1	Time *t*					
	13	14	15	16	17	18
13	0.4000000	0.6000000	0.0000000	0.0000000	0.0000000	0.0000000
14	0.4000000	0.2000000	0.2000000	0.0000000	0.2000000	0.0000000
15	0.0000000	0.2000000	0.0000000	0.6000000	0.0000000	0.2000000
16	0.0000000	0.0000000	0.1818182	0.7272727	0.0909091	0.0000000
17	0.0000000	0.0000000	0.5000000	0.0000000	0.5000000	0.0000000
18	0.0000000	0.0000000	1.0000000	0.0000000	0.0000000	0.0000000
Mean value (% of GDP)	12.6203712	13.2601224	14.5172306	15.2616211	16.2209818	17.8937361

Source: author´s estimates.

Table 6 shows the values adopted by the transition matrices, T^{K}_{col} and T^{q}_{cr}, once they reach the steady state at the *k-th* and *q-th* projection periods, respectively. In the case of Colombia, k = 53, and in the case of Costa Rica, q = 60. These transition matrices are not invertible and, therefore, no fundamental matrix exists for each country. In steady state, the probability mass of the ratio of central

Table 6
Transition Matrices for the Ratio of Revenues to GDP, in steady state

Colombia (n ≥ 53)

Time *t* - 1	Time *t*						
	11	12	13	14	15	16	17
11	0,07161	0,26256	0,22613	0,12563	0,08794	0,16960	0,05653
12	0,07161	0,26256	0,22613	0,12563	0,08794	0,16960	0,05653
13	0,07161	0,26256	0,22613	0,12563	0,08794	0,16960	0,05653
14	0,07161	0,26256	0,22613	0,12563	0,08794	0,16960	0,05653
15	0,07161	0,26256	0,22613	0,12563	0,08794	0,16960	0,05653
16	0,07161	0,26256	0,22613	0,12563	0,08794	0,16960	0,05653
17	0,07161	0,26256	0,22613	0,12563	0,08794	0,16960	0,05653

Costa Rica (n ≥ 60)

Time *t* - 1	Time *t*					
	13	14	15	16	17	18
13						
14	0,06897	0,10345	0,20690	0,45517	0,12414	0,04138
15	0,06897	0,10345	0,20690	0,45517	0,12414	0,04138
16	0,06897	0,10345	0,20690	0,45517	0,12414	0,04138
17	0,06897	0,10345	0,20690	0,45517	0,12414	0,04138
18	0,06897	0,10345	0,20690	0,45517	0,12414	0,04138

Source: author´s estimates.

government revenues to GDP is concentrated in the interval (10%, 13%], in Colombia; and in the interval (13%,16%], in Costa Rica. Independently of the initial state, the expected value of the revenue ratio, after reaching the steady state, is the same; 13.09% for Colombia, and 14.95% for Costa Rica. However, the path to the equilibrium is different for each initial state.

As explained before, there are 7 possible paths for Colombia, and 6 for Costa Rica, each one corresponding to a particular initial state. Given the path of expected revenues, and the values of the other variables and parameters included in Table 3, the ratio of debt to GDP evolves according to (1). Graph 2 shows the time elapsed until reaching the debt limit of 50% for the fastest, the slowest and the average path in each country, for different initial debt ratios. The average path is not a path in itself, but simply a construct for purposes of presentation of the average behavior of the observed paths. For initial ratios of debt of 10%, there are paths in both countries that never hit the debt limit, but there is at least one path that does. For initial debt ratios between 10% and 35%, Colombia hits the debt limit faster than Costa Rica, but from 40% on, the difference in time for the three paths in both countries is small. Thus, for example, for an initial debt ratio of 40%, in Colombia the fastest path hits the debt limit in 4 periods, while in Costa Rica it does so in 5 periods. Those numbers are 1 and 2, respectively, for an initial debt ratio of 45%. On the other hand, there are substantial differences between different paths, for the same initial debt ratio, independently of what that initial ratio is, as illustrated by the comparison of the levels of the three bars corresponding to the fastest, the slowest, and the average path, in both countries.

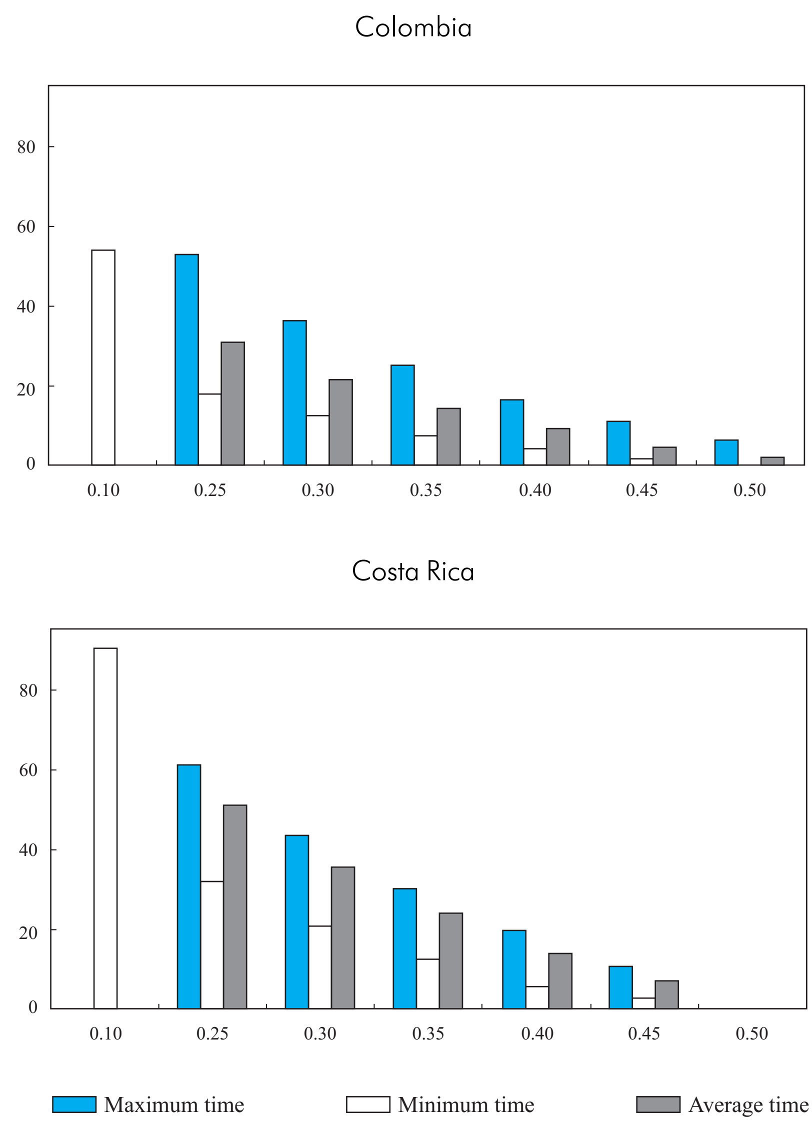
Graphs 3 and 4 show the forecast of the debt ratio, for the same three types of paths and for different initial debt ratios. It is interesting to note in Graph 3, that in the first 5 years (periods) of the projection horizon there are no substantial differences between the two countries, regardless of the initial level of debt, in the case of the fastest path, even though such differences exist in the case of the slowest paths. The differences are more notorious in a horizon of 25 years, but only for initial ratios of debt below 40%. Graph 4 shows that for the initial levels of debt of 10% there are decreasing paths of debt, in both countries, as well as increasing paths (particularly, the fastest path) to the debt limit; but it will take a relatively long period of 54 years in Colombia, and 91 years in Costa Rica to hit the debt limit, starting from that initial level. For an initial debt ratio of 10%, in the long run the average path stabilizes at a considerably lower level than the debt limit since, as explained, it corresponds to the average of all possible paths; some of them reach the debt limit, while others go to zero. On the contrary, for an initial debt ratio of 30%, all of the paths reach the debt limit, but in Colombia the slowest path decreases in the first 6 years.

Instead of using the expected value for the revenue ratios obtained from (5), it could be simulated a series of revenues, one observation for each projection period,

Graph 2

Time to Reach the Debt Limit (50%)

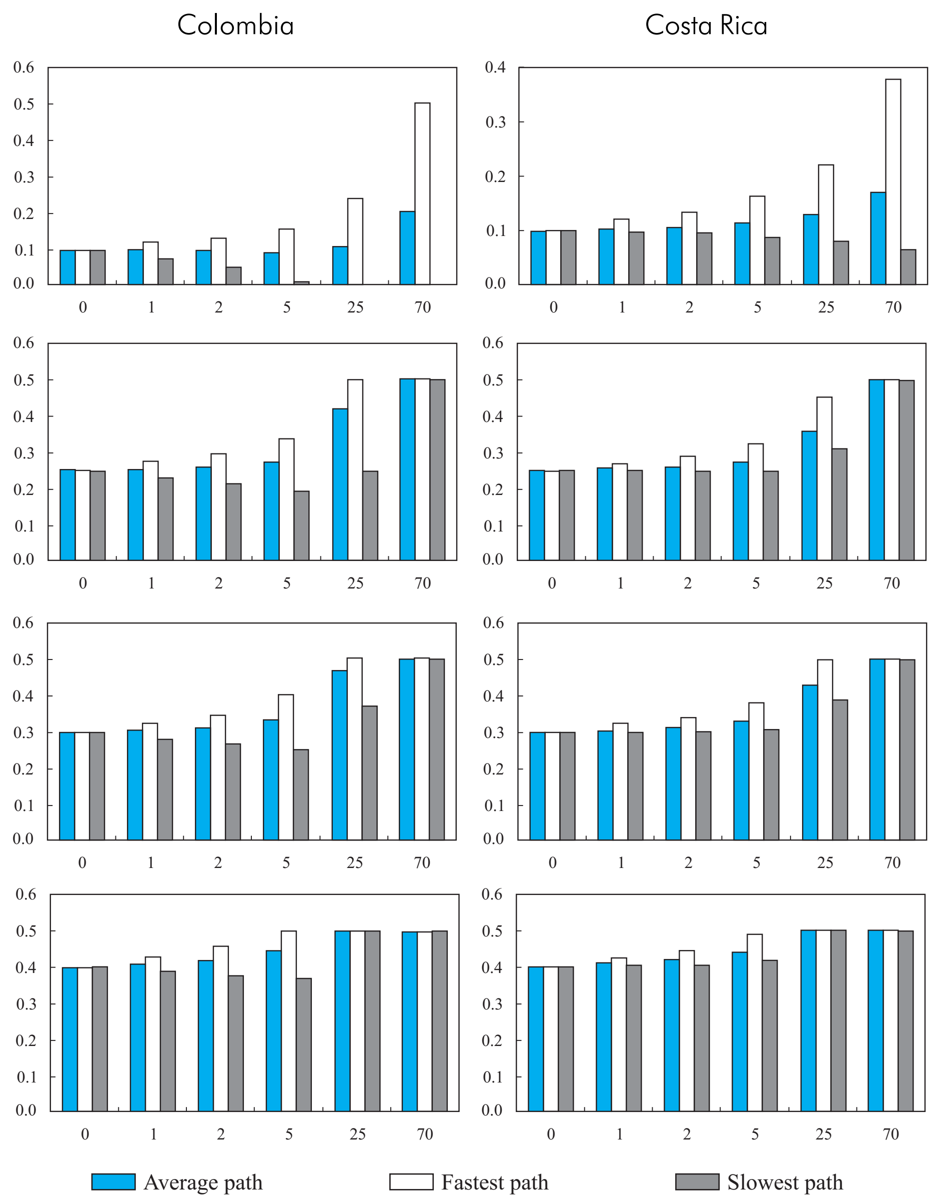
(Number of Periods)



Source: author´s estimates.

Graph 3

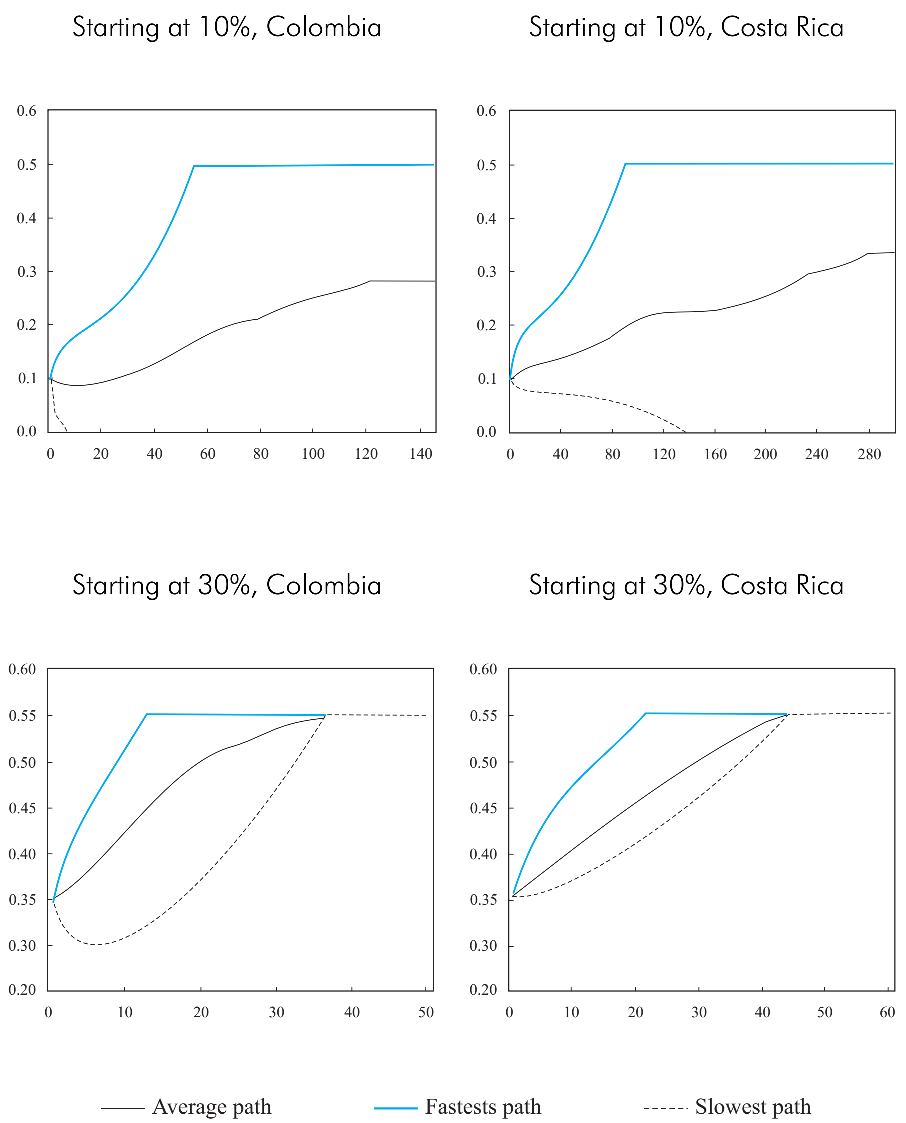
Forecast of the Debt to GDP Ratio, by Periods



Source: author´s estimates.

Graph 4

Forecast of the Debt to GDP, by Periods



Source: author´s estimates.

using the cummulative probability distribution obtained from the transition matrix. Graph 5 shows the results of simulating a sequence of random realizations of revenues, from realizations of a uniform distribution, which are allocated to the different states by means of the cummulative distribution function obtained from the transition matrix for each country. In this way, for each initial state, the shock will be allocated to one of the possible states[13] . The shocks are common to the two countries, and there are as many shocks (projection periods) as needed to stabilize the average path in the long run. Once the shock is produced, there is only one state that corresponds to it, for each initial state; which is equivalent to say that the shock identifies one state that occurs with probability one, for each initial state; as different from the previous forecast experiment, where for each projection period q, the i-th row of the transition matrix, T^q, consisted of the vector of probabilities allocated to the possible but known states, for each initial state i. The graph shows that under those random shocks the fastest path in each country hits the debt limit in both countries, for the two initial debt ratios (10% and 30%). Nevertheless, now there exists a decreasing path of debt, even for the initial ratio of 30%.

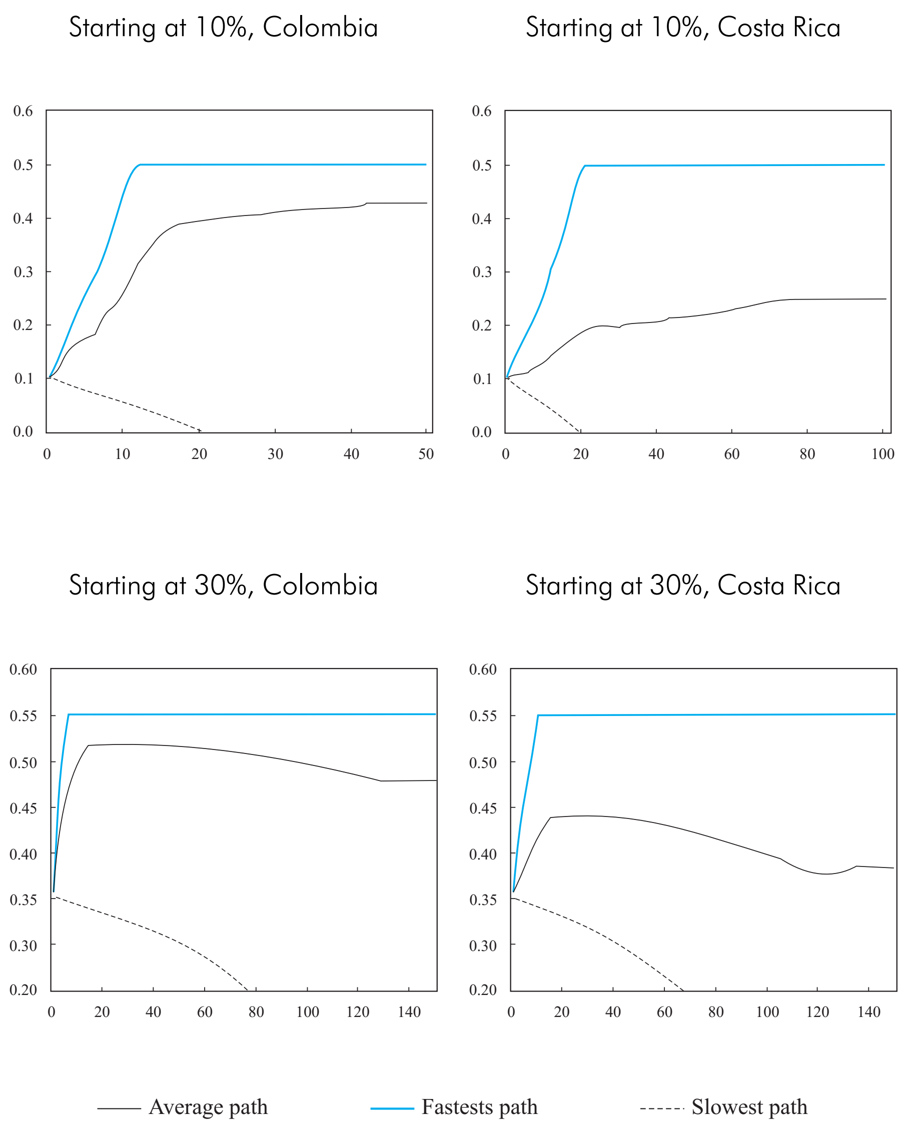
Another way of simulating the effect of random shocks on the debt ratio is equivalent to the IMF's methodology, where a sample of random shocks were simulated. In the case of this particular methodology, the random shocks will be added to the expected value of the revenue ratio obtained from (5). The first row of the block each country of Table 7 contains the expected value of the debt ratio for the period 2004-2008, using the Markov chain, for the initial debt ratio observed in 2003 (51.3% for Colombia, and 40.1% for Costa Rica) and given also the initial state of revenues observed in that year (17 and 15). In addition, from a random sample of shocks to the revenue output ratio, of size 20.000 for each year, drawn from a standard normal distribution, the behavior of the debt ratio was simulated[14] . Table 7 also contains the summary statistics of the distribution obtained from that simulation. These results can be compared to the ones reported in Table 2, from

[13] Given the random realization of w, drawn from a uniform distribution, and an initial state i, if $\sum_{j=1}^{k-1} P_{i,j} < w \leq \sum_{j=1}^{k} P_{i,j}$, then the shock identifies state k-th, where $P_{i,j}$ is the probability of shifting from state i (in time t-1), to state j (in time t). Once the state k-th is identified for the initial state i, the mean value of that state is allocated to the realization of the shock w.

[14] We are simulating the revenue output ratio given by $t = E[t] + \varepsilon$, where $E[t]$ was obtained from the Markov process, as explained, and $\varepsilon \sim N(0,I)$. Given these 20.000 simulated values of t, for each year of the projection period, we simulate the evolution of the corresponding debt ratio.

Graph 5

Simulated Series of the Debt to GDP Ratio, by Periods



Source: author´s estimates.

Table 7

Forescated Debt Ratio for the Initial Level and State of 2003

	2003	2004	2005	2006	2007	2008
Colombia						
Expected value from the Markov chain	51.3	50.3	49.5	49.5	49.8	50.5
Average of simulated ratios		53.1	55.1	57.1	59.2	61.3
95.0% value		54.8	57.5	60.1	62.6	65.3
97.5% value		55.1	57.9	60.7	63.3	66.0
Maximum value		62.7	64.9	65.3	68.7	71.2
Minimum value		49.4	49.3	50.0	51.0	50.7
Standard deviation		1.0	1.5	1.8	2.1	2.4
Costa Rica						
Expected value from the Markov chain	40.1	40.2	40.9	41.3	41.9	42.4
Average of simulated ratios		40.8	41.5	42.2	43.0	43.7
95.0% value		42.4	43.9	45.2	46.3	47.5
97.5% value		42.7	44.3	45.7	47.0	48.2
Maximum value		50.3	51.2	50.2	52.5	53.3
Minimum value		37.0	35.7	35.3	34.9	33.4
Standard deviation		1.0	1.4	1.8	2.1	2.3

Source: author´s estimates.

the IMF´s methodology. For illustration purposes, the average of the simulated debt ratios is also included. The shocks for both countries are the same, and have zero mean; however, it can be observed that the difference between the average of the simulated ratios and the expected value obtained from the Markov chain is much larger in the case of Colombia than in Costa Rica due in part to the larger initial ratio of debt and also, to the greater value of the ratio of the gross real interest rate and the gross rate of output growth (in the case of the first country), which magnifies the effects of the disturbances increasingly over time following the first projection year.

In comparison with the results of Table 2, the debt ratios simulated with this methodology has much lower variance for both countries; the mean value of the forecasted ratios is also much lower for the entire projection period, as are the limits of the 95% and the 97.5% confidence intervals, for both countries. Of course, since the initial debt ratio is 51.3% in the case Colombia, it exceeds the "debt limit" of this methodology since the beginning of the projection period and, from this perspective, debt is not sustainable in this country. On the contrary, in Costa Rica it is sustainable not only because the mean of the simulated ratios is smaller than 50%, but also because the limits of the confidence intervals are smaller than that "debt limit". But beyond the limitations that may be present in the definition of the "debt limit", what seems attractive of this methodology is the use of Markov chains for forecasting the revenue ratio which, as explained, allows to incorporate all available historical sequence of the behavior of the variable(s) of interest, even though at the cost of constraining the future values of those variables to its observed past sequence.

IV. TRANSITORY AND PERMANENT COMPONENTS OF DEBT: THE USE OF MARKOV'S CHAINS FOR THE CYCLICAL COMPONENTS OF THE MAIN MACRO VARIABLES

As has been documented elsewhere, emerging economies are characterized by large fluctuations in fiscal revenues and outlays that often exhibit a pro-cyclical behavior with respect to output. In other cases, explicit fiscal policy decisions seek to counter balance fluctuations in output as a mechanism to smooth consumption and output, generating fiscal surpluses in the ascending part of the cycle, and deficits in the downturn. A key complementary policy instrument in those cases has been the adoption of an annual fiscal target for the structural balance, distinct from the currently observed fiscal balance.

This section proposes a methodology to assess fiscal sustainability that incorporates the explicit treatment of the structural and the cyclical components of the primary fiscal balance as well as of the other forcing variables for the accumulation of debt. Due to large macroeconomic fluctuations in emerging economies, which have been analyzed extensively in the literature[15], fiscal revenues and outlays exhibit large fluctuations between the downturn and the recovery of output, affecting public debt.

Fiscal policy is sometimes used actively to attempt to balance the effect on output of foreign shocks, usually deepening the deterioration of the current account,

15 See, for example, Mendoza & Oviedo (2003); and IMF (2003 b).

increasing the cost of foreign financing, and enlarging the debt ratio. In other instances, in recognition of the negative effects on international capital markets of active counter-cyclical fiscal policy, authorities simply contemplate the deterioration of revenues, while at the same time try to cut outlays and increase tax rates, as well as the revenue base with a long lag, due to the political difficulties involved in adopting expenditure corrective measures and fiscal reforms. This also translates into the deterioration of the debt ratio, as long as international and domestic capital markets remain open for public financing. In both cases, the result is usually a deterioration of the debt situation of the country. Counter-cyclical and sustainable fiscal policy has also been adopted in some emerging economies, but in order to be effective, a basic requirement is to start with its implementation at the peak of the cycle, in order to generate the savings that will be used in the downturn.

The incorporation into the analysis of debt sustainability of the cyclical behavior of output, government revenues, and expenditures may provide a completely different perspective on the proper measure of tolerable bounds for the government´s debt ratio, as long as the cyclical component of these variables becomes effectively large in (some) emerging economies. In particular, in the hypothetical case in which the whole primary balance behaves cyclically, it would be natural to expect that a negative current outcome will be compensated in the future by a positive result, and then, the increase in debt that results in the downturn must be matched by the subsequent reduction coming from the application of future fiscal savings into the repayment of the debt. In that extreme case, international financial institutions could provide the required financing during the slump, with no default risk on the part of the indebted country.

Specifically, denoting by an asterisk the permanent component of a variable, and by the letter *c* its cyclical component[16], equation (1) can be written, in terms of the two components of the forcing variables, in the following way:

$$(6) \qquad d_t = \left\{ \left[\frac{R_t^*}{\gamma_t^*} \left(\frac{\gamma_t^*}{\gamma_t^*} \right) \right] d_{t-1} - cb_t^* \right\} + \left\{ \left[\frac{R_t^c}{\gamma_t^c} \left(\frac{\gamma_t^c}{\gamma_t^c} \right) \right] d_{t-1} - cb_t^c \right\}$$

$$= \{ \lambda_t^* d_{t-1} - cb_t^* \} + \{ \lambda_t^c d_{t-1} - cb_t^c \}$$

[16] It is assumed that anyone of the forcing variables, at time t, X_t, has the two components, permanent and cyclical, such that $X_t = X_t^* + X_t^c + \eta_t$, where η is a vector of i.i.d. random disturbances.

Where the expression in the first bracket of the right hand side can be seen as the permanent component of the current debt ratio, and the second one as its cyclical component, such that $d_t = d_t^* + d_t^c$.

In order to estimate the permanent and transitory components of the primary balance, it is applied the same methodology followed by the European Commission (2000) in the estimation of the annual target for the structural deficit of the public sector. In particular, to obtain the cyclical or transitory component of government primary outlays and revenues, the two following equations are estimated:

$$(7) \qquad g = c_1 + \varepsilon_g B_y + \mu , \qquad\qquad t = c_2 + \varepsilon_t B_y + \upsilon$$

Where B_y is the output gap[17], defined as the percentage difference between the currently observed and the permanent output. The structural components of expenditures, revenues, and the primary balance are estimated as:

$$(8) \qquad g^* = g - \hat{\varepsilon}_g B_y, \qquad t^* = t - \hat{\varepsilon}_t B_y, \qquad cb^* = cb - (\hat{\varepsilon}_t - \hat{\varepsilon}_g)_t B_y$$

In order to obtain the permanent component of output, the Hodrick & Prescott filter was used, with a smoothing parameter equal to 100, for the sample 1950-2010, using for 2004-2010 the expected output growth rates included in the IMF (2004) report[18], for the case of Colombia; and our own estimates, for the case of Costa Rica[19] (Graph 6).

In order to estimate equation (7), outlays and expenditures are expressed as proportions of the permanent output. The sample for estimation is much shorter than the one for output, since Colombia only has information available on the central government´s primary balance for 1979-2003; and Costa Rica, for 1973-2003. In the case of Colombia, neither the ratio of revenues to permanent output, nor the ratio of primary expenditures are stationary variables, while the output gap

---

17 $B_y \equiv Y - Y^* / Y^*$.

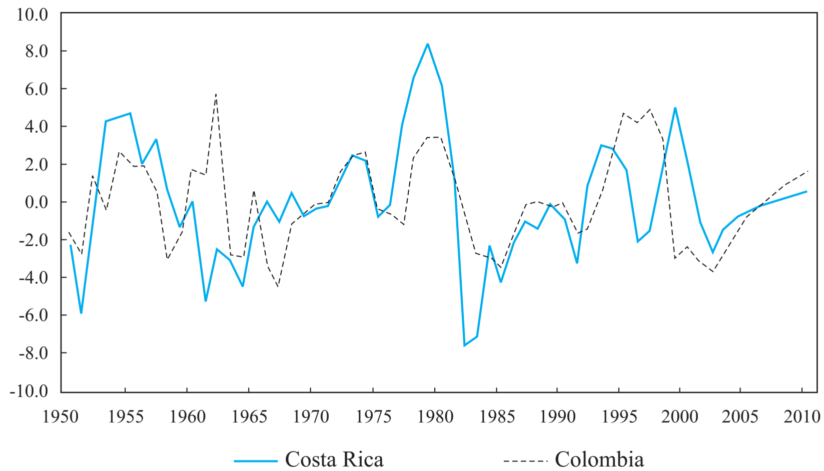
18 Given that in this IMF´s report the rate of growth for 2003 was 2.8, while in reality the economy grew at 3.6% in that year, the estimates of the IMF were adjusted for the projection period 2004-2010 in the following way: 3.9% for 2004 and 4% from 2005 to 2010, instead of 3.3% for 2004; 3.7 for 2005; 3.9% for 2006; and 4% from 2007 to 2010.

19 The estimated real rates of output growth for Costa Rica are: 5.3% for 2004 (the economy grew at 5.9% in 2003), and 5% from 2005 to 2010.

Graph 6

Output Gap: 100 * (*y* - *y**) / *y**

(Percentage)



Source: author´s estimates.

is. In the case of Costa Rica, all three variables are stationary. To reach stationarity in the residuals, some dummy variables were included in the regression. The results are shown in Table 8.

The coefficient that measures the sensitivity of revenues, as a proportion of permanent output, to the output gap are statistically equal to zero in both countries; but the sensitiveness of expenditures is equal to about -0.21 in Colombia, and 0.32 in Costa Rica. The negative sign in the case of Colombia implies that expenditures follow a counter-cyclical macroeconomic behavior in this country, in the sense that when output falls (below its permanent level, for example), expenditure increases; while in Costa Rica, it behaves pro-cyclically[20]. As a result, the primary

[20] These results differ from the ones obtained in Mora (2002), with an estimation sample that went until 2001. In that case, revenues and expenditures for Colombia were statistically equal to zero for Colombia, with the cyclical component being equal to zero. In the case of Costa Rica, both

Table 8

Sensitivity of Revenues to the Output Gap

Colombia

Dependent Variable: TRDCO_YP
Method: Least Squares
Sample (adjusted): 1972 2003
Included observations: 32 after adjusting endpoints
Convergence achieved after 10 iterations
TRDCO_YP = C(1) + C(2) * @TREND + C(3) * BREYCO + C(4) * DUMGCO + C(5) * DUMTCO + [AR(1) = C(6)]

	Coefficient	Std. Error	*t*-Statistic	Prob.
C(1)	0.085606	0.013825	6.192294	0.000000
C(2)	0.001090	0.000349	3.119289	0.004400
C(3)	-0.056500	0.081620	-0.692234	0.494900
C(4)	0.026287	0.006911	3.803568	0.000800
C(5)	-0.015965	0.007253	-2.201155	0.036800
C(6)	0.553668	0.193496	2.861392	0.008200

R-squared	0.740938	Akaike info criterion	-6.696163
Adjusted R-squared	0.691118	Schwarz criterion	-6.421337
Mean dependent var	0.126344	Log likelihood	113.138600
S.D. dependent var	0.014076	Durbin-Watson stat	1.757423
S.E. of regression	0.007823	Inverted AR Roots	0.550000
Sum squared resid	0.001591		

Costa Rica

Dependent Variable: TRDCR_YP
Method: Least Squares
Sample (adjusted): 1972 2003
Included observations: 32 after adjusting endpoints
TRDCR_YP = C(20) + C(21) * @TREND + C(22) * BREYCR + C(23) * DUMGCO + C(24) * DUMTCR

	Coefficient	Std. Error	*t*-Statistic	Prob.
C(20)	0.119303	0.005990	19.916810	0.000000
C(21)	0.000713	0.000153	4.663903	0.000100
C(22)	-0.017996	0.042621	-0.422235	0.676200
C(23)	0.029450	0.007867	3.743439	0.000900
C(24)	-0.022911	0.008747	-2.619178	0.014300

R-squared	0.656503	Sum squared resid	0.001605
Adjusted R-squared	0.605615	Akaike info criterion	-6.749730
Mean dependent var	0.146175	Schwarz criterion	-6.520709
S.D. dependent var	0.012279	Log likelihood	112.995700
S.E. of regression	0.007711	Durbin-Watson stat	1.621259

TRDCO_YP: Ratio of central government revenues to permanent output for Colombia. **BREYCO:** Output gap for Colombia. **GPCO_YP:** Ratio of central government primary expenditure to permanent output for Colombia. **DUMGCO, DUM_CO and DUMGCO2:** Dummy variables. **TRDCR_YP:** Ratio of central government revenues to permanent output for Costa Rica. **BREYCR:** Output gap for Costa Rica. **GPCR_YP:** Ratio of central government primary expenditure to permanent output for Costa Rica. **DUMGCO, DUMTCR:** Dummy variables.
Source: author´s estimates.

Table 8 (continuation)

Sensitivity of the Primary Expenditure to the Output Gap

Colombia

Dependent Variable: GPCO_YP
Method: Least Squares
Sample (adjusted): 1980 2003
Included observations: 24 after adjusting endpoints
Convergence achieved after 54 iterations
GPCO_YP = C(10) + C(12) * BREYCO + C(13) * DUM_CO + C(14) * DUMGCO + C(15) * DUMGCO2 + [AR(1) = C(16)]

	Coefficient	Std. Error	*t*-Statistic	Prob.
C(10)	0.144741	0.008674	16.686240	0.000000
C(12)	-0.207257	0.106265	-1.950374	0.066900
C(13)	-0.038780	0.007317	-5.299941	0.000000
C(14)	-0.026757	0.007281	-3.674738	0.001700
C(15)	0.012023	0.006327	1.900345	0.073500
C(16)	0.787296	0.162689	4.839267	0.000100

R-squared	0.828328	Akaike info criterion	-6.597630
Adjusted R-squared	0.780641	Schwarz criterion	-6.303116
Mean dependent var	0.139448	Log likelihood	85.171560
S.D. dependent var	0.017156	Durbin-Watson stat	1.551254
S.E. of regression	0.008035	Inverted AR Roots	0.790000
Sum squared resid	0.001162		

Costa Rica

Dependent Variable: GPCR_YP
Method: Least Squares
Sample (adjusted): 1973 2003
Included observations: 31 after adjusting endpoints
GPCR_YP = C(30) + C(32) * BREYCR

	Coefficient	Std. Error	*t*-Statistic	Prob.
C(30)	0.147696	0.002549	57.947700	0.000000
C(32)	0.317987	0.069298	4.588670	0.000100

R-squared	0.420648	Sum squared resid	0.005789
Adjusted R-squared	0.400670	Akaike info criterion	-5.618881
Mean dependent var	0.148791	Schwarz criterion	-5.526365
S.D. dependent var	0.018250	Log likelihood	89.092650
S.E. of regression	0.014129	Durbin-Watson stat	2.055630

TRDCO_YP: Ratio of central government revenues to permanent output for Colombia. **BREYCO:** Output gap for Colombia. **GPCO_YP:** Ratio of central government primary expenditure to permanent output for Colombia. **DUMGCO, DUM_CO and DUMGCO2:** Dummy variables. **TRDCR_YP:** Ratio of central government revenues to permanent output for Costa Rica. **BREYCR:** Output gap for Costa Rica. **GPCR_YP:** Ratio of central government primary expenditure to permanent output for Costa Rica. **DUMGCO, DUMTCR:** Dummy variables.
Source: author´s estimates.

balance, whose sensitivity to the gap is, according to (7), equal to 0.21 and -0.32, respectively, also follows a counter-cyclical behavior in Colombia, and a pro-cyclical behavior in Costa Rica[21] .

These coefficients are used to estimate the cyclical and permanent components of the primary balance, as a proportion of permanent output, according to (7). The other four variables needed to estimate the two parts of the right hand side of equation (6) are the permanent and cyclical components of the gross real rate of interest, and of output growth. The sum of the two components of each variable must be equal to the variable itself. The Hodrick & Prescott filter was used to decompose the gross real rate of interest[22] . The rate of growth of the permanent and cyclical components of output, was estimated from the results of the decomposition of GDP. The gross rate of growth of the permanent component of output, weighted by the share of that structural component on output is equal to γ^*; while the gross rate of the cyclical component of output weighted by the share on output of that cyclical component is equal to γ^c.

Graph 7 shows the cyclical component of output as a proportion of GDP, as well as its moving average of order 10 for both countries. For the available sample, two complete cycles are clearly demarked for the case of Colombia, the first one with about a 19 years length, and the other with length of about 16 years; obviously, under the assumption that the forecasted rates of output growth for 2004-2010 are reliable. In the case of Costa Rica, one complete cycle is observed, with a length of 19 years.

---

variables were sensitive and positive, but with a larger coefficient for expenditures, which implied a counter-cyclical characteristic of the primary balance. The addition of new observations, and also, the sample used for estimation of the permanent output, are in the base of those differences, since the results obtained from the filter of Hodrick & Prescott are very sensitive to the data at the end of the sample. The smoothing parameter of the filter of Hodrick & Prescott was set at the value of 100 (l=100) in both estimations.

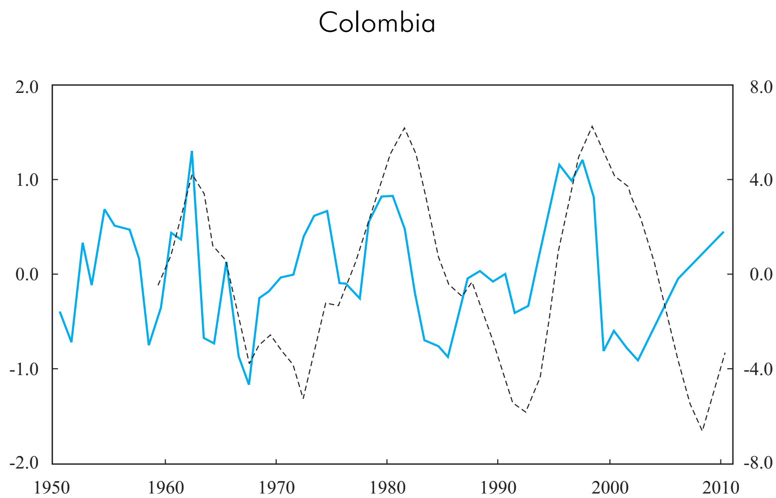
21 However, the confidence interval at the 5% significance level is [-0.4325, 0.0180] for Colombia, and [0.1763, 0.4597] for Costa Rica, which implies that the sign an then, the counter-cyclical behavior, in the case of Colombia, changes in the vicinity of the upper bound.

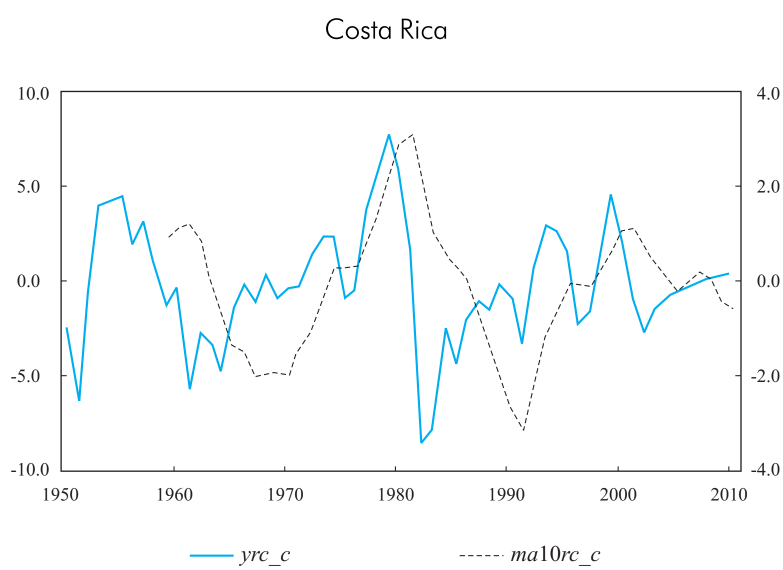
22 As in the case of GDP, a forecast of the real interest rate was made for the two countries. For Colombia, it was supposed to fall from 8.03% in 2003, to 7% in 2004; 6% in 2005; 5.5% in 2006-2007; and 5% since 2008 to 2010. For the case of Costa Rica, it was also supposed to fall from 4.1% in 2003, to 4% for 2005-2007; and 3.5% for 2008-2010. The nominal rate for 2003 was estimated as the ratio between interest payments in 2003 and the value of the outstanding debt at the end of 2002. The real interest rate was obtained as the ratio of the gross nominal rate and the gross inflation rate of the GDP´s deflactor.

Graph 7

Cyclical Component of Output and Moving Average of Order 10

(Percentage of GDP)





Source: author´s estimates.

However, in practice the observed cycles of output are not completely similar and symmetrical along time, and the relationship between the primary balance and the output gap is not always stable. Therefore, to illustrate the application of the proposed methodology to assess the sustainability of debt, two complementary approaches will be followed. In the first one, the previous information regarding the estimated parameters of the economies will be used to derive the structural and the cyclical components of the debt ratio. In the second approach, a perfectly symmetrical cycle for the transitory component of the primary balance will be imposed, and the structural component of the primary balance will be derived subtracting from the projected primary balance its cyclical component.

For the projection period, the cyclical and structural components of γ and R, up to 2010 are available from the results of the Hodrick & Prescott decomposition. Also, from the estimated output gap for this period, the cyclical component of the primary balance may be derived applying the econometric results of Table 8. In the case of the primary balance, it will be assumed that it follows a Markov process throughout the entire projection period. Given the forecasted value of the primary balance and its cyclical component, the structural component is derived as the difference of these other two. Table 9 contains the results in terms of the estimated structural and cyclical components of the debt ratio[23] .

As it may be observed in Table 9, the cyclical component of debt for this two countries is negligible. This is due to the small share of the cyclical component of the forcing variables of the debt equation like, for example, in the case of the primary balance, which is associated with the small coefficient of sensibility of the primary balance to the output gap, that was shown in Table 8. The same is true for the case of the variables entering into the cyclical component of the l coefficient of the debt equation, also shown in Table 9.

On the other hand in this methodology, for R^* larger than γ^*, it will be said that the debt is sustainable if the current structural component is smaller than or equal to the steady state level of the structural component of the debt ratio:

[23] It is worth mentioning that the application of the debt equation (1) to the historical data does not allow to replicate the reported behavior. For this reason, in Table 9 the application of that equation for the decomposition of the debt ratio between the structural and the cyclical components is only made for the projection period, even though all the forcing variables have been decomposed in those two components for the historical data.

$$(9) \qquad d^*_t \leq \left(\frac{\gamma^*}{R^* - \gamma^*}\right)(t^* - g^*)$$

Where the asterisk indicates the structural component of the corresponding variable. As is shown in the last column of Table 9, in the case of Colombia the debt ratio outstanding in the projection period is sustainable for 2004, but not sustainable for 2005-2008, simply because the average structural balance projected for 2006-2008 is negative, while in 2005 the structural component of debt (53.6%) is larger than the limit (27%). On the contrary, in the case of Costa Rica it is sustainable for the entire projection period, even if that country incurs in a (moderate) primary deficit, since the structural gross rate of growth is larger than the gross interest rate.

Regarding the projection of the primary balance as a Markov process, Table 10 shows the transition matrix corresponding to the historical information for both countries, as well as the mean value in each interval. For the case of Colombia, the transition matrix converges to the steady state after 43 periods, and in the case of Costa Rica, after 48. In steady state, the probability vector, which is also shown in Table 10, is the same, independently of the initial state. Finally, the last row of

Table 9
Forecasted Structural and Cyclical Components of the Debt Ratio

	Debt (percentage of GDP)			Coefficient (lambda)			Primary balance (percentage of GDP)		Debt limit for the structural component (percentage of GDP)
	Total	Structural Component	Cyclical Component	Total	Structural Component	Cyclical Component	Cyclical	Structural	
Colombia									
Initial debt ratio (2003)	51.3								
2004	52.8	52.1	0.7	1.03	1.02	0.01	-0.4184	0.4378	74.9
2005	53.6	53.6	0.0	1.02	1.02	0.00	-0.3770	0.3570	27.0
2006	54.7	55.0	-0.3	1.01	1.02	-0.01	-0.3363	-0.3437	0.0
2007	55.9	56.2	-0.3	1.01	1.02	0.00	-0.2768	-0.4293	0.0
2008	56.7	57.3	-0.6	1.01	1.02	-0.01	-0.2600	-0.4745	0.0
2009	57.6	58.1	-0.6	1.01	1.01	0.00	-0.2380	-0.6150	0.0
2010	58.4	58.9	-0.5	1.01	1.01	0.00	-0.2211	-0.6689	0.0
Costa Rica									
Initial debt ratio (2003)	40.1								
2004	38.7	38.7	0.0	0.99	0.98	0.01	0.3579	0.6739	125.9
2005	37.3	37.3	0.0	0.99	0.99	0.00	0.1845	0.9353	151.0
2006	36.2	36.2	0.0	0.99	0.99	0.00	0.2128	0.6568	145.7
2007	35.2	35.2	0.1	0.99	0.99	0.00	0.1722	0.6109	154.7
2008	34.2	34.3	-0.1	0.99	0.99	0.00	0.1490	0.5507	164.9
2009	33.3	33.4	0.0	0.99	0.99	0.00	0.1352	0.5134	184.4
2010	32.6	32.6	0.0	0.99	0.99	-0.01	0.1252	0.4862	228.5

Source: author´s estimates.

each block shows the expected primary balance after convergence of the transition matrix, which is the same independently of the initial state.

In order to illustrate the application of this methodology to the case of economies with a greater sensitivity to the cyclical components of the forcing variables than the

Table 10

Transition Matrices for the Primary Balance as a Share of GDP (%): Probability Estimates of Shifting From State *i* (Row), in *t* - 1, to State *j* (Column), in Time *t*

Colombia

Time *t* - 1	Time *t*					
	-2.98	**-1.49**	**0.00**	**1.49**	**4.47**	**5.96**
-2.98	0.50	0.50	0.00	0.00	0.00	0.00
-1.49	0.25	0.50	0.13	0.13	0.00	0.00
0.00	0.00	0.33	0.33	0.17	0.00	0.17
1.49	0.00	0.00	0.50	0.50	0.00	0.00
4.47	0.00	0.00	1.00	0.00	0.00	0.00
5.96	0.00	0.00	0.00	0.00	1.00	0.00
Steady state	0.17	0.33	0.25	0.17	0.04	0.04
Mean value (% of GDP)	-3.53	-2.07	-0.66	0.50	3.79	5.07
Expected value	-0.99	-0.99	-0.99	-0.99	-0.99	-0.99

Costa Rica

Time *t* - 1	Time *t*				
	-2.83	**-1.42**	**0.00**	**1.42**	**2.83**
2.98	0.33	0.33	0.33	0.00	0.00
-1.49	0.25	0.25	0.25	0.25	0.00
0.00	0.20	0.00	0.40	0.40	0.00
1.49	0.00	0.08	0.08	0.46	0.38
2.98	0.00	0.00	0.00	0.80	0.20
Steady state	0.07	0.08	0.13	0.48	0.23
Mean value (% of GDP)	-3.52	-0.61	-0.37	0.71	2.31
Expected value	0.53	0.53	0.53	0.53	0.53

Source: author´s estimates.

one estimated in Table 8, the expected values of the primary balance derived from the Markov process specific to Colombia and Costa Rica, described previously will be combined with a simulated perfect cyclical component of this variable, together with the estimated structural and cyclical components of the remaining forcing variables. In particular, in equation (6), it will be imposed the condition that the share of the estimated structural component of the gross rate of output growth is 75%, as will also be the case for the structural component of the gross interest rate. The average of the structural gross rates for the period 2000-2010, under the previous assumption, will be taken to estimate the λ-coefficients. The previous two assumptions result in a larger λ-coefficient for the transitory component of the debt ratio than the ones shown in Table 9. In addition, instead of estimating the transition matrix for the cyclical component of the primary balance, a transition matrix that describes a perfect cycle and that is shown in Table 11, will be imposed for both countries. There is no

Table 11

Simulated Transition Matrix for the Cyclical Component of the Primary Balance as a Share of GDP (%): Probability of Shifting From State *i* (Row), in *t* - 1, to State *j* (Column), in Time *t*

Colombia													
Time *t* - 1	**Time *t***												
	-1.96	**-1.31**	**-0.65**	**0.00**	**0.65**	**1.31**	**1.96**	**1.31**	**0.65**	**0.00**	**-0.65**	**-1.31**	**-1.96**
-1.96	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-1.31	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.65	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.65	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.31	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
1.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
1.31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
0.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
-0.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
-1.31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
-1.96	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Mean value (% of GDP)	-1.96	-1.31	-0.65	0.00	0.65	1.31	1.96	1.31	0.65	0.00	-0.65	-1.31	-1.96

Source: author´s estimates.

steady state for this matrix. The expected structural component of the primary balance is obtained as the difference between the two Markov processes, the one corresponding to the primary balance, and the other for its cyclical component.

Graph 8 shows the forecasted level of debt for 100 periods ahead and for different initial debt ratios. It may be observed that since the structural component of the primary balance was obtained as the difference between the forecasted Markov process corresponding to the primary balance, and the forecasted Markov process for the cyclical component, the first of which converges to some specific value, while the second one always fluctuates, this structural component also describes a cycle. For any initial debt ratio, in the case of Colombia the debt ratio explodes, while in Costa Rica converges to zero, due to the forecasted negative primary balance for the first country, and the positive one for the second country. Also, the proposed debt limit fluctuates. In Graph 8 only the path for each country with the largest number of periods in which the debt is sustainable is shown, according to the proposed criteria. In the case of the primary balance, the path corresponding to the initial observed condition for 2003 was chosen.

Graph 9 shows the projected primary balance and its cyclical component, for the mentioned paths that were considered to make the calculation included in Graph 8. As mentioned, in Colombia the primary balance converges to a negative ratio of -0.99%, while in the case of Costa Rica it converges to a positive ratio of 0.53%. Nevertheless, the cyclical component for both countries is the same, and the structural component, being the difference of the previous two components, is the mirror image of the cyclical component.

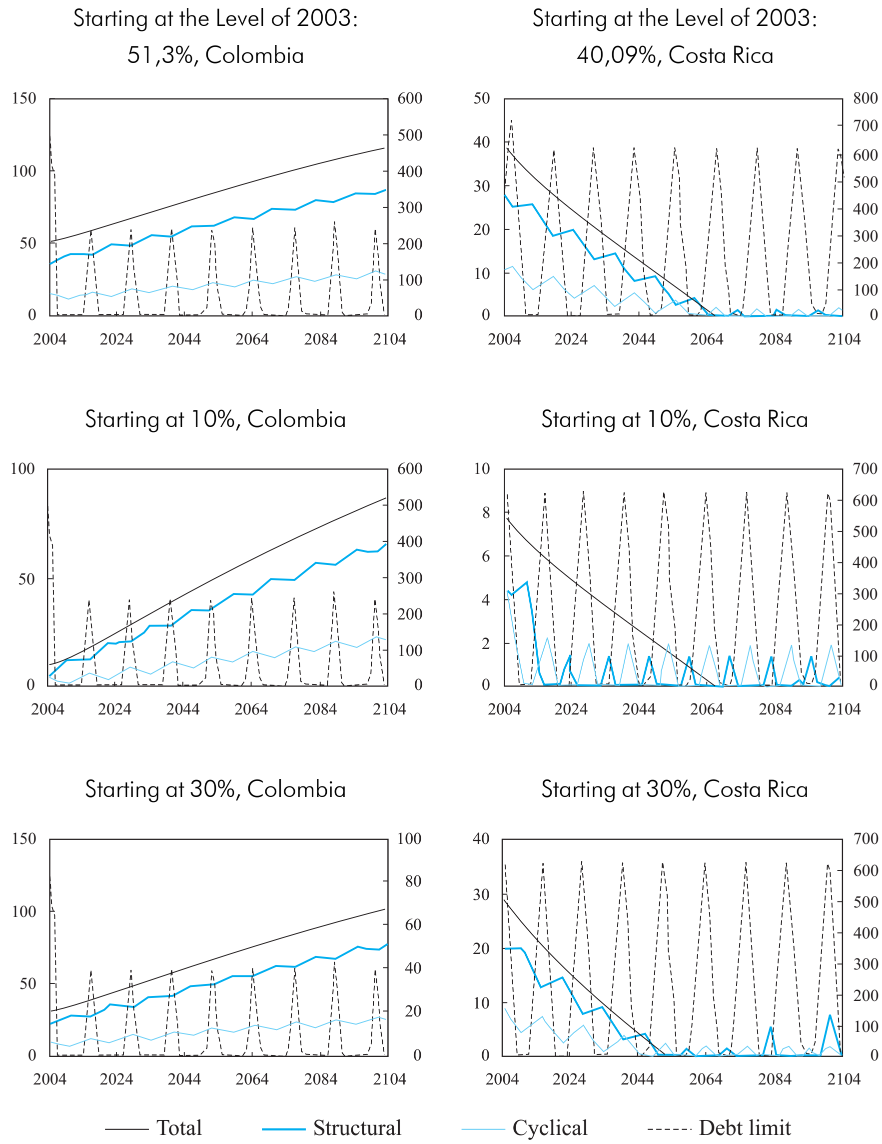
Finally, Graph 10 shows the number of periods, out of 100, for which the debt position is sustainable, according to the proposed criteria of equation (8), as well as with the classical criteria consisting of using the steady state debt ratio as the limit, i.e. equations (4) or (9), but with the observed forcing variables instead of the minimum primary balance (eq. (4)), or the structural components (eq. (9)). In Graph 10, different initial debt ratios are considered, one of which corresponds to the observed debt ratio in 2003 (51.3% for Colombia, and 40.1% for Costa Rica). Additionally, from the different paths, the one with the maximum, and the one with the minimum number of periods where debt is sustainable, are shown in the graph.

It may be observed in Graph 10 that for the case of Colombia, according to the classical criteria, none of the 100 periods presents a sustainable debt position,

Graph 8

Forecast of the Debt to GDP Ratio

(Debt Limit)

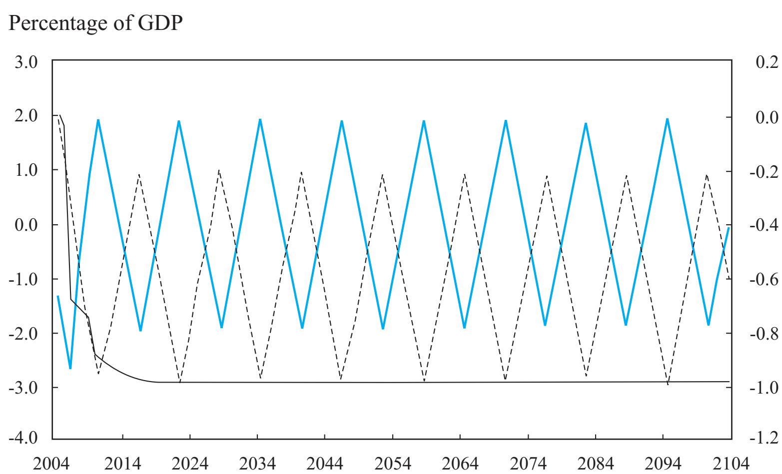


Source: author´s estimates.

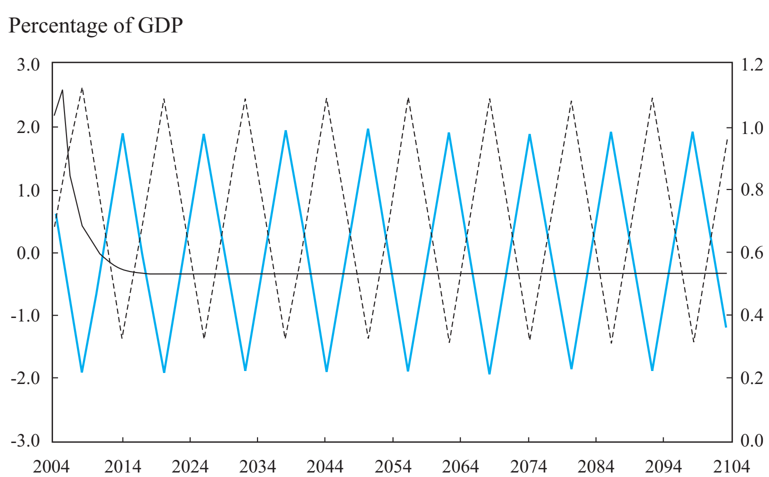
Graph 9

Forecast of the Primary Balance and its Components

Colombia



Costa Rica



Structural (right scale) Cyclical (right scale) Primary Balance

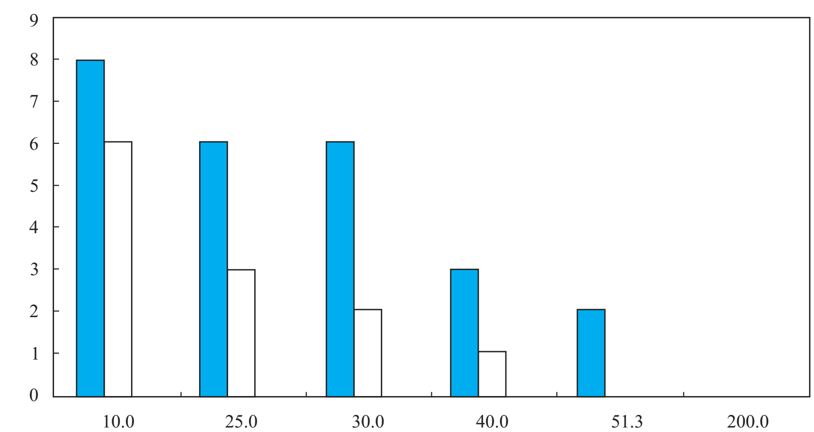
Source: author´s estimates.

Graph 10

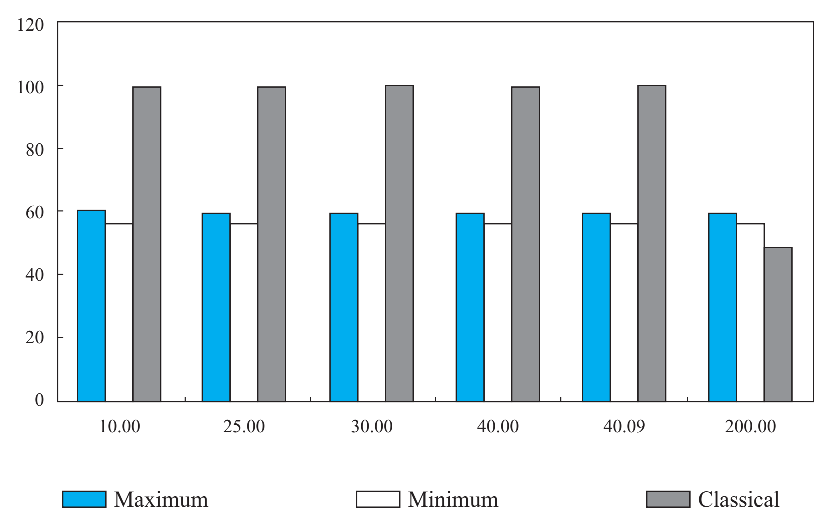
Number of Periods Where Debt is Sustainable, by Starting Debt to GDP Ratio

(Percentage)

Colombia



Costa Rica



Source: author´s estimates.

independently of the initial debt ratio; but from the point of view of the proposed indicator, there exist paths where debt is sustainable in several periods. Thus, for example, with an initial debt ratio of 10%, one path exhibits as many as 8 periods with this characteristic, even though there are paths where that number is only 6, for the same debt ratio. As expected, the number of periods where debt is sustainable decreases as the initial debt ratio increases. In the case of Costa Rica, the classical criteria identifies all 100 periods as periods where debt is sustainable, up to the observed debt ratio in 2003 (40.09%); while that number is 60, or 61, at the most, for different initial debt ratios, according to the proposed criteria. Only at an initial debt ratio of 200%, the classical criteria identifies periods (51) where debt is unsustainable, while the proposed criteria identifies 43 such periods, at the most, at that initial debt ratio.

Thus, even though the results obtained from the application of this methodology, in terms of the sustainability of debt, are very similar to the ones obtained from the other two, namely that the tendency of debt is not sustainable in Colombia, while it is in Costa Rica, a further qualification of this observation is gained by evaluating the effect of the transitory component of debt.

As in the cases of the IMF's methodology, or the one proposed by Mendoza & Oviedo, it could also be simulated a sample of random shocks for each of the projection periods in order to evaluate its incidence on the debt ratio and, also on its sustainability according to the proposed criteria. That could be done in the future. However, the characteristic aspect of this proposed methodology has already been expressed with the information contained in graphs 8 and 10, which is that even if the tendency of debt is explosive, as in the case of Colombia, in some periods it might be sustainable if a proper interpretation of the effect of its cyclical component is taken into account. And in the case of Costa Rica, not all of the periods in which the debt does not explode corresponds to periods where debt is sustainable, from the point of view of the structural indicator.

V. ESTIMATION OF THE PROBABILITY OF DEFAULT

In the three methodologies presented so far the interest rate paid by outstanding debt is assumed to be independent of the debt position of the country, while in reality the international capital market reacts very fast, through changes in the interest rate, to any event affecting the prospects of fiscal sustainability. In this section the interest rate is treated as an endogenous variable and is a function of the probability of default of the debt in each country.

As discussed in Eaton & Gersovitz (1981), when a government defaults its debt with international lenders, and as different from private agents, there is no explicit legal mechanism to compensate creditors to the extent that assets allow, and the concept applied to private agents according to which bankruptcy occurs when net worth is negative, is meaningless in the case of governments. Of course, there are costs for governments that repudiate debt, the most important of which could be the exclusion of future borrowing, which impedes the government of using foreign financing to mitigate periods of low income relative to trend. The decision of defaulting the debt corresponds to a situation when the benefits of default, which increase with the size of debt, are larger than the costs, associated with the variability and growth rate of the country's income, as well as with other variables determining its future demand for debt. These authors, explicitly model the behavior of the macroeconomic variables determining that demand for debt in the future.

The approach that will be followed in this section to identify a fiscal configuration of default is much simpler, and is derived from the standard criteria used to evaluate debt sustainability. In fact, implicit in the derivation of the debt limit for debt sustainability, like for example the debt limits proposed in (4), or in (9), is the imposition of a transversality condition that rules out any path of exploding debt in the future[24]. Therefore, only current debt positions smaller than the debt limit are said to be sustainable. Levels of debt larger than the limit explode and, in this sense, are not sustainable. Therefore, the criteria that will be used to identify a situation of default is the magnitude of the probability of exceeding the debt limit in any year:

$$\theta_t = Pr\ [\ d^s_t > d^*_t\ ] \qquad (10)$$

where θ_t is the probability of default, d^s_t is the structural component of the debt output ratio at time t, and d^*_t is the debt limit, which is the same structural debt limit defined in (9).

[24] For the derivation of an indicator of debt sustainability having the same form as (4) or (9), and in which the transversality condition to rule out explosive paths of debt is explicitly imposed see, for example, Talvi & Végh (1998).

On the other hand, to incorporate the probability of default in the determination of the interest rate, it assumed, as in Eaton & Gersovitz (1981), that lenders will only provide loans at an interest rate, r, large enough as to guarantee them an expected return at least as high as the risk free interest rate, r^f:

$$(11) \qquad (1 - \theta_t)\,(1 + r_t)\, d_t = (1 - r_t^f)\, d_t$$

where the left hand side corresponds the expected value of the gross return of debt after discounting the probability of default, and the right hand side is the gross return from debt at the risk free interest rate. Solving (11) for (1+r), it is clear that the interest rate increases with the probability of default.

The incorporation of the probability of default into the analysis also requires a modification of the debt limit, since instead of the (structural component of the) interest rate, it must be considered the (structural component of the) risk free interest rate:

$$(12) \qquad d^* \equiv \left(\frac{\gamma^*}{R^{f,*} - \gamma^*}\right)\,(t^* - g^*)$$

this is because, according to (11), the interest rate increases with the probability of default, and the debt limit corresponds to the (structural) debt ratio where fiscal policy is 100% sustainable or, in other words, where the probability of default is zero.

The substitution of (12) and (11) into (10) gives:

$$(13) \qquad \theta_t = Pr\left[\left(\frac{d^*_{t-1}}{cb^*_t}\right)\left(\frac{(1 + r_t^{*,f}) - (1 + g_t^*)}{1 + g_t^*}\right) > (1 - \theta_t)\right]$$

which is a useful expression to highlight a particular characteristic of the estimation process that will be followed for the probability of default. Since that probability appears in both sides of (13), an iterative process will be developed to estimate it. In the first iteration, an initial value of θ will be set and the probability that the structural component of debt exceeds the structural debt limit will be computed. If the resulting estimate of θ is different from the initially assumed value, then in the second iteration this resulting value will be taken as the initial value and the process of estimation of the probability of default will be repeated. Under some conditions, this process will converge.

There are two different elements affecting the convergence of that iterative process. According to (1), or (6), the larger the interest rate (or its structural component) is, the faster the accumulation of debt will be. And since, from (11), the interest rate increases with the probability of default, the larger the initial value of q in the iterative process, the larger will be the estimated probability that the debt ratio exceeds the debt limit. Therefore, there is an upward slopping relationship between debt and the probability of default. However, the debt limit itself is affected by the relative magnitude of the (structural component of the) risk free interest rate with respect to the (structural component of the) rate of growth of output. Even if it is likely that the interest rate be larger than the output growth, the risk free interest rate may be considerably smaller. In that case, in expression (13) the left hand side inside the probability bracket will fall as debt accumulates, and the resulting estimated probability will also decrease. Therefore, the second element exhibits a downward slopping relationship between debt (lagged debt) and the probability of default. The two components (the upward, and the downward slopping) may intersect. For this reason, under those circumstances, the iterative process may converge.

In the particular case of the fiscal programs that will be analyzed for the two countries in order to estimate the probability of default, and to deal with the interest rate as an endogenous variable, that will be precisely the situation. The main characteristics of these programs are shown in Table 12.

The forecast of the primary balance, its cyclical and structural components, is made in exactly the same way as in the previous section. Particularly, the primary balance is assumed to follow a Markov process characterized by the transition matrix already shown in Table 10, using historical information. The forecast is made for the initial state observed state in of 2003. However, in this section, around this expected value of the primary balance, a sample of 10.000 random shocks drawn from a standard normal distribution is simulated. Table 12 also shows the resulting mean value of the primary balance obtained from this random

[25] The initial state for the cyclical component of the primary balance corresponds to the one associated with the slowest path of accumulation of debt, in the previous section.

simulation. The cyclical component of the primary balance[25] describes a perfect cycle and the structural component is obtained as the difference between the simulated primary balance and its cyclical component. Both, the expected structural component and the mean of its simulated value are reported in Table 12.

Table 12 also reports the assumed risk free gross interest rate, which is the same for both countries. It is interesting to note that had the real interest rate in local

Table 12

Forecast of the Main Forcing Variables of the Accumulation of Debt

	2003	2004	2005	2006	2007	2008	2009	2010
Colombia								
Expected primary balance ratio **(*)**	-0.18	0.02	-0.02	-0.68	-0.71	-0.73	-0.85	-0.89
Mean of the simulated primary balance ratio **(*)**		0.02	-0.04	-0.69	-0.71	-0.75	-0.83	-0.90
Expected cyclical component of the primary balance ratio **(*)**		-1.96	-1.31	-0.65	0.00	0.65	1.31	1.96
Expected structural component of the primary balance ratio **(*)**		1.98	1.29	-0.03	-0.71	-1.39	-2.16	-2.85
Mean of the simulated structural component of the primary balance ratio **(*)**		1.98	1.27	-0.03	-0.71	-1.40	-2.14	-2.86
Risk free gross annual interest rate		1.01	1.01	1.01	1.01	1.01	1.01	1.01
Gross rate of output growth		1.04	1.04	1.04	1.04	1.04	1.04	1.04
Costa Rica								
Expected primary balance ratio **(*)**	1.59	1.03	1.12	0.87	0.78	0.70	0.65	0.61
Mean of the simulated primary balance ratio **(*)**		1.03	1.10	0.86	0.78	0.69	0.67	0.60
Expected cyclical component of the primary balance ratio **(*)**		-1.96	-1.31	-0.65	0.00	0.65	1.31	1.96
Expected structural component of the primary balance ratio **(*)**		2.99	2.43	1.52	0.78	0.05	-0.66	-1.35
Mean of the simulated structural component of the primary balance ratio **(*)**		2.99	2.41	1.52	0.78	0.03	-0.64	-1.36
Risk free gross annual interest rate		1.01	1.01	1.01	1.01	1.01	1.01	1.01
Gross rate of output growth		1.05	1.05	1.05	1.05	1.05	1.05	1.05

(*) Percentage.
Source: author's estimates.

currency been estimated as the treasury's bill interest rate adjusted by the devaluation of the currency and deflacted by inflation of the GDP deflactor, the resulting estimate would had been much larger than the real interest rate effectively paid on outstanding debt, which is estimated as the ratio of interest payments in one year, and the level of outstanding debt in the previous year. The reason may be that part of the debt, particularly that part denominated in local currency which is sometimes subscribed by enforcing of public institutions, pays a much lower effective interest rate. For this reason, the estimated 1% for the risk free rate applied to total central government debt seems quite reasonable.

In addition, Table 12 also reports the estimated rate of growth of output, which is the same estimate already mentioned, taken from the IMF, in the case of Colombia, but adjusted according to the last observed results for 2003; and from own estimates, in the case of Costa Rica.

The structural components of the rate of growth of output, the gross risk free real interest rate, and the gross real interest rate, were also estimated as in the previous section; roughly speaking, they correspond to the 75% of the corresponding variable, as explained in that section.

It is important to note that the main difference between Colombia and Costa Rica, in terms of their forecasted fiscal programs, lies in the predicted value of their primary balance, specially its structural primary balance (which determines the debt limit defined in (12)); as well as in the rate of growth of output, which is 1 percentage point larger in the case of Costa Rica, for 2005-2010, and 1.4 pp larger in 2004. In turn, the difference in the forecasted primary balance corresponds to the transition matrix and to the mean value in each state, which capture the historical behavior of this variable in the two countries, as already explained. The results of the simulations, in terms of both, the probability of default, and the endogenously estimated interest rate, are shown in Table 13.

It may be observed in Table 13 that in the case of both countries the probability of default, together with the interest rate, increase along time. However, in Colombia the probability of default becomes 100% in 2007, while in Costa Rica it occurs in 2009. That probability is very small in Costa Rica from 2004-2006, as well as in Colombia in 2004-2005. The basic characteristic of those two periods is the positive sign and large size of the structural primary balance. Soon after the structural primary balance becomes negative, the probability of default increases drastically.

Of course, the previous estimates are conditional to the realization of the predictions reported in Table 13. The authorities in both countries have ample space to undertake the necessary reforms to avoid the realization of the primary deficits contemplated in Table 13, since in the near future (two years) the fiscal position does not seem to be particularly complicated in any of the two countries. What the estimations do highlight in both countries is the need of fiscal reforms that avoid the repetition of the past behavior of the primary balance.

VI. CONCLUSIONS

- Colombia and Costa Rica are very different in one aspect from the point of view of debt sustainability, which is the difficulty that has had the first country to reach positive primary balances, even though a substantial reduction in

Table 13

Estimated Probability of Default and Endogenous Real Interest Rate

(Percentage)

	Probability of default	Real interest rate
Colombia		
2004	2.0	3.1
2005	8.6	10.5
2006	44.8	83.1
2007	100.0	-
2008	100.0	-
2009	100.0	-
2010	100.0	-
Costa Rica		
2004	0.1	1.1
2005	0.8	1.8
2006	6.3	7.8
2007	21.1	28.1
2008	48.8	97.2
2009	100.0	-
2010	100.0	-

Source: author's estimates.

the primary deficit has been achieved in the last 4 years. Since outstanding debt has to be paid with the present discounted value of the future primary balances, in order to be sustainable, in a general sense, Colombia is in an urgent need to reach that level in its fiscal results. In addition, the relatively low rate of output growth in Colombia affects negatively the evolution of debt. Given this basic difference between the two countries, the application of the three alternative methodologies explored in this paper allows to identify other aspects that are rather common to the two countries in terms of the characteristics of the evolution of debt.

- The use of the VAR models in the IMF's methodology may produce a large variance of the forecasted debt ratio if the data available for estimation purposes does not allow a reasonable fitness of the equations for the forcing variables in the debt equation. Additionally, since the historical inertia of some of the forcing variables is captured in the coefficients corresponding to the structure of lags, the forecasted value of some of the forcing variables, like for example, inflation, adopt strange negative values in the near future, which are followed by large positive values. This additional variability is not present when the stochastic characteristics of the main variables is captured by a Markov's chain.

- However, in the IMF's methodology, the simulation of the effect of the stochastic shocks on each of the forcing variables, and through this channel, on the debt ratio, has a great analytical importance for the evaluation of fiscal sustainability, since it allows to quantify the effect of uncertainty on the evolution of debt, as well as to measure the sensitivity of this variable to that unpredictable component. The debt limits corresponding to the levels of significance derived from the cummulative distribution of the debt ratio constitute valuable instruments to be incorporated into the analysis of this effect. Its application to Colombia and Costa Rica produces a mean value of the debt ratio that decreases for Colombia, increases for Costa Rica and has a much larger variance for the second country, due to the lower quality of the econometric results obtained in the estimation of the VAR with annual data (Costa Rica), instead of quarterly data (Colombia).

- In Mendoza & Oviedo's methodology, the "debt limit" may represent a very strict condition for debt sustainability, specially in the case of highly volatile revenues, since it corresponds to the steady state debt ratio under the worst possible realization of revenues. However, the substitution of the initially proposed minimum expenditure, for that consistent with a debt ratio of 50% relaxes that criteria, but then it is not clear which one of the two implied definitions of the debt limit is more relevant in order to measure the government's commitment to repay.

- The use of Markov's chains proposed in Mendoza & Oviedo to incorporate the uncertainty on government's revenues seems to be a very useful instrument for forecasting and simulation purposes, since it captures the historical sequence in the behavior of revenues, while at the same time providing estimates with a much lower variance than the one obtained with VAR models. This is true even if a sample of random shocks is used for forecasting purposes. In this case the range of the simulated debt ratios is much lower than in the VAR models while, at the same time, the dispersion around the expected value, and the variance of the simulated ratios, increase along the projection period. However, one of the possible limitations of this approach is that the behavior of the relevant variables is constrained to follow the same sequence observed in the past.

- The methodology to assess fiscal sustainability proposed in this paper may be appealing in the case of economies characterized by large cyclical fluctuations of output, and government's revenues and expenditures. In these cases, the cyclical component should exhibit and increasing phase, in the downturn of output, followed by a decreasing period in the ascending part of the cycle and, for this reason, balances out along the whole cycle. Therefore, fiscal sustainability must be assessed with respect to the structural component of debt.

- However, the use of historical information to estimate the cyclical components of the forcing variables, and from these results, the transition matrix for the primary balance, for forecasting purposes, might be misleading, since the observed cycles are not perfectly symmetrical and are subject to changes along time. In particular, the application of the methodology to Colombia and Costa Rica results in very small cyclical components of the debt ratio. For analytical purposes it seems to be desirable to impose a transition matrix for the primary balance that reproduces a perfect cycle and evaluate its incidence on the decomposition of the debt ratio between its cyclical and structural components.

- The results of simulating a perfect cycle for the transitory component of the primary balance in Colombia and Costa Rica identify a larger number of periods in which the debt ratio is sustainable, than under the classical approach.

- The estimation of the probability of default and the treatment of the interest rate as an endogenous variable, which was done in the last part of the paper sheds additional light on the interpretation of the results of incorporating uncertainty into the analysis of debt sustainability. In particular, for the cases of Colombia and Costa Rica, the results of estimating this probability indicate that if no fiscal reforms are undertaken, the debt may explode in three years, in the first country, and in six years in the second country. However, in the near future, the authorities in both countries have ample space to undertake the necessary reforms to avoid the realization of the primary deficits observed in the past. The estimations do highlight in both countries the need of those fiscal reforms.

REFERENCES

Eaton, Jonathan; Gersovitz, Mark (1981). "Debt with Potential Repudiation: Theoretical and Empirical Analysis", en *Review of Economic Studies*, XLVIII, p. 289-309.

European Commission (2000). "Public Finances in EMU-2000: Report of the Directorate General for Economic and Financial Affairs", 24 May 2000, European Commission, ECFIN/339/00

IMF (2003 a). "Sustainability Assessments-Review of Applications and Methodological Refinements", June.

IMF (2003 b). "World Economic Outlook-Chapter 3", September.

IMF (2004). "Colombia: Second Review Under the Stand-By Arrangements and request for Waiver of Performance Criteria-Staff Report", IMF, *Country Report* No 04/15, January.

Karlin, Samuel; Taylor, Howard M. (1975). *A first course in stochastic processes*, Second Edition, Academic Press, Inc.

Mendoza, Enrique; Oviedo, Marcelo (2003). "Public Debt, Fiscal Solvency, and Macroeconomic Uncertainty in Emerging Markets", Mimeo, University of Maryland and University of Iowa.

Mora, Humberto (2002). "Sostenibilidad de la deuda del Gobierno central en los países miembros del FLAR", *Coyuntura Económica*, December and *Boletín del FLAR*, December.

Kemeny, John G.; Snell J. Laurie (1960). "Finite Markov Chains", The University Series in Undergraduate Mathematics, D. Van Nostrand Company, Inc., Toronto and London.

Talvi, Ernesto; Vegh, Carlos (1998). "Fiscal Policy Sustainability: a Basic Framework", BID, December.

World Bank (2003). "Brazil: Stability for Growth and Poverty Reduction", en *Report* No. 25278-BR.

Comentarios sobre el texto *Assessing fiscal sustainability with alternative metodologies*

Álvaro Concha *

En primer lugar quisiera manifestarle mis agradecimientos a los editores invitados de la revista *ESPE*, Olga Lucía Acosta y Hernán Rincón, por la invitación que me hicieron para participar en este seminario, como comentarista del interesante documento que Humberto Mora nos ha descrito.

Como él nos lo ha dicho, el propósito del documento es aplicar y analizar los resultados de tres metodologías alternativas para estudiar el problema de la sostenibilidad de la deuda, particularmente la del Gobierno Nacional Central, en los casos de Colombia y de Costa Rica.

En la investigación, Humberto utiliza de una manera interesante procedimientos econométricos para incorporar de manera explícita, en las variables que determinan la evolución de la deuda, factores de incertidumbre y volatilidad.

Con este instrumental, replica las metodologías utilizadas por el Fondo Monetario Internacional y por los autores Enrique Mendoza y Marcelo Oviedo, para analizar el comportamiento de la deuda y evaluar bajo estos enfoques metodológicos su sostenibilidad.

* Director de Estudios Económicos de la Asociación Bancaria y de Entidades Financieras. Correo electrónico: Aconcha@asobancaria.com

I. EL PRIMER ENFOQUE

En el desarrollo de la primera metodología, además de hacer evidente el problema que enfrentan los analistas de las finanzas públicas en Colombia, en materia de carencia e integridad de los datos, Humberto estima los valores esperados de las variables que determinan la evolución de deuda en la ecuación clásica de su comportamiento como porcentaje del PIB, para posteriormente simular la probabilidad de distribución de la razón deuda a PIB, generando para tal efecto choques aleatorios sobre las variables "forzadas" y que se asocian con el crecimiento real de la economía, al balance primario como porcentaje del PIB, y a las tasas reales de interés.

Como bien lo menciona Humberto, el uso de modelos VAR, para estimar las relaciones de deuda a PIB, en el caso colombiano, produce una gran varianza en las proyecciones debido a las propiedades estadísticas de algunas de las variables que afectan la autocovarianza de los residuos en los modelos VAR.

II. EL SEGUNDO ENFOQUE

En el segundo desarrollo metodológico, expuesto por Humberto, y que sigue la primera aproximación analítica del estudio de Mendoza y Oviedo, se incluye el problema de la incertidumbre en el componente de los ingresos del Gobierno y su relación con la sostenibilidad de la deuda.

Para tal efecto, después de analizar la dinámica de los ingresos, y teniendo en cuenta de manera indirecta las inflexibilidades que afronta el Gobierno para ajustar sus gastos en situaciones de crisis, define los límites de deuda financiable bajo escenarios alternativos de volatilidad, tomando para efectos del análisis, el límite que se relaciona con el peor de los escenarios.

Posteriormente y para evaluar la dinámica de la deuda, supone que los ingresos siguen en el futuro procesos de Markov. El uso de la información histórica le permite definir los estados de las cadenas, para posteriormente evaluar el número de períodos en los que se alcanzan los límites de deuda bajo el supuesto de diferentes relaciones iniciales de deuda a PIB.

III. EL TERCER ENFOQUE

En la última parte del documento, Humberto se concentra en el análisis de la sostenibilidad fiscal, incorporando de manera explícita el tratamiento de los componentes estructurales y cíclicos de las variables que determinan la dinámica de la deuda a través del tiempo.

En primera instancia estima estos componentes para el balance primario, utilizando para tal efecto el enfoque de la brecha del producto. En esta etapa y para el cálculo de los componentes cíclicos y permanentes del producto así como de las tasas reales de interés, recurre al procedimiento tradicional de los filtros de Hodrick & Prescott. Posteriormente y para efectos de la realización de las proyecciones, asume que el balance primario sigue, como en el caso de la segunda metodología, procesos de Markov durante todo el período de estimación.

La utilización de los componentes cíclicos y permanentes de cada una de las variables que determinan la evolución de la deuda le permiten a su vez deducir el componente cíclico y permanente de la deuda en sí misma y, definir como deuda sostenible aquella en la que el componente estructural de un período en particular, es menor o igual al componente estructural de la deuda en estado estacionario.

IV. UNA NOTA SOBRE EL PROBLEMA DE LOS DATOS

En el desarrollo de su documento, Humberto manifiesta de forma explícita algunas de las fortalezas y debilidades de la utilización de modelos econométricos para lograr una mejor comprensión del problema de la sostenibilidad de la deuda.

Deja en claro el problema que afrontan los econometristas cuando se incluyen en las series de tiempo los resultados de las diferentes variables macro de una economía en crisis, como la que experimentó el país entre 1998 y el 2000, así como el problema de la carencia y consistencia de las cifras fiscales de Colombia.

V. OTRA NOTA SOBRE LAS CADENAS DE MARKOV

Quisiera mencionar en este punto las posibles observaciones que harían los econometristas a los procedimientos que incluye en su investigación Humberto, y en particular a la asimilación de los procesos de Markov, en lo que respecta a la dinámica de los determinantes de la deuda.

Aunque Humberto Mora no lo menciona en el escrito, es posible suponer que las cadenas de Markov que se utilizan son homogéneas y continuas (aun cuando las observaciones sean discretas en el tiempo). Si esto es así, y para que un proceso markoviano sea homogéneo, la matrices de probabilidades deben cumplir con una serie de condiciones necesarias de compatibilidad, tal vez lo que Humberto llama invertibles, y que él reconoce que las suyas no lo son.

Las dificultades que se presentan para probar estadísticamente si los datos corresponden a un proceso markoviano "compatible" y relacionadas con el hecho de que las matrices no son libres y están acotadas, imponen la necesidad de utilizar métodos como los bayesianos, sobre otras opciones como la estimación de las probabilidades mediante el método de máxima verosimilitud.

La verdad es que cualquier modelo de ingresos fiscales daría que la relación de ingresos a PIB es altamente persistente y muy asociada con el ciclo. Si esto es así, ¿por qué las matrices de transición son homogéneas? ¿El Estado en t+1 no depende no solo del Estado en t sino también de los observados en t-1, t-2 y t-n?

Quisiera resaltar que el trabajo en mi concepto es una aplicación muy valiosa para lograr una mejor aproximación al tema de la sostenibilidad fiscal bajo escenarios de volatilidad e incertidumbre, fenómenos comunes en nuestras economías.

La riqueza de estos enfoques metodológicos se relaciona precisamente con la posibilidad de proyectar y simular bajo diferentes escenarios de incertidumbre las posibles tendencias de la deuda, sin dejar de lado la evolución histórica de sus determinantes.

Si bien es cierto que existe un cierto grado de discrecionalidad por parte de las autoridades económicas para alcanzar determinados balances primarios, la verdad es que esta discrecionalidad pierde importancia cuando se fijan objetivos en materia de deuda pública, debido a la volatilidad del resto de variables que juegan un papel determinante en su evolución, fenómeno que le da una mayor relevancia al trabajo.

VI. APRECIACIONES SOBRE EL PROBLEMA DE LA SOSTENIBILIDAD DE LAS FINANZAS PÚBLICAS EN COLOMBIA

Finalmente, quisiera exponer ante este auditorio mis apreciaciones acerca del problema de la sostenibilidad de las finanzas públicas en Colombia.

A pesar de los esfuerzos que ha realizado el Gobierno en los últimos años para evitar la crisis de nuestro sistema pensional, las reservas del ISS se agotarán en el tercer trimestre del presente año. Desafortunadamente la mecha lenta de la bomba pensional se ha agotado, y la necesidad de mitigar sus efectos nos obligan a impulsar reformas que aunque van a implicar importantes sacrificios en el corto plazo, le darán a las generaciones futuras la posibilidad de mejores condiciones de vida y tranquilidad con respecto al problema de la sostenibilidad fiscal.

En esta legislatura el ejecutivo debe tramitar una adición presupuestal por un valor de $950 mil millones para financiar la brecha existente en el ISS. De esta manera, las transferencias a esa entidad totalizarán en el 2004 un valor de $2 billones, y se estima que para el año 2005 estos aportes bordearán los $4,2 billones.

Ante este panorama, las autoridades económicas han manifestado de manera explícita su intención de presentar ante el Congreso de la República un nuevo paquete de reformas que atacarían el problema desde dos frentes: reduciendo la carga pensional en cabeza del Estado y aumentando los ingresos de la nación.

Por el lado de las pensiones la propuesta incluye la eliminación gradual de los regímenes especiales con la excepción de militares, la imposición de un tope máximo a las pensiones de 25 salarios mínimos, el adelanto del período de transición previsto para 2014, y finalmente, la eliminación de la mesada 14 para los nuevos pensionados.

Debido a que los efectos de una nueva transformación del sistema pensional sólo se harían evidentes en el futuro, se ha planteado la urgente necesidad de solucionar el problema en el corto plazo, fortaleciendo los ingresos a través de una nueva reforma tributaria. Esta, además de contemplar la posibilidad de gravar las pensiones superiores a cuatro salarios mínimos, se concentraría en la ampliación de la base del IVA, incluyendo los bienes que hoy no están gravados con este impuesto, con la excepción de la educación, la salud y los servicios públicos.

Considero que este paquete de reformas debe ser tenido en cuenta en el Congreso de la República y creo que una solución soportada en la ampliación de la base del IVA con una tarifa generalizada, que afecte de la menor manera posible el ingreso de las clases menos favorecidas, modificaría nuestro sistema de tributación de manera estructural, haciéndolo más equitativo y balanceado.

El IVA es un impuesto progresivo, equitativo y eficiente, pues se aplica solamente en función de la capacidad de pago de las personas. Así lo demuestran diversos estudios, que han comprobado que los estratos más altos son los que soportan la mayor carga del impuesto con relación a sus ingresos.

Por otra parte, la progresividad de un impuesto se asocia con mayores tarifas a mayores niveles de ingreso. En el caso del IVA, esta debe entenderse en un sentido más amplio, no sólo en relación con la estructura del recaudo, sino también con la forma en que se distribuyen y utilizan dichos recursos en inversión social dirigida a las clases menos favorecidas.

Las conclusiones de la Misión del Ingreso Público demostraron que el IVA genera menos distorsiones sobre el crecimiento económico, que otras opciones como el impuesto a la renta, al patrimonio o a los movimientos financieros. De otra parte, existe unanimidad en que se presenta una mejora en la eficiencia económica del impuesto, reduciendo la evasión, cuando se aplican tasas uniformes de tributación a la mayor parte de la base. Esto implica el uso limitado de productos excluidos y exentos, aun cuando su existencia se justifique plenamente por razones de distribución del ingreso.

Considero que ante el complejo horizonte fiscal que enfrenta nuestro país, las soluciones no deben dar espera. La pasividad, o un nuevo aplazamiento del problema, serían las peores decisiones.

La dinámica del gasto pensional implica un aumento del déficit del Gobierno que se refleja en la evolución de la deuda pública. De acuerdo con nuestras estimaciones, de no aprobarse una reforma que ataque el creciente pago de pensiones o que compense este con una nueva reforma tributaria, la deuda pública se acercaría peligrosamente a 60% del PIB en el corto plazo. Este nivel, que contrasta con los límites que nos ha expuesto Humberto en su documento, tendría efectos perversos e impredecibles en los mercados internacionales, y más grave aún, se derrumbaría la confianza de los consumidores y empresarios colombianos, que vienen empujando la dinámica de nuestro crecimiento económico.

Así las cosas, el Gobierno está abocado a implementar las reformas necesarias con celeridad y prontitud, y en esta acción creo que debemos apoyarlo. El diagnóstico es claro y las soluciones, que nunca están libres de costos, se encuentran disponibles.