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Working Paper No. 58
July 2008

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A Vindication of Responsible Parties*

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July, 2008

Abstract

Electoral platform convergence is perceived unfavorably by both the popular press and many academic scholars. This paper provides a formal account of these perceived negative effects. We show that when parties do not know voters' preferences perfectly, voters prefer some platform divergence to the convergent policy outcome of competition between opportunistic, office-motivated, parties. We characterize when voters prefer responsible parties (which weight policy positively in their utility function) to opportunistic ones. Voters prefer responsible parties when office benefits and concentration of moderate voters are high enough relative to the ideological polarization between parties. In particular, with optimally-chosen office benefits, responsible parties improve welfare.

*We thank Becky Morton and Joao Santos for valuable conversations. All mistakes are our own.
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1 Introduction

Since the seminal papers of Hotelling (1929), Downs (1957), and Black (1958), spatial competition models have greatly advanced our understanding of elections and campaigning. The central prediction is the median voter theorem: Given voters with single-peaked preferences over a unidimensional policy space and two office-motivated parties who are perfectly informed about voter preferences, both parties locate at the median voter's preferred policy in the unique equilibrium. In particular, there is perfect policy convergence. This insight extends to many variations of the basic model. For example, policy convergence occurs when parties are policy-motivated but perfectly informed about voter preferences, or office-motivated and imperfectly informed about voter preferences.[1]

Platform convergence is not perceived favorably by the popular press, nor by many academic scholars. To wit, it is often argued that there is "not enough choice" between candidates, and that "they are all the same." Indeed, a manifesto calling for "responsible parties" presented in 1950 by the Committee on Political Parties of the American Political Science Association, which included the most influential political scientists of the day, was based on the premise that office-motivated parties do not provide the electorate enough choice. Their opening statement reads, "Popular government of a nation [...] requires political parties that provide the electorate with a proper range of choice between alternatives of action." (Committee on Political Parties (1950), page 15). Practical proposals are also presented on how differentiated party platforms should be formed and how parties should insure their implementation by elected candidates (pages 50–56). Page (1978), page 21, observes that "[Many] American political scientists, most notably Woodrow Wilson and E.E. Schattschneider, have [...] called for parties *to provide the electorate with sufficient choice"* (our italics).

Despite the wide-spread negative perception of policy convergence, we are not aware of

[1]Models that induce platform separation feature policy-motivated candidates with uncertain voter preferences (Wittman (1983), Calvert (1985)), office-motivated candidates with asymmetric information about voter preferences (Bernhardt, Duggan, and Squintani (2007, 2008)), platform-motivated candidates (Callander and Wilkie (2003), Kartik and McAfee (2006)), heterogeneity in candidate valence (Aragones and Palfrey (2002), Groseclose (2001)) or personality (Gul and Pesendorfer (2006)), the threat of entry by a third candidate (Palfrey (1984)), or citizen-candidate models where candidates cannot commit to policies (Osborne and Slivinsky (1996), Besley and Coate (1997)).

any theoretical explanation of the supposed negative effects of platform convergence. Indeed, under conventional assumptions, it is easy to draw the contrary welfare implication that policy convergence *maximizes* voter welfare. To elaborate, if voters have symmetric, single-peaked, risk-averse preferences, then they all prefer the known median policy to an election between two differentiated parties who win with equal probability and, hence, must be located symmetrically about the median policy. More generally, dropping symmetry, risk aversion alone implies that a majority of voters prefer convergence to the median to an election with two differentiated parties that produce an electoral tie. This insight extends to spatial models of electoral competition that derive policy divergence in equilibrium (e.g., the citizen-candidate models of Osborne and Slivinsky (1996) or Besley and Coate (1997)).

This note provides a formal account of the wide-spread unfavorable view of platform convergence and of claims that office-motivated parties do not provide voters with enough choice. We show that in a model where voters' preferences are not perfectly known by parties, some divergence in platforms is beneficial to *all* voters. If there is slight dispersion in party platforms around the median of the distribution of the median voter, then each platform individually targets the median less accurately; but collectively, the platform closest to the realized position of the median voter is more accurate. Thus, the message of the Downsian model is fully reversed: Differentiated platforms raise voter welfare. To gain intuition for this result, consider a simple example where the median policy may take two values, $-x$ and x, with equal probability. If the parties converge at the median of the distribution of medians, say 0, then the median voter incurs a loss independently of the median policy realization. Instead, if the left party chooses platform $-x$, and the right party chooses platform x, then the median voter can match the realized median policy with the appropriate party and achieve her bliss point. Our analysis extends this simple example to general distributions of voters exhibiting positive correlation across voter ideal points, and it shows that voters *unanimously* prefer some party platform dispersion.

Having concluded that some policy divergence unambiguously improves welfare, we give conditions under which the equilibrium policy outcome of competition between opportunistic parties (which are purely office-motivated) is worse than the outcome from responsible parties (which accrue some utility from policy) When the median policy is unknown, opportunistic parties' platforms converge to the median of the distribution of the median voter's position, as candidates fail to internalize the externality of providing voters choice. Responsible parties, in contrast, trade off the probability of winning the election against the policy realized in equilibrium, and they choose platforms closer to their preferred policies than the median of the median policy distribution—in equilibrium, they differentiate policies.

We identify conditions under which responsible parties provide voters higher ex-ante welfare than opportunistic parties by identifying when the distance between the parties' equilibrium platforms is positive but below a welfare-improving threshold. We show that the dispersion in equilibrium policies of responsible parties rises with the degree of ideological polarization and falls with the benefits from office and with the concentration of moderate voters. Indeed, if office benefits are too great or moderate voters are too concentrated, or parties are too ideologically similar, strategic incentives induce the parties to converge at the median of medians, replicating the equilibrium with opportunistic parties.

Thus, the economies for which responsible parties are welfare improving are described by a two-dimensional set in the space of parameters: essentially, all voters prefer responsible parties when the degree of ideological polarization between the parties is not too great relative to the level of office benefit and the concentration of moderate voters. In particular, there is always a range of office benefits for which responsible parties improve welfare. Indeed, given any degree of ideological polarization between parties and concentration of moderate voters, there is a level of office benefits that maximizes ex-ante welfare for all voters; and provided parties are sufficiently polarized, the optimal assignment of office benefits can achieve the socially optimal level of platform dispersion. Further, the welfare-improving threshold of office benefits increases with uncertainty about the location of the median voter, reinforcing the case for responsible parties.

On the route to our welfare result, we give general conditions for existence and uniqueness of equilibrium in the party location game, complementing the analyses of Wittman (1983) and Calvert (1985). We also show that provided the separation between the parties' platforms is not too great, all voters benefit from increases in the degreee of ideological polarization between parties, decreases in the concentration of moderate voters, or decreases in office benefits.

2 Analysis

We first consider the standard Downsian model, in which parties know the bliss point of the median voter, and show that a majority of voters are hurt by divergence from the median policy. The remainder of the paper addresses the situation in which parties are uncertain about the location of the median voter: In sharp contrast, voters unanimously prefer some divergence of party platforms, up to a welfare-improving threshold, to platforms that converge to the median of the distribution of the median voter's ideal policy. We later provide conditions under which the separation between the equilibrium policies of responsible parties respects this threshold and increases the ex ante welfare of all voters.

We suppose that voter preferences are described by symmetric and strictly concave utility functions defined on a one-dimensional space. A voter is indexed by her preferred policy θ, and when policy z is adopted, her utility is $u(\theta, z) = w(|\theta - z|)$, where w is twice-differentiable, strictly decreasing, and strictly concave, i.e., $w' < 0$ and $w'' < 0$. Because a voter's preferences are symmetric around her bliss point, each voter votes for the party whose platform is closest to her preferred policy, voting for the parties with equal probabilities when indifferent (or when the parties choose the same platform). We initially assume that the median μ of the distribution over θ is known and, without loss of generality, normalized to zero.

Let $W_\theta(x_L, x_R)$ represent the expected welfare of voter θ when parties use strategies x_L and x_R, where without loss of generality, we assume that $x_L \leq x_R$. We first compare the median convergent platforms, $x_L = 0 = x_R$, with any symmetric divergent platforms, $x_R = -x_L = x > 0$. Because platforms are symmetrically located around the median $\mu = 0$, each party wins with probability $1/2$. From risk aversion, we conclude that convergence to the median policy $\mu = 0$ is optimal for all voters. That is, because $w(|\cdot|)$ is strictly concave,

$$\begin{aligned}
W_\theta(-x, x) &= \frac{1}{2} w(|-x-\theta|) + \frac{1}{2} w(|x-\theta|) \\
&< w\left(\left|\frac{1}{2}(-x-\theta) + \frac{1}{2}(x-\theta)\right|\right) \\
&= w(|-\theta|) = \frac{1}{2} w(|-\theta|) + \frac{1}{2} w(|-\theta|) = W_\theta(0,0).
\end{aligned}$$

Now suppose that departures from the median are asymmetric, i.e., that $x_L \neq -x_R$, and suppose without loss of generality that $0 < |x_L| < |x_R|$. It follows that welfare is:

$$W_\theta (x_L, x_R) = w (|x_L - \theta|).$$

Then, by definition of the median, $W_\theta (0,0) > W_\theta (x_L, x_R)$ for a strict majority of voters θ. Obviously, if either $x_L = 0$ or $x_R = 0$, but $x_L \neq -x_R$, then each voter θ's welfare $W_\theta (x_L, x_R)$ coincides with $W_\theta (0,0)$. Summarizing, we have proved the following result.

Proposition 1 *Assume that parties know the median policy. Then compared to platforms that converge to the median policy, i.e., $x_L = x_R = 0$:*

1. *Any other platform pair (x_L, x_R) symmetric around the median policy, i.e., with $-x_L = x_R = x > 0$, strictly reduces the expected utility of all voters.*
2. *Any asymmetric pair (x_L, x_R) where neither party adopts the median policy, i.e., with $x_L \neq 0$ and $x_R \neq 0$, strictly reduces the expected utility of a majority of voters.*
3. *Voter utility is unchanged by any pair where at least one party adopts the median policy.*

In sum, platform divergence from a known median policy always hurts a majority of voters.

Henceforth, we consider a setting where the median voter is unknown to the parties. As is standard in the literature on Bayesian games, we model the location of the median voter as a random variable. We decompose the bliss point of a voter v as follows:

$$\theta_v = \delta_v + \mu + \varepsilon_v.$$

The term δ_v is fixed and represents the ex-ante difference between the bliss point of voter v and the median. We assume that δ_v is distributed symmetrically around zero across voters, with connected and bounded support. The term μ represents a common shock that shifts all voters' bliss points in the same way, and it is distributed with connected support according to the symmetric, continuously differentiable density f, i.e., $f(-\mu) = f(\mu)$ for all μ, with the implication that $f'(0) = 0$ and, since the support of f is connected, $f(0) > 0$. Finally, ε_v is an idiosyncratic shock that may change the position of voter v relative to the median.[2]

[2] All of the arguments of the paper go through when the ε_v term is dropped.

We assume that ε_v is distributed independently of μ with connected support according to a symmetric density, g, so that $g(\varepsilon_v) = -g(-\varepsilon_v)$ for all ε_v. We assume that there is no aggregate uncertainty in the distribution of idiosyncratic shocks ε_v. Hence, the median of the ex-post distribution is μ with probability one, but the identities of the ex-ante and ex-post median voters differ with probability one.

Given platforms (x_L, x_R), the ex-ante welfare of voter v is obtained by integrating over both the common shock μ to the electorate and the idiosyncratic shock ϵ_v. Parties do not see these shocks prior to their choices of platforms. Therefore, their platforms are treated as fixed in the calculation of ex-ante welfare. As highlighted in the introduction, however, the outcome of the election does depend on preferences, as the winning party will be the party located closest to the realized median. The ex-ante welfare of voter v, denoted $W_v(x_L, x_R)$, is defined formally as follows:

$$\begin{aligned} W_v(x_L, x_R) &= \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{[x_L+x_R]/2} w(|x_L - \mu - \delta_v - \varepsilon_v|) f(\mu)\, d\mu \right. \\ &\quad \left. + \int_{[x_L+x_R]/2}^{\infty} w(|x_R - \mu - \delta_v - \varepsilon_v|) f(\mu)\, d\mu \right] g(\varepsilon_v)\, d\varepsilon_v. \end{aligned}$$

If we view $W_v(-x, x)$ as a function of x, then it is the integral of strictly concave functions and is therefore itself strictly concave. We now show that voters unanimously prefer some differentiation of parties symmetrically around the median of medians.

Proposition 2 *Assume that the median policy is unknown to the parties. Then there exists $\bar{x} > 0$ such that $W_v(-x, x) > W_v(0, 0)$, for all $x \in (0, \bar{x})$ and all v. That is, voters unanimously prefer symmetrically differentiated platforms $(x_L = -x, x_R = x)$ to platform convergence at the median of medians,* 0.

The appendix to the paper contains the proofs of Proposition **??** and other results not proved in the text.

The preceding result implies that when the median voter is unknown, platform convergence hurts all voters. In this case, convergent platforms cannot perfectly target the median policy, even if they converge to the median of the median policy distribution. Introducing slight dispersion in party platforms, individual platforms target the unknown median less accurately. Collectively, however, the platform closest to the realized median can be more accurate than the median of the median policy distribution. If platform divergence is bounded above by the welfare-improving threshold, $\bar{x}$, then divergence in fact increases voter welfare. Thus, the message of the Downsian model is fully reversed: Platform convergence hurts *all* voters.

We now determine conditions under which voters prefer responsible parties to opportunistic, purely office-motivated, parties. In equilibrium, opportunistic parties choose platform $x = 0$, the median of the distribution of median voters. We model responsible parties as having mixed policy and office motivations, à la Wittman (1984) or Calvert (1985). For expositional simplicity, we restrict attention to parties L and R with symmetric bliss points $-\pi$ and π, respectively, so that $\pi > 0$ measures the degree of ideological polarization between parties. For simplicity, assume $\pi/2$ lies in the support of f. The party that wins office also receives a benefit $b \in \Re_+ \cup \{\infty\}$. We focus on symmetric equilibria, with $-x_L = x_R = x$. Hence, if policy x is implemented, then the utility of party L is $w(|-\pi - x|)$ if it loses the election and $w(|-\pi - x|) + b$ if it wins; and the utility of party R is $w(|\pi - x|)$ if it loses the election and $w(|\pi - x|) + b$ if it wins. The case $b = 0$ captures purely policy-motivated parties, and $b = \infty$ represents purely office-motivated parties. Parties are expected utility maximizers, and to ensure that the parties' expected payoffs are single-peaked over the relevant range, we assume

(A1) For all $\mu \in [0, \pi/2]$,

$$-\frac{f'(\mu)}{2} \leq \frac{f(\mu)^2}{1 - F(\mu)}.$$[3]

To guarantee that the equilibrium policy x increases in π, we assume

(A2) For all $x \in (0, \pi)$,

$$\frac{w''(\pi - x)}{w'(\pi - x)} > \frac{w'(x + \pi) - w'(\pi - x)}{w(x + \pi) - w(\pi - x)}.$$

Note that (A2) is satisfied when utilities are quadratic, i.e., $w(|x|) \equiv -|x|^2$.

Next, we characterize the equilibrium platforms of the game between the parties. Our result complements the analysis of Wittman (1983), who considers candidates who maximize

[3]This condition is satisfied if f is single-peaked and $1-F$ is log concave on $[0, \pi/2]$. Then $-f(\mu)/(1-F(\mu))$ is decreasing, which implies that the derivative has sign $-(1 - F(\mu))f'(\mu) - f(\mu)^2 \leq 0$, which implies (A1).

a "weighted mandate" (p.151), and Calvert (1985). Calvert (1985) establishes a continuity result for responsible parties, but does not give conditions for existence, uniqueness, nor characterize equilibrium for the case of mixed motives. We establish existence and uniqueness of a symmetric equilibrium.[4] We give a threshold level of office benefit, $b = -w'(\pi)/f(0)$, such that if the benefits of office exceed the threshold, then convergence at the median of medians, 0, is sustained as the unique equilibrium, extending Calvert's (1985) result on "estimated medians" to the case of sufficiently high office benefits. More interestingly, when benefits lie below the threshold, the equilibrium is characterized by the first-order condition of the parties, allowing us to derive comparative statics on the parameters that describe the economy. Intuitively, $-f(0)b$ represents the marginal cost to party R of a small move from the $(0,0)$ platform pair, while $w'(\pi)$ represents the marginal gain: convergence at 0 can be supported as an equilibrium if and only if the marginal cost of a deviation offsets the marginal gain.

Proposition 3 *Under (A1), there exists a unique symmetric equilibrium $(-x, x)$, and this satisfies $x \in [0, \pi)$. In particular, $x = 0$ if $b \geq -w'(\pi)/f(0)$, and $x > 0$ is the unique solution to*

$$\frac{w'(\pi - x)}{w(x+\pi) - w(\pi - x) - b} = f(0) \tag{1}$$

if $b < -w'(\pi)/f(0)$. Adding (A2), if equilibrium platforms are interior, i.e., $x \in (0, \pi)$, then responsible parties adopt more extreme policy positions as (a) the ideological polarization π grows, (b) the density $f(0)$ of the distribution of the median μ at zero falls, or (c) the benefits from holding office fall. That is, $\partial x/\partial \pi > 0$, $\partial x/\partial f(0) < 0$, and $\partial x/\partial b < 0$.

A corollary establishes conditions under which the welfare of every voter is higher with responsible parties than with opportunistic parties, vindicating appeals for responsible parties. From Proposition **??**, there is a threshold $\bar{x}$ such that if the symmetric equilibrium $(-x, x)$ in the game between responsible parties satisfies $0 < x < \bar{x}$, then all voters prefer the divergent responsible party platforms to the convergent platforms offered by opportunistic parties. In particular, when $b < -w'(\pi)/f(0)$, equilibrium is characterized by the first-order conditions, and the symmetric equilibrium satisfies $x > 0$. A straightforward continuity argument then

[4] Saporiti (2008), Proposition 2, derives a related result on existence and uniqueness in a model with probabilistic voting.

yields how changes in the parameters decribing the economy affect the symmetric equilibrium policy choices. Since $w'(\pi)/f(0)$ is negative, an implication is that there is always a non-degenerate range of office benefits for which responsible parties are beneficial.

Corollary 1 *Under (A1) and (A2), there is a threshold $\bar{x} > 0$ such that if the symmetric equilibrium $(-x, x)$ satisfies $0 < x < \bar{x}$, then all voters prefer responsible parties to opportunistic parties. Further, if $b + w'(\pi)/f(0)$ is negative and close enough to zero, then the symmetric equilibrium indeed satisfies $0 < x < \bar{x}$.*

The comparative statics results of Proposition **??** have further implications for voter welfare. For each voter v, let x_v maximize $W_v(-x, x)$, and let $x^* = \inf_v x_v$ be the smallest of the ex-ante bliss points of the voters. Then Proposition **??**, with concavity of $W_v(-x, x)$, implies that $W_v(-x^*, x^*) > W_v(0, 0)$ and $\frac{d}{dx} W_v(-x, x) > 0$ at all policies in $[0, x^*)$. By Proposition **??**, using the chain rule, we therefore have

$$\frac{\partial W_v}{\partial \pi} = \frac{dW_v}{dx}\frac{\partial x}{\partial \pi} > 0, \qquad \frac{\partial W_v}{\partial f(0)} = \frac{dW_v}{dx}\frac{\partial x}{\partial f(0)} < 0, \quad \text{and} \qquad \frac{\partial W}{\partial b} = \frac{dW_v}{dx}\frac{\partial x}{\partial b} < 0,$$

for all $x \in [0, x^*)$ and all voters v. In particular, if the equilibrium dispersion of the parties' platforms is not too great, then a small increase in the level of polarization increases the divergence between equilibrium platforms and hence increases the welfare of all voters; while increases in the concentration of moderate voters or office benefits have the opposite effect.

We now conduct a more detailed welfare analysis by imposing a parametric form for the utility functions of voters and parties, allowing us to derive an expression for the maximal welfare-improving threshold from Proposition **??** and to completely characterize when responsible parties raise welfare. Specifically, we assume

(A3) Voters and parties have quadratic utility, i.e., $w(|x|) = -|x|^2$ for all x.

The next lemma provides the foundations for the analysis, revealing that all voters share the *same* ex-ante ordering over platform pairs that are symmetric around the ex-ante median policy of zero, yielding unambiguous welfare comparisons. The result applies, in particular, to symmetric equilibria of the game between the parties.

Lemma 1 *Under (A3), each voter v's expected utility from symmetric platform pairs* $(-x, x)$ *is a fixed amount* δ_v^2 *less than the expected utility of the ex-ante median voter:*

$$W_{\delta_v}(-x, x) = -\delta_v^2 + W_0(-x, x).$$

Hence, we can drop the subscript on $W(\cdot)$ when deriving the optimal dispersion in party platforms from the perspective of voters. The ex-ante median voter's welfare is

$$W(-x, x) = -2\int_0^{\infty}\int_{-\infty}^{\infty}(-x+\mu-\varepsilon)^2 g(\varepsilon)\, d\varepsilon f(\mu)\, d\mu,$$

and differentiating with respect to x yields

$$\frac{d}{dx}W(-x, x) = -4\int_0^{\infty}\int_{-\infty}^{\infty}(-x+\mu-\varepsilon)\, g(\varepsilon)\, d\varepsilon f(\mu)\, d\mu = 0$$
$$\Rightarrow\ x = E[\mu-\varepsilon|\mu>0] = E[\mu|\mu>0].$$

Using mean-variance analysis, we can write $W(-x, x) = -2[(x - E[\mu-\epsilon])^2 - V[\mu-\epsilon]$. In particular, $W(-x, x)$ is an affine transformation of a quadratic function and is strictly concave as a function of x. We therefore have the following result.

Proposition 4 *Under (A3), the symmetric platform pair* $(-x, x)$ *that maximizes the welfare of all voters is given by* $x = E[\mu|\mu>0]$. *Thus, policy positions up to* $E[\mu|\mu>0]$ *increase voter welfare, and policy positions beyond* $E[\mu|\mu>0]$ *decrease voter welfare.*

In order to determine precise conditions under which voters unanimously prefer responsible parties to opportunistic parties, we must first sharpen our equilibrium characterization. With quadratic utilities, we can solve the first-order condition (**??**) in Proposition **??** to derive a closed-form expression for equilibrium platforms:

Corollary 2 *Under (A1) and (A3), the unique symmetric equilibrium of the game between the parties is given by* $x = 0$ *if* $b \geq 2\pi/f(0)$, *and it is given by*

$$x = \frac{2\pi - bf(0)}{4f(0)\pi + 2}$$

whenever $b < 2\pi/f(0)$.

With this result in hand, we can state our final welfare result, which completely characterizes the conditions under which responsible parties increase voter welfare. From the foregoing analysis, we know that $W(-x, x)$ is a concave, quadratic function of x: It is therefore symmetric about its maximizer, $E[\mu|\mu > 0]$. Thus, the unique solution to the indifference condition $W(0, 0) = W(-\bar{x}, \bar{x})$ is $\bar{x} = 2E[\mu|\mu > 0]$, which pins down the maximal threshold satisfying the conditions of Proposition **??** and Corollary **??**. Thus, with quadratic preferences, we see that by all voters prefer responsible parties to opportunistic ones when the degree of ideological polarization is not too great relative to the level of office benefits and concentration of moderate voters. We also see that the scope for improvement of voter welfare by responsible parties increases with the level of uncertainty, through $E[\mu|\mu \geq 0]$, regarding the location of the median voter.

Proposition 5 *Under (A1) and (A3), responsible parties provide higher welfare to all voters than opportunistic parties in equilibrium if and only if:*

$$0 < \frac{2\pi - bf(0)}{4f(0)\pi + 2} < 2E[\mu|\mu \geq 0]. \tag{2}$$

These results imply that the class of economies in which responsible parties increase voter welfare is described by a two-dimensional manifold of parameters $(\pi, b, f(0))$ such that

$$\frac{2\pi - bf(0)}{4f(0)\pi + 2} = 2E[\mu|\mu \geq 0].$$

We can then explicitly calculate the parameter values under which responsible parties raise voter welfare. For example, suppose that $1 > 4f(0)E[\mu|\mu \geq 0]$, so that the welfare comparison depends on the degree of polarization, π, and office benefits, b. Then for high enough π, we have $\frac{2\pi}{4f(0)\pi+2} > 2E[\mu|\mu \geq 0]$, and we can calculate the range of office benefits such that responsible parties increase voter welfare:

$$\frac{2\pi[1 - E[\mu|\mu \geq 0](4f(0) + \frac{1}{\pi})]}{f(0)} < b < \frac{2\pi}{f(0)}.$$

These insights also shed light on the issue of optimal institutional design: a propitious choice of office benefit of $b = \frac{\pi[2-4f(0)E[\mu|\mu\geq 0]]-2E[\mu|\mu\geq 0]}{f(0)}$ can achieve exactly the socially-optimal policy locations from Proposition **??**. Our analysis suggests, in this context, that office benefits should be higher when parties are more polarized, in order to induce platform moderation on the optimal policy, $x = E[\mu|\mu \geq 0]$.

3 Conclusion

The central prediction of spatial models of electoral competition is the median voter theorem. However, platform convergence is unfavorably perceived by the popular press, and by many academic scholars. It is often argued that there is "not enough choice" between candidates (Committee on Political Parties (1950)). We provide a formal account of the supposed negative effects of platform convergence in a rational-choice theoretic setting. We show that in a model where voter preferences are not perfectly known, voters unanimously prefer some amount of platform divergence to the policy convergent outcome. Further, we give general conditions for existence and uniqueness of equilibrium in the game between the parties, extending the analyses of Wittman (1983) and Calvert (1985). We determine the conditions under which responsible parties provide all voters higher welfare than opportunistic parties—this occurs when the degree of ideological polarization is not too great relative to the level of office benefit and the concentration of moderate voters—and we present comparative statics on party platforms and voter welfare. Specifically, when the separation between the parties' platforms is not too great, the welfare of all voters increases with an increase in ideological polarization, a decrease in the concentration of moderate voters, or a decrease in the level of office benefit. Finally, we characterize the level of office benefits that maximize ex-ante voter welfare.

A Proofs of Propositions

Proof of Proposition ??. Consider the welfare of any voter v with parameter δ:

$$W_\delta(-x, x) = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{0} w(|-x-\mu-\varepsilon-\delta|) f(\mu) \, d\mu + \int_{0}^{\infty} w(|x-\mu-\varepsilon-\delta|) f(\mu) \, d\mu \right] g(\varepsilon) \, d\varepsilon.$$

Differentiating with respect to x, yields

$$\begin{aligned}
\frac{d}{dx}W_\delta(-x,x) &= \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{0} w'(|-x-\mu-\varepsilon-\delta|)\frac{\partial}{\partial x}|-x-\mu-\varepsilon-\delta| f(\mu)\,d\mu\right. \\
&\quad \left.+\int_{0}^{\infty} w'(|x-\mu-\varepsilon-\delta|)\frac{\partial}{\partial x}|x-\mu-\varepsilon-\delta| f(\mu)\,d\mu\right] g(\varepsilon)\,d\varepsilon \\
&= \int_{-\infty}^{+\infty}\left[\int_{0}^{\infty}\left[w'(|-x+\mu-\varepsilon-\delta|)\frac{\partial}{\partial x}|-x+\mu-\varepsilon-\delta|\right.\right. \\
&\quad \left.\left.+w'(|x-\mu-\varepsilon-\delta|)\frac{\partial}{\partial x}|x-\mu-\varepsilon-\delta|\right] f(\mu)\,d\mu\right] g(\varepsilon)\,d\varepsilon.
\end{aligned}$$

Define

$$\gamma(x,\mu,\varepsilon) = w'(|-x+\mu-\varepsilon-\delta|)\frac{\partial}{\partial x}|-x+\mu-\varepsilon-\delta| + w'(|x-\mu-\varepsilon-\delta|)\frac{\partial}{\partial x}|x-\mu-\varepsilon-\delta|.$$

Hence, because we integrate over the positive reals, $\mu > 0$, there are three cases:

Case 1: $\mu-\delta-\varepsilon > 0$ and $-\mu-\delta-\varepsilon > 0$. Then $|\mu-\delta-\varepsilon| > |-\mu-\delta-\varepsilon|$, because $\mu > 0$, and $\gamma(0,\mu,\varepsilon) = -w'(|\mu-\varepsilon-\delta|) + w'(|-\mu-\varepsilon-\delta|)$.

Case 2: $\mu-\delta-\varepsilon > 0$ and $-\mu-\delta-\varepsilon < 0$. Then $\gamma(0,\mu,\varepsilon) = -w'(|\mu-\varepsilon-\delta|) - w'(|-\mu-\varepsilon-\delta|)$.

Case 3: $\mu-\delta-\varepsilon < 0$ and $-\mu-\delta-\varepsilon < 0$. Then $|-\mu-\delta-\varepsilon| > |\mu-\delta-\varepsilon|$, because $\mu > 0$, and $\gamma(0,\mu,\varepsilon) = w'(|\mu-\varepsilon-\delta|) - w'(|-\mu-\varepsilon-\delta|)$.

In all cases, $w' < 0$ and $w'' < 0$ imply $\gamma(0,\mu,\varepsilon) > 0$.

Let Δ be a compact set containing δ_v for all voters v. We have shown that $\frac{\partial}{\partial x}W_\delta(-x,x) > 0$ at $x = 0$ for all δ. Since $W_\delta(-x,x)$ is concave in x, $\arg\max_x W_\delta(-x,x)$ is uniquely defined, and therefore $\arg\max_x W_\delta(-x,x) > 0$ for all δ. By the theorem of the maximum, $\arg\max_x W_\delta(-x,x)$ is continuous in δ, and therefore it attains a minimum, $x^* > 0$, on the compact set Δ. Given $\delta \in \Delta$, concavity of $W_\delta(-x,x)$ then implies that $W_\delta(-x,x) > W_\delta(0,0)$ for all $x \in (0,x^*]$. Therefore, for all v and all $x \in (0,x^*]$, we have $W_v(-x,x) > W(0,0)$. ■

Proof of Proposition ??. Given locations $x_L < x_R$, party R wins the election whenever $\mu > [x_L + x_R]/2$, so she wins with probability $1 - F([x_L + x_R]/2)$. Given $x_L = x_R$, party R wins with probability one half, creating a payoff discontinuity. Suppose that L chooses a location $-x < 0$. Because $\pi \geq 0$, i.e., the bliss point of R is positive, party R will never choose a location $x_R < -x$. Hence, party R maximizes:

$$U(-x,x_R) = \begin{cases} w(|-x-\pi|)F([-x+x_R]/2) & \text{if } -x < x_R \\ +[w(|x_R-\pi|)+b][1-F([-x+x_R]/2)] & \\ w(|-x_R-\pi|)+\frac{1}{2}b & \text{if } -x = x_R, \end{cases}$$

which is differentiable whenever $-x < x_R$ or $-x = x_R = 0$. Differentiating, we obtain:

$$\frac{\partial}{\partial x_R} U(-x, x_R) = w(|-x-\pi|) \frac{f([-x+x_R]/2)}{2}$$
$$+w'(|x_R-\pi|) \frac{\partial |x_R-\pi|}{\partial x_R}[1-F([-x+x_R]/2)] + [w(|x_R-\pi|)+b][-\frac{f([-x+x_R]/2)}{2}].$$

We first establish that the platform x_R maximizing $U(-x, x_R)$ is such that $x_R \leq \pi$. In case $x_R > \pi$, define $x' = \min\{x, \pi\}$, and note that $F([-x+x_R]/2) \leq 1/2$ and

$$\begin{aligned}
&U(-x,x') - U(-x,x_R) \\
&= w(x+\pi)[F([-x+x']/2) - F([-x+x_R]/2)] \\
&\quad +[w(\pi-x')+b][1-F([-x+x']/2)] - [w(|\pi-x_R|)+b][1-F([-x+x_R]/2)] \\
&= [w(\pi-x')+b-w(x+\pi)][F([-x+x_R]/2) - F([-x+x']/2)] \\
&\quad +[w(\pi-x') - w(|\pi+x_R|)][1-F([-x+x_R]/2)].
\end{aligned}$$

Further, $w(\pi - x') + b > w(x+\pi)$, and either $F([-x+x_R]/2) > F([-x+x']/2)$ or $F([-x+x_R]/2) < 1$. We conclude that $U(-x,x') > U(-x,x_R)$, so x_R is not a maximizer, as claimed.

We now show that given $x \in [0,\pi]$, there is at most one solution to the first-order condition $\frac{\partial}{\partial x_R} U(-x,x_R) = 0$ on the interval $(-x,\pi]$ (or on $[-x,\pi]$ if $-x=0$), and if there is one, then it is the unique maximizer of $U(-x,x_R)$ on $(-x,\pi]$ (or on $[-x,\pi]$ if $-x=0$). Consider:

$$\begin{aligned}
&\frac{\partial^2}{\partial x_R^2} U(-x,x_R) \\
&= w(|-x-\pi|)\frac{f'([-x+x_R]/2)}{4} + w''(|x_R-\pi|)\left(\frac{\partial|x_R-\pi|}{\partial x_R}\right)^2 [1-F([-x+x_R]/2)] \\
&\quad -w'(|x_R-\pi|)\frac{\partial|x_R-\pi|}{\partial x_R}\frac{f([-x+x_R]/2)}{2} + w'(|x_R-\pi|)\frac{\partial|x_R-\pi|}{\partial x_R}[-\frac{f([-x+x_R]/2)}{2}] \\
&\quad +[b+w(|x_R-\pi|)][-\frac{f'([-x+x_R]/2)}{4}] \\
&= [w(x+\pi)-b-w(\pi-x_R)]\frac{f'([-x+x_R]/2)}{4} + w''(\pi-x_R)[1-F([-x+x_R]/2)] \\
&\quad +w'(\pi-x_R) f([-x+x_R]/2).
\end{aligned}$$

Rewriting the first derivative as

$$\frac{\partial U(-x,x_R)}{\partial x_R} = [w(x+\pi)-b-w(\pi-x_R)]\frac{f([-x+x_R]/2)}{2} - w'(\pi-x_R)[1-F([-x+x_R]/2)],$$

we substitute the first-order condition into the second derivative to obtain

$$
\begin{aligned}
&\left.\frac{\partial^2}{\partial x_R^2} U\left(-x, x_R\right)\right|_{\frac{\partial}{\partial x_R} U(-x, x_R)=0} \\
= \;& \left[w\left(x+\pi\right)-b-w\left(\pi-x_R\right)\right] \frac{f'\left(\left[-x+x_R\right]/2\right)}{4}+w''\left(\pi-x_R\right)\left[1-F\left(\left[-x+x_R\right]/2\right)\right] \\
& +\left[w\left(x+\pi\right)-w\left(\pi-b-x_R\right)\right] \frac{f\left(\left[-x+x_R\right]/2\right)}{2[1-F\left(\left[-x+x_R\right]/2\right)]} f\left(\left[-x+x_R\right]/2\right) \\
< \;& \frac{1}{2}\left[w\left(x+\pi\right)-b-w\left(\pi-x_R\right)\right]\left[\frac{f'\left(\left[-x+x_R\right]/2\right)}{2}+\frac{f\left(\left[-x+x_R\right]/2\right)^2}{\left[1-F\left(\left[-x+x_R\right]/2\right)\right]}\right] \\
\propto \;& -\left[\frac{f'\left(\left[-x+x_R\right]/2\right)}{2}+\frac{f\left(\left[-x+x_R\right]/2\right)^2}{\left[1-F\left(\left[-x+x_R\right]/2\right)\right]}\right] \leq 0,
\end{aligned}
$$

where the last inequality follows from (A1). We have shown that every solution to the first-order condition satisfies the second-order sufficient condition for a strict local maximizer. Therefore, by continuity, the solutions are locally isolated. Consider any such solution x_1, and if x_1 is not unique, suppose there is a solution greater than x_1; in particular, let x_2 be the next solution, i.e., if y solves (**??**) and $y < x_2$, then $y \leq x_1$. Assume without loss of generality that $U(-x, x_1) \geq U(-x, x_2)$. Since x_2 is a strict local maximizer, it follows that $\min\{U(-x, y) \mid y \geq [x_1, x_2]\} < U(-x, x_2)$. But then this minimum must be achieved at some $y \in (x_1, x_2)$, and y must solve the first-order condition, a contradiction. If there is a solution to the first-order condition, then a similar argument implies that it is the unique maximizer, as claimed.

It follows that given $x \in [0, \pi]$, there is a unique best response $r(x)$ in $[0, \pi]$ for party R. Indeed, if z and z' are distinct best responses, then $z, z' \in \{0, \pi\}$, for if $z \in (0, \pi)$, then z satisfies the first order condition and, by the above argument, is the unique best response for party R. Thus, 0 and π are both best responses. Then $U(-x, \cdot)$ achieves a minimum, say z^*, on $[0, \pi]$, but then z^* satisfies the first order condition and is the unique best response, a contradiction. That the mapping $r\colon [-\pi, \pi] \to [-\pi, \pi]$ so-defined is continuous follows directly from an application of the theorem of the maximum. By Brower's theorem, r admits a fixed point, and we claim that $(-x, x)$ is a symmetric equilibrium. By symmetry of the parties, it is immediate that $-x$ is a best response to x for party L in $[-\pi, 0]$. It remains to be shown that party R cannot deviate profitably to a platform y such that $-x \leq y < 0$. We have argued that $r(x)$ is the unique best response in $(-x, \pi]$, so we need only verify that $y = -x$ is not profitable. Consider the sequence $\{y_n\}$ defined by $y_n = y + \frac{1}{n}$. Since $y < 0$ and F and w are continuous, it follows that $1 - F(y_n) \to 1 - F(y) > 1/2$ and $w(-y_n + \pi) \to w(y + \pi)$, and then

$$U(-x, y) = w(-y + \pi) + \frac{1}{2}b \leq w(-y + \pi) + [1 - F(y)]b = \lim_{n \to \infty} U(-x, y_n).$$

Since $U(-x, y_n) < U(-x, r(x))$ for all n, we conclude that $U(-x, y) \leq U(-x, r(x))$, as required.

We now restrict attention to symmetric equilibria and impose $x_L = x_R = x$. The first-order condition characterizing a symmetric equilibrium $(-x, x)$ with $x \in (0, \pi)$ is

$$\frac{\partial}{\partial x_R} U(-x, x_R)\bigg|_{x_R = x} = w\left(x + \pi\right) \frac{f\left(0\right)}{2} - w'\left(\pi - x\right)\left(1/2\right) - \left[w\left(\pi - x\right) + b\right] \frac{f\left(0\right)}{2} = 0,$$

or equivalently,

$$\frac{w'\left(\pi - x\right)}{w\left(x + \pi\right) - w\left(\pi - x\right) - b} = f\left(0\right). \tag{3}$$

Note that the left-hand side of (**??**) is strictly decreasing, so it has at most one solution. Evaluated at $x = \pi$, the left-hand side of (**??**) equals zero, so $\frac{\partial}{\partial x_R} U(-\pi, x_R)|_{x_R = \pi} < 0$, and we conclude that $(-\pi, \pi)$ cannot be an equilibrium. We consider three remaining cases. If $-w'(\pi)/b < f(0)$, then the left-hand side of (**??**) evaluated at $x = 0$ is less than $f(0)$, and (**??**) has no solution, and

$$\frac{\partial}{\partial x_R} U\left(0, x_R\right)\bigg|_{x_R = 0} = -b \frac{f\left(0\right)}{2} - w'\left(\pi\right)/2 < 0,$$

so $x_R = x_L = 0$ is the unique symmetric equilibrium. If $-w'(\pi)/b = f(0)$, then $x = 0$ is the unique solution of (**??**), and $(0, 0)$ is the unique symmetric equilibrium. Finally, if $-w'(\pi)/b > f(0)$, then the left-hand side of (**??**) evaluated at $x = 0$ is greater than $f(0)$, while evaluated at π it is equal to zero and less than $f(0)$. Therefore, by the intermediate value theorem, (**??**) as a positive solution, which is unique and then characterizes the unique symmetric equilibrium.

Provided that $x \in (0, \pi)$, the equation defining the equilibrium is

$$\phi\left(\pi, x, f\left(0\right), b\right) \equiv -w'\left(\pi - x\right) + f\left(0\right)\left[w\left(x + \pi\right) - w\left(\pi - x\right) - b\right] = 0.$$

The comparative statics follow, as:

$$\begin{aligned}
\frac{\partial x}{\partial \pi} &= -\frac{\phi_\pi\left(\pi, x, f\left(0\right), b\right)}{\phi_x\left(\pi, x, f\left(0\right), b\right)} = -\frac{-w''\left(\pi - x\right) + f\left(0\right)\left[w'\left(x + \pi\right) - w'\left(\pi - x\right)\right]}{w''\left(\pi - x\right) + f\left(0\right)\left[w'\left(x + \pi\right) + w'\left(\pi - x\right)\right]} \\
&\propto -w''\left(\pi - x\right) + f\left(0\right)\left[w'\left(x + \pi\right) - w'\left(\pi - x\right)\right] \\
&= -w''\left(\pi - x\right) + \frac{w'\left(\pi - x\right)}{w\left(x + \pi\right) - w\left(\pi - x\right)}\left[w'\left(x + \pi\right) - w'\left(\pi - x\right)\right] \\
&\propto \frac{w''\left(\pi - x\right)}{w'\left(\pi - x\right)} - \frac{w'\left(x + \pi\right) - w'\left(\pi - x\right)}{w\left(x + \pi\right) - w\left(\pi - x\right)} > 0,
\end{aligned}$$

where the inequality follows from (A2). Further,

$$\begin{aligned}
\frac{\partial x}{\partial f\left(0\right)} &= -\frac{\phi_{f(0)}\left(\pi, x, f\left(0\right), b\right)}{\phi_x\left(\pi, x, f\left(0\right), b\right)} = -\frac{w\left(x + \pi\right) - w\left(\pi - x\right) - b}{w''\left(\pi - x\right) + f\left(0\right)\left[w'\left(x + \pi\right) + w'\left(\pi - x\right)\right]} \\
&\propto w\left(x + \pi\right) - w\left(\pi - x\right) - b < 0,
\end{aligned}$$

and finally,

$$\frac{\partial x}{\partial b} = -\frac{\phi_b\left(\pi, x, f\left(0\right), b\right)}{\phi_x\left(\pi, x, f\left(0\right), b\right)} = -\frac{-f\left(0\right)}{w''\left(\pi - x\right) + f\left(0\right)\left[w'\left(x+\pi\right) + w'\left(\pi - x\right)\right]} \propto -f\left(0\right) < 0.$$

■

Proof of Lemma ??.

$$\begin{aligned}
W_{\delta_v}(x) &= \int_{-\infty}^{\infty}\left[-\int_{-\infty}^{0}\left(-x-\theta_v\right)^2 f\left(\mu\right)d\mu - \int_{0}^{\infty}\left(x-\theta_v\right)^2 f\left(\mu\right)d\mu\right]g\left(\varepsilon\right)d\varepsilon \\
&= \int_{-\infty}^{\infty}\left[-\int_{-\infty}^{0}\left(-x-\mu-\delta_v-\varepsilon\right)^2 f\left(\mu\right)d\mu - \int_{0}^{\infty}\left(x-\mu-\delta_v-\varepsilon\right)^2 f\left(\mu\right)d\mu\right]g\left(\varepsilon\right)d\varepsilon \\
&= \int_{-\infty}^{\infty}\left[-\int_{0}^{\infty}\left(-x+\mu-\delta_v-\varepsilon\right)^2 f\left(\mu\right)d\mu - \int_{0}^{\infty}\left(x-\mu-\delta_v-\varepsilon\right)^2 f\left(\mu\right)d\mu\right]g\left(\varepsilon\right)d\varepsilon \\
&= -\int_{0}^{\infty}\int_{-\infty}^{\infty}\left(\left(-x+\mu-\varepsilon\right)^2 - 2\delta_v\left(-x+\mu-\varepsilon\right)+\delta_v^2\right)g\left(\varepsilon\right)d\varepsilon f\left(\mu\right)d\mu \\
&\quad -\int_{0}^{\infty}\int_{-\infty}^{\infty}\left(\left(x-\mu-\varepsilon\right)^2 - 2\delta_v\left(x-\mu-\varepsilon\right)+\delta_v^2\right)g\left(\varepsilon\right)d\varepsilon f\left(\mu\right)d\mu \\
&= -\int_{0}^{\infty}\int_{-\infty}^{\infty}\left(\left(-x+\mu-\varepsilon\right)^2 - 2\delta_v\left(-x+\mu-\varepsilon\right)+\delta_v^2\right)g\left(\varepsilon\right)d\varepsilon f\left(\mu\right)d\mu \\
&\quad -\int_{0}^{\infty}\int_{-\infty}^{\infty}\left(\left(x-\mu+\varepsilon\right)^2 - 2\delta_v\left(x-\mu+\varepsilon\right)+\delta_v^2\right)g\left(\varepsilon\right)d\varepsilon f\left(\mu\right)d\mu \\
&= -2\int_{0}^{\infty}\int_{-\infty}^{\infty}\left(-x+\mu-\varepsilon\right)^2 g\left(\varepsilon\right)d\varepsilon f\left(\mu\right)d\mu - \delta_v^2 \\
&= -\delta_v^2 + W_0(x).
\end{aligned}$$

■

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