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Working Paper No. 57
June 2008

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June 15, 2008

Abstract

We study a dynamic game of incomplete information in which two political parties contest elections with endogenously formed reputations regarding the preferences that prevail within each party. Party preferences exhibit serial correlation and change with higher probability following defeat in elections. We show that when partisans care sufficiently about office, extreme policies are pursued with positive probability by the government if the ruling party is perceived relatively more extreme than the opposition. In equilibrium such policies occur when (a) both parties are perceived to be more extreme than a fixed benchmark level, and (b) elections are close in that both parties have similar reputations. Two qualitatively different equilibrium dynamics are possible depending on the relative speed with which preferences of parties in government or in the opposition change: One produces regular government turnover and extreme policies along the path of play, another involves a strong incumbency advantage and policy moderation.

1. Introduction

In the canonical model of two candidate electoral competition, the two contenders for office make platform announcements that the electorate takes at face value. In the classic Hotelling (1929), Downs (1957) version of this model equilibrium platforms converge to the median when the policy space has one dimension under the assumption that candidates are motivated by the pursuit of office. In part to address the criticism that actual elections do not result in identical policy platforms by the candidates, platform divergence is obtained in equilibrium under the alternative

*Earlier versions of this paper were presented at NYU, Cornell University, the University of Crete, the Athens University of Economics and Business, the 2005 MPSA, and the 2005 SED conferences.

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assumption that candidates have policy motivations and face electoral uncertainty (e.g., Wittman (1983), Calvert (1985)). These models and their variants generate ideals with trivial dynamics, since they imply either persistent policy convergence or (partial) divergence over time. At least with regard to partisan competition, empirical observation suggests otherwise. For instance, Downsian convergence seems to be a fair approximation of British politics in the 50's and 60's. But this era of the "politics of consensus" Kavanagh and Morris (1994) came to an end in the 80's with the governments of Margaret Thatcher.

Besides their variable success in explaining policy extremism and their inability to account for different degrees of policy moderation over time, there is another count on which static models of electoral competition are at odds with empirical observation. In these models both candidates are in principle able to be competitive in elections since, barring a non-anonymous resolution of voters' indifference, a candidate can perform at least as well as the opponent by adopting an identical platform. Yet, we often observe two party systems in which one of the two parties contesting for power is widely perceived to have a small or no chance of winning office, often over multiple successive elections. This seems to have been the case for the Tories in Britain in the second part of the 1990's, but a similar inability of the Labour party to compete effectively against the Tories occurred in the mid 1980's. Our objective in this paper is to develop a dynamic model of two party competition which allows for equilibrium dynamics that are consistent with these empirical observations.

The model we develop shares features with many static models of electoral competition. First, we combine the premises of the models of Downs and Wittman and assume candidates with a mix of office and policy motivations. Second, as is the case in the citizen-candidate models of Osborne and Slivinski (1996) and Besley and Coate (1997), we focus on credible policy choices by the governing parties and dispense with *cheap talk* policy platform declarations. Third, while the citizen-candidate models focus on individual candidacies, we model electoral competition between political parties which we assume are populated by individuals with different policy preferences as is the case, for example, in the party equilibrium model of Roemer (2000). In particular, we assume that individuals within the party with different policy preferences battle for control of the party in each period and the outcome of this battle is probabilistic.

We depart from the above models, though, in that we assume that the true preferences that prevail within each party are (at least partly) private information. In effect, party preferences cannot be credibly communicated to other actors in the absence of policy platform commitment, except possibly via costly policy choices while the party is in office. Hence, in lieu of party platform declarations, we assume each political party enters the electoral arena with an endogenously formed reputation. These reputations are the beliefs of the electorate about the preferences that prevail within each party and reflect the accumulated history of electoral outcomes and policy choices that have transpired prior to the elections. We also assume that political parties, like all organizations, exhibit inertia. We formalize this idea by assuming that party preferences are positively serially correlated: Extremists (Moderates) have a higher probability of prevailing within the party if the party was controlled by extremists (moderates) in the previous period. Thus, when a party reveals that it is controlled by extremists by implementing an extreme policy, it damages its reputation for several electoral cycles. These assumptions define a stochastic game of incomplete information in which players' strategies (those of parties and the voter) are conditioned on parties' joint reputations in any given period, and these reputations in turn are rationally updated given past actions.

A first finding from the analysis of this model is that it is inconsistent with Downsian convergence to the median. No matter how much weight parties place in office there does not exist a robust equilibrium in which parties in government implement moderate policies with probability one independent of their combined reputations. In fact, in the case when parties are impatient or place significant emphasis on policy relative to office the only equilibrium involves party types implementing their ideal policy independent of the electorate's beliefs about the two parties. The most interesting case, though, on which we devote the bulk of the analysis concerns the situation when partisans assign significant weight on office utility relative to policy. We report three main findings under this assumption. First, we find that the policy choice of the parties in government depends on the joint reputation levels of both competing parties. In particular, the governing party (on or off the equilibrium path) pursues an extreme policy with positive probability when it has a relative reputational disadvantage compared to the opposition party. Second, extreme policies are observed along the equilibrium path if (a) both parties are perceived to be extreme with a probability that exceeds a benchmark reputation level, and (b) following *close* elections, that is, elections in which both parties have similar reputations. Finally, with regard to policy and electoral dynamics, the equilibrium is consistent with two radically different patterns of competition over time. For some values of the parameters that regulate the probability with which party preferences change, equilibrium dynamics are characterized by moderate policies and a strong incumbency advantage for the government, as the government maintains a persistent reputational advantage over the opposition party. But there also exist configurations of these parameters for which extreme policies and alternation of parties in government are a regular equilibrium phenomenon that occurs infinitely often along the path of play. In this case, the opposition's reputation gradually improves relative to that of the government and the government is induced to pursue extreme policies with positive probability upon losing its reputational advantage over the opposition party.

A number of other models explicitly study the dynamics of two party competition but assume complete information. Kramer (1977) and Wittman (1977) assume that in each period the incumbent is committed to the policy pursued in the previous period, while the challenger myopically chooses a vote or policy maximizing platform, respectively. Harrington (1992) and Aragones, Palfrey and Postlewaite (2007) study equilibrium models of repeated elections with an explicit focus on party reputations. The term reputation has a different meaning in these studies than the one we adopt in the present analysis. In particular, a reputation in the context of these studies is the belief about the policy to be chosen by the candidates in equilibrium. In contrast, a reputation in the present study is the belief of the electorate about the preferences that prevail within the party. Harrington (1992) and Aragones, Palfrey and Postlewaite (2007) establish the range of policy choices or reputations other than the candidate's ideal policy that are consistent with equilibrium. These equilibrium reputations are built on a form of history dependent strategies such that upon observing a choice different than the one dictated by a candidate's reputation, the voters expect that candidate to switch to pursuing her ideal policy for ever after. Alesina (1988) and Duggan and Fey (2006) study a different type of history dependent strategies in repeated election models in which candidates have policy and office motivations, respectively. They characterize the set of subgame perfect equilibria which are consistent with a wide range of policy platform choices by the parties in Alesina (1988), and include all possible equilibrium policy outcomes in Duggan and Fey (2006). Dixit, Grossman and Gul (2000), characterize efficient subgame perfect equilibria in a model in which parties' re-election probabilities follow an exogenous Markov process conditional on the incumbent's policy choice. Besides the fact that we study a model with incomplete information, another difference with these studies is that we focus on Markovian equilibria, so that the different players (partisans, voters, etc.) do not coordinate on complex history dependent strategies.

Among models with incomplete information, Alesina and Cukierman (1990) study a two period model under the assumption that the candidates' preferences in the second period are serially correlated with their first period preferences, as is assumed in the present infinite period model. Their analysis focuses on the first period policy choice, emphasizing the strategic incentive of the candidates to choose a moderate policy in order to win reelection. Thus, their study does not involve the type of dynamic analysis we pursue. Repeated elections under incomplete information about candidate preferences are studied by Duggan (2000), Bernhardt, Dubey and Hughson (2004), and Banks and Duggan (2008).[1] These models are suitable for the study of elections in which the candidates are individuals so that it is plausible to assume, as these authors do, that challengers to the incumbent are drawn from an identical pool of possible candidates over time. On the other hand, the assumption that challengers are drawn from a stationary distribution seems inappropriate for partisan candidacies because inertia within party organizations will typically imply that past choices by the challenger party contain information about the reputation of that party.

The remainder of this paper is organized as follows. In section 2 we describe the model. In section 3 we study simple equilibria of this model such that party strategies are independent of the configuration of party reputations. The main equilibrium analysis appears in section 4, where we analyze the case in which parties value office significantly compared to policy. We discuss equilibrium properties and equilibrium dynamics for that case in section 5. In section 6 we extend the analysis to allow for probabilistic elections. We conclude in section 7.

2. Model

We consider two parties and an electorate that interact over an infinity of periods $t = 0, 1, \ldots$. We model the electorate as a pivotal or median voter, M, and denote a generic party by P, which is either a left-wing party ($P = L$) or a right-wing party ($P = R$). We use $-P$ to denote the party in opposition of party P. Each of the two parties contains individuals with two different ideological convictions, call them *moderates* and *extremists*, who vie for control of the party in each period. We define the *type* $\tau \in \{e, m\}$ of the party in a given period as one of the two groups,

[1] Other models of incomplete information, such as Ferejohn (1986), focus on the fact that the incumbent's action while in office is unobserved (hidden action) and can be traced to Barro (1973). Rogoff (1990), Banks and Sundaram (1993), and Ashworth (2005) combine aspects of both models.

extremists or moderates, whose preferences prevail within the party in that period. We assume that a party of type τ that is in opposition in period t is of the same type in period $t+1$ with probability $\pi^o_\tau \in [0,1]$. Similarly, if the governing party is of type τ in period t it is of the same type in period $t+1$ with probability $\pi^g_\tau \in [0,1]$. We assume that these transition probabilities satisfy

$$\pi^g_\tau > \pi^o_\tau,\ \tau \in \{e,m\}, \text{ and} \tag{1}$$

$$\pi^o_e > 1 - \pi^o_m. \tag{2}$$

Inequality (1) states that parties are more likely to change type while in opposition than while the party is in government, since electoral defeat naturally triggers internal shifts of power within the losing parties. Inequality (2), in conjunction with (1), amounts to the assumption that party types are positively serially correlated so that parties are more likely to be of type τ if they were of the same type in the previous period.

Parties know the realization of their own type in each period, but other players cannot directly verify the type that prevails within each party. Instead, players rationally update beliefs about the probability that (other) parties are moderate or extreme. We will refer to these beliefs as the *reputation* of each party which at the beginning of each period t we represent by a pair of probabilities $b^t = (b^t_L, b^t_R) \in [0,1]^2$. Thus, for example, probability b^t_L represents the belief of voter M (and party R) in period t that party L is extreme. The initial party reputations b^0 are exogenously given. Party reputations reflect uncertainty by extra-partisan players regarding the outcome of the internal battle for control of the party between extremists and moderates due to, for example, irreducible uncertainty about the preferences of influential decision makers within the party who may have strategically concealed their true ideological convictions from the public in order to gain prominence within the party, shifting internal party alliances that take place behind the scenes and permit losing ideological groups to exercise control on party decisions, or perhaps due to the stochastic nature of the process via which older partisans retire and younger party members with unknown preferences gain influence within the party. While parties may implicitly take (unmodeled) actions that optimally mitigate some of the sources of this uncertainty, we assume that the effect of any such actions is already captured by the probabilities π^o_τ and π^g_τ so that the reputation of each party reflects the residual uncertainty of individuals outside the party about the type of the party that cannot be credibly reduced by publicly observed actions of partisans except, possibly, through the policy choice of the party when in government.

Each period in the game represents a complete political cycle. First, elections take place in which voter M chooses whether to reelect previous period's government or not. Then the party/type elected in government implements a policy $x^t \in X$. Extreme types of party P can choose between their favorite policy x_e^P and a moderate policy x_m^P, while moderate types of either party always implement the moderate policy of their party x_m^P.[2] Thus, in general, there are four possible policies given by $X = \{x_e^L, x_m^L, x_m^R, x_e^R\}$. Following the policy choice of the governing party, which is publicly observed, nature chooses a new type for each party, players update their beliefs, and the game moves to the next period. In that period, the voter elects a new government, the governing party implements a policy, new partisan types are realized, etc.

If policy $x_\tau^P \in X$ is implemented in period t, then voter M's payoff in that period is given by $v_\tau^P \in \mathbb{R}$, while extremists of party R receive $r_\tau^P \in \mathbb{R}$ and extremists of party L receive $l_\tau^P \in \mathbb{R}$.[3] We preserve the symmetry of the game by setting $l_\tau^P = r_\tau^{-P}$ and $v_\tau^L = v_\tau^R$ for $\tau \in \{e, m\}$ and $P \in \{L, R\}$. We assume that $v_m^P > v_e^P$ and $r_e^R > r_m^R \geq r_m^L > r_e^L$, that is, the voter prefers moderate policies and extremists of each party prefer the respective partisan policy most, moderate policies next, and they least prefer the partisan policy of the other party. Note that by assuming $r_m^R \geq r_m^L$ we do not preclude the possibility that $x_m^L = x_m^R$ is a common policy. This permits a convergence to the median equilibrium to occur. But we also allow $x_m^L \neq x_m^R$ so that there may exist residual partisanship even if the moderates are the prevailing group within each party. Parties receive additional office payoff when they control the government so that extreme partisans receive utility $G > 0$ when their *party* is in government.[4] We assume that the voter is strategic but cares only about the policy outcome in the current period. Partisan types are (potentially) more farsighted and care about the electoral and policy outcome in two periods, the current period t as well as period $t + 1$. The weight parties place on the outcome of the next period is given by a discount factor $\delta \in [0, 1]$.

We will focus our attention to equilibria in strategies that are appropriately Markovian.

[2] While we do not allow moderates to choose between the available policies, this is the behavior that would arise endogenously in an equilibrium of the type we characterize.

[3] Since moderate partisan types always pursue the same action, we only specify payoffs for the voter and the two extreme partisan types (left and right).

[4] While the office payoff G is independent of the type that prevails within the party, we can easily accommodate different office payoffs for extreme partisans depending on the type that controls the party when the party is in office.

Given the structure of the model, the relevant strategic environment for the party in government is summarized by the reputations of the two parties.[5] Thus, a strategy for an extreme type e of governing party P is given by a function $\sigma_P : [0,1]^2 \to [0,1]$, $P \in \{L, R\}$. Accordingly, $\sigma_P(b)$ is the probability that extreme type e of party P implements policy x_e^P when party reputations are given by $b \in [0,1]^2$. We express the choice of the voter between the two parties as a reelection strategy $\sigma_M : [0,1]^2 \times \{L, R\} \to [0,1]$, so that this strategy is conditioned on the pair of beliefs about the two parties as well as the identity of the incumbent government party. Accordingly, the voter reelects the incumbent party in government P by setting $\sigma_M(b, P) = 1$, and elects the opposition party by setting $\sigma_M(b, P) = 0$. We denote a strategy profile for all three players by $\sigma = (\sigma_M, \sigma_L, \sigma_R)$.

Players update their beliefs regarding the extremism of the two parties at two stages within a period. First, players update beliefs about the type of the party in government after observing its policy choice. Let $\beta(b^t, x; \sigma)$, where $\beta : [0,1]^2 \times X \to [0,1]$, denote the updated belief of the electorate about the type of the governing party $P \in \{L, R\}$ after observed policy $x \in X$ and party reputations $b^t \in [0,1]^2$ in period t and strategies given by σ. The second change in party reputations between periods t and $t+1$ occurs due to the possibility of internal ideological shifts within the two parties. Define a transition function $T_g : [0,1] \to [1 - \pi_m^g, \pi_e^g]$ for the reputation of the governing party, parameterized by the probabilities π_τ^g as $T_g(b) = \pi_e^g b + (1 - \pi_m^g)(1 - b)$, and similarly define $T_o : [0,1] \to [1 - \pi_m^o, \pi_e^o]$ for the opposition party as $T_o(b) = \pi_e^o b + (1 - \pi_m^o)(1 - b)$. Now, if beliefs at the beginning of period t are given by $b^t \in [0,1]^2$, if the party in government in period t is P^t, and if this party implements a policy $x^t \in X$, then the beliefs of the electorate in period $t+1$ are given by $b^{t+1} = b'(b^t, x^t; \sigma)$, where the coordinate of function $b' : [0,1]^2 \times X \to [0,1]^2$ that corresponds to party P takes the form

$$b'_P(b^t, x^t; \sigma) = \begin{cases} T_g(\beta(b^t, x^t; \sigma)) & \text{if} \quad P = P^t \\ T_o(b_P^t) & \text{if} \quad P \neq P^t. \end{cases} \tag{3}$$

Let $V(b, P; \sigma)$ be the expected payoff of voter M from electing party P when party reputations are given by $b \in [0,1]^2$ and strategies are denoted by σ, and let $U_P(b, x_\tau^P; \sigma)$ denote the expected payoff of party P from implementing a policy x_τ^P while in government in a period with party reputations b. Explicit expressions for these expected payoffs are provided in equations (16)

[5] For a dynamic game in which players' Markov strategies are conditioned on beliefs in a similar fashion, see Mailath and Samuelson, 2001.

and (17) in the Appendix. We can now state the definition of the equilibrium concept which is a version of Markov Perfect Bayesian Nash equilibrium:

Definition 1 *An* equilibrium *is a triple of strategies* $\sigma^* = (\sigma^*_M, \sigma^*_L, \sigma^*_R)$ *such that*

$$\sigma^*_M(b, P) = \begin{cases} 1 & \text{if} \quad V(b, P; \sigma^*) > V(b, -P; \sigma^*) \\ 0 & \text{if} \quad V(b, P; \sigma^*) < V(b, -P; \sigma^*) \end{cases} \tag{4}$$

and

$$\sigma^*_P(b) = \begin{cases} 1 & \text{if} \quad U_P(b, x_P; \sigma^*) > U_P(b, x^P_m; \sigma^*) \\ 0 & \text{if} \quad U_P(b, x^P_e; \sigma^*) < U_P(b, x^P_m; \sigma^*) \end{cases} \tag{5}$$

for all $b \in [0,1]^2$ *and all* $P \in \{L, R\}$*, and a reputations updating rule* b' *that satisfies (3) and is such that the updating function* β *satisfies*

$$\beta(b, x; \sigma) = \begin{cases} \dfrac{(1-\sigma^*_P(b))b_P}{1-\sigma^*_P(b)b_P} & \text{if} \quad x = x^P_m,\ b_P\sigma^*_P(b) < 1, \\ 1 & \text{if} \quad x = x^P_e. \end{cases} \tag{6}$$

In (6) we require players to use Bayes' rule to update their beliefs about the governing party after observing its policy choice, but we also effectively restrict certain out of equilibrium beliefs by assuming that other players believe the government party is extreme with probability one after observing an extreme policy even when its strategy dictates a moderate policy with probability one ($\sigma^*_P(b) = 0$). We do not restrict out of equilibrium beliefs in cases when a moderate policy is observed and $b_P\sigma^*_P(b) = 1$, but the flexibility allowed by the absence of any such restriction is of no consequence for the equilibrium analysis that follows. Indeed, such a restriction is redundant if $\pi^g_e < 1$ since then we cannot have party P with reputation $b_P = 1$ in periods other than the very first along any path of play. We refine the equilibrium concept as follows:

Definition 2 *An equilibrium with strategies* σ^* *is* robust *if there exists an* $\bar{\varepsilon} > 0$, $\bar{\varepsilon} < \frac{1}{2}$*, such that for each* $\varepsilon \in (0, \bar{\varepsilon})$*, the voting strategy* σ^*_M *satisfies (4) when party strategies are perturbed according to*

$$\sigma^{\varepsilon *}_P(b) = \begin{cases} 1-\varepsilon & \text{if} \quad \sigma^*_P(b) > 1-\varepsilon \\ \varepsilon & \text{if} \quad \sigma^*_P(b) < \varepsilon \\ \sigma^*_P(b) & \quad \text{otherwise.} \end{cases} \tag{7}$$

Informally an equilibrium is *robust* if the voter's strategy is a best response even if party strategies involve trembling. The objective of the refinement is to resolve the electorate's possible indifference in a manner that is responsive to its beliefs about the relative extremism of the two parties. Such indifference may arise, for example, in cases when both parties are expected to pursue a moderate policy with probability one following the election. If parties may tremble as in (7), though, the voter strictly prefers that between the two parties that is perceived less extreme. There is a more direct (and apparently more restrictive) manner to impose such a refinement. In particular, we define an *intuitive equilibrium* as follows:

Definition 3 *An equilibrium with strategies* σ^* *is* intuitive *if the voting strategy* σ^*_M *satisfies*

$$\sigma^*_M(b, P) = \begin{cases} 1 & \text{if} \quad b_P < b_{\text{-}P} \\ 0 & \text{if} \quad b_P > b_{\text{-}P}. \end{cases} \tag{8}$$

With the solution concept clarified, we proceed to the analysis of the game. First we consider analogues of pooling and separating equilibria for this game. In such equilibria extreme partisan types pursue the same policy (moderate or extreme, respectively) independent of party reputations, hence we call these equilibria simple. The chief equilibrium results are contained in section 4, where we consider robust equilibria that are not simple and involve parties that place high weight in office (high G) and in the future (high δ). Then, in section 5 we discuss equilibrium dynamics and other equilibrium properties.

3. Simple Equilibria

In this section we consider two simple types of equilibria in which parties' strategy does not depend on party reputations. First, in Proposition 1 we give a precise range of parameters for which extreme partisan types implement extreme policies whenever in power, independent of party reputations $b \in [0, 1]^2$. We have:

Proposition 1

(i) An equilibrium with party strategies satisfying $\sigma_P(b) = 1$ *for all* $b \in [0, 1]^2$*,* $P \in \{L, R\}$*, exists*

if and only if

$$\delta \leq \frac{r_e^R - r_m^R}{G + r_m^R - r_m^L + \pi_e^o(r_m^L - r_e^L) + \pi_e^g(r_e^R - r_m^R)}. \tag{9}$$

(ii) Every equilibrium with party strategies that satisfy $\sigma_P(b) = 1$ for all $b \in [0,1]^2$, $P \in \{L, R\}$ is robust and intuitive.

(iii) If inequality (9) is strict then party strategies satisfy $\sigma_P(b) = 1$ for all $b \in [0,1]^2$, $P \in \{L, R\}$, in every equilibrium.

Part (iii) of the proposition establishes that when condition (9) holds with the inequality strict all equilibria of the game involve extreme partisan types pursuing their ideal policy, independent of the electorate's beliefs. This is despite the fact that in these equilibria parties implementing extreme policies lose the election with probability one, as is implied by the fact that these equilibria are intuitive by part (ii) of the proposition. Thus, according to condition (9) such punishment is not sufficient to induce moderation when either (a) parties are impatient (low δ), or (b) parties place low value to office (low G), or (c) the loss in utility due to the policies pursued by the opposition party controlling the government is small (low $r_m^R - r_m^L$), or (d) when the ability of extremists to maintain control of their party following electoral defeat is small (low π_e^o, π_e^g).

One may conjecture that when these conditions are reversed we may instead obtain a simple 'pooling' equilibrium in which extreme partisan types always imitate the moderate partisan types by pursuing a moderate policy. It is possible to construct such equilibria (for high enough G & δ) exploiting voters' indifference, but these equilibria are not robust. Indeed we can show that there does not exist a robust 'pooling' equilibrium:

Proposition 2 *There does not exist a robust equilibrium such that $\sigma_P(b) = 0$ for all $b \in [0,1]^2$, $P \in \{L, R\}$.*

Thus the analogue to a convergence-to-the-median result is not attainable in this dynamic game in a robust equilibrium. The reasoning behind Proposition 2 is straightforward. If all party types choose moderate policies independent of the electorate's beliefs, then the electorate is indifferent between the two parties. Then, in a robust equilibrium the voter elects that between the two parties that has a (strictly) better reputation. Thus, a party that is in government, is controlled by extremists, and is perceived to be more extreme than the opposition even after implementing a moderate policy has no incentive to pursue such a moderate policy. This party faces electoral defeat independent of its policy choice, so types in control of the party might as well pursue their ideal policy.

In combination, Propositions 1 and 2 imply that when condition (9) fails a robust equilibrium must involve some configuration of reputations at which there is a positive probability of extreme policies pursued by extreme types as well as reputations at which such types choose a moderate policy with positive probability. We take the analysis of such more interesting equilibria in the next section.

4. Equilibrium with Office Motivations

Our goal in this section is to establish an equilibrium when condition (9) fails and parties are sufficiently patient and motivated predominantly by office considerations (high G). When parties are motivated by office they are willing to pursue a moderate policy even when they are controlled by extremists, if such a policy secures them reelection. Note that for any strategies σ a governing party's reputation cannot deteriorate following a choice of a moderate policy (i.e., we have $\beta(b_P, x_m^P; \sigma) \leq b_P$). Hence, the updated pair of party reputations determined by (3) ensures that the incumbent party will enjoy a better reputation than the opposition in the following election if $T_g(b_P) < T_o(b_{-P})$, since

$$b'_P(b, x_m^P; \sigma) \leq T_g(b_P) < T_o(b_{-P}) = b'_{-P}(b, x_m^P; \sigma).$$

Thus, in an intuitive equilibrium the governing party P will pursue a moderate policy with probability one for all party reputations $b \in [0,1]^2$ such that $T_g(b_P) < T_o(b_{-P})$. Such a (relative) reputational advantage and a moderate policy lead to reelection with probability one, and partisans that care a lot about office will follow a moderate policy in all such cases.

The situation is rather different when the governing party P is in power with party reputations b in the set $\mathcal{B}_P$ defined by

$$\mathcal{B}_P = \{b \in [0,1]^2 : T_g(b_P) > T_o(b_{-P})\}.$$

If the incumbent party pursues a moderate policy with probability one at such reputations, the policy of the government conveys no information to the electorate regarding the government's type (i.e., $\beta(b_{P}, x_m^P; \sigma) = b_{P}$). Hence, party reputations in the upcoming election satisfy $b'_P(b, x_m^P; \sigma) = T_g(b_P) > T_o(b_{\text{-}P}) = b'_{\text{-}P}(b, x_m^P; \sigma)$ and party P loses the election despite its attempt to appear moderate. Thus, pursuing a moderate policy with probability one at these reputation levels is not part of an intuitive equilibrium. Similarly, it is not an equilibrium for extremists of party P to implement an extreme policy with probability one. If such an extreme policy were pursued by extremists with probability one, then extremists would have an incentive to deviate and implement a moderate policy instead which would convince the electorate that the party is moderate and would secure them reelection. Thus, the only possibility for equilibrium policy making by party P at reputation levels $b \in \mathcal{B}_P$ is a mixed strategy. As we show in Propositions 3 and 6, by allowing the voter to also use a mixed strategy we can establish that such a robust equilibrium exists. The equilibrium is such that the governing party's mixture probability between moderate and extreme policies makes it barely competitive against its opponent at the elections when the realization of the party's randomization is a moderate policy.

Proposition 3 *Assume*

$$\delta > \frac{r_e^R - r_m^R}{G + r_m^R - r_m^L}. \tag{10}$$

(i) There exists a robust and intuitive equilibrium such that party strategies satisfy

$$\sigma_P(b) = \begin{cases} \dfrac{T_g(b_P) - T_o(b_{\text{-}P})}{b_P(\pi_e^g - T_o(b_{\text{-}P}))} & \text{if} \quad T_g(b_P) > T_o(b_{\text{-}P}) \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

for each $P \in \{L, R\}$.

(ii) The probability of an extreme policy in this equilibrium by a government of party P, $b_P\sigma_P(b)$, (weakly) increases with the governing party's reputation b_P and (weakly) decreases with the reputation of the opposition, $b_{\text{-}P}$.

(iii) The equilibrium probability of an extreme policy in a period with party reputations b is positive

if and only if

$$b \in \mathcal{B}_e = \mathcal{B}_L \cap \mathcal{B}_R \subset (b^*, 0]^2, \text{ where } b^* = \frac{\pi_m^g - \pi_m^o}{\pi_m^g - \pi_m^o + \pi_e^g - \pi_e^o}.$$

(iv) For a fixed reputation level b_P, the equilibrium probability of an extreme policy by party P is maximum when party $-P$'s reputation equals that of party P ($b_P = b_{\text{-}P}$) and decreases with the absolute difference between $b_{\text{-}P}$ and b_P.

The equilibrium in Proposition 3 holds for arbitrarily large values of the office payoff G, as long as parties place some weight in the future ($\delta > 0$). Thus, no matter how much weight parties place in office, there exists a configuration of party reputations that makes it worthwhile for extreme partisan types to pursue extreme policies when in government. Figure 1(a) displays a contour plot of the equilibrium probability of an extreme policy choice by extreme partisans of party L in the space of party reputations $[0, 1]^2$. As we have already discussed, a party must be perceived *relatively* more extreme than it's opposition in order for it to pursue extreme policies with positive probability. Note that from the perspective of the electorate the expected probability that, say, party L will pursue an extreme policy given reputations $b \in [0, 1]^2$ is equal to $b_L \sigma_L(b)$. In Figure 1(b) we plot this probability. This graph illustrates the comparative static in part (ii) of Proposition 3, that is, that the probability of an extreme policy by the governing party increases with that party's reputation level and decreases with that of the opposition party. The more disadvantaged the government party is, the more surprising a moderate policy by such a government must be in order for such a moderate policy to convince the electorate that the governing party is as likely to be moderate as the opposition, rendering the subsequent election competitive. Thus, the worse the governing party's reputation is relative to the opposition, the more likely it is that that party will pursue an extreme policy in equilibrium.

The fact that the party strategies in the equilibrium of Proposition 3 are such that governments may pursue extreme policies for some configurations of party reputations is not sufficient to produce extreme policies in equilibrium. Indeed, the voter can ensure that a moderate policy prevails with probability one at reputation levels such that one of the two competing parties pursues a moderate policy with probability one. Hence, since the voter prefers moderate policies, a positive probability of extreme policies arises in equilibrium only when the voter is forced to choose the

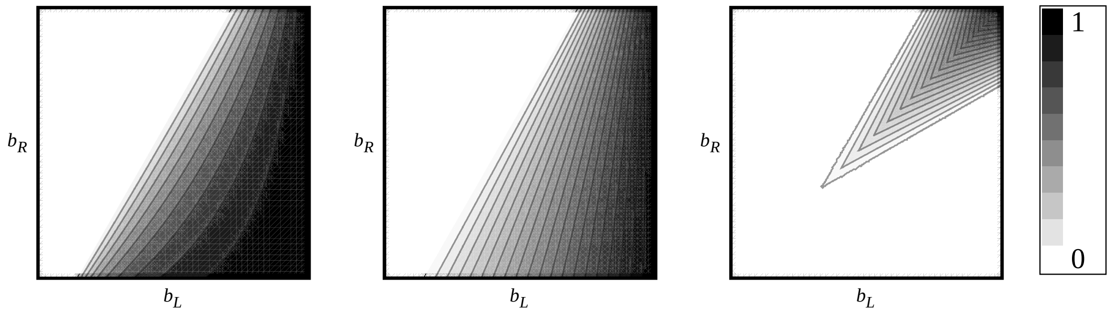

Figure 1: Party Strategies and Probability of Extreme Policies.

best between two evils, that is, for reputation levels such that both parties would pursue extreme policies with positive probability if elected. As stated in part (iii) of Proposition 3, the set of such reputation levels, $\mathcal{B}_e$, lies in the upper quadrant of the space of party reputations. By part (iv) of Proposition 3 we conclude that two conditions produce extreme policies in equilibrium: (a) The two parties are perceived to be relatively extreme (both have reputations above level b^* defined in part (iii) of Proposition 3), and (b) the two parties have similar reputation levels. If we interpret the proximity of the two parties' reputation levels as a proxy for the closeness of the election, then the second of the above two conditions states that extreme policies emerge with higher probability after *close elections*. When the parties' reputations are at high levels and close to each other, the opposition is likely to enjoy an improved reputation in the next period. This prospect places the government at a relative disadvantage, and it is the anticipation of this future electoral disadvantage that produces extreme policies in equilibrium. Figure 1(c) plots the probability of an extreme policy by the party elected in government as a function of the two parties' reputation levels.

While Proposition 3 establishes the existence of a robust equilibrium, it leaves open the possibility that additional equilibria may exist. In the next Proposition we show that under an additional condition the equilibrium of Proposition 3 is essentially *unique* in the class of intuitive equilibria.

Proposition 4 *Assume (10) and*

$$(r_e^R - r_m^R) > \frac{\delta(T_g(\pi_e^o) - T_o(\pi_e^g))}{\pi_e^g - T_o(\pi_e^g)}(r_m^L - r_e^L). \tag{12}$$

Then party strategies satisfy (11) in every intuitive equilibrium.

Condition (12) is a mild sufficient condition for uniqueness that is guaranteed to hold if, for example, $T_g(\pi_e^o) \leq T_o(\pi_e^g)$ or if $(r_e^R - r_m^R) > \delta(r_m^L - r_e^L)$. In sum, in this section we established a robust intuitive equilibrium when partisans are primarily office motivated. We showed that this equilibrium produces extreme policies in equilibrium when both parties are perceived to be relatively extreme and after close elections. Whether the combination of these two conditions on party reputations arises frequently in equilibrium depends on the dynamics on party reputations induced by this equilibrium. We turn to a study of these dynamics in the next section.

5. Reputation and Policy Dynamics

In order to understand the dynamics induced by the equilibrium in Proposition 3, we must determine the direction and rate of change of the reputation of the government and the opposition along the path of play. Towards that end observe first that, irrespective of equilibrium strategies, the reputation of the opposition party adjusts monotonically towards a (non-equilibrium) steady-state level

$$b^o = \frac{1 - \pi_m^o}{2 - \pi_m^o - \pi_e^o},$$

where b^o uniquely solves the equation $T_o(b^o) = b^o$. Second, if we assume that there is a positive probability that party preferences change while the party is in government (i.e., if $\pi_m^g + \pi_e^g < 2$) and that the government's policy choice is not informative about its type, then the governing party's reputation monotonically adjusts towards a level given by

$$b^g = \frac{1 - \pi_m^g}{2 - \pi_m^g - \pi_e^g},$$

where in this case b^g uniquely solves $T_g(b^g) = b^g$. In order to combine these two remarks to determine the reputation dynamics induced in equilibrium, we first establish a lemma concerning the two reputation levels b^o and b^g.

Lemma 1 *For all π_e^o, π_m^o and all π_e^g, π_m^g such that $\pi_e^g + \pi_m^g < 2$:*

(i) If $b > b^o$ then $b > T_o(b) > b^o$ and if $b < b^o$ then $b < T_o(b) < b^o$.

(ii) If $b > b^g$ then $b > T_g(b) > b^g$ and if $b < b^g$ then $b < T_g(b) < b^g$.

(iii) $b^g > b^o > b^*$ if and only if

$$\frac{1-\pi_e^g}{1-\pi_e^o} < \frac{1-\pi_m^g}{1-\pi_m^o}. \tag{13}$$

Parts (i) and (ii) of Lemma 1 establish the monotonic convergence of reputations towards the non-equilibrium steady-state levels b^o and b^g, respectively. Condition (13) in part (iii) provides a criterion that ranks the non-equilibrium steady-state reputation levels b^o and b^g. The ratio $\frac{1-\pi_e^g}{1-\pi_e^o}$ is the relative probability with which parties switch from being extreme to being moderate when in government versus when in opposition, while the ratio $\frac{1-\pi_m^g}{1-\pi_m^o}$ represents the relative probability with which parties switch from being moderate to being extreme when in government versus when in opposition. Thus, part (iii) of Lemma 1 states that the non-equilibrium steady-state reputation for parties in opposition, b^o, is lower than that for parties in government, b^g, when parties in government tend to switch from being moderate to extreme with higher relative frequency than the relative frequency of switching from being extreme to being moderate.

The next step in our analysis is to show that, despite the fact that b^o and b^g constitute non-equilibrium steady-state levels of reputation, these quantities constrain the possible long-term dynamics on party reputations under the equilibrium of Proposition 3 as follows:

Lemma 2 *Assume party strategies given by (11). For every (possibly non-equilibrium) voting strategy σ_M, for every initial reputations $b^0 \in [0,1]^2$, and every sequence of party reputations $b^0, b^1, \ldots, b^t, \ldots$ induced by (3) and (6):*

(i) If $\pi_e^g + \pi_m^g < 2$ then

$$\liminf_{t\to+\infty} \left(\min\left\{b_L^t, b_R^t\right\}\right) \geq \min\left\{b^o, b^g\right\} \text{ and } \limsup_{t\to+\infty} \left(\min\left\{b_L^t, b_R^t\right\}\right) \leq \max\left\{b^o, b^g\right\}.$$

(ii) If $\pi_e^g = \pi_m^g = 1$ then

$$\liminf_{t\to+\infty} \left(\min\left\{b_L^t, b_R^t\right\}\right) \geq \min\left\{b_L^0, b_R^0, b^o\right\} \text{ and } \limsup_{t\to+\infty} \left(\min\left\{b_L^t, b_R^t\right\}\right) \leq \max\left\{\min\left\{b_L^0, b_R^0\right\}, b^o\right\}.$$

Note that Lemma 2 allows for voter strategies that may differ from the equilibrium strategies in Proposition 3. In combination with part (iii) of Lemma 1, Lemma 2 permits a detailed characterization of equilibrium dynamics. In particular, condition (13) of Lemma 1 provides a succinct criterion to categorize the dynamics that are possible in the equilibrium of Proposition 3 into three main cases which we now discuss. These cases are illustrated in Figure 2.

Case I ($\pi_e^g + \pi_m^g < 2$ and $b^g > b^o > b^*$): Policy Extremism

When condition (13) holds then both non-equilibrium steady-state reputation levels of the opposition and governing parties b^o and b^g, respectively, exceed the reputation level b^* characterized in part (iii) of Proposition 3. As a result, by part (i) of Lemma 2 we conclude that party reputations are absorbed in the right-top quadrant of the space of reputations defined by $(b^*, 1]^2$. Furthermore, we will argue that starting from any reputations in $(b^*, 1]^2$ the government party is eventually guaranteed to impelement extreme policies with positive probability along the path of play. Substantively, these extreme policies arise because condition (13) guarantees that in spite of any initial reputational advantage for the government, the opposition party eventually enjoys a competitive reputation level. Thus, a pattern of policy making emerges whereby governments may maintain a better reputation than the opposition by pursuing moderate policies in their initial terms in office, but that initial advantage dissipates and governments face competitive elections and pursue extreme policies with positive probability in subsequent terms until they are replaced by the opposition party. In the next few paragraphs we provide a more formal demonstration of this argument.

Note that for any party reputations $b^t \in (b^*, 1]^2$, if the governing party in period t implements a moderate policy with probability one (i.e., if $b^t \notin \mathcal{B}_P$), then party reputations must reach the set $\mathcal{B}_e$ defined in part (iii) of Proposition 3 in a finite number of k periods. First, since party P is in government and $b^t \notin \mathcal{B}_P$ we have $b_P^t \leq b_{\text{-}P}^t$ and $T_g(b_P^t) \leq T_o(b_{\text{-}P}^t)$ by the definition of the set $\mathcal{B}_P$. If set $\mathcal{B}_e$ is not reached in k periods, then $b_P^{t+k} = T_g^k(b_P^t) \leq T_o^k(b_{\text{-}P}^t) = b_{\text{-}P}^{t+k}$ where T_g^k and T_o^k are the k-times compositions of the mappings T_g and T_o, respectively, and party P is reelected with probability one and implements a moderate policy with probability one in all periods $t+1, \ldots, t+k$.

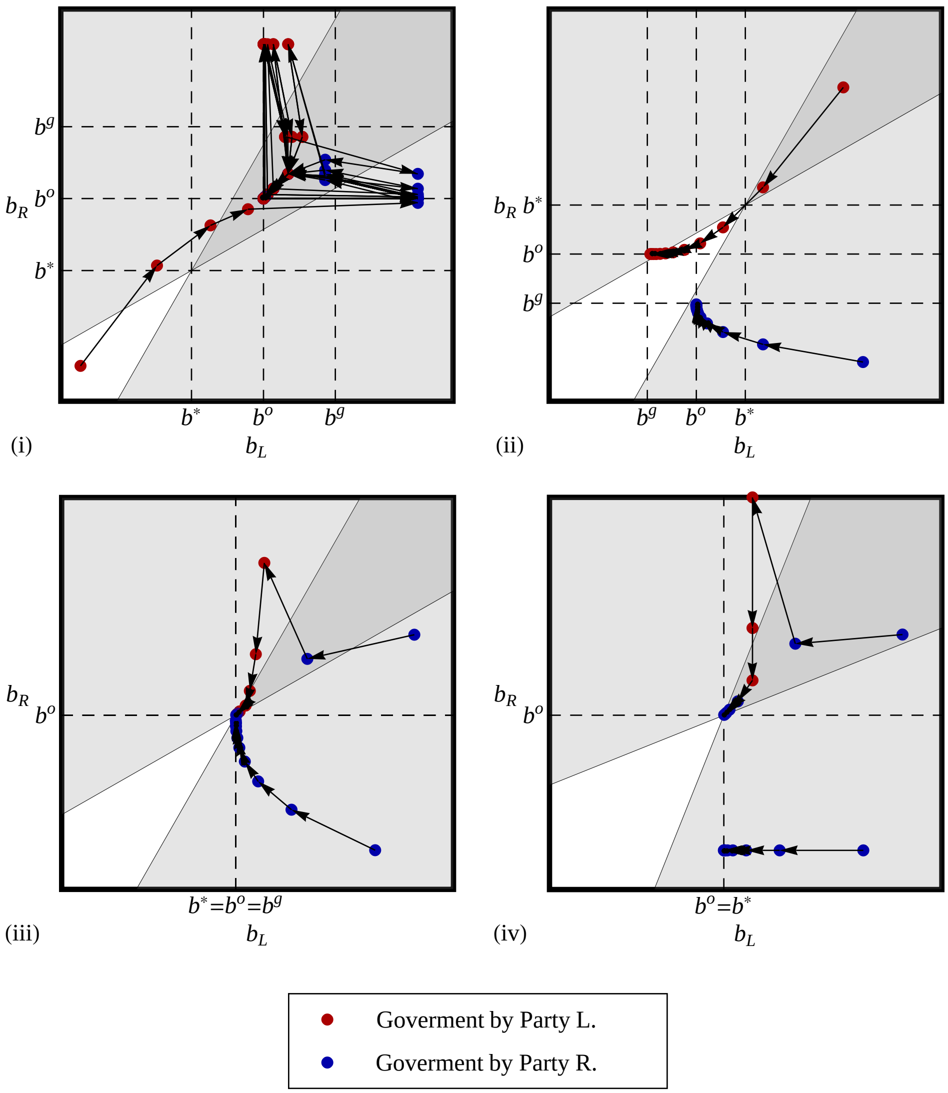

Figure 2: Equilibrium Dynamics.

But by Lemma 1 we have

$$\lim_{k \to +\infty} T_g^k(b_P^t) = b^g > b^0 = \lim_{k \to +\infty} T_o^k(b_{\text{-}P}^t),$$

so that there must exist finite k such that $b^{t+k} \in \mathcal{B}_e$. As we already discussed, when condition (13) holds, the opposition party may lose a number of elections but it eventually overcomes its initial reputation disadvantage to the government party thus inducing extreme types of the government party to pursue extreme policies with positive probability. It follows from the above and by part (iii) of Proposition 3 that extreme policies and alternation of the party in government occur infinitely often along the path of play.

Next, note that when party P is elected in government following an extreme policy by party $-P$, it is facing an opposition with a reputation level that exceeds the level b^o (i.e., $b_{\text{-}P} = \pi_e^g > b^o$) and this implies that both party reputations eventually exceed the level b^o. This is because the opposition party's reputation remains above b^o in all periods before it is re-elected in government (by part (i) of Lemma 1), while the governing party's reputation either matches the reputation of the opposition party (by part (iii) of Lemma 4 in the appendix) or it adjusts monotonically towards $b^g > b^o$ following moderate policies by the government or it becomes equal to $\pi_e^g > b^o$ following an extreme policy by the government. It follows that party reputations are absorbed in $[b^o, 1]^2 \subset (b^*, 1]^2$ once the minimum reputation of the parties satisfies $\min\{b_L^t, b_R^t\} > b^o$. Now consider any period t with reputations $b^t \in [b^o, 1]^2$ and assume that moderate policies occur in all periods $t, t+1, ..., t+\bar{k}$ for some finite $\bar{k}$. If $\bar{k}$ is large enough, then by our earlier arguments it must be that $b^{t+k} \in \mathcal{B}_e$ for some $k < \bar{k}$. As a result, for all periods $t+k+1, ..., t+\bar{k}$ in which the government pursues a moderate policy, we have from Lemma 1 and part (iii) of Lemma 4 in the Appendix that

$$b_P^{t+k+1} = b_{\text{-}P}^{t+k+1} > b_P^{t+k+2} = b_{\text{-}P}^{t+k+2} > \ldots > b_P^{t+\bar{k}} = b_{\text{-}P}^{t+\bar{k}} > b^o.$$

Thus, invoking parts (ii), (iii), and (iv) of Proposition 3 we conclude that the probability of an extreme policy in all periods $t' = t+k+1, ..., t+\bar{k}$ satisfies $b_P^{t'} \sigma_P(b^{t'}) \geq b^o \sigma_P(b^o, b^o) > 0$. It follows that the probability that all policies in periods $t+1, \ldots, t+\bar{k}$ are moderate is smaller than $[1 - b^o \sigma_P(b^o, b^o)]^{\bar{k}-k}$, a quantity that goes to zero as $\bar{k}$ increases. Since this is true for any initial period t with reputations $b^t \in [b^o, 1]^2$, the above argument establishes part (i) of Proposition 5 which we state at the end of this section. In particular, an extreme policy is guaranteed to occur with probability that gets close to one within any fixed window of consecutive time periods, for a large enough such window of consecutive time periods. Thus, when condition (13) is met, the equilibrium in Proosition 3 induces alternation of parties in government and extreme policies that occur at a rate that does not dissipate in the long-run.

Case II ($\pi_e^g + \pi_m^g < 2$ and $b^g < b^o < b^*$): Policy Moderation & Incumbency Advantage

Unlike the equilibrium dynamics we characterized in the previous case, part (i) of Lemma 2 now implies that when $b^g < b^o < b^*$ party reputations eventually lie outside the subset$[b^*, 1]^2$ with probability one. As a consequence, by part (iii) of Proposition 3, governments implement a moderate policy with probability one. It is also the case that for party reputations $b \notin [b^*, 1]^2$, the government party wins reelection with probability one by pursuing a moderate policy. As a result, in this case we obtain dynamics such that either party reputations converge to $(b^g, b^o) \notin \mathcal{B}_L$ and a government by party L implements a moderate policy and wins reelection perpetually, or party reputations converge to $(b^o, b^g) \notin \mathcal{B}_R$ and party R implements a moderate policy and wins reelection perpetually, that is, the equilibrium exhibits policy moderation and an extreme form of incumbency advantage.

Case IIIa ($\pi_e^g + \pi_m^g < 2$ and $b^g = b^o = b^*$)

In the special case when the ratios in condition (13) are exactly equal, the equilibrium induces two qualitatively different dynamics that depend on initial reputations. If initial reputations satisfy $b^0 \notin (b^*, 1]^2$, then by part (i) of Lemma 2 we conclude that $b^t \notin (b^*, 1]^2$ for all periods t. Thus, by part (iii) of Proposition 3, policies remain moderate with probability one along the path of play and the government is re-elected with probability one. Lemma 2 then ensures that party reputations converge to the pair (b^o, b^o). If, on the other hand, initial reputations satisfy $b^0 \in (b^*, 1]^2$ then party reputations may reach the set $\mathcal{B}_e$ along the path of play, so that extreme policies are possible in equilibrium. Nevertheless, the minimum party reputation converges towards the long-run level b^o by Lemma 2 so that for any $\epsilon > 0$ there exists a $\bar{t}$ such that for all periods $t > \bar{t}$ extreme policies can occur in equilibrium only for party reputations $b^t \in \mathcal{B}_e \cap [b^*, b^* + \epsilon]^2$. But the maximum probability of an extreme policy in set $\mathcal{B}_e \cap [b^*, b^* + \epsilon]^2$ is given by $\sigma_P(b^* + \epsilon, b^* + \epsilon)$ by part (iv) of Proposition 3 hence (since $\lim_{\epsilon \to 0} \sigma_P(b^* + \epsilon, b^* + \epsilon) = 0$) a moderate policy prevails with probability one in the long-run in this case. Furthermore, party reputations converge in probability to the pair (b^o, b^o).

Case IIIb ($\pi_e^g = \pi_m^g = 1$ and $b^o = b^*$)

This case is otherwise identical to case IIIa, except for two possible differences. First, the minimum party reputation in this subcase may converge to a level lower than b^o when initial party reputations satisfy $b^0 \notin [b^*, 1]^2$. In particular, in those cases the party in government (say party P) enjoys a reputation that is lower to that of the opposition, $b_P < b_{\text{-}P}$, and implements a moderate policy with probability one. Thus, since $\pi_e^g = \pi_m^g = 1$, the reputation of the government party remains constant at its initial level $b_P^t = b_P^0 = T_g(b_P^0), t > 0$, while the opposition party's reputation converges to $b^* = b^o$. Second, for initial party reputations $b^0 \in (b^*, 1]^2$, arguments similar to those used in case I demonstrate that the set of reputations $\mathcal{B}_e$ is reached infinitely often along the path of play. Nevertheless, as was shown in subcase IIIa, the probability of an extreme policy goes to zero over time and joint party reputations converge in probability to the pair (b^o, b^o). Specifically, by part (ii) of Lemma 2 we conclude that the minimum party reputation does not increase along the path of play, since the governing party's reputation (say b_P^t) remains constant when $b^t \notin \mathcal{B}_P$, while when $b^t \in \mathcal{B}_P$ the minimum reputation decreases to $\min\{b_P^{t+1}, b_{\text{-}P}^{t+1}\} = b_{\text{-}P}^{t+1} = T_o(b_{\text{-}P}^t) < T_g(b_P^t) = b_P^t$, so that $\lim_{t \to +\infty} \min\{b_L^t, b_R^t\} = b^o$.

With the above analysis we have effectively shown the following proposition which summarizes the dynamics of the equilibrium in Proposition 3.

Proposition 5 *Let b^t, P^t, x^t, $t = 0, 1, ...$ be the sequence of reputations, parties in government, and policies, respectively, induced by the equilibrium in Proposition 3.*

(i) If $b^g > b^o > b^*$ then there exists $\bar{t}$ such that party reputations satisfy $b^t \in [b^o, 1]^2$ for all periods $t > \bar{t}$. In addition, for all $\epsilon > 0$ there exists $\bar{k}$ such that for all periods $t > \bar{t}$ the sequence of policies $\{x^{t+1}, ..., x^{t+\bar{k}}\}$ satisfies

$$Prob[\{x^{t+1}, ..., x^{t+\bar{k}}\} \cap \{x_e^L, x_e^R\} = \emptyset] < \epsilon. \tag{14}$$

(ii) *If* $b^g < b^o < b^*$ *then there exists* $\bar{t}$ *such that moderate policies prevail with probability one* $(\sigma_{P^t}(b^t) = 0)$ *for all periods* $t > \bar{t}$*, and either party reputations converge to* (b^g, b^o) *with a government by party* L *(i.e.,* $\lim_{t\to+\infty} b^t = (b^g, b^o)$ *and* $\lim_{t\to+\infty} P^t = L$*) or party reputations converge to* (b^g, b^o) *with a government by party* R *(i.e.,* $\lim_{t\to+\infty} b^t = (b^o, b^g)$ *and* $\lim_{t\to+\infty} P^t = R$*).*

(iii) *If* $b^o = b^*$ *and if initial reputations* $b^0 \in (b^o, 1]^2$ *then the probability of extreme policies goes to zero in the long-run and party reputations converge in probability to* (b^o, b^o) *(i.e.,* $\lim_{t\to+\infty} \sigma_{P^t}(b^t) = 0$ *and* $p\lim_{t\to+\infty} b^t = (b^o, b^o)$*). If initial reputations satisfy* $b^0 \in [0,1]^2 \setminus (b^o, 1]^2$ *then extreme policies are pursued with probability zero,* $\sigma_{P^t}(b^t) = 0$*, in all periods* t*, and:*

  (a) *If* $\pi_e^g + \pi_m^g < 2$ *then party reputations converge to* (b^o, b^o) *with one of the two parties perpetually in power (i.e.,* $\lim_{t\to+\infty} b^t = (b^o, b^o)$*, and* $\lim_{t\to+\infty} P^t = L$ *or* $\lim_{t\to+\infty} P^t = R$*).*

  (b) *If* $\pi_e^g = \pi_m^g = 1$ *then either party reputations converge to* (b_L^0, b^o) *with a government by party* L *(i.e.,* $\lim_{t\to+\infty} b^t = (b_L^0, b^o)$ *and* $\lim_{t\to+\infty} P^t = L$*) or party reputations converge to* (b^o, b_R^0) *with a government by party* R *(i.e.,* $\lim_{t\to+\infty} b^t = (b^o, b_R^0)$ *and* $\lim_{t\to+\infty} P^t = R$*).*

Parts (i), (ii), and (iii) of Proposition 5 correspond to the three cases I, II, and III, respectively, that we identified in the discussion of the dynamics induced by the equilibrium of Proposition 3. Note that the integer $\bar{k}$ in part (i) of the proposition is a function of ϵ, but can be chosen independent of the period t once party reputations are absorbed in the subset $[b^o, 1]^2$ of party reputations. Since policies are moderate with probability one in the long run in cases (ii) and (iii) of Proposition 5, the equilibrium in Proposition 3 is consistent with two radically different patterns of policy-making by the government depending on the values of the transition probabilities π_τ^o and π_τ^g. This indeterminacy suggests an obvious direction for future research, that is, in order to identify the relevant equilibrium dynamic we must further study the process of internal party competition and the forces that determines the transition probabilities π_τ^o and π_τ^g. As an empirical matter, these quantities can be inferred from the likelihood over observed data induced by the equilibrium of the model, a task pursued by Kalandrakis and Spirling (2008).

6. Probabilistic Elections

The model we have considered so far constitutes a clean benchmark on the basis of which we can evaluate the consequences of introducing more complicated assumptions. In this section we consider one such extension, namely the possibility of probabilistic elections. It is reasonable to assume that events out of the control of the two political parties may influence the outcome of the electoral campaign and give a critical electoral advantage to one of the two competing parties. Such exogenous events can be both favorable to the incumbent government (*e.g.*, a victorious war or success in foreign policy) or the opposition (*e.g.*, scandals involving the government, etc.). These events may simply represent a temporary swing in the electorate's ideological convictions. To incorporate this possibility in the model, we assume that in each period there is an (exogenous) probability $s < \frac{1}{2}$ of an electoral surprise so that the incumbent government is reelected with probability

$$(1 - s)\sigma_M(b, P) + s(1 - \sigma_M(b, P)).$$[6]

As a result, if voter M chooses to re-elect the incumbent government it is re-elected with probability $1-s$, while if voter M chooses not to re-elect the government it is re-elected with probability s and we have $1 - s > s$ by the restriction that $s < \frac{1}{2}$. In the next proposition we show that under this assumption and a condition on players' patience and office payoff which is analogous to condition (10) we obtain a robust, intuitive equilibrium with the same party strategies as those obtained in the equilibrium of Proposition 3:

Proposition 6 *Assume probabilistic elections with $s \in [0, \frac{1}{2})$ and*

$$\delta \geq \frac{(r_e^R - r_m^R) + \delta s \left(\pi_e^o(r_m^L - r_e^L) + \pi_e^g(r_e^R - r_m^R)\right)}{(1 - 2s)(G + r_m^R - r_m^L)}. \tag{15}$$

Then there exists a robust and intuitive equlibrium with party strategies given by (11).

Condition (15) is a sufficient (not necessary) condition for existence of equilibrium that is guaranteed to be satisfied for large enough G, that is, if parties care sufficiently about office. Note that when it comes to players' strategies the equilibrium in Proposition 6 is essentially identical to that in Proposition 3. Thus, the main effect of the introduction of probabilistic elections is a modification of the dynamics induced by this equilibrium. One clear difference in these dynamics has to do with the pattern of alternation of parties in government over time. As we established in parts (ii) and (iii)-(b) of Proposition 5, Proposition 3 permits dynamics such that the governing

[6]A slightly more complicated assumption in the same spirit is to assume s is an appropriate function of the electorate's beliefs $b \in [0, 1]^2$. This can be implemented in the analysis to follow, without any gain in insight.

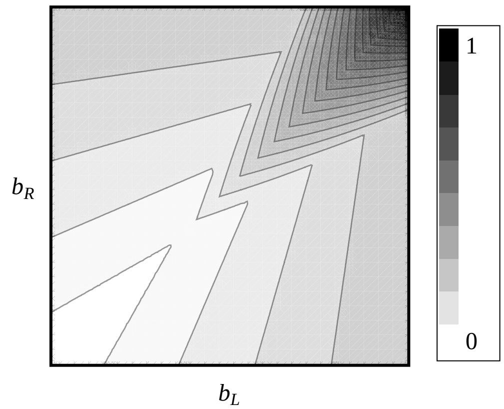

Figure 3: Probability of an Extreme Policy when $s > 0$.

party wins reelection with probability one in all periods. Obviously, once we assume probabilistic elections ($s > 0$) such strong incumbency advantage is no longer possible as the government must lose the election with probability that is at least as large as s in any equilibrium. This also implies that there is positive probability that a party may remain in government even after implementing an extreme policy.

The second implication of allowing for probabilistic elections has to do with changes in equilibrium reputation and policy dynamics, since it is now possible that extreme policies occur in equilibrium even for party reputations outside the set $\mathcal{B}_e$ defined in part (iii) of Proposition 3. In Figure 3 we depict the equilibrium probability of an extreme policy being implemented for different party reputations assuming $s > 0$. As is evident by a comparison with the corresponding probability when $s = 0$ depicted in Figure 1(c), the probability of an extreme policy is positive if and only if party reputations satisfy $b \in \mathcal{B}_L \cup \mathcal{B}_R$ when we assume $s > 0$, whereas in Proposition 3 this was the case only for the smaller subset of party reputations $\mathcal{B}_e = \mathcal{B}_L \cap \mathcal{B}_R$. Nevertheless, extreme policies occur at reputations $b \in (\mathcal{B}_L \cup \mathcal{B}_R) \setminus \mathcal{B}_e$ only after an electoral surprise, so that the probability of such policies when $b_P > b_{\text{-}P}$ is given by $sb_P\sigma_P(b) < s$. Note that Lemma 2 still applies when it comes to the equilibrium of Proposition 6, so that we can now derive the following extension of Proposition 5:

Proposition 7 *Assume $s > 0$ and let b^t, P^t, x^t, $t = 0, 1, ...$ be the sequence of reputations, parties in government, and policies, respectively, induced by the equilibrium in Proposition 6.*

(i) If $b^g > b^o > b^*$ then the conclusions of part (i) of Proposition 5 hold.

(ii) If $b^g < b^o < b^*$ then

(a) If $T_g(b^o) < T_o(b^g)$ there exists $\bar{t}$ such that for all periods $t > \bar{t}$ moderate policies prevail with probability one, $\sigma_{P^t}(b^t) = 0$, and party reputations satisfy $b^t \in [b^g, b^o]^2$.

(b) If $T_g(b^o) > T_o(b^g)$ there exists $\bar{t}$ such that $\min\{b^t_L, b^t_R\} \in [b^g, b^o]$ for all periods $t > \bar{t}$, and the probability of an extreme policy satisfies

$$0 < \limsup_{t \to +\infty} Prob[x^t \in \{x^L_e, x^R_e\}] < s.$$

(iii) If $b^o = b^*$ then the probability of an extreme policy converges in probability to zero and party reputations converge in probability to (b^o, b^o) (i.e., $\operatorname{plim}_{t\to+\infty} \sigma_{P^t}(b^t) = 0$ and $\operatorname{plim}_{t\to+\infty} b^t = (b^o, b^o)$).

According to Proposition 7, the introduction of probabilistic elections qualifies the equilibrium dynamics described in Proposition 5 in two main respects. First, while moderate policies prevail with probability one in Case II of the previous section, there now exists a subset of transition probabilities π^g_τ and π^o_τ that violate condition (13) of Lemma 1 for which extreme policies may occur with positive probability even in the long-run. This case is described in part (ii)-(b) of Proposition 7. To understand the difference in the two propositions, note that in part (ii) of Proposition 5 party reputations converge to either (b^g, b^o) and party L wins with probability one, or to (b^o, b^g) and party R wins with probability one. But with probabilistic elections party R may win the election with probability s at reputations (b^g, b^o). Thus, if $(b^g, b^o) \in \mathcal{B}_R \Leftrightarrow T_g(b^o) > T_o(b^g)$, then an extreme policy may be pursued at (or near) reputations (b^g, b^o) after a surprise victory by party R, and the same is true for reputations (b^o, b^g) and a surprise victory by party L. Since reputations converge to levels (b^g, b^o), (b^o, b^g) along paths of play without surprise electoral outcomes, extreme policies are occurring infinitely often along the path of play. This establishes part (ii)-(b) of Proposition 7. On the other hand, in the cases covered by part (ii)-(a) of Proposition 7, surprise electoral victories and alternation of parties in government do not produce extreme policies because neither party pursues an extreme policy with positive probability at reputations levels $b \in [b^g, b^o]^2$. The two subcases of case II of the previous section (when $b^g < b^o < b^*$) are depicted graphically in Figure 4.

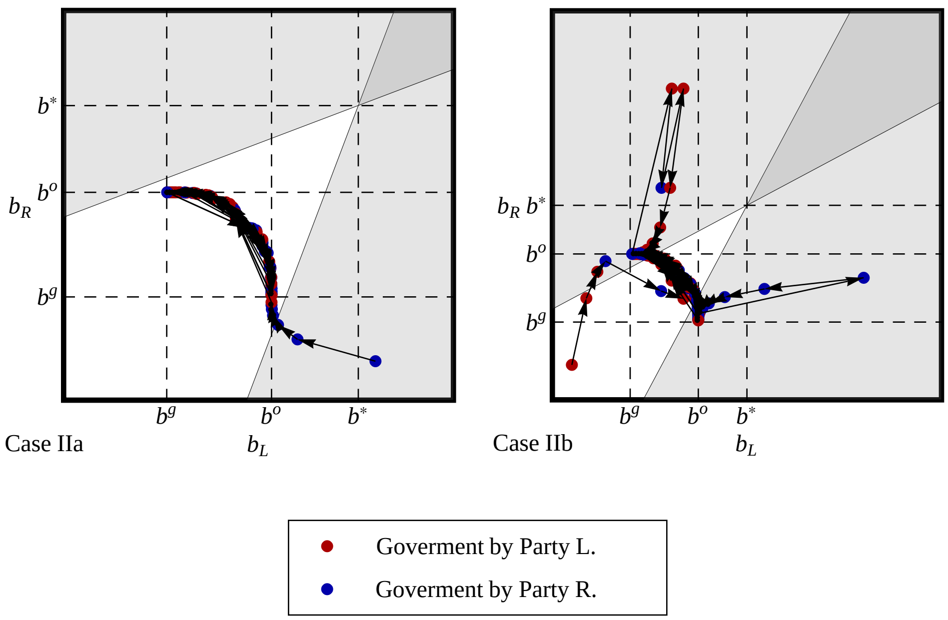

Figure 4: Probabilistic Elections and Dynamics in Case II.

The second main difference in the equilibrium dynamics of Proposition 6 compared to Proposition 3 is that when $b^o = b^*$ and $\pi_e^g = \pi_m^g = 1$ (i.e., case IIIb of the previous section) the long-run steady-state levels of party reputations no longer depend on initial conditions. In particular, contrary to the conclusion of part(iii)-(b) of Proposition 5, when initial party reputations are given by $b^0 \in [0,1]^2 \setminus [b^o, 1]^2$, the party with the minimum reputation, say party P, may lose the election (with probability $s > 0$) so that party P's reputation adjusts to a higher level $T_o(b_P^0) > b_P^0$. Since such surprise electoral outcomes are guaranteed to occur along the path of play, party reputations eventually converge to $(b^o, b^o) \in [0,1]^2$.

On the other hand, this change in reputation dynamics does not alter the policy dynamics we established in part (iii) of Proposition 5 as it is still the case that moderate policies prevail with probability one in the long-run. Similarly, despite the fact that extreme policies are possible for a subset of transition probabilities in case II of the previous section when we introduce probabilistic elections, case I (when $b^g > b^o > b^*$) still produces more policy extremism (and government turnover) in equilibrium. This is because in case (ii)-(b) of Proposition 7 policy extremism and government turnover occur due to electoral surprises, while policy extremism and alternation of the parties in government are not constrained by the value of the probability of electoral surprise, s, when $b^g > b^o > b^*$. In sum, probabilistic elections introduce realism into the model but do not alter the main conclusions of the previous section.

7. Conclusions

We have developed a model of two-party competition based on the assumption that political parties enter the electoral arena with endogenously formed reputations regarding the prevailing policy preferences within each party. These reputations shape the electorate's expectations about the policies that are likely to be pursued by each party, instead of relying on campaign promises in order to infer the policies of future governments. The second basic premise of the model is that party preferences exhibit inertia so that absent credible actions by the party in government party reputations improve or deteriorate gradually. From these two simple premises we built a dynamic model of two party competition in which equilibrium government policies are dependent not only on the incumbent party's reputation, but also on the opposition party's reputation.

We showed that in robust equilibria in which parties care sufficiently about office, the ruling party pursues extreme policies when it has a relatively worse reputation compared to the opposition. Barring electoral surprises in which parties with worse reputation win the election, extreme policies occur in equilibrium when (a) both parties' reputations are above some benchmark level and (b) elections are close, that is, both parties have similar reputations. These equilibrium strategies are consistent with two radically different electoral and policy dynamics. One possible pattern of dynamics involves regular government turnover and the recurrence of party reputations such that the party that wins the election implements extreme policies with positive probability. The second pattern of dynamics involves predominantly moderate policies and a strong incumbency advantage for the governing party. Either dynamic may prevail depending on the relative speed with which parties switch preferences while in government or in the opposition, according to condition (13). Thus, the different types of equilibrium dynamics that are permitted by the present analysis suggest that in order to fully understand the nature of two party competition we must further study the manner in which competition between different ideological groups is resolved *within* political parties. This shift of focus from inter-party competition to intra-party competition would allow us to develop insights on the forces that determine the relative size of quantities that are exogenous in the present analysis, such as the transition probabilities π_τ^g and π_τ^o.

Besides prompting a shift of focus from inter-party competition to intra-party competition, the present model leaves a number of other open avenues for improvement. One such improvement involves an increase of the time horizon which affects the strategic calculations of political actors. Perhaps the most important extension of the current model, though, would be to enrich the policy/type space by allowing more than two policy choices and party types per party. More party types and policy choices open the possibility for richer dynamics such that, for example, the governing party pursues a relatively extreme policy even when it has a large reputational advantage over the opposition party. If the opposition party has a really bad reputation, then the governing party may be able to afford to pursue such intermediate policies and maintain an (smaller) advantage over the opposition. In particular, the government's worse reputation due to the fact that it does not pursue the most moderate policy need not come at the cost of losing the elections. The restriction to only two party types that we impose in the present analysis does not permit this type of policy making by the government, since any extreme policy choice renders the updated reputation of the governing party worse than that of the opposition party under this assumption.

Appendix

In this appendix we prove Propositions 1, 2, 3, 4, and 6 and Lemmas 1 and 2. Before we proceed, we derive expressions for players' expected payoffs. The expected payoff of the voter M from a government by party P in a period with party reputations b is

$$V(b, P; \sigma) = b_P \sigma_P(b)(v_e^P - v_m^P) + v_m^P. \tag{16}$$

Also, the expected payoff of party P from implementing a policy x_τ^P while in government in a period with party reputations b is given by

$$U_P(b, x_\tau^P; \sigma) = r_\tau^R + G + \delta \left(\begin{array}{c} \sigma_M(b', P) \left[\pi_e^g \sigma_P(b')(r_e^R - r_m^R) + r_m^R + G\right] \\ +(1 - \sigma_M(b', P)) \left[b'_{-P}\sigma_{-P}(b')(r_e^L - r_m^L) + r_m^L\right] \end{array} \right), \tag{17}$$

where $b' = b'(b, x_\tau^P; \sigma)$. Observe that the expected utility calculation in (17) reflects the uncertainty of extreme types of party P regarding the type prevailing in the next period both in the opposition party, $-P$, as well as within their own party, P. We start with Proposition 1:

Proof of Proposition 1 We start by showing part (ii). Consider an equilibrium with strategies $\sigma = (\sigma_M, \sigma_L, \sigma_R)$, where party strategies satisfy $\sigma_P(b) = 1$ for all $b \in [0,1]^2$, $P \in \{L, R\}$. If we perturb party strategies by $\varepsilon < \frac{1}{2}$, we have $\sigma_P^\varepsilon(b) = 1 - \varepsilon$. Define the profile of strategies $\sigma^\varepsilon = (\sigma_M, \sigma_L^\varepsilon, \sigma_R^\varepsilon)$. Note that the original equilibrium strategy profile is obtained when $\varepsilon = 0$, i.e., we have $\sigma^0 = \sigma$. We use (16) to calculate voter's expected utility when strategies σ^ε are used and obtain $V(b, P; \sigma^\varepsilon) = b_P(1-\varepsilon)(v_e^P - v_m^P) + v_m^P$, $P \in \{L, R\}$. We now have

$$V(b, P; \sigma^\varepsilon) > V(b, -P; \sigma^\varepsilon) \Leftrightarrow b_P < b_{\text{-}P}, \text{ for all } \varepsilon \in [0, \tfrac{1}{2}).$$

Since the above holds for $\varepsilon = 0$ and $\sigma^0 = \sigma$ is an equilibrium, we must also have

$$b_P < b_{\text{-}P} \Leftrightarrow V(b, P; \sigma^0) > V(b, -P; \sigma^0) \Rightarrow \sigma_M(b, P) = 1 - \sigma_M(b, -P) = 1,$$

so we conclude that the voter's strategy σ_M satisfies (8). Thus equilibrium σ is intuitive and it is also robust since it satisfies (4) with perturbed strategies σ^ε for all $\varepsilon \in [0, \frac{1}{2})$.

Next, we show part (iii). Suppose that (9) is satisfied strictly and there exists an equilibrium σ^* with $\sigma_P^*(b) < 1$ for some $b \in [0,1]^2$ to get a contradiction. Then we must have $U_P(b, x_m^P; \sigma^*) \geq U_P(b, x_e^P; \sigma^*)$ for these beliefs. Note that logically there exists a lower bound on expected utilities which must satisfy $U_P(b, x_e^P; \sigma) \geq r_e^R + G + \delta(T_o(b_{\text{-}P}) r_e^L + (1 - T_o(b_{\text{-}P})) r_m^L)$, for all σ. Since $T_o(y) \leq \pi_e^o$ for all $y \in [0,1]$ and $r_m^L > r_e^L$, the lower bound inequality above implies

$$U_P(b, x_e^P; \sigma) \geq r_e^R + G + \delta(\pi_e^o r_e^L + (1 - \pi_e^o) r_m^L), \tag{18}$$

for all $b \in [0,1]^2$ and all σ. Similarly, we obtain

$$U_P(b, x_m^P; \sigma) \leq r_m^R + G + \delta(\pi_e^g r_e^R + (1 - \pi_e^g) r_m^R + G), \tag{19}$$

for all $b \in [0,1]^2$ and all σ. Now, with simple algebra, the strict version of (9) yields

$$\delta < \frac{r_e^R - r_m^R}{\pi_e^o(r_m^L - r_e^L) + \pi_e^g(r_e^R - r_m^R) + G + r_m^R - r_m^L} \Leftrightarrow$$

$$r_m^R + G + \delta(\pi_e^g r_e^R + (1-\pi_e^g) r_m^R + G) < r_e^R + G + \delta(\pi_e r_e^L + (1-\pi_e) r_m^L),$$

and the latter implies, using (18) and (19), that

$$\delta < \frac{r_e^R - r_m^R}{\pi_e^o(r_m^L - r_e^L) + \pi_e^g(r_e^R - r_m^R) + G + r_m^R - r_m^L} \Rightarrow U_P(b, x_e^P; \sigma) > U_P(b, x_m^P; \sigma),$$

for all $b \in [0,1]^2$ and all σ. The latter part contradicts the working hypothesis that $U_P(b, x_m^P; \sigma^*) \geq U_P(b, x_e^P; \sigma^*)$ for some $b \in [0,1]^2$, proving part (iii).

It remains to prove part (i). First, we verify that there exists an equilibrium with $\sigma_P(b) = 1$ for all $b \in [0,1]^2$, $P \in \{L, R\}$, when (9) holds. Note that (18) and (19) yield

$$(9) \Rightarrow U_P(b, x_e^P; \sigma) \geq U_P(b, x_m^P; \sigma), \tag{20}$$

for all $b \in [0,1]^2$ and all σ, so that strategy $\sigma_P(b) = 1$ for all $b \in [0,1]^2$ is a (weak) best response independent of the voting strategy σ_M. As a result, in order to establish existence of equilibrium we only need specify a voting strategy that satisfies (4). We know from part (ii) that the voting strategy must satisfy (8). If we set arbitrary values for $\sigma_M(b, P)$ for $b \in [0,1]^2$ such that $b_L = b_R$, then the resultant strategy profile constitutes a robust and intuitive equilibrium whenever (9) is true.

Lastly, we need to show that when (9) fails there cannot exist an equilibrium with these party strategies. Suppose instead that (9) fails and there exists an equilibrium σ^* with $\sigma_P^*(b) = 1$ for all $b \in [0,1]^2$, $P \in \{L, R\}$, to get a contradiction. Consider a period t with party P of type e in government. We shall show that there exist beliefs $b^t \in [0,1]^2$ with $b_P^t < 1$ for which party P has a profitable deviation from strategy σ_P^*. If the governing party follows its strategy and implements a policy x_e^P, we have that $b^{t+1} = b'(b^t, x_e^P; \sigma^*)$ which by (6) and (3) translates to beliefs $b_P^{t+1} = \pi_e^g$ and $b_{-P}^{t+1} = T_o(b_{-P}^t)$ in period $t+1$. Using (1) and (2) we then conclude that $b_P^{t+1} > b_{-P}^{t+1}$ for all b_{-P}^t. Furthermore, by part (ii) the voter's strategy σ_M^* satisfies (8), hence $\sigma_M^*(b^{t+1}, P) = 0$ and the expected payoff of type e of party P implementing policy x_e^P is given by

$$U_P(b^t, x_e^P; \sigma^*) = r_e^R + G + \delta(T_o(b_{-P}^t)(r_e^L - r_m^L) + r_m^L).$$

On the other hand, a one-period deviation to a moderate policy x_m^P results in beliefs $b^{t+1} =$

$b^{'}(b^t, x^P_m; \sigma^*)$ in period $t+1$ which by (6) and (3) take the values $b^{t+1}_P = 1 - \pi^g_m$ and $b^{t+1}_{-P} = T_o(b^t_{-P})$. We now have that $b^{t+1}_P = 1 - \pi^g_m < b^{t+1}_{-P} = T_o(b^t_{-P})$ for all b^t_{-P} by (1) and (2). Thus, implementing x^P_m leads to $\sigma^*_M(b^{'}(b^t, x^P_m; \sigma^*), P) = 1$, and accrues payoff

$$U_P(b^t, x^P_m; \sigma^*) = r^R_m + G + \delta(\pi^g_e(r^R_e - r^R_m) + r^R_m + G).$$

Since σ^* is an equilibrium we must have $U_P(b^t, x^P_e; \sigma^*) \geq U_P(b^t, x^P_m; \sigma^*))$ for all $b^t \in [0,1]^2$, which (substituting for the expected utilities computed above) satisfies

$$U_P(b^t, x^P_e; \sigma^*) \geq U_P(b^t, x^P_m; \sigma^*)) \Leftrightarrow \delta \leq \frac{r^R_e - r^R_m}{T_o(b^t_{-P})(r^L_m - r^L_e) + \pi^g_e(r^R_e - r^R_m) + G + r^R_m - r^L_m}.$$

But we have assumed that (9) is violated, i.e., there exists $\epsilon > 0$ such that

$$\frac{r^R_e - r^R_m}{\pi^o_e(r^L_m - r^L_e) + \pi^g_e(r^R_e - r^R_m) + G + r^R_m - r^L_m} + \epsilon = \delta,$$

so that we obtain

$$\frac{r^R_e - r^R_m}{T_o(b^t_{-P})(r^L_m - r^L_e) + \pi^g_e(r^R_e - r^R_m) + G + r^R_m - r^L_m} \geq \frac{r^R_e - r^R_m}{\pi^o_e(r^L_m - r^L_e) + \pi^g_e(r^R_e - r^R_m) + G + r^R_m - r^L_m} + \epsilon,$$

which is false for any b^t_{-P} sufficiently close to 1. This is a contradiction emanating from the hypothesis that σ^* is an equilibrium. ■

We continue with the proof of Proposition 2.

Proof of Proposition 2 Assume there exists a robust equilibrium σ^* with strategies $\sigma^*_P(b) = 0$, for all $b \in [0,1]^2$, $P \in \{L, R\}$. We first show that this equilibrium must also be intuitive, i.e., the voting strategy σ^*_M must satisfy (8). Consider perturbed party strategies according to $\sigma^{\varepsilon *}_P(b) = \varepsilon$, $P \in \{L, R\}$, with $\varepsilon > 0$, and compare the voter's expected utility for reputations b with $b_P < b_{-P}$ when players use strategies $\sigma^{\varepsilon *} = (\sigma^*_M, \sigma^{\varepsilon *}_L, \sigma^{\varepsilon *}_R)$. We get

$$\begin{aligned} b_P &< b_{-P} \Rightarrow \\ b_P\varepsilon(v^P_e - v^P_m) + v^P_m &> b_{-P}\varepsilon(v^{-P}_e - v^{-P}_m) + v^{-P}_m \Rightarrow \\ V(b, P; \sigma^{\varepsilon *}) &> V(b, -P; \sigma^{\varepsilon *}). \end{aligned}$$

Since the above holds for all $\varepsilon > 0$ and σ^* is a robust equilibrium, we conclude that we must have $\sigma^*_M(b, P) = 1$. Thus, the robust equilibrium σ^* must be intuitive as we wished to show.

In order to prove the proposition we will now show that for the robust and intuitive equilibrium with strategies σ^* there exist party reputations such that σ^*_P is not a best response. Trivially there exist reputations $b^t \in [0,1]^2$ such that $T_g(b^t_P) > T_o(b^t_{\text{-}P})$. Consider a government by party P in period t with party reputations given by $b^t \in [0,1]^2$. Given strategy $\sigma^*_P(b^t) = 0$ and applying (3), beliefs in period $t+1$ along the equilibrium path are given by $b^{t+1}_{\text{-}P} = T_o(b^t_{\text{-}P})$ and $b^{t+1}_P = T_g(b^t_P)$, the latter because the posterior belief obtained from (6) following a moderate policy is $\beta(b^t, x^P_m; \sigma^*) = b^t_P$. Since we have assumed $T_g(b^t_P) > T_o(b^t_{\text{-}P})$, we have from (8) and the fact that the equilibrium with strategies σ^* is intuitive that $\sigma^*_M(b^{t+1}, P) = 0$. If party P implements an extreme policy x^P_e in period t instead, then since $\beta(b, x^P_e; \sigma^*) = 1$ we obtain $b^{t+1}_P = T_g(1) = \pi^g_e > b^{t+1}_{\text{-}P} = T_o(b^t_{\text{-}P})$, where the last inequality follows from (1) and (2). Thus, when beliefs in period t are given by b^t such that $T_g(b^t_P) > T_o(b^t_{\text{-}P})$, we have $\sigma^*_M(b^{t+1}, P) = 0$ whether party P pursues a moderate or an extreme policy. Substituting in the expected utility expression (17) we get

$$U_P(b^t, x^P_m; \sigma^*) = r^P_m + G + \delta r^{\text{-}P}_m < r^P_e + G + \delta r^{\text{-}P}_m = U_P(b^t, x^P_e; \sigma^*).$$

We conclude that there exist $b^t \in [0,1]^2$ such that $\sigma^*_P(b^t) = 0$ is not a best response, and σ^* cannot be part of a robust equilibrium. ■

Before we prove Proposition 3, we prove two lemmas.

Lemma 3 *Consider party strategy σ_P given by (11) and reputations $b \in [0,1]^2$ such that $T_g(b_P) > T_o(b_{\text{-}P})$. Then,*

(i) $\dfrac{\partial \sigma_P(b)}{\partial b_P} > 0$ *and* $\dfrac{\partial b_P \sigma_P(b)}{\partial b_P} > 0,$ *and*

(ii) $\dfrac{\partial b_P \sigma_P(b)}{\partial b_{\text{-}P}} \leq 0$ *and* $\dfrac{\partial b_P \sigma_P(b)}{\partial b_{\text{-}P}} \leq 0.$

Proof. To show part (i) we compute

$$\frac{\partial \sigma_P(b)}{\partial b_P} = \frac{T_o(b_{\text{-}P}) - (1 - \pi^g_m)}{b^2_P(\pi^g_e - T_o(b_{\text{-}P}))} > 0,$$

since $T_o(b_{\text{-}P}) \geq 1 - \pi_m^o > 1 - \pi_m^g$ by (1); and,

$$\frac{\partial b_P \sigma_P(b)}{\partial b_P} = \frac{\pi_e^g + \pi_m^g - 1}{\pi_e^g - T_o(b_{\text{-}P})} > 0,$$

since the numerator is positive by (1) and (2). Part (ii) is similarly obtained since

$$\frac{\partial \sigma_P(b)}{\partial b_{\text{-}P}} = \frac{(T_g(b_P) - \pi_e^g)(\pi_e^o + \pi_m^o - 1)}{b_P(\pi_e^g - T_o(b_{\text{-}P}))^2} \leq 0, \text{ and}$$

$$\frac{\partial b_P \sigma_P(b)}{\partial b_{\text{-}P}} = \frac{(T_g(b_P) - \pi_e^g)(\pi_e^o + \pi_m^o - 1)}{(\pi_e^g - T_o(b_{\text{-}P}))^2} \leq 0,$$

because $T_g(b_P) \leq \pi_e^g$. ■

The next Lemma is:

Lemma 4 *Consider any equilibrium strategy profile σ and reputations $b \in [0,1]^2$.*

(i) If $b_P \sigma_P(b) < 1$ then $b'_P(b, x_m^P; \sigma) < b'_{\text{-}P}(b, x_m^P; \sigma) \Leftrightarrow b_P \sigma_P(b) > \frac{T_g(b_P) - T_o(b_{\text{-}P})}{(\pi_e^g - T_o(b_{\text{-}P}))}$.

(ii) $b'_P(b, x_e^P; \sigma) > b'_{\text{-}P}(b, x_e^P; \sigma)$.

(iii) If party strategies are given by (11) and $b_P < 1$ then $b'_P(b, x_m^P; \sigma) = b'_{\text{-}P}(b, x_m^P; \sigma)$.

Proof. To show part (i) note that, since $\sigma_P(b) b_P < 1$, we have from (6) and (3) that

$$\begin{aligned}
b'_P(b, x_m^P; \sigma) &< b'_{\text{-}P}(b, x_m^P; \sigma) \Leftrightarrow \\
T_g(\beta(b, x_m^P; \sigma)) &< T_o(b_{\text{-}P}) \Leftrightarrow \\
T_g\left(\frac{(1 - \sigma_P(b)) b_P}{1 - \sigma_P(b) b_P}\right) &< T_o(b_{\text{-}P}) \Leftrightarrow \\
\pi_e^g \left(\frac{(1 - \sigma_P(b)) b_P}{1 - \sigma_P(b) b_P}\right) + (1 - \pi_m^g)\left(1 - \frac{(1 - \sigma_P(b)) b_P}{1 - \sigma_P(b) b_P}\right) &< T_o(b_{\text{-}P}) \Leftrightarrow \\
T_g(b_P) - \pi_e^g \sigma_P(b) b_P &< (1 - \sigma_P(b) b_P) T_o(b_{\text{-}P}) \Leftrightarrow \\
T_g\left(b_P\right) - T_o\left(b_{\text{-}P}\right) &< \left(\pi_e^g - T_o(b_{\text{-}P})\right) \sigma_P(b) b_P \Leftrightarrow \\
b_P \sigma_P(b) &> \frac{T_g(b_{\text{-}P}) - T_o(b_{\text{-}P})}{(\pi_e^g - T_o(b_{\text{-}P}))}.
\end{aligned}$$

For part (ii) we have from (6) and (3) that

$$b'_P(b, x_e^P; \sigma) = T_g(\beta(b, x_e^P; \sigma)) = T_g(1) = \pi_e^g > T_o(b_{\text{-}P}) = b'_{\text{-}P}(b, x_e^P; \sigma),$$

by (1) and (2).

Finally, part (iii) follows immediately from part (i) and (11). ■

We now prove Proposition 3.

Proof of Proposition 3 Part (i) follows from Proposition 6 since condition (10) implies condition (15) when $s = 0$. Part (ii) follows from Lemma 3. For part (iii), note that (b^*, b^*) constitutes the unique solution of the system of linear equations

$$\begin{aligned} T_g(b_L) - T_o(b_R) &= 0 \\ T_g(b_R) - T_o(b_L) &= 0, \end{aligned}$$

for the unknowns (b_L, b_R). Furthermore, the equation $T_g(b_P) = T_o(b_{\text{-}P})$ is linear in parties' reputations so that $\mathcal{B}_P$ is formed as the intersection of the open half-space defined by $T_g(b_P) > T_o(b_{\text{-}P})$ and the unit square $[0,1]^2$. Note that reputations (b^*, b^*) lie at the boundary of $\mathcal{B}_e$, reputations $(1,1) \in \mathcal{B}_e$ since $T_g(1) = \pi_e^g > \pi_e^o = T_o(1)$ by (1) and (2), and reputations $(b^*, 1), (1, b^*) \notin \mathcal{B}_e$ since $T_g(b^*) = T_o(b^*) < \pi_e^o = T_o(1)$. As a result, the set $\mathcal{B}_e = \mathcal{B}_L \cap \mathcal{B}_R \subset [b^*, 1]^2$. Part (iv) follows from part (ii) and the fact that a voter pursuing an intuitive equilibrium induces an equilbrium probability of an extreme policy equal to $\min\{b_L \sigma_L(b), b_R \sigma_R(b)\}$. ■

Next we prove Proposition 4.

Proof of Proposition 4 We prove the Proposition in two steps. In the first step we show that parties' equilibrium probability of pursuing an extreme policy cannot be larger than that implied by the strategies in (11).

1. *For any intuitive equilibrium σ', party strategies σ'_P satisfy $\sigma'_P(b) \leq \sigma^*_P(b)$ for all $b \in [0,1]^2$, where $\sigma^*_P(b)$ is given by (11).* Obvious for $b \in [0,1]^2$ such that $b_P = 1$, since $\sigma^*_P(b) = 1$ in these cases. For reputations $b \in [0,1]^2$ with $b_P < 1$, assume that $\sigma'_P(b) > \sigma^*_P(b)$ in order to get a contradiction. Since $b_P < 1$, from part (i) of Lemma 4 we have that $\sigma'_P(b) > \sigma^*_P(b) = \frac{T_g(b_P) - T_o(b_{\text{-}P})}{b_P(\pi_e^g - T_o(b_{\text{-}P}))} \Rightarrow b'_P(b, x_m^P; \sigma') < b'_{\text{-}P}(b, x_m^P; \sigma')$. Similarly from part (ii) of Lemma 4 we conclude that $b'_P(b, x_e^P; \sigma') > b'_{\text{-}P}(b, x_e^P; \sigma')$. Since the voting strategy satisfies (8), then $\sigma'_M(b'(b, x_e^P; \sigma'), P) = 0$ and $\sigma'_M(b'(b, x_m^P; \sigma'), P) = 1$. We now have, using (17), that

$$\begin{aligned} U_P(b, x_m^P; \sigma') > U_P(b, x_e^P; \sigma') &\Leftrightarrow \\ r_m^R + G + \delta(\pi_e^g \sigma'_P(b'(b, x_m^P; \sigma'))(r_e^R - r_m^R) + r_m^R + G) &> \\ r_e^R + G + \delta(T_o(b_{\text{-}P})\sigma'_{\text{-}P}(b'(b, x_e^P; \sigma'))(r_e^L - r_m^L) + r_m^L) &\Leftrightarrow \\ r_m^R - r_e^R + \delta(G + r_m^R - r_m^L) > -\delta \left(\begin{array}{c} \pi_e^g \sigma'_P(b'(b, x_m^P; \sigma'))(r_e^R - r_m^R) \\ +T_o(b_{\text{-}P})\sigma'_{\text{-}P}(b'(b, x_e^P; \sigma'))(r_m^L - r_e^L) \end{array} \right). \end{aligned}$$

The right-hand side of the above is less than or equal to zero, while the left-hand side satisfies

$$r_m^R - r_e^R + \delta(G + r_m^R - r_m^L) > 0 \Leftrightarrow (10).$$

Thus, if $\sigma'_P(b) > \sigma^*_P(b) \geq 0$, we conclude that $U_P(b, x_m^P; \sigma') > U_P(b, x_e^P; \sigma')$, which contradicts the assumption that σ' constitute equilibrium strategies. Thus we must have $\sigma'_P(b) \leq \sigma^*_P(b)$.

We conclude the proof by showing:

2. *For any intuitive equilibrium σ', party strategies σ'_P satisfy $\sigma'_P(b) \geq \sigma^*_P(b)$ for all $b \in [0,1]^2$, where $\sigma^*_P(b)$ is given by (11).* The claim is obviously true for $b \in [0,1]^2$ such that $T_g(b_P) \leq T_o(b_{\text{-}P})$ since $\sigma^*_P(b) = 0$ in those cases. Thus, it remains to consider reputations $b \in [0,1]^2$ such that $T_g(b_P) > T_o(b_{\text{-}P})$, and we will prove the claim by contradiction as in step 1. By part (i) of Lemma 4 we conclude that if $\sigma'_P(b) < \sigma^*_P(b) = \frac{T_g(b_P) - T_o(b_{\text{-}P})}{b_P(\pi_e^g - T_o(b_{\text{-}P}))}$, then $b'_P(b, x_m^P; \sigma') > b'_{\text{-}P}(b, x_m^P; \sigma')$. We also have from part (ii) of Lemma 4 that $b'_P(b, x_e^P; \sigma') > b'_{\text{-}P}(b, x_e^P; \sigma')$. We conclude that if $\sigma'_P(b) < \sigma^*_P(b)$ we have $\sigma'_M(b'(b, x_m^P; \sigma'), P) = \sigma'_M(b'(b, x_e^P; \sigma'), P) = 0$, since σ'_M constitutes an equilibrium strategy that satisfies (8). Thus the expected payoff from pursuing either policy, x_τ^P, $\tau \in \{e, m\}$, satisfies

$$U_P(b, x_\tau^P; \sigma^*) = r_\tau^R + G + \delta(T_o(b_{\text{-}P})\sigma'_{\text{-}P}(b'(b, x_\tau^P; \sigma'))(r_e^L - r_m^L) + r_m^L). \tag{21}$$

From step 1 and the fact that $\pi_e^o \geq T_o(b_{\text{-}P})$ we have

$$\sigma'_{\text{-}P}(b'(b, x_e^P; \sigma')) \leq \frac{T_g(T_o(b_{\text{-}P})) - T_o(\pi_e^g)}{T_o(b_{\text{-}P})(\pi_e^g - T_o(\pi_e^g))} \leq \frac{T_g(\pi_e^o) - T_o(\pi_e^g)}{T_o(b_{\text{-}P})(\pi_e^g - T_o(\pi_e^g))}.$$

The above inequality along with (12) and (21) yield:

$$
\begin{aligned}
r_e^R - r_m^R &> \frac{\delta(T_g(\pi_e^o) - T_o(\pi_e^g))}{(\pi_e^g - T_o(\pi_e^g))}(r_m^L - r_e^L) \Rightarrow \\
r_e^R - r_m^R &> \delta T_o(b_{\text{-}P})\sigma'_{\text{-}P}(b'(b, x_e^P; \sigma'))(r_m^L - r_e^L) \Rightarrow \\
r_e^R - r_m^R &> \delta T_o(b_{\text{-}P})(\sigma'_{\text{-}P}(b'(b, x_e^P; \sigma')) - \sigma'_{\text{-}P}(b'(b, x_m^P; \sigma'))(r_m^L - r_e^L) \Leftrightarrow \\
r_m^R + \delta T_o(b_{\text{-}P})\sigma'_{\text{-}P}(b'(b, x_m^P; \sigma'))(r_e^L - r_m^L) &< r_e^R + \delta T_o(b_{\text{-}P})\sigma'_{\text{-}P}(b'(b, x_e^P; \sigma'))(r_e^L - r_m^L) \Leftrightarrow \\
U_P(b, x_m^P; \sigma') &< U_P(b, x_e^P; \sigma').
\end{aligned}
$$

Thus, since $U_P(b, x_m^P; \sigma') < U_P(b, x_e^P; \sigma')$ we cannot have $\sigma'_P(b) < \sigma_P^*(b) = \dfrac{T_g(b_P) - T_o(b_{\text{-}P})}{b_P(\pi_e^g - T_o(b_{\text{-}P}))} \leq 1$. ■

We now prove Lemmas 1 and 2.

Proof of Lemma 1

(i) We have $b > T_o(b) \Leftrightarrow b > \pi_e^o b + (1 - \pi_m^o)(1 - b) \Leftrightarrow b > \frac{1-\pi_m^o}{2-\pi_m^o-\pi_e^o} \Leftrightarrow b > b^o$. Also $T_o(b) > b^o \Leftrightarrow \pi_e^o b + (1 - \pi_m^o)(1 - b) > \frac{1-\pi_m^o}{2-\pi_m^o-\pi_e^o} \Leftrightarrow b > \frac{1-\pi_m^o-(1-\pi_m^o)(2-\pi_m^o-\pi_e^o)}{(2-\pi_m^o-\pi_e^o)(\pi_e^o+\pi_m^o-1)} \Leftrightarrow b > \frac{1-\pi_m^o}{2-\pi_m^o-\pi_e^o}$, where we have made use of the fact that $\pi_e^o + \pi_m^o - 1 > 0$ from (2).

(ii) Same as in part (i) *mutatis mutandis.*

(iii) We start with the first inequality, for which we have

$$
\begin{aligned}
b^g > b^o \Leftrightarrow \frac{1 - \pi_m^g}{(1 - \pi_e^g) + (1 - \pi_m^g)} &> \frac{1 - \pi_m^o}{(1 - \pi_e^o) + (1 - \pi_m^o)} \Leftrightarrow \\
(1 - \pi_m^g)((1 - \pi_e^o) + (1 - \pi_m^o)) > (1 - \pi_m^o)((1 - \pi_e^g) + (1 - \pi_m^g)) &\Leftrightarrow \frac{1 - \pi_e^g}{1 - \pi_e^o} < \frac{1 - \pi_m^g}{1 - \pi_m^o}.
\end{aligned}
$$

We proceed similarly for the second inequality, so now we have

$$
\begin{gathered}
b^o > b^* \Leftrightarrow \frac{1 - \pi_m^o}{(1 - \pi_e^o) + (1 - \pi_m^o)} > \frac{(1 - \pi_m^o) - (1 - \pi_m^g)}{(1 - \pi_e^o) - (1 - \pi_e^g) + (1 - \pi_m^o) - (1 - \pi_m^g)} \Leftrightarrow \\
(1 - \pi_m^o)((1 - \pi_e^o) - (1 - \pi_e^g) + (1 - \pi_m^o) - (1 - \pi_m^g)) > \\
(1 - \pi_m^o)((1 - \pi_e^o) + (1 - \pi_m^o)) - (1 - \pi_m^g)((1 - \pi_e^o) + (1 - \pi_m^o)) \Leftrightarrow \\
-(1 - \pi_m^o)(1 - \pi_e^g) > -(1 - \pi_m^g)(1 - \pi_e^o) \Leftrightarrow \\
\frac{1 - \pi_e^g}{1 - \pi_e^o} < \frac{1 - \pi_m^g}{1 - \pi_m^o}. \blacksquare
\end{gathered}
$$

Proof of Lemma 2 Consider party reputations $b^t \in [0,1]^2$ at the beginning of period t. We distinguish three cases:

Case 1, a government by party $P \in \{L, R\}$ and $T_g(b^t_P) \leq T_o(b^t_{-P})$: Then by (11) party P implements a moderate policy x^P_m and $b^{t+1}_P = T_g(b^t_P)$, $b^{t+1}_{-P} = T_o(b^t_{-P})$, hence $\min\{b^{t+1}_L, b^{t+1}_R\} = T_g(b^t_P) = \min\{T_g(b^t_P), T_o(b^t_{-P})\}$.

Case 2, a government by party $P \in \{L, R\}$, $T_g(b^t_P) > T_o(b^t_{-P})$, and party P implements a moderate policy, x^P_m: Then by part (iii) of Lemma 4 we have $b^{t+1}_P = b^{t+1}_{-P} = T_o(b^t_{-P})$. Hence, $\min\{b^{t+1}_L, b^{t+1}_R\} = T_o(b^t_{-P}) = \min\{T_g(b^t_P), T_o(b^t_{-P})\}$.

Case 3, a government by party $P \in \{L, R\}$, $T_g(b^t_P) > T_o(b^t_{-P})$, and party P implements an extreme policy, x^P_e: In this case by (6) and (1) and (2) we conclude that $b^{t+1}_P = \pi^g_e > b^{t+1}_{-P} = T_o(b^t_{-P})$. Hence, $\min\{b^{t+1}_L, b^{t+1}_R\} = T_o(b^t_{-P}) = \min\{T_g(b^t_P), T_o(b^t_{-P})\}$.

Hence, in all three cases $\min\{b^{t+1}_L, b^{t+1}_R\} = \min\{T_g(b^t_P), T_o(b^t_{-P})\}$, where P is the governing party. Now we have

$$\begin{aligned}
\min\{b^{t+1}_L, b^{t+1}_R\} &= \min\{T_g(b^t_P), T_o(b^t_{-P})\} \Rightarrow \\
\min\{b^{t+1}_L, b^{t+1}_R\} &\geq \min\left\{\min\{T_g(b^t_L), T_o(b^t_R)\}, \min\{T_g(b^t_R), T_o(b^t_L)\}\right\} \Leftrightarrow \\
\min\{b^{t+1}_L, b^{t+1}_R\} &\geq \min\{T_g(\min\{b^t_L, b^t_R\}), T_o(\min\{b^t_L, b^t_R\})\}, \qquad (22)
\end{aligned}$$

where we make use of parts (i) and (ii) of Lemma 1. Similarly we have

$$\begin{aligned}
\min\{b^{t+1}_L, b^{t+1}_R\} &= \min\{T_g(b^t_P), T_o(b^t_{-P})\} \Rightarrow \\
\min\{b^{t+1}_L, b^{t+1}_R\} &\leq \max\left\{\min\{T_g(b^t_L), T_o(b^t_R)\}, \min\{T_g(b^t_R), T_o(b^t_L)\}\right\} \Rightarrow \\
\min\{b^{t+1}_L, b^{t+1}_R\} &\leq \max\{T_g(\min\{b^t_L, b^t_R\}), T_o(\min\{b^t_L, b^t_R\})\}. \qquad (23)
\end{aligned}$$

Part (i) now follows from (22) and (23) and parts (i) and (ii) of Lemma 1. We similarly obtain part (ii), noting that if $\pi^g_e = \pi^g_m = 1$ we have $T_g(b) = b$. ■

Before we prove Proposition 6, observe that with probabilistic voting the expected payoff of party P from implementing a policy x^P_τ while in government in a period with party reputations

$b \in [0,1]^2$ is given by

$$U_P(b, x_\tau^P; \sigma) = r_\tau^R + G + \delta\begin{pmatrix} ((1-s)\sigma_M(b', P) + s(1-\sigma_M(b', P)))(\pi_e^g \sigma_P(b')(r_e^R - r_m^R) + r_m^R + G) \\ +(s\sigma_M(b', P) + (1-s)(1-\sigma_M(b', P)))(b'_{\text{-}P}\sigma_{\text{-}P}(b')(r_e^L - r_m^L) + r_m^L) \end{pmatrix}, \quad (24)$$

where $b' = b'_P(b, x_\tau^P; \sigma)$ is given by (3). We now proceed to the last proof.

Proof of Proposition 6 We will establish the existence of an intuitive equilibrium with party strategies σ_P^* given by (11), and any voting strategy σ_M^* that satisfies (8) and

$$\sigma_M^*(b, P) = \frac{r_e^R - r_m^R + \delta\begin{pmatrix} s\pi_e^g(\sigma_R^*(b_R, \pi_e^g) - \sigma_R^*(b))(r_e^R - r_m^R) \\ +(1-s)b_R(\sigma_R^*(b) - \sigma_R^*(\pi_e^g, b_R))(r_m^L - r_e^L) \end{pmatrix}}{\delta(1-2s)(G + r_m^R - r_m^L + \sigma_R^*(b)(\pi_e^g(r_e^R - r_m^R) + b_R(r_m^L - r_e^L)))}, \quad (25)$$

for reputations $b = (b_L, b_R)$ such that $b_L = b_R \in [1 - \pi_m^o, \pi_e^o]$. Let σ^* denote a profile of such strategies and specify out-of-equilibrium beliefs $\beta(b, x_m^P; \sigma^*)$ for reputations $b \in [0,1]^2$ with $b_P = 1$ to $\beta(b, x_m^P; \sigma^*) = T_o(b_{\text{-}P})$.[7] The proof now proceeds in five steps. First, we show that the probability specified in (25) is well-defined.

1. *$\sigma_M^*(b) \in [0,1]$ for all $b \in [0,1]^2$ such that $b_P = b_{\text{-}P} \in [1 - \pi_m^o, \pi_e^o]$.* Assume such reputations b. First, note that

$$r_e^R - r_m^R + \delta\begin{pmatrix} s\pi_e^g(\sigma_R^*(b_R, \pi_e^g) - \sigma_R^*(b))(r_e^R - r_m^R) \\ +(1-s)b_R(\sigma_R^*(b) - \sigma_R^*(\pi_e^g, b_R))(r_m^L - r_e^L) \end{pmatrix} \geq 0 \Rightarrow \sigma_M^*(b) \geq 0.$$

But the former inequality is true since both $\sigma_R^*(b_R, \pi_e^g) \geq \sigma_R^*(b)$ and $\sigma_R^*(b) \geq \sigma_R^*(\pi_e^g, b_R)$, because $\sigma_R^*(b)$ is weakly increasing in b_R and weakly decreasing in b_L by Lemma 3. Furthermore, we have

$$\sigma_M^*(b) \leq 1 \Leftrightarrow \delta \geq \frac{(r_e^R - r_m^R) + \delta\begin{Bmatrix} \pi_e^g(s\sigma_R^*(b_R, \pi_e^g) - (1-s)\sigma_R^*(b))(r_e^R - r_m^R) \\ +b_R(s\sigma_R^*(b) - (1-s)\sigma_R^*(\pi_e^g, b_R))(r_m^L - r_e^L) \end{Bmatrix}}{(1-2s)(G + r_m^R - r_m^L)},$$

and the latter (using the fact that $b_L = b_R \in [1 - \pi_m^o, \pi_e^o]$) is implied by (15), completing the proof of this step.

[7] A range of values for $\beta(b, x_m^P; \sigma^*)$ is consistent with equilibrium. Once more, observe that such beliefs with $b_P = 1$ never occur along the path of play when $\pi_e^g < 1$.

2. *The voter's strategy, σ^*_M, is a best response.* Note that by symmetry we have

$$V(b, P; \sigma^*) = V(b, -P; \sigma^*),$$

for all $b \in [0,1]^2$ such that $b_P = b_{\text{-}P}$, so that any value of $\sigma^*_M(b) \in [0,1]$ forms part of a best response for such reputations. Furthermore, the party strategy $\sigma^*_P(b)$ is weakly increasing in b_P and weakly decreasing in $b_{\text{-}P}$ by Lemma 3. Since $\sigma^*_P(b) = \sigma^*_{\text{-}P}(b)$ if $b_P = b_{\text{-}P}$, we have

$$b_P < b_{\text{-}P} \Rightarrow \sigma^*_{\text{-}P}(b) \geq \sigma^*_P(b) \Leftrightarrow V(b, P; \sigma^*) \geq V(b, -P; \sigma^*),$$

and σ^*_M that satisfies (8) is a best response.

We show that party strategies constitute best responses in Steps 3 and 4.

3. *If $b \in [0,1]^2$ is such that $T_g(b_P) < T_o(b_{\text{-}P})$, then $\sigma^*_P(b)$ is a best response.* By part (ii) of Lemma 4, we have that if party P implements policy x^P_e, $b'_P(b, x^P_e; \sigma^*) = T_g(1) = \pi^g_e > T_o(b_{\text{-}P}) = b'_{\text{-}P}(b, x^P_e; \sigma^*)$. Thus, $\sigma^*_M(b'(b, x^P_e; \sigma^*), P) = 0$ by (8). We conclude using (24) that

$$U_P(b, x^P_e; \sigma^*) = r^R_e + G + \delta \begin{pmatrix} s(\pi^g_e \sigma^*_P(b'(b, x^P_e; \sigma^*))(r^R_e - r^R_m) + r^R_m + G) \\ +(1-s)(T_o(b_{\text{-}P})\sigma^*_{\text{-}P}(b'(b, x^P_e; \sigma^*))(r^L_e - r^L_m) + r^L_m) \end{pmatrix}.$$

Since $\sigma^*_P(b) = 0$, we also have from (6) that $\beta(b, x^P_m; \sigma^*) = b_P$. As a consequence, if party P implements a moderate policy, $b'_P(b, x^P_m; \sigma^*) = T_g(b_P) < T_o(b_{\text{-}P}) = b'_{\text{-}P}(b, x^P_m; \sigma^*)$, hence $\sigma^*_M(b'(b, x^P_m; \sigma^*), P) = 1$ from (8). So, using (24), the expected payoff from pursuing a moderate policy is

$$U_P(b, x^P_m; \sigma^*) = r^R_m + G + \delta \begin{pmatrix} (1-s)(\pi^g_e \sigma^*_P(b'(b, x^P_m; \sigma^*))(r^R_e - r^R_m) + r^R_m + G) \\ +s(T_o(b_{\text{-}P})\sigma^*_{\text{-}P}(b'(b, x^P_m; \sigma^*))(r^L_e - r^L_m) + r^L_m) \end{pmatrix}.$$

We now compare the expected payoffs from the two policy choices and we obtain after a bit of algebra that

$$U_P(b, x^P_m; \sigma^*) \geq U_P(b, x^P_e; \sigma^*) \Leftrightarrow$$

$$\delta \geq \frac{(r^R_e - r^R_m) + \delta \begin{pmatrix} \pi^g_e \left(s\sigma^*_P(b'(b, x^P_e; \sigma^*)) - (1-s)\sigma^*_P(b'(b, x^P_m; \sigma^*))\right)(r^R_e - r^R_m) \\ T_o(b_{\text{-}P})\left(s\sigma^*_{\text{-}P}(b'(b, x^P_m; \sigma^*)) - (1-s)\sigma^*_{\text{-}P}(b'(b, x^P_e; \sigma^*))\right)(r^L_m - r^L_e) \end{pmatrix}}{(1-2s)(G + r^R_m - r^L_m)}$$

But the last inequality is implied by (15) since $\pi_e^o \geq T_o(b_{\text{-}P})$ and

$$\delta s \left(\pi_e^g(r_e^R - r_m^R) + \pi_e^o(r_m^L - r_e^L)\right) \geq$$
$$\delta \begin{pmatrix} \pi_e^g \left(s\sigma_P^*(b'(b, x_e^P; \sigma^*)) - (1-s)\sigma_P^*(b'(b, x_m^P; \sigma^*))\right)(r_e^R - r_m^R) \\ T_o(b_{\text{-}P})\left(s\sigma_{\text{-}P}^*(b'(b, x_m^P; \sigma^*)) - (1-s)\sigma_{\text{-}P}^*(b'(b, x_e^P; \sigma^*))\right)(r_m^L - r_e^L) \end{pmatrix}.$$

We conclude that (15) implies that $U_P(b, x_m^P; \sigma^*) \geq U_P(b, x_e^P; \sigma^*)$ as desired.

4. *If $b \in [0,1]^2$ is such that $T_g(b_P) \geq T_o(b_{\text{-}P})$, then $\sigma_P^*(b)$ is a best response.* As in the previous step, by part (ii) of Lemma 4 we have that if party P implements policy x_e^P then $b_P'(b, x_e^P; \sigma^*) > b_{\text{-}P}'(b, x_e^P; \sigma^*)$. Thus, $\sigma_M^*(b'(b, x_e^P; \sigma^*), P) = 0$ by (8). Note that part (iii) of Lemma 4 and (11) ensure that if $b_P < 1$ we have $b_P'(b, x_m^P; \sigma^*) = b_{\text{-}P}'(b, x_m^P; \sigma^*) = T_o(b_{\text{-}P})$ and the same is true by the specified out-of-equilibrium beliefs when $b_P = 1$. Thus, $\sigma_M^*(b'(b, x_m^P; \sigma^*))$ is given by (25). Now we make use of (24) and of the symmetry $\sigma_R^*(b_L, b_R) = \sigma_L^*(b_R, b_L)$ to deduce

$$U_P(b, x_e^P; \sigma^*) = U_P(b, x_m^P; \sigma^*) \quad \Leftrightarrow$$
$$r_e^R + G + \delta \begin{pmatrix} s(\pi_e^g \sigma_P^*(b'(b, x_e^P; \sigma^*))(r_e^R - r_m^R) + r_m^R + G) \\ +(1-s)(T_o(b_{\text{-}P})\sigma_{\text{-}P}^*(b'(b, x_e^P; \sigma^*))(r_e^L - r_m^L) + r_m^L) \end{pmatrix} = r_m^R + G$$
$$+\delta \begin{pmatrix} ((1-s)\sigma_M^*(b', P) + s(1 - \sigma_M^*(b', P)))(\pi_e^g \sigma_P(b')(r_e^R - r_m^R) + r_m^R + G) \\ +(s\sigma_M^*(b', P) + (1-s)(1 - \sigma_M^*(b', P)))(b_{\text{-}P}'\sigma_{\text{-}P}(b')(r_e^L - r_m^L) + r_m^L) \end{pmatrix} \quad \Leftrightarrow$$
$$\sigma_M^*(b', P)\left(\delta(1-2s)(G + r_m^R - r_m^L + \sigma_R^*(b')(\pi_e^g(r_e^R - r_m^R) + b_R'(r_m^L - r_e^L)))\right) =$$
$$r_e^R - r_m^R + \delta \begin{pmatrix} s\pi_e^g(\sigma_R^*(b_R', \pi_e^g) - \sigma_R^*(b'))(r_e^R - r_m^R) \\ +(1-s)b_R'(\sigma_R^*(b') - \sigma_R^*(\pi_e^g, b_R'))(r_m^L - r_e^L) \end{pmatrix} \quad \Leftrightarrow \quad (25),$$

where $b' = (b_L', b_R') = b'(b, x_m^P; \sigma^*) = (T_o(b_{\text{-}P}), T_o(b_{\text{-}P}))$. We conclude that party strategy $\sigma_P^*(b)$ is a best response.

With Steps 2 to 4 we have established the existence of an intuitive equilibrium with strategies σ^* that satisfy (11) and (8). It remains to show that this equilibrium is also robust. This we show in a last step:

5. *Equilibrium σ^* is robust.* To show that σ^* is robust, consider party strategies, $\sigma_P^{*\varepsilon}$, $P \in \{L, R\}$, that are obtained from σ_P^* for some $\varepsilon > 0$ according to (7). Let $\sigma^{\varepsilon *} = (\sigma_M^*, \sigma_L^{\varepsilon *}, \sigma_R^{\varepsilon *})$. Note that we still have $V(b, P; \sigma^{\varepsilon *}) = V(b, -P; \sigma^{\varepsilon *})$ if $b_P = b_{\text{-}P}$, and that (by Lemma 3) $\sigma_P^{\varepsilon *}$ is weakly increasing in b_P, and weakly decreasing in $b_{\text{-}P}$ for all $\varepsilon \in (0, \frac{1}{2})$, so that $b_P > b_{\text{-}P} \Rightarrow \sigma_P^{*\varepsilon}(b) \geq \sigma_{\text{-}P}^{*\varepsilon}(b)$. Thus, for all $\varepsilon \in (0, \frac{1}{2})$ we must have:

$$\begin{aligned} b_P &> b_{\text{-}P} \Rightarrow \\ b_P \sigma_P^{*\varepsilon}(b) &> b_{\text{-}P} \sigma_{\text{-}P}^{*\varepsilon}(b) \Rightarrow \\ V(b, P; \sigma^{\varepsilon *}) &< V(b, -P; \sigma^{\varepsilon *}). \end{aligned}$$

We conclude that σ^* is a robust equilibrium, completing the proof of this step and of the proposition. ■

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