Non-Fully Strategic Information Transmission

Marco Ottaviani and Francesco Squintani

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Non-Fully Strategic Information Transmission*

Marco Ottaviani
London Business School
Economics Subject Area†

Francesco Squintani
University of Rochester
Department of Economics‡

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Abstract

Building on Crawford and Sobel's (1982) general communication model, this paper introduces the possibility that players are non-strategic. The sender might be honest, truthfully reporting private information, or the receiver might be naive, blindly implementing the sender's recommendations. In contrast to the predictions of the fully-strategic model, we show that equilibrium communication is inflated but detailed, and that the equilibrium outcome is biased in an ex-ante sense. Our findings are relevant to understanding communication by financial analysts and academic evaluators.

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†Regent's Park, London NW1 4SA, United Kingdom.

‡Harkness Hall, University of Rochester, 14627 NY, USA.

1 Introduction

This paper studies communication among players with heterogeneous strategic sophistication. As in Crawford and Sobel's (1982) model, a privately informed sender recommends an action to a receiver with partially aligned preferences. In their model, both players are aware that the sender would like to manipulate the receiver's actions. Here we introduce the possibility that players may be non-strategic, e.g., the sender may be honest or the receiver naive. Whereas an honest sender make truthfully reports, a naive receiver blindly follows the sender's recommendation or erroneously believes that the sender is honest.[1]

If non-strategic behavior occurs with strictly positive probability, we find that in equilibrium: (i) all information is transmitted, (ii) communication is encoded in an inflated language, (iii) the action taken by the receiver (i.e., outcome) is ex-ante biased, and (iv) a biased third party has incentive to give biased preferences to the sender.[2] These predictions are qualitatively different from those obtained in the fully-strategic model where in equilibrium: (i) some information is necessarily lost due exclusively to strategic reasons, (ii) the language is arbitrary, (iii) the outcome is unbiased, and (iv) the bias is self defeating.

Our results are pertinent to the recent debate on the independence and bias of financial analysts. Fed by accounts in the financial press, a growing body of evidence has confirmed that financial analysts' recommendations tend to be overoptimistic. The explanations proposed for this in the empirical finance literature impute the recommendations' bias to conflicts of interest between analysts and investors.[3] The evidence confirms that biased analysts

[1] If both players are strategic with probability one, we return to the communication model of Crawford and Sobel (1982). If instead the sender is strategic and the receiver is naive with probability one, the receiver effectively commits to take the action recommended by the sender. Dessein (2002) compares the performance of these two polar cases, communication and delegation.

[2] We also show that an increase in the fraction of strategic players results in more inflated communication.

[3] First, analysts might prefer to release favorable forecasts which tend to generate more investment banking business. For instance, Michealy and Womack (1999) show that the recommendations of brokerage analysts working for the IPO lead underwriter are significantly more favorable than those of non-underwriters. (See also Dugar and Nathan (1995), Francis and Soffer (1997), Lin and McNichols (1998), and Womack (1996)). Second, positively biased forecasts might result in higher brokerage commissions for the trading arms of the firms for which they work for (Konrad and Greising (1989)). Third, analysts might release optimistic forecasts in order to obtain more accurate information from the management of the firms they follow (Lim (2001)).

give biased advice and that investors are at least partly deceived by the biased advice received. Inspection of financial reports suggests that financial analysts transmit as detailed information as possible.[4]

These facts are consistent with the predictions of our simple model of non-fully strategic information transmission. When allowing the receiver to be of either a strategic or a naive type, the sender's equilibrium communication is inflated. As a result, the final choice of the strategic receiver is unbiased and the choice of the naive receiver is biased. This implies that the financial firms have a clear incentive to give biased incentives to their financial advisers, regardless of whether the analysts' bias is publicly observable.

It is difficult to reconcile these regularities with the predictions of fully-strategic models of communication. If investors are aware of the analysts' bias, they should be able to undo the bias contained in the reports they receive. This implies that investors' decisions should not reflect any bias. As shown by Crawford and Sobel (1982), the only effect of the bias is to reduce the amount of information that the adviser can communicate in equilibrium. Communication is noisy but the actions taken are on average unbiased.[5] As a result, the bias in the advisers' objectives is self defeating, and financial firms that can credibly commit to a level of bias would then have no reason to give biased incentives to analysts.[6]

However, in reality firms seem to avoid both disclosure rules and institutional mechanisms, such as strict separation of brokerage and investment banking divisions (so-called

[4] Communication costs impose a natural limit on the amount of information transmitted. In practice information is often summarized in categorical rankings (such as buy, sell and hold) but it is substantiated by detailed reports.

[5] Morgan and Stocken (2001) have recently considered what happens when the level of the bias is unknown, but on average positive. In this case, the action taken is typically biased, but not necessarily in the same direction as the adviser's bias. In other papers in this area (e.g., Trueman 1994, and Ottaviani and Sorensen 2002), the analysts want to convince investors of their expertise in forecasting, rather than being concerned with the investment decisions induced by their reports.

[6] To elaborate on this point, consider the following two-stage game, in which the sender is hired by a principal who would like the receiver to take a biased action. In the first stage, the principal chooses the level of bias for the sender. In the second stage, the fully-strategic sender-receiver game is played. Clearly, if the first-stage choice of bias is publicly observable, the principal finds it optimal to give the agent no bias. If instead bias is unobservable, then it can be shown that in equilibrium the principal gives the agent her same level of bias. The resulting payoff for the principal in the second case is lower than in the first. This implies that the principal would like to take steps to make the sender's bias publicly observable and then commit to give the sender unbiased instructions.

"Chinese walls") that would allow them to commit to unbiased instructions. These measures are instead imposed by regulators of the financial retail industry in order to prevent naive investors from following overoptimistic recommendations and thus suffering excessive losses.[7] Further, the quality of the services provided by financial advisers is actively monitored and subject to various forms of regulation.[8] Congressional hearings are also currently underway in the US regarding proposed reforms to reduce the conflict of interest leading analysts to give biased recommendations.[9] These biased statements are widely believed to influence individual investors' decisions, a fact inconsistent with the presumption of fully-strategic receivers.

Our predictions may also yield new insights into the inflation in Grade Point Averages (GPA) and letters of recommendation, a phenomenon that has recently received much attention both in news reports and in the academic press.[10] The widely publicized report prepared by Rosovsky and Hartley (2002) for the American Academy of Arts and Sciences[11] documents "an upward shift in the GPA of students without a corresponding increase in student achievement," that "began in the 1960s and continued through, at least, the mid-1990s."[12] The report documents a similar, or worse, trend in the inflation of recommendation letters.[13]

The explanations for GPA inflation presented in the report can be grouped into three basic categories. The most compelling of these blames the rise of "consumerism", i.e. universities operating like businesses for student clients. College and universities compete fiercely to get

[7] Following the amendment of securities laws in 1988, the Securities and Exchange Commission (SEC) issued guidelines and the National Association of Securities Dealers and the New York Stock Exchange issued a joint memorandum in 1991 endorsing Chinese walls. Congress is also considering implementing more stringent disclosure rules for financial analysts (see Schroeder 2001 and Schack 2001).

[8] See Black (1997) for a legal analysis of the regulation of financial services in the UK. In particular, Chapter 4 gives an overview of recent developments in the regulation of retail financial products.

[9] In the wake of the recent Enron scandal, congressional committees are also currently discussing the accuracy of financial statements and the independence of auditors. Cf. MSNBC, Congress launches Enron hearings, http://www.msnbc.com/news/688458.asp?cp1=1

[10] The debate has been spurred by a series of articles by Patrick Healy in the Boston Globe published between September 2001 and January 2002 on GPA inflation at Harvard. Since then the debate has broadened to include a variety of academic institutions ranging from Ivy League schools to State Colleges.

[11] Among other accounts in the academic press, see Koretz and Berends (2001), and Sabot and Wakeman (1991).

[12] Rosovsky and Hartley (2002), page 4.

[13] Rosovsky and Hartley (2002), page 16.

and retain students generating a significant pressure on faculty to meet the expectations and wishes of their students. As good grades improve the individual's chances in competing on job and graduate school markets, students demand high marks from their teachers. This pressure on teachers has been magnified by the introduction and use of student evaluations, and by the increased tendency for non-tenured adjunct faculty to undertake teaching duties.[14]

Our model interprets student evaluations as communications from faculty (senders) to prospective employers and graduate schools (receivers). Because of "consumerism," a strategic sender has an incentive to induce the false belief that her students perform better than they actually do. However not all players are strategic. A fraction of academics report their students' performances honestly, and a fraction of receivers naively believe GPAs and recommendation letters. Our results predict that academic evaluations are inflated, and yet are as detailed as possible. In fact, while each course grade is expressed on a fairly coarse scale, the GPA delivers finely calibrated information. Through direct inspection, we observe that recommendation letters are usually very detailed, and at the same time very inflated.

Moreover, our analysis suggests simple learning/evolutionary explanations for the trend of GPA inflation. In the presence of some grade inflation and consumerism, honest senders are penalized in comparison with strategic ones. This presents honest senders the choice of either marginalization or of accepting the logic of strategic communication. Following our results, as the fraction of honest senders decreases, academic communication must become progressively more inflated. This, in turn, penalizes even further honest senders, thus generating a self-sustaining process of grade inflation.[15] Alternatively, in the presence of grade inflation, naive employers fare worse than strategic ones. Again, as the fraction of non-strategic players

[14] A second explanation presented in the report attributes grade inflation during the 1960s to the Vietnam War, as poor students could be forcibly drafted into the armed forces. However, this cannot explain GPA inflation after the early 1970s. The third explanation provocatively highlights attempts by universities to retain minority students whose preparation for higher level education may be inadequate. Against this, Rosovsky and Hartley report a study showing that African American students perform less well in college than white students with equivalent SAT scores. This seriously undermines the idea of faculty favoritism of minorities.

[15] As reported in Rosovsky and Hartley (2002, p. 11): "It is most important to stress that, once started, grade inflation has a self-sustaining character: it becomes systemic, and it is difficult for faculty to opt out of the system."

decreases, communication must become even more inflated.

Finally, this paper's approach allows us to build a first step towards a fully behavioral "disequilibrium" theory of strategic communication and persuasion.[16] We achieve this by abandoning the assumption that players' beliefs are in equilibrium, and by instead allowing that they hold dispersed beliefs on the opponents' strategy. Specifically, we assume that receivers, while strategic, hold a belief that concentrates mass on the strategies that lie between truth telling and the actual equilibrium strategy of the sender.[17] We also briefly compare the predictions of our theory with data from experimental studies on communication.

The paper is organized as follows. Section 2 discusses the related literature. For expositional reasons, we analyze first in Section 3 the case of naive receivers. Section 4 then extends the model to the case of honest senders, disequilibrium beliefs, and private information bias. Section 5 concludes.

2 Related Literature

The closest methodological contributions are Benabou and Laroque (1992) and Crawford (2001), who show that in simple binary models, the introduction of honest senders or of naive receivers may modify communication in equilibrium. Building on the work of Sobel (1985), Benabou and Laroque study a binary-state, binary-action repeated game. They show that in the presence of the honest sender, the strategic sender may truthfully report the state of the world for long periods of time in order to establish a reputation for honesty. This reputation is then subsequently used to manipulate the receiver. In a one-shot version of their game however, unless the sender is honest with probability at least one half, there exists only the babbling equilibrium in which no information is transmitted. This suggests that fully-revealing communication may arise solely due to reputational concerns. Quite to

[16] For a first look on psychological theories of communication, see Griffin (2000).

[17] Our "disequilibrium" model is closely related the concept of cursed equilibrium introduced by Eyster and Rabin (2000) to explain the winner's curse in auctions, and to the concept of random belief equilibrium developed by Friedman and Mezzetti (2002) in normal-form games. While these concepts presume that "equilibrium" beliefs are unbiased, we assume that the receiver's belief is biased, as it concentrates all mass between the actual "equilibrium" strategy and the truth-telling one.

the contrary, by studying Crawford and Sobel's model in its full generality, we show that a fully-revealing equilibrium exists *for any* positive probability of non-strategic behavior.[18]

Crawford (2001) is motivated by Operation Fortitude in the Second World War in which the Allies' deployment of a few troops on the Channel deceived the Germans into diverting relevant resources from the defence of Normandy. He studies communication with non-strategic players prior to an asymmetric matching-pennies game, and hence his model is appropriate to deal with misrepresentation of *intentions* rather than information. As in Benabou and Laroque, if non-strategic players are not likely enough, there is only a babbling equilibrium in the communication game. Interestingly, in his main case, both the strategic and the naive receiver are deceived by the sender's misrepresentation of intentions. In our equilibrium, instead, only the naive receiver believes the sender's message, whereas the strategic one corrects it for communication inflation.

Morgan and Stocken (2001) consider the problem of financial analysts' communication when their bias is private and unobservable. Unlike our model, all players in their game are fully strategic. Depending on parameter values they obtain either non-responsive or semi-responsive equilibria. In either case, the divergence of interests between the analyst and the client makes communication noisy and reduces the amount of information that can be communicated in equilibrium. In the semi-responsive equilibrium, unbiased analysts can credibly communicate bad news, but cannot credibly convey good news. It can be easily shown that the equilibrium actions taken by the receiver in their model are typically biased, but not necessarily in the same direction as the bias of the sender.

Variants of Crawford and Sobel's model have been widely applied to a number of social, political, and economic situations. Austen-Smith (1989) applied cheap talk equilibrium models to legislative debate, Matthews (1989) applied them to political bargaining between the President and the Congress, Stein (1989) to macroeconomic policy announcements, Gross-

[18] Olszewski (2001) studies a one-shot model in which the sender has mixed incentives and concern for both the receiver's choice and the receiver's belief about her own accuracy. When the latter component dominates the former, he shows that truthful communication occurs under some regularity assumptions.

man and Helpman's (2001) Chapter 4 to lobbying, Dorazselski, Gerardi and Squintani (2001) to voting games, and Morris (2001) to political correctness. Battaglini (2002) has extended Crawford and Sobel's model to allow for two senders and multi-dimensional action and message spaces, and has shown how to enforce full-revelation in equilibrium.

3 Naive Receivers

3.1 The Model

Our fully-strategic communication model closely follows Crawford and Sobel (1982), but is extended to the case of unbounded state space and signal support. After being privately informed of the state of the world $x \in \mathbb{R}$, with cumulative distribution function $F \in \mathcal{C}^2$, a sender (S) sends a message $m \in \mathbb{R}$ to a receiver (R). Upon receiving the message, R takes a payoff-relevant action $y \in \mathbb{R}$. The von Neuman-Morgenstern utilities of the players are $U^S(y, x, b) \in \mathcal{C}^2$, and $U^R(y, x) \in \mathcal{C}^2$, where $b \in \mathbb{R}$ is common knowledge among the players. For each $i = S, R$, player i's utility has a unique maximum y^i such that $U_1^i(y^i, m) = 0$, it satisfies $U_{11}^i < 0$ as well as the single-crossing condition $U_{12}^i > 0$. A message strategy is a family $(\nu(\cdot|x))_{x \in \mathbb{R}}$, where for each x, $\nu(\cdot|x)$ is a c.d.f. on the message space. An action strategy is a function $s : m \mapsto y$ (since $U_{11}^R < 0$, the receiver does not ever play a mixed strategy). When $\nu(\cdot|x)$ is degenerate for all x, we represent $(\nu(\cdot|x))_{x \in \mathbb{R}}$ by means of a function $\mu : x \mapsto m$.

We modify the fully-strategic communication model to account for the possibility of naive receivers. Following a standard approach,[19] the simplest manner to model the naive receiver's behavior is to introduce a "crazy" type who always chooses to match her action with the sender's message.[20] Formally, in an expanded communication game Γ_α, with probability $1 - \alpha$, the receiver is strategic and her payoff is $U^R(y, x)$, with probability α, the receiver

[19] See for instance Fudenberg, Kreps and Levine (1988) or Kajii and Morris (1997).

[20] In the next section, we also present alternative ways to model naive receivers. For instance, a naive receiver may falsely believe that the sender always tells the truth. It is easy to verify that there is no fully-revealing equilibrium in a model where naive receiver is known to believe that the sender's incentives coincide with her own ones.

is naive and her payoff is $U^{RN}(y,m,x) = 1$ if $y = m$, and else $U^{RN}(y,m,x) = 0$. Given the players' equilibrium strategies, the equilibrium outcome is represented by the family $(\xi(\cdot|x))_{x\in\mathbb{R}}$, where for any x, $\xi(\cdot|x)$ is a measure on the actions space. When $\xi(\cdot|x)$ is degenerate for all x, we represent $(\xi(\cdot|x))_{x\in\mathbb{R}_+}$ by means of a function $\zeta : x \mapsto y$.

3.2 The Fully-Revealing Inflated-Communication Equilibrium

The most celebrated result of the fully-strategic model is Lemma 1 of Crawford and Sobel (1982). It states that if $y^S(x,b) \neq y^R(x)$ for all x, then there must be an $\varepsilon > 0$ such that $|u - v| \geq \varepsilon$, for any pair of actions u, v played in equilibrium. When the state space is unbounded, the same result requires a slightly stronger condition.[21] If there is an $\epsilon > 0$ such that for any x, $\left|y^S(x,b) - y^R(x)\right| \geq \epsilon$, then there is $\varepsilon > 0$ such that $|u - v| \geq \varepsilon$, for any pair of actions u, v played in equilibrium.

The main finding of this subsection is that when the receiver may be naive there always exists an equilibrium in which the sender fully reveals the state to rational receivers. For any $\alpha > 0$, the game Γ_α has an equilibrium where the message strategy is invertible. Remarkably, this occurs independently of the prior signal distribution. In these fully-revealing equilibria the sender exaggerate the state of the world even beyond her own bias. Communication is inflated, yet detailed. For ease of exposition but without loss of generality, we restrict attention to the case where there is an $\epsilon > 0$ such that for any x, $y^S(x,b) - y^R(x) \geq \epsilon$.

Informally the result is explained by hypothesizing that for any state of the world x, in equilibrium the sender reports the message m that reveals the state x according to an invertible message function μ, and by checking that she has no incentive to deviate. Since the sophisticated receiver correctly de-biases the sender's message, and determines $x = \mu^{-1}(m)$, the sender has an incentive to add more bias to the report $\mu(x)$. But if she does so, the naive receiver will believe her and damage her, as long as $\mu(x)$ is already above the sender's bliss point $y^S(x,b)$. For any $\alpha > 0$, since the sender's utility is strictly concave, it is possible

[21] The proof of this result is omitted as it is an immediate extension of Lemma 1 in Crawford and Sobel (1989).

to find $\mu(x)$ large enough so that the rate at which the sender's utility drops because of the naive receiver's response is fast enough to make up for the gain achieved through the response of the sophisticated sender. It is left to show that the constructed function μ is in fact invertible. By the Implicit Function Theorem, this follows from the assumptions that $U^S_{11} < 0$ (concavity) and that $U^S_{12} > 0$ (single-crossing property).

Before stating and proving our general result, we introduce its main features with a simple explicit example.

Example 1 *Say that the receiver and sender have quadric utilities with bliss points respectively x and $x+b$, formally $U^R(y,x) = -(y-x)^2$ and $U^S(y,x,b) = -(y-(x+b))^2$. Suppose that in equilibrium, the sender adopts an invertible function μ as her communication strategy. When a message m is sent, the strategic receiver correctly infers the state $\mu^{-1}(m)$ and the naive receiver plays the action m regardless of strategic considerations. Hence the sender will not deviate from the strategy μ only if for any x,*

$$\mu(x) \in \arg\max_m -(1-\alpha)\left(\mu^{-1}(m) - (x+b)\right)^2 - \alpha(m-(x+b))^2.$$

The first order condition, is

$$-2(1-\alpha)\left(\mu^{-1}(m) - x - b\right)\left(\mu^{-1}(m)\right)' - 2\alpha(m-x-b) = 0.$$

By substituting $\mu^{-1}(m)$ with x and m with $\mu(x)$ we obtain the differential equation

$$-2(1-\alpha)(x-x-b) - 2(\mu(x) - x - b)\mu'(x)\alpha = 0,$$

with the linear strictly-increasing solution

$$\mu(x) = x + \frac{b}{\alpha}.$$

This strategy may be interpreted as revealing the actual state of the world, but inflating the communication by an amount b/α. The factor by which communication is inflated is inversely proportional to the fraction of naive receivers in the population.

We are now ready to state our result in its full generality.

Theorem 1 *For any $\alpha > 0$, the game Γ_α has an equilibrium where the communication strategy μ is an invertible function, and hence $s \circ \mu(x) = x$ for any x. In any such an equilibrium, $\mu(x) > y^S(x, b)$ for any x.*

Proof. Given the rational receiver's equilibrium choice s, for any $x \in \mathbb{R}$, in equilibrium, the sender must choose

$$\mu(x) \in \arg\max_{m \in \mathbb{R}} \alpha U^S(m, x, b) + (1-\alpha) U^S\left(y^R(s(m)), x, b\right).$$

Suppose that s is differentiable, the first order condition for the sender's program is

$$\alpha U_1^S(m, x, b) + (1-\alpha) U_1^S\left(y^R(s(m)), x, b\right) y^{R\prime}(s(m)) s'(m) = 0. \tag{1}$$

Suppose that $s' > 0$, and that $s \circ \mu(x) = x$, so implies that $s = \mu^{-1}$. Hence we can rewrite condition (1) as

$$\alpha U_1^S(\mu, x, b) \mu' + (1-\alpha) U_1^S\left(y^R(x), x, b\right) y_1^R(x) = 0. \tag{2}$$

This expression is an ordinary differential equation in μ, and by Peano's fundamental existence theorem[22], it admits a local solution ϕ for any pair (x_0, m_0) such that the function

$$-\frac{(1-\alpha) U_1^S\left(y^R(x), x, b\right) y_1^R(x)}{\alpha U_1^S(m, x, b)}$$

is continuous in a neighborhood of (x_0, m_0). This means that equation (2) has a local solution ϕ for any (x_0, m_0) such that $U_1^S(m_0, x_0, b) \neq 0$.

For any x, since $y^S(x, b) > y^R(x)$, and $U_{11}^S < 0$, it follows that $U_1^S\left(y^R(x), x, b\right) > 0$. Since $U_{12}^R > 0$ it follows that $y_1^R(x) > 0$. It also follows that the second term in equation (2) is positive. So we suppose that $U_1^S(m_0, x_0, b) < 0$, and hence that $m_0 > y^S(x_0, b)$. By Dini's implicit function theorem, $\phi'(x_0) > 0$.

[22] For this and for any further quoted result from the qualitative theory of Ordinary Differential Equations, see e.g., Hurewitz (1963).

Since x_0 is arbitrary, we can set $x_0 = 0$. Picard's extension theorem allows ϕ to be continuously and differentiably extended for any $x \in (0, \bar{x})$ and any $\bar{x} > 0$ unless there is an $x' \in (0, \bar{x})$ such that $\lim_{x \to x'} \phi(x) = \infty$. Suppose by contradiction that there is such an x', and take the infimum of these x'. Then $\lim_{x \to x'} \phi'(x) = +\infty$ and $\lim_{x \to x'} U_1^S(\phi(x), x, b) < 0$ because $\lim_{x \to x'} \phi(x) > y^S(x', b)$ which is finite because $y^S(\cdot)$ is a continuous function. Since $y^R(\cdot)$ is a continuous function, $\lim_{x \to x'} U_1^S\left(y^R(x), x, b\right)$ is finite. But this means that there is a small $\varepsilon > 0$ such that ϕ cannot satisfy equation (2) on the set $(x' - \varepsilon, x')$. This is a contradiction as it violates Picard's extension theorem and the supposition that x' is the infimum. We have concluded that ϕ can be continuously and differentiably extended on the whole set $\mathbb{R}_+$; the extension on $\mathbb{R}_-$ is analogously undertaken, and we let $\mu = \phi$.

Since $U_1^S(m, x, b) = 0$ implies that $\mu' \to \infty$, it must be the case that $U_1^S(\mu(x), x, b) < 0$ for any $x \in \mathbb{R}$, and hence equation (2) implies that $\mu' > 0$. Thus we can let $s = \mu^{-1}$, and have that $s' > 0$. Since μ is differentiable, so is s.

We are left to show that s satisfies the second order condition:

$$\begin{aligned}
&\alpha U_{11}^S(m, x, b) + (1-\alpha) U_{11}^S\left(y^R(s(m)), x, b\right)\left(y^{R\prime}(s(m))\right)^2 (s'(m))^2 \\
&+ (1-\alpha) U_1^S\left(y^R(s(m)), x, b\right) y^{R\prime\prime}(s(m)) (s'(m))^2 \\
&+ (1-\alpha) U_1^S\left(y^R(s(m)), x, b\right) y^{R\prime}(s(m)) s''(m) < 0,
\end{aligned}$$

equivalently we will show that μ satisfies:

$$\begin{aligned}
&\alpha U_{11}^S(m, x, b) + (1-\alpha) U_{11}^S\left(y^R(x), x, b\right)\left(y^{R\prime}(x)\right)^2 \frac{1}{(\mu'(x))^2} \\
&+ (1-\alpha) U_1^S\left(y^R(x), x, b\right) y^{R\prime\prime}(x) \frac{1}{(\mu'(x))^2} \\
< \;& (1-\alpha) U_1^S\left(y^R(x), x, b\right) y^{R\prime}(x) \frac{\mu''(x)}{(\mu'(x))^3} = -\alpha U_1^S(\mu, x, b) \frac{\mu''(x)}{(\mu'(x))^2},
\end{aligned} \tag{3}$$

where the last equality comes from equation (2).

By applying the Implicit Function Theorem to equation (2), we obtain:

$$\mu'' = -\frac{\left(\begin{array}{c} \alpha U_{12}^S(\mu, x, b)\,\mu' + (1-\alpha) U_{12}^S\left(y^R(x), x, b\right) y^{R\prime}(x) + \\ (1-\alpha) U_{11}^S\left(y^R(x), x, b\right)\left(y^{R\prime}(x)\right)^2 + (1-\alpha) U_1^S\left(y^R(x), x, b\right) y^{R\prime\prime}(x) \end{array}\right)}{\alpha U_1^S(\mu, x, b)},$$

substituting into inequality (3) and simplifying we obtain

$$\alpha U_{11}^{S}\left(m,x,b\right)<\frac{\alpha U_{12}^{S}\left(\mu,x,b\right)\mu'+\left(1-\alpha\right)U_{12}^{S}\left(y^{R}\left(x\right),x,b\right)y^{R\prime}\left(x\right)}{\left(\mu'\left(x\right)\right)^{2}},$$

this inequality is satisfied because $U_{11}^{S}<0$, $U_{12}^{S}>0$, $\mu'>0$ and $y^{R\prime}>0$.

In order to prove the second claim, consider equation (2) again:

$$\alpha U_{1}^{S}\left(\mu,x,b\right)\mu'+\left(1-\alpha\right)U_{1}^{S}\left(y^{R}\left(x\right),x,b\right)y^{R\prime}\left(x\right)=0.$$

Note that $y^{R\prime}\left(x\right)>0$, and that $\mu'>0$, since $y^{S}(x,b)>y^{R}(x)$, it follows that $U_{1}^{S}\left(y^{R}\left(x\right),x,b\right)>0$, hence it must always be the case that $U_{1}^{S}\left(\mu,x,b\right)<0$, that requires $\mu(x)>y^{S}(x,b)$. ∎

Remark 1 *A close inspection of our result and its proof reveals that it is not necessary for the receiver to be truly naive, in order to establish the existence of the fully-revealing inflated-communication equilibrium. It is enough that the sender believes (maybe wrongly) that the receiver could be naive. More importantly, one need not stop this line of argument to one iteration of high-order beliefs. For instance, a simple argument (available upon request) shows that a fully-revealing equilibrium also exists when the receiver believes that the sender believes that the receiver may be naive. This suggests that a fully-revealing equilibrium exists whenever introducing any arbitrary collection of types that hold any arbitrary high-order belief that the receiver is naive. We elaborate further on these considerations later in the paper.*

We conclude this part of the section by looking more closely at the case for small α. On the one hand, it is easy to see that Theorem 1 implies that when $\alpha\to0$, for any x, the equilibrium outcome $\xi_{\alpha}\left(\cdot|x\right)$ approximates the distribution degenerate on x. In this sense, the equilibrium for negligible α is very close to the receiver's optimal outcome where $\zeta\left(x\right)=x$ is established for every x. On the other hand, it is easy to show that, for $\alpha\to0$, message inflation explodes. In order to communicate the state of the world, the sender will use in equilibrium more and more inflated messages.

Proposition 1 *For any $\alpha > 0$, let μ_α be the set of sender's equilibrium strategies of game Γ_α such that $s \circ \mu(x) = x$ for any x. For any x, as $\alpha \to 0$, $\mu_\alpha(x) \to \infty$ and $\xi_\alpha(\cdot|x)$ converges weakly to the distribution degenerate on x.*

Proof. From the proof of Theorem 1, we obtain $\xi_\alpha(x|x) \geq 1-\alpha$. Hence $\xi_\alpha(\cdot|x)$ converges weakly to the distribution degenerate on x. We have concluded in the proof of Theorem 1 that $U_1^S\left(y^R(x), x, b\right) > 0$ and that $U_1^S(\mu(x), x, b) < 0$. As $\alpha \to 0$, equation (2) is thus satisfied only if $U_1^S(\mu(x), x, b) \to -\infty$. Since $U_{12}^S > 0$, this is possible only if $\mu(x) \to \infty$. ■

3.3 The Partitional Equilibria

In the model of Crawford and Sobel (1982) equilibrium is fully characterized in terms of partitions of the state space. Their Theorem 1 states that any equilibrium outcome can be expressed by a function ζ that partitions the state space into a collection $A = \{(a_n, a_{n+1})\}_{n \in N}$ of intervals, for some countable index set N. For any n, any component (a_n, a_{n+1}), and any $x \in (a_n, a_{n+1})$, it is the case that $\zeta(x) = \arg\max_{y \in \mathbb{R}} \int_{a_n}^{a_{n+1}} U^R(y,x) f(x) dx$, and that $U^S(\zeta(a_n, a_{n+1}), a_n, b) = U^S(\zeta(a_{n-1}, a_n), a_n, b)$. For any $n' \neq n$, moreover, $Supp(\nu(a_n, a_{n+1})) \cap Supp(\nu(a_{n'}, a_{n'+1})) = \emptyset$.

This equilibrium specification leaves free the choice of the support of the message strategy ν. A well known feature of these equilibria in the fully-strategic model, is that they can also be constructed with all messages on the path. When introducing naive receivers, the structure of the message strategy ν is severely limited. For any interval (a_n, a_{n+1}), the support of $\nu(a_n, a_{n+1})$ must be a singleton set, which must be different for each n.

Lemma 1 *In any equilibrium identified by any partition A, for any n, there is a unique m_n such that $m_n \in Supp(\nu(a_n, a_{n+1}))$, and for any $n' \neq n$, $m_n \neq m_{n'}$.*

Proof. For any $x \in (a_n, a_{n+1})$, and $m \in Supp(\nu(a_n, a_{n+1}))$, in fact, the sender's utility is

$$U_*^S(m, x, b) = \alpha U^S(m, x, b) + (1-\alpha) U^S(\zeta(a_n, a_{n+1}), x, b),$$

Since $U_{12}^S > 0$, there cannot be any $m' \neq m$ that yields the sender $U_*^S(m', x, b) = U_*^S(m, x, b)$ for any $x \in (a_n, a_{n+1})$. We let m_n denote the message associated with $Supp(\nu(a_n, a_{n+1}))$.

If there were $n' \neq n$ with $m_n = m_{n'}$, then it would be the case that $\zeta(a_n, a_{n+1}) = \zeta(a_{n'}, a_{n'+1})$ contradicting the definition of equilibrium identified by A. ■

The above result introduces an unavoidable weakness in any partitional equilibrium. Since all messages $m \notin \{m_n\}_{n \in N}$ are not on the equilibrium path, the receiver's beliefs upon receiving one of these messages are not pinned down by the equilibrium conditions. In order to avoid sender's deviations, however, partitional equilibrium may require quite special receiver's belief with respect to off-path messages. In the refinement literature started by Cho and Kreps (1987), the plausibility of an equilibrium is judged by the plausibility of the off-path beliefs that are required to support it.

We will show that the off-path beliefs required by partitional equilibrium are not very reasonable, as they violate a simple restriction on the language adopted by the players off the equilibrium path. In order to introduce our restriction, note that in any equilibrium, the communication established on path determines what information can be transmitted, and how it can be transmitted; let us dub these quantities as the *meaning* of communication. We do not believe that an equilibrium is very plausible if off the equilibrium path, it requires the receiver to assign a different or more informative meaning to communication.

In the specific, when a partitional equilibrium is played, the receiver knows that the only information that the sender can feasibly provide is to identify to which interval (a_n, a_{n+1}) the state x belongs. We say that the equilibrium is *meaning-preserving* if for any n, any off-path message $m \in (a_n, a_{n+1})$ conveys only the information that the state $x \in (a_n, a_{n+1})$. As a result, the strategic receiver acts as if the message m_n were sent.

Definition 1 *A partitional equilibrium is meaning preserving if for any* n, *and any* $m \in (a_n, a_{n+1})$, $m \notin \{m_n\}_{n \in N}$, *it is the case that* $s(m) = \zeta(a_n, a_{n+1})$.

It is immediate to see that the proposed refinement has no bite on the equilibria of the fully-strategic model where all messages are on path, and in this sense it is to be considered less demanding that refinements such as neologism proofness (Farrell 1993) and communication proofness (Matthews, Okuno-Fujiwara and Postlewaite 1998). When allowing for naive receivers, however, we can show that there are no partitional equilibrium that satisfy our meaning-preserving requirement on off-path beliefs.

Proposition 2 *For any $\alpha > 0$, the game Γ_α does not admit any meaning-preserving partitional equilibrium.*

Proof. Take a candidate meaning-preserving partitional equilibrium: for any n, and any $m \in (a_n, a_{n+1})$, $m \notin \{m_n\}_{n \in N}$, it is the case that $s(m) = \zeta(a_n, a_{n+1})$. Pick an x such that $y^S(x,b) \in (a_n, a_{n+1})$, and $y^S(x,b) \notin \{m_n\}_{n \in N}$. Since

$$\alpha U^S(m_n, x, b) < \alpha U^S\left(y^S(x,b), x, b\right),$$

and $s\left(y^S(x,b)\right) = \zeta(a_n, a_{n+1}) = s(m_n)$, it follows that

$$\alpha U^S(m_n, x, b) + (1-\alpha) U^S(\zeta(a_n, a_{n+1}), x, b) < \alpha U^S(m, x, b) + (1-\alpha) U^S(s(m), x, b),$$

and hence the sender will deviate from the choice $\mu(x) = m_n$ to send message m. ■

We conclude by noting that in the fully-revealing equilibrium studied in the first part of the section, all messages are on path. Therefore that equilibrium survives all refinements (such as the one we have introduced in this section) based on off-path beliefs restrictions.

4 Extensions

The purpose of this section is two-fold. First, we want to show that the main insight from the case of the naive receiver case, can be extended to several alternative specifications of non-strategic behavior. The fully-revealing equilibrium with naive receivers is generated by some receivers being (at least partially) manipulated by the sender's message. We will show that this occurs also when the sender may be honest, or when the receiver's beliefs do not coincide with the sender's equilibrium strategy. Second, this section compares the analysis with the case where the sender's objectives are private information, and with some probability may be aligned with the receiver's.

4.1 Honest Sender

The construction that accounts for the possibility of a honest sender is the mirror image of the construction for the naive receiver. In the game Γ_α with probability $1-\alpha$, the sender's payoff is $U^S(y,x,b)$, with probability α, the sender is honest: her payoff is $U^{SN}(y,m,x,b)=1$ if $m=x$, and else $U^{SN}(y,m,x,b)=0$; so that she always sends the truthful message $m=x$. We maintain the notation ν for the strategy of the rational type of receiver, and without loss of generality we say that there is an $\epsilon>0$ such that for any x, $y^S(x,b)-y^R(x)\geq\epsilon$.

As in the previous section, for any $\alpha>0$, we show that, under a simple regularity condition, the game Γ_α has an equilibrium where the rational sender plays an invertible communication strategy.[23] The reason why such an equilibrium exists is more complex than for the case of naive receivers. Here, the receiver is rational, however she knows that with some probability the sender is honest. If the rational sender plays a fully-revealing message strategy, the receiver will play an action that can be represented as the convex combination between the optimal choice when believing the sender's message and the optimal choice when decoding the rational sender's message. On standpoint of the rational sender, this "perturbation" in the receiver's strategy is equivalent to the perturbation due to the introduction of naive receivers, and hence guarantees the existence of fully-revealing equilibrium. As for the case of the naive receiver, the fully-revealing equilibrium with a honest sender type must display inflation.

Theorem 2 *Say that $y_1^S(x,b)\geq y^{R\prime}(x)$. For any $\alpha>0$, the game Γ_α admits an equilibrium where the function μ is invertible, and hence $s\circ\mu(x)=x$ for any x. In any such an equilibrium, $\mu(x)>y^S(x,b)$ for any x.*

[23] The regularity condition requires that the slope of the receiver's optimal function $y_R(x)$ is not larger than the slope of the sender's optimal function $y_1^S(x,b)$, for any x. See the statement of Theorem 2.

Proof. Suppose that the rational sender's strategy μ is strictly increasing, differentiable and say that $\mu(x) > x$, for every x. Given message m, the receiver knows that if the sender is a rational type, then the state is $x = \mu^{-1}(m)$. If the sender is honest, instead, the state is $x = m$. Hence the receiver's optimal strategy is:

$$s(m) \in \arg\max_y \alpha U^R(y,m) + (1-\alpha)\, U^R(y, \mu^{-1}(m)). \tag{4}$$

A first order condition is:

$$0 = \alpha U_1^R(y,m) + (1-\alpha)\, U_1^R(y,x)$$

which implicitly defines a unique solution $y(m,x,\alpha) \in \left(y^R(x), y^R(m)\right)$. Since $U_{12}^R > 0$ $U_{11}^R < 0$ and $\alpha > 0$, it follows that $y_2 < y^{R\prime}$. Since $U_{11}^R < 0$, the associated second-order condition is satisfied.

Given the receiver's equilibrium choice s, for any $x \in \mathbb{R}$, in equilibrium, the rational sender must choose:

$$\mu(x) \in \arg\max_{m\in\mathbb{R}} U^S\left(y(m,x,\alpha), x, b\right).$$

A first order condition is:

$$0 = U_1^S\left(y(\mu,x,\alpha), x, b\right)\left(y_1(\mu,x,\alpha) + y_2(\mu,x,\alpha)\frac{1}{\mu'}\right). \tag{5}$$

Suppose that $y_1 + y_2/\mu' > 0$, then equation (5) simplifies as:

$$0 = U_1^S\left(y(\mu,x,\alpha), x, b\right), \quad \text{i.e. } y(\mu,x,\alpha) = y^S(x,b), \tag{6}$$

which yields a finite solution for every x, because $y^S(\cdot,b)$ is a continuous function. The induced function μ is continuous and differentiable, and satisfies $y_1\mu' + y_2 = y_1^S > 0$, hence insuring that $y_1 + y_2/\mu' > 0$.

In order to show that $\mu' > 0$, notice that

$$\mu' = \frac{y_1^S - y_2}{y_1} \geq \frac{y^{R\prime} - y_2}{y_1} > 0,$$

where the first inequality follows from $y_1^S \geq y^{R\prime}$, and the second from $y^{R\prime} > y_2$. The condition $y^R(x) < y(\mu(x), x, \alpha)$ and $\mu(x) > x$, for every x, are satisfied because $y^S(x, b) > y^R(x)$, the condition $y(\mu(x), x, \alpha) < y^R(\mu(x))$ is satisfied by construction.

We finally need to establish that $s = \mu^{-1}$ satisfies the following second order condition for any m,

$$U_1^S(y(m, s(m), \alpha), x, b) \left(\begin{array}{c} y_{11}(m, s(m), \alpha) + y_{12}(m, s(m), \alpha) s'(m) + y_{21}(m, s(m), \alpha) s'(m) \\ + y_{22}(m, s(m), \alpha)(s'(m))^2 + y_2(m, s(m), \alpha) s''(m) \end{array} \right) +$$
$$U_{11}^S(y(m, s(m), \alpha), x, b)(y_1(m, s(m), \alpha) + y_2(m, s(m), \alpha) s'(m))^2 < 0.$$

This condition is satisfied since $U_{11}^S < 0$, and since $y(m, s(m), \alpha) = y(\mu(x), x, \alpha)$, which implies that $U_1^S(y(m, s(m), \alpha), x, b) = 0$.

Since for any x, the value $\mu(x)$ coincides with the m satisfying:

$$\begin{aligned} 0 &= \alpha U_1^R(y, m) + (1 - \alpha) U_1^R(y, x) \\ 0 &= U_1^S(y, x, b). \end{aligned}$$

The second equation defines $y^S(x, b)$. Since $y^S(x, b) - y^R(x) \geq \epsilon$, and $U_{12}^S > 0$, it must be the case that $U_1^R(y^S(x, b), x) < 0$. This implies that $U_1^R(y^S(x, b), m) > 0$, and hence that $\mu(x) > y^S(x, b)$. ∎

As for the case of naive receiver (see Remark 1), one can show that existence of the fully-revealing equilibrium does not require that the sender may be truly honest, it is enough that the receiver believes (maybe erroneously) that the sender may be honest. In fact, one can reinterpret such a mystified receiver's belief as defining a type of naive sender, which is formally different from the naive type introduced in the previous section, but that plays an analogous role in the construction of the fully-revealing equilibrium. Again, one need not stop this line of argument to one iteration of high-order beliefs. For instance, it can be shown (details available upon request) that a fully-revealing equilibrium exists also when the sender (possibly mistakenly) believes that the receiver believes that the sender may be honest.

As an antidote to the abstractness of the above results, we conclude this part of the section with our recurrent quadratic loss example. A pleasing feature of the analysis is that the equilibrium strategy coincides with the equilibrium strategy of Example 1 which refers to the case of naive receiver.

Example 2 *Say that the receiver's and the sender's utilities are* $U^R(y,x) = -(y-x)^2$ *and* $U^S(y,x,b) = -(y-(x+b))^2$. *Suppose that the sender employs an invertible message strategy* μ. *The receiver's optimal strategy is:*

$$s(m) \in \arg\max_y -\alpha(m-y)^2 - (1-\alpha)(\mu^{-1}(m)-y)^2,$$

which has the unique solution $s(m) = \alpha m + (1-\alpha)\mu^{-1}(m)$.

For any x, *the sender will not deviate from the strategy* $\mu(x)$ *only if:*

$$\mu(x) \in \arg\max_m -(\alpha m + (1-\alpha)\mu^{-1}(m) - (x+b))^2.$$

The first order condition yields:

$$-2(\alpha\mu + (1-\alpha)x - (x+b))(\alpha + (1-\alpha)\frac{1}{\mu'}) = 0,$$

this ordinary differential equation admits the same linear solution as the case for the naive receiver:

$$\mu(x) = x + \frac{b}{\alpha}.$$

4.2 Disequilibrium Beliefs

This part of the section develops a behavioral "disequilibrium" approach to communication, and shows how the analysis of the previous subsection may be reinterpreted to derive a solution in this context. This is achieved by abandoning the assumption that players' beliefs are in equilibrium, and by presuming that players may hold dispersed beliefs about the opponents' strategy. Specifically, we will say that the receiver, instead of correctly anticipating the sender's strategy, formulates a belief that consists of a distribution over possible sender's strategies. Further, we suggest that the receiver may be partially persuaded by the sender's message even though she knows that the sender may not be telling the truth. This is formalized by assuming that the receiver will hold a belief that concentrates mass on the strategies that lie between the truth telling strategy and the actual equilibrium strategy.[24]

Consider the space of distributions $\Delta(M)$, generated by the sender's pure strategy set $M \subseteq \{\mu : \mathbb{R} \to \mathbb{R}\}$ and the Borel σ-algebra $\mathcal{B}(M)$.[25] Suppose that the sender plays a specific strategy $\mu \in M$. The receiver holds *dispersed* equilibrium beliefs $\gamma^{\mu} \in \Delta(M)$ that attach positive probability only to strategies that are "in-between" the actual strategy μ and the truth-telling strategy i such that $i(x) = x$. Specifically, given the measure $\gamma \in \Delta(0,1)$ we say that $\gamma^{\mu}(B) = \gamma\{\alpha : \alpha\mu + (1-\alpha)i \in B\}$, for any $B \in \mathcal{B}(M)$, and denote by α the random variable associated to γ. The quantity α may be interpreted as the *persuasion* factor that determines how easily the receiver is persuaded by the arguments of the sender.

Definition 2 *The pair* (μ, s) *is a* γ*-dispersed equilibrium if for any* m, $s(m)$ *maximizes* $U^R(s(m), x)$ *under belief* γ^{μ}, *and if for any* x, $\mu(x)$ *maximizes* $U^S(s \circ \mu(x), x, b)$.

The analysis for the case of the honest sender serves to show that this disequilibrium model allows for a solution displaying precise, yet inflated communication. Again, we assume that there is an $\epsilon > 0$ such that for any x, $y^S(x,b) - y^R(x) \geq \epsilon$, and $y_1^S(x,b) \geq y^{R\prime}(x)$.

Theorem 3 *For any* γ, *the game* Γ *has a* γ*-dispersed equilibrium* (μ, s) *such that* μ *is an invertible function. In this equilibrium,* $\mu(x) > y^S(x,b)$.

Proof. The result follows from the proof of Theorem 2, once noticed that for any message m,

$$y(m,x,b) \in \arg\max_y \int_{\mathbb{R}} U^R(y,x) dB_m(x),$$

[24] The assumption that the receiver holds a belief that concentrates mass close to truth-telling differentiates our approach from those of Friedman and Mezzetti (2000) and of Rabin and Eyster (2000), who assume that in equilibrium each player holds an unbiased belief.

[25] For the purpose of this paper, we would like M to include all $\mathcal{C}^2$ (including unbounded ones) and all step functions $\mu : \mathbb{R} \to \mathbb{R}$. An appropriate topology to endow M with a suitable Borel σ-algebra $\Delta(M)$ is then generated, for instance, by the metric $< \mu, \mu' >= \int_{-\infty}^{+\infty} -(\mu(x) - \mu'(x))^2 d\Phi(x)$, where Φ is the standard Normal c.d.f.

where B_m denotes the cumulative distribution function associated to the belief β_m such that $\beta_m(B) = \gamma^\mu(\{f : f(x) = m, x \in B\})$, for any $B \in \mathcal{B}(\mathbb{R})$. By construction, it follows that $y(m,x,b) \in \left(y^R(x), y^R(m)\right)$, and the arguments in the proof of Theorem 2 apply. ■

It may be worth testing the empirical implications of this disequilibrium theory against those of the fully-strategic equilibrium theory of Crawford and Sobel using experimental data. First, our theory predicts that the receiver's action should on average be biased, rather than unbiased as in the equilibrium theory. Second, our theory predicts that the actions taken conditional on the state are heterogeneous, rather than constant as in the equilibrium theory. In the only available experimental study of strategic information transmission, Dickhaut, McCabe, and Mukherjee (1995) considered a model with quadratic preferences and uniform prior distribution on four states. In the data collected there seem to be a positive bias in the average action taken, at least for low states, and a substantive amount of heterogeneity in the conditional action distribution.

4.3 Uncertain Bias

The first result of this part of the section is that it is not enough that the sender's incentives are aligned with those of the receiver's, in order to achieve a fully-revealing equilibrium. As long as the sender is surely strategic and may be biased, even an unbiased type of sender will not be able to credibly communicate her information.[26] For simplicity, we model bias uncertainty by letting the sender's bias by 0 with probability p, and $b > 0$ with probability $1 - p$. We let Γ^p be the game where all players are strategic, but the sender bias is b with probability $1 - p$. Let the unbiased sender's strategy be μ_0 and the biased sender strategy by μ_1.

Proposition 3 *For any p, there is no equilibrium in the game Γ^p for which μ_0 and μ_1 are both invertible.*

[26] This result is also reported by Morgan and Stocken (2001) for the case of quadratic utilities.

Proof. Suppose to the contrary that both μ_0 and μ_1 are invertible. The receiver chooses s that satisfies:

$$s(m) \in \arg\max_y pU^R(y, \mu_0^{-1}(m)) + (1-p)\, U^R(y, \mu_1^{-1}(m)),$$

this yields the first-order condition:

$$0 = U_1^R(s(m), \mu_0^{-1}(m))p + U_1^R(s(m), \mu_1^{-1}(m))(1-p). \tag{7}$$

The biased and unbiased senders respectively choose:

$$\mu_1(x) \in \arg\max_m U^S(s(m), x, b), \text{ and } \mu_0(x) \in \arg\max_m U^S(s(m), x, 0)$$

which yields the first-order conditions

$$U_1^S(s(\mu_1(x)), x, b)s'(\mu_1(x)) = 0, \text{ and } U_1^S(s(\mu_0(x)), x, 0)s'(\mu_0(x)) = 0.$$

This means that $\mu_1(x)$ must be such that $s(\mu_1(x)) = y^S(x,b)$ and that $\mu_0(x)$ must be such that $s(\mu_0(x)) = y^S(x,0) = y^R(x)$. Substituting in the Equation (7), we obtain the contradiction

$$0 = U_1^R(y^R(x), x)p + U_1^R(y^S(x,b), x)(1-p) \propto U_1^R(y^S(x,b), x) \neq 0,$$

where the last inequality follows from $|y^S(x,b) - y^R(x)| \geq \epsilon > 0$. ∎

We now show that our results with non-strategic players do not change when allowing for uncertain bias. We denote by $\Gamma_{\alpha S}^p$ the game where the sender may be honest with probability α, and whenever she is strategic, her bias is b with probability $1-p$; we denote by $\Gamma_{\alpha R}^p$ the analogous game where the receiver may be naive with probability α.

Theorem 4 *For any $\alpha > 0$, both games $\Gamma_{\alpha S}^p$ and $\Gamma_{\alpha R}^p$ have an equilibrium where both μ_0 and μ_1 are invertible.*

Proof. We consider first the case of naive receiver. Suppose that μ_0 and μ_1 are strictly increasing. The strategic receiver's strategy s satisfies Equation (7). Since $U_{11}^R < 0$ and $U_{12}^R > 0$, it must be the case that $s(m) \in \left(\mu_0^{-1}(m), \mu_1^{-1}(m)\right)$, and that $s(m)$ is continuous, differentiable and strictly increasing. Thus the biased and unbiased senders choose respectively μ_1 and μ_0 such that:

$$\begin{aligned} \mu_1(x) &\in \arg\max_{m\in\mathbb{R}} (1-\alpha)U^S(s(m), x, b) + \alpha U^S(m, x, b), \\ \text{and } \mu_0(x) &\in \arg\max_{m} (1-\alpha)U^S(s(m), x, b) + \alpha U^S(m, x, b); \end{aligned}$$

which respectively yield the first-order conditions:

$$\begin{aligned} &(1-\alpha)U_1^S(s(\mu_1), x, b)s'(\mu_1) + \alpha U_1^S(\mu_1, x, b), \\ &\text{and } (1-\alpha)U_0^S(s(\mu_0), x, 0)s'(\mu_0) + \alpha U_1^S(\mu_0, x, 0). \end{aligned}$$

The same arguments as in the proof of Theorem 1 allows us to conclude that this ODE system has a solution μ_0, μ_1 that identifies the fully-revealing equilibrium.

For the case of the honest sender, suppose again that μ_0 and μ_1 are strictly increasing. Hence the receiver's optimal strategy is:

$$s(m) \in \arg\max_y \alpha U^R(y, m) + (1-\alpha)\, pU^R(y, \mu_0^{-1}(m)) + (1-\alpha)\,(1-p)U^R(y, \mu_1^{-1}(m)),$$

which yields the solution $y(m, x)$. The unbiased and biased sender's first-order conditions are:

$$\begin{aligned} 0 &= U_1^S\left(y(\mu_1, x, b), x\right)\left(y_1(\mu_1, x) + y_2(\mu_1, x)\, 1/\mu_1'\right) \\ 0 &= U_1^S\left(y(\mu_0, x, 0), x\right)\left(y_1(\mu_0, x) + y_2(\mu_0, x)\, 1/\mu_0'\right). \end{aligned}$$

The same arguments as those in the proof of Theorem 2 allow us to conclude that this ODE system has a solution μ_0, μ_1 that identifies the fully-revealing equilibrium. ■

Again to make our general results concrete, we conclude the section with the analysis of the quadratic loss example.

Example 3 *Say that* $U^R(y,x) = -(y-x)^2$ *and* $U^S(y,x,b) = -(y-(x+b))^2$, *and that the receiver may be naive with probability* α. *The strategic receiver's strategy* s *satisfies:*

$$s(m) \in \arg\max_y -p(\mu_0^{-1}(m)-y)^2 - (1-p)(\mu_1^{-1}(m)-y)^2,$$

i.e. $s(m) = p\mu_0^{-1}(m) + (1-p)\mu_1^{-1}(m)$. *Thus the biased and unbiased senders first-order conditions are:*

$$-2\alpha(\mu_1(x)-(x+b)) - 2(1-\alpha)(p\mu_0^{-1}(\mu_1(x)) + (1-p)x) - (x+b))(p\mu_0'(x) + (1-p)\mu_1'(x)) = 0,$$
$$-2\alpha(\mu_0(x)-x) - 2(1-\alpha)(px + (1-p)\mu_1^{-1}(\mu_0(x)) - x)(p/\mu_0'(x) + (1-p)/\mu_1'(x)) = 0,$$

this system admits the linear solution $\mu_0 = x+b(1-p)(1-\alpha)/\alpha$ *and* $\mu_1 = x+b(1-p+\alpha p)/\alpha$.

In the case of the honest sender, instead, $s(m) = \alpha m + p(1-\alpha)\mu_0^{-1}(m) + (1-p)(1-\alpha)\mu_1^{-1}(m)$, *and the senders' first-order conditions are:*

$$-2(\alpha\mu_1(x) + p(1-\alpha)\mu_0^{-1}(\mu_1(x)) + (1-p)(1-\alpha)x - (x+b))(\alpha + \tfrac{p(1-\alpha)}{\mu_0'(x)} + \tfrac{(1-p)(1-\alpha)}{\mu_1'(x)}) = 0,$$
$$-2(\alpha\mu_0(x) + p(1-\alpha)x + (1-p)(1-\alpha)\mu_1^{-1}(\mu_0(x)) - x)(\alpha + \tfrac{p(1-\alpha)}{\mu_0'(x)} + \tfrac{(1-p)(1-\alpha)}{\mu_1'(x)}) = 0$$

which again admit the solution $\mu_0 = x+b(1-p)(1-\alpha)/\alpha$ *and* $\mu_1 = x+b(1-p+\alpha p)/\alpha$.

5 Conclusion

We have formulated a communication model in which players are possibly non strategic. Our model allows for the sender to be honest (and so truthfully report private information regardless of strategic considerations), or for the receiver to be naive (and so either blindly implementing the sender's recommendation or believing that the sender is honest). We show that in these instances equilibrium communication is fully revealing, but is encoded in an inflated language. These results may serve to build a simple disequilibrium theory of persuasion, where the receiver, while strategic and rational, is partially persuaded by the sender and hence unable to correctly invert the inflated communication language.

Our results hold for many forms of professional communication, including financial advice and academic evaluations. With financial advice, optimistic recommendations can be imputed to conflicts of interests between analysts and investors. Investors are at least partly deceived by such advice. Inspection of financial reports also indicates that, abstracting from communication costs, the advice is as detailed as possible. With academic communications, "grade inflation" is well documented. While each course grade is expressed on a fairly coarse scale, the GPA communicated to prospective employers and graduate programs delivers information on a very fine scale. Recommendation letters are usually very detailed, but at the same time very inflated.

Our results suggest that to obtain inflated yet detailed communication it suffices that a player may think that her opponent may be non-strategic. It should be possible to extend our results by allowing for any list of Mertens-Zamir (1985) universal types that capture a high-order belief that a player may be non-strategic. In principle, it is not difficult to sketch a framework that specializes Mertens and Zamir's construction to the problem at hand. However the construction will be somewhat cumbersome to analyze, as it implies the introduction of a continuum of "crazy" types to Crawford and Sobel's communication game. Since this exercise is not crucial for the point made in this paper, we postpone it to future research.

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