Dynamics of the Presidential Veto: A Computational

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Dynamics of the Presidential Veto: A Computational Analysis[*]

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Abstract

We specify and compute equilibria of a dynamic policy-making game between a president and a legislature under insitutional rules that emulate those of the US Constitution. Policies are assumed to lie in a two-dimensional space in which one issue dimension captures systemic differences in partisan preferences, while the other summarizes non-partisan attributes of policy. In any period, the policy choices of politicians are influenced by the position of the status quo policy in this space, with the current policy outcome determining the location of the status quo in the next period. Partisan control of the legislature and presidency changes probabilistically over time. We find that politicians strategically compromise their ideal policy in equilibrium, and that the degree of compromise increases when the opposition party is more likely to take control of the legislature in the next period, while politicians become relatively more extreme when the opposition party is more likely to control the presidency. We measure gridlock by (the inverse of) the expected distance of enacted policies from the status quo in the long run, and we show that both gridlock and the long run welfare of a representative voter are maximized when government is divided without a supermajority in the legislature. Under unified government, we find that the endogeneity of the status quo leads to a non-monotonic effect of the size of the legislative majority on gridlock; surprisingly, under unified government, gridlock is higher when the party in control of the legislature has a supermajority than when it has a bare majority. Furthermore, a relatively larger component of policy change occurs in the non-partisan policy dimension when a supermajority controls the legislature. We conduct constitutional experiments, and we find that voter welfare is minimized when the veto override provision is abolished and maximized when the presidential veto is abolished.

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1   Introduction

The ability of the US President to wield a veto over legislation approved by Congress is one of the key ingredients in the checks and balances system envisioned by the authors of the US Constitution. The chief executive's ability to veto policies increases the possibility of legislative stalemate, or "gridlock," especially under divided government when the partisan affiliation of the president differs from that of the majority party in Congress, a frequent occurrence in recent US history. While divided government and the prospect of policy gridlock have traditionally stirred skepticism and/or concern among scholars as to the desirability of the presidential veto, recent scholarship has taken a more neutral or even conditionally positive outlook on the welfare implications of the veto. On the one hand, Mayhew (1991) has presented empirical evidence that casts doubt on the expectation that the incidence of gridlock increases with divided government in the US context. Mayhew's finding does not necessarily contradict theoretical analyses of the role of the veto in the lawmaking process, since the impact of the veto on gridlock or policy stability is generally contingent on the location of the status quo policy (e.g., Krehbiel (1998), or Tsebelis (2002) in a more general context). But even if legislative stalemate does occur under divided government, the inability to change the status quo policy is an undesirable consequence of the veto only if the status quo itself constitutes an undesirable policy.

The preceding paragraph highlights the fact that the conclusions of any analysis of the effect of the veto on policy stability and welfare depend heavily on the assumed location of the status quo policy, which is itself determined by the past decisions of policy makers. Thus, in lieu of making ad hoc assumptions as to the location of the status quo, in this paper we analyze the effect of the veto by studying the inherently dynamic process via which the location of the current status quo is determined by the policy implemented in the previous period. Our goal is to understand the effects of the presidential veto in a context where there is room for meaningful coalition formation across parties and between legislators and the president. Moreover, we aim to evaluate the effects of specific institutional details, such as the provision for the legislative override of an executive veto. We therefore imbed our analysis in an institutionally detailed, infinite-horizon model of policy making among multiple politicians in a two-dimensional space of policies—a setting that presents daunting challenges to analytical solution. Indeed, since credible analysis of the impact of the veto requires us to understand the interactions among a multiplicity of political phenomena, we are forced at some point to abandon analytical methods. After reviewing some known analytical results on the model, we undertake a numerical equilibrium analysis of the dynamics of the presidential veto.

We analyze stationary equilibria of a model with a five-member legislature and a president.[1] Two political parties are represented with variable size of representation in the legislature, and one of the two parties controls the presidency in any given period. We assume that the majority party in the legislature controls the legislative agenda (Cox and

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[1]We assume five legislators for computational reasons; that is the minimum (odd) number that allows us to distinguish between bare majority and non-unanimous, supermajority coalitions.

McCubbins (2005)) and a member of that party is randomly drawn to propose a policy. This policy proposal is subject to the approval of the legislature and the president: if a majority of three legislators, with the president, approve the proposal, then it is enacted; similarly, the proposal is enacted if it is approved by a supermajority of four legislators, with or without the president. Otherwise, a status quo policy remains in place for the current period. We assume a two-dimensional policy space, with one dimension capturing systematic conflict due to partisan differences along a traditional left-right continuum, while the second dimension captures political disagreement that is not systematically represented by the two parties, e.g., foreign policy or some other non-partisan policy dimension. To facilitate the numerical analysis of the model, we assume that legislators belonging to the same party have ex ante identical preferences over policy, with a common, fixed stage ideal point, and we introduce heterogeneity among party members by subjecting stage utilities (per-period payoffs) to random shocks that encapsulate exogenous shocks to preferences of electoral constituencies that are uncorrelated over time, perhaps due to variation of an economic variable or news coverage of different political issues. Thus, the realized stage preferences of Democratic legislators in any period are distributed to the one side of the policy space, with Republican legislators preferences similarly distributed to the other. We model presidential preferences similarly, but we assume that presidential politicians (as competitors in a general election) are on average more moderate than their legislative counterparts. Control of the legislature and presidency in the next period is governed by an exogenous Markov process which we calibrate to US data, and policy in the current period determines a status quo for policy negotiations in the next period, creating non-trivial dynamic incentives for the politicians. We prove existence of and solve for a symmetric, stationary political equilibrium (SSPE) in pure strategies.

We find that the strategic incentives of the players in the computed SSPE imply moderation of their underlying policy preferences, but we we also find that the degree of this moderation correlates significantly with the configuration of the partisan control of the legislature and the presidency. First, politicians tend to moderate in equilibrium, in the sense that the ideal policy outcome of a legislator or president in a given period, induced by expectations of future play, is closer to the median ideal point than his underlying myopic ideal point as in, e.g., Baron (1996). This strategic moderation can be understood as a result of the role of the status quo in political decision making: if one party, say the Democrats, impose a policy to the far left of the median, then they risk the possibility that the Republicans will take control of the agenda in the following period and move policy to the far right of the policy space, since a status quo to the far left creates the scope for a far-right agenda setter to move policy much closer to his ideal point. To mitigate this possibility, a Democratic agenda setter would prefer to pass a policy in the center/left of the policy space, trading a relatively worse outcome in the present period with a better expected stream of policies in the future. Second, in line with the above reasoning, the incentive to compromise in the future increases with the probability that the opposing party will take control of the legislature. In addition, and perhaps counter-intuitively, we find that the incentive to compromise is mitigated somewhat when it is likely that the opposing party will control the presidency in the next period. In that case, considering again the perspective of a Democratic legislator,

the attractiveness of policies in the center of the space decreases because the veto of a Republican president would constrain policies to lie in the center of the policy spectrum in the next period, and prevent moves to the center/left. In turn, the different degrees by which politicians moderate in anticipation of future control of the legislature and the Presidency are reflected in the policies proposed and passed in equilibrium.

Perhaps in sharp contrast to numerous voices of concern over the consequences of divided government on the performance of the US political system, we find that a representative voter's welfare is actually maximized when government is divided without a supermajority in the legislature. This finding is consistent with the observed historical frequency of this configuration of partisan control of the two institutions. Consistent with expectations, we find also that gridlock is maximized in such situations of divided government. We identify a non-monotonic relation between the incidence of gridlock and the type of government due to the endogeneity of the status quo: under unified government, gridlock is higher when the party in control of the legislature has a supermajority than when it has a bare majority. This is due to endogeneity of the status quo. When the party in control of the legislature has a supermajority, it is likely to have been in control of the legislature in the previous period and to have implemented a favorable policy. Thus, the current status quo is likely to be more favorable, leaving a smaller scope for movement in policy. Furthermore, for the same reason, the type of policy change that occurs under unified government with supermajority control of the legislature is systematically different, with a relatively larger component of policy change occurring in the non-partisan (as opposed to the ideological) policy dimension. Finally, we conduct constitutional experiments and find that the representative voter's welfare is maximized when the veto override provision is abolished and minimized when the presidential veto is abolished. The current institutional setting produces intermediate levels of welfare.

A number of studies employ bargaining models in order to study conditions under which an executive may exercise veto power, as in Matthews (1989) and Groseclose and McCarty (2001), Cameron (2000), or to evaluate the effect of presidential veto on spending (Primo (2006)) or the distribution of pork barrel policies (McCarty (2000)). Alesina and Rosenthal (1996) study a model of US policy making and elections allowing for the separate partisan composition of the legislature and the presidency. Alesina et al. (1993) model and structurally estimate the determination of economic policy as well as the elections of the president and the legislature in the US. Roemer et al. (2007), employ numerical methods for the analysis of a multidimensional equilibrium model of two-party competition in the US. Roemer (2006) studies the dynamics of inequality in human capital assuming each period's policy is determined by a static two-party competition model. The present paper belongs in a strand of recent literature on legislative policy making with endogenous status quo and far-sighted players.[2] Duggan and Kalandrakis (2007) provide a general existence result along with other properties of the equilibrium for a model on which the present analysis is based. Dixit et al. (2000) analytically study the effect of a supermajority requirement in a dynamic

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[2]E.g., Epple and Riordan (1987), Baron (1996), Kalandrakis (2004, 2007), Gomez and Jehiel (2005), Penn (2005), Cho (2005), Bernheim et al. (2006), Fong (2006), Diermeier and Fong (2007), Baron et al. (2007), Battaglini and Coate (2007a,b).

model of two-party competition in one dimension. Unlike the present study, they assume that the party, modeled as a unitary actor, is a dictator over policy when it controls a (super)majority, and they study efficient subgame perfect equilibria of the resultant two-player game. Except in special settings in which it is possible to explicitly characterize equilibria, equilibrium analysis in dynamic models of the type we consider often requires recourse to numerical methods as used, for example, by Baron and Herron (2003), Penn (2005), and Battaglini and Palfrey (2007).

In what follows, we first present the model, define SSPE, and state certain properties of our equilibrium concept in Section 2. In Section 3, we describe the numerical algorithm we use for the computation of equilibrium, and specify the values of the model parameters used for numerical analysis. We discuss the equilibrium analysis of the baseline model in Section 4, evaluating the relation between equilibrium preferences and the configuration of partisan control of the political institutions of the legislature and the presidency, the occurrence of gridlock, the modes of equilibrium winning coalitions, and voter welfare. Finally, in Section 5, we perform constitutional experiments evaluating the effect of removing the veto or the veto override provisionon welfare and equilibrium quantities.

2 Model of Presidential Veto

A Dynamic Framework A president and a legislature of size n, n odd, bargain in order to determine a policy $x_t \in X = [-\bar{x}, \bar{x}]^2, \bar{x} > 0$, in each of an infinity of periods $t = 1, 2, \ldots$. We model the interaction between these two institutions as a dynamic game among $2n + 2$ players in the set $N = \mathcal{D} \cup \mathcal{R}$, where

$$\mathcal{D} = \{1, ..., n+1\} \quad \text{and} \quad \mathcal{R} = \{-1, ..., -(n+1)\}$$

partition the players into two political parties, the *Democratic party* and the *Republican party*, respectively. The legislature in period t is a subset $\mathcal{L}_t \subseteq \{1, \ldots, n, -1, \ldots, -n\}$ of size n, each player i, $-n \leq i \leq n$, being a potential legislator, and the presidency in period t, $\mathcal{P}_t \in \{n+1, -(n+1)\}$, is held by one of two potential presidents. We will often use the convention of writing the Democrat presidential politician as $D = n + 1$, and write $R = -(n+1)$ for her Republican counterpart. We capture, among other interpretations, the case in which the legislature includes a representative, either Democrat or Republican, from each of n single member districts, $1, \ldots, n$. We write d_t for the number of active Democrats in the legislature in period t, so that $n - d_t$ is the number of active Republicans. Thus, d_t and $n - d_t$ represent the strength of the representation of the Democratic party and the Republican party, respectively, in the legislature. The *political state,* summarizing the partisan composition of the legislature and the presidency, is given by $(\mathcal{L}_t, \mathcal{P}_t)$, and we assume that partisan control of these two institutions changes probabilistically over time according to an exogenous Markov process.

Following Duggan and Kalandrakis (2007), we model legislative interaction as follows. Each period t begins with a publicly observed state $s_t = (\mathcal{L}_t, \mathcal{P}_t, q_t, \theta_t, j_t)$. Here, $(\mathcal{L}_t, \mathcal{P}_t)$ is

the political state, $q_t \in X$ is the status quo policy, $\theta_t = (\theta_{1,t}, \ldots, \theta_{n+1,t}, \theta_{-1,t}, \ldots, \theta_{-(n+1),t}) \in \Theta \subseteq \Re^{2(2n+2)}$ is a vector of preference shocks, and $j_t \in \mathcal{L}_t$ is the agenda setter for the period. Play of the game proceeds as follows. The setter j_t proposes a policy $y \in X$, and the active legislators, $\mathcal{L}_t$, and the current president, $\mathcal{P}_t$, either approve or disapprove of the proposal. The proposal passes if it receives the approval of a winning coalition, and it fails otherwise, where for now we let $\mathcal{W}(s)$ denote the collection of winning coalitions (to be described later in more detail) in state s. The policy for period t, denoted x_t, is y if the proposal passes and is q_t otherwise. Each player i's stage utility is quadratic with an added linear noise term,

$$u_i(x_t, \theta_{i,t}) = -||x_t - \hat{x}_i||^2 + \theta_{i,t} \cdot x_t,$$

where $\hat{x}_i \in \mathbb{R}^2$ is player i's fixed ideal point and $\theta_{i,t}$ is the player's utility shock. Note that the preference shock $\theta_{i,t}$ is equivalent to perturbing the ideal point of player i by $\frac{1}{2}\theta_{i,t}$.[3] We assume that the potential Democratic legislators share the same underlying ideal policy position, i.e., $\hat{x}_i = (-x_L, 0)$, $x_L > 0$, for all $i = 1, \ldots, n$; and similarly, we have $\hat{x}_i = (x_L, 0)$ for all $i = -1, \ldots, -n$. We similarly specify the presidential politicians' fixed ideal points as $\hat{x}_{n+1} = (-x_P, 0)$ and $\hat{x}_{-(n+1)} = (x_P, 0)$, $x_P > 0$. Thus, the two parties are symmetrically located across each other on the first policy dimension which captures systematic policy disagreement, e.g., on a left-right dimension, while the second dimension represents a policy domain in which there is no systematic partisan disagreement such as, e.g., foreign policy.

Finally, after the policy in period t is implemented, a new state s_{t+1} for period $t + 1$ is randomly drawn. The transition probabilities for the process governing the political state are given by $p[\mathcal{L}_{t+1}, \mathcal{P}_{t+1}|\mathcal{L}_t, \mathcal{P}_t]$; the status quo q_{t+1} for period $t + 1$ is drawn from X according to the density $g(\cdot|x_t)$; and a new vector θ_{t+1} is drawn from Θ according to the density $f(\cdot)$. We assume that each θ_i is distributed uniformly on $[-\bar{\theta}, \bar{\theta}]^2$, and that the preference shocks of the players are independently distributed. These preference shocks ensure that the effective ideal policies of Democratic politicians in any period are non-trivially distributed to the left of the policy space, with Republican politicians' preferences similarly distributed to the right. We assume that the density $g(\cdot|x)$ is uniform on $[-a+bx_1, a+bx_1] \times [-a+bx_2, a+bx_2]$, where $a > 0$ and $b \in (0, 1)$ are constants satisfying $a + b\bar{x} \leq \bar{x}$. This noise on the status quo, which we assume is small in our numerical analysis, captures shocks to the policy making environment, including uncertainty about the implementation of policy by bureaucrats in the future, future legal interpretations of statutes, etc. The agenda setter in period $t + 1$ is drawn uniformly, with probability $r_j[\mathcal{L}_{t+1}]$, from the members of the majority party in the legislature. That is, the probability that legislator i is recognized to make a proposal is

$$r_j[\mathcal{L}_{t+1}] = \begin{cases} \frac{1}{d_{t+1}} & \text{if } j \in \mathcal{D} \cap \mathcal{L}_{t+1} \text{ and } d_{t+1} > \frac{n}{2}, \\ \frac{1}{n-d_{t+1}} & \text{if } j \in \mathcal{R} \cap \mathcal{L}_{t+1} \text{ and } d_{t+1} < \frac{n}{2}. \end{cases}$$

The above procedure is then repeated in period $t + 1$, and so on. Payoffs in the dynamic game are given by the expected discounted sum of stage utilities, as is standard, and we

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[3] To see this, note that $-||x - \hat{x}_i||^2 + \theta_i \cdot x = -||x - (\hat{x}_i + \frac{1}{2}\theta_i)||^2 + \theta_i \cdot \hat{x} + \frac{1}{4}\theta_i \cdot \theta_i$, which is just the sum of a term, $\theta_i \cdot \hat{x} + \frac{1}{4}\theta_i \cdot \theta_i$, constant in x and the quadratic utility with ideal point $\hat{x}_i + \frac{1}{2}\theta_i$.

denote the common discount factor by $\delta \in [0, 1)$. In what follows, when time is not germane to the discussion, we drop time subscripts and write $s = (\mathcal{L}, \mathcal{P}, q, \theta, j)$ for a generic state.

We consider an institutional setting that mimics the veto clause (Article I, Section 7) of the US Constitution as our baseline voting rule: a law can be passed by a majority of the legislature with the approval of the president or, failing that, with the vote of two-thirds of the legislature.

Baseline Veto A coalition $C \subseteq N$ is winning in state s if and only if

$$\left[|C \cap \mathcal{L}| > \frac{n}{2} \text{ and } \mathcal{P} \in C \right] \quad \text{or} \quad |C \cap \mathcal{L}| \geq \frac{2n}{3}.$$

We also examine the effects of weakening or strengthening the veto relative to the baseline institution by considering two alternative specifications of the set of winning coalitions. The second institutional setting amounts to pure majority rule, eliminating the veto altogether.

No Veto A coalition $C \subseteq N$ is winning in state s if and only if

$$|C \cap \mathcal{L}| > \frac{n}{2}.$$

In the third institutional setting, we strengthen the president's veto power by removing the legislative override.

Absolute Veto A coalition $C \subseteq N$ is winning in state s if and only if

$$\left[|C \cap \mathcal{L}| > \frac{n}{2} \text{ and } \mathcal{P} \in C \right].$$

Stationary Political Equilibrium A strategy in the game consists of two components giving the proposals of legislators when acting as agenda setter and the approval or disapproval of legislators and the president after a policy is proposed. While these choices can conceivably depend on histories arbitrarily, we seek subgame perfect equilibria in which political actors use simple strategies. We therefore focus on pure stationary Markov strategies. For players $i = \pm 1, \ldots, \pm n$, a strategy is therefore $\sigma_i = (\pi_i, \alpha_i)$, where $\pi_i \colon S \to X$ is legislator i's proposal strategy, and $\alpha_i \colon S \times X \to \{0, 1\}$ is i's approval strategy. We write $\alpha_i(s, y) = 1$ if i approves of proposal y given state s, and $\alpha_i(s, y) = 0$ if i disapproves. For the presidential players $i = \pm(n+1)$, a strategy $\sigma_i = \alpha_i$ simply registers approval or disapproval. In fact, we are able to strengthen stationarity and henceforth impose that proposal and approval strategies are independent of the agenda setter: if states s and s' satisfy $(\mathcal{L}, \mathcal{P}) = (\mathcal{L}', \mathcal{P}')$, $q = q'$, and $\theta = \theta'$, then $\pi_i(s) = \pi_i(s')$ and $\alpha_i(s, y) = \alpha_i(s', y)$.[4] We let

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[4] The first of these restrictions, that $\pi_i(s) = \pi_i(s')$, has no consequences for the payoffs of the players, since i's proposal only goes to the floor when $i = j_t$.

$\sigma = (\sigma_1, \ldots, \sigma_{n+1}, \sigma_{-1}, \ldots, \sigma_{-(n+1)})$ denote a stationary Markov strategy profile. Define the *political approval set* at state s as

$$A(s;\sigma) \;\;=\;\; \left\{ y \in X : \{i \in \mathcal{L} \cup \{\mathcal{P}\} : \alpha_i(s,y) = 1\} \in \mathcal{W}(s) \right\}, \tag{1}$$

so that given strategies σ, a policy proposal will pass if and only if it lies in $A(s;\sigma)$. Given our focus on stationarity, we henceforth omit time subscripts where appropriate.

Given strategies σ, we define politician i's dynamic preferences over policies by

$$U_i(x, \mathcal{L}, \mathcal{P}, \theta_i; \sigma) = (1 - \delta) u_i(x, \theta_i) + \delta v_i(x, \mathcal{L}, \mathcal{P}; \sigma), \tag{2}$$

where $v_i(x, \mathcal{L}, \mathcal{P}; \sigma)$ is i's expected discounted payoff, calculated at the beginning of next period, conditional on policy outcome x and political state $(\mathcal{L}, \mathcal{P})$ in the current period. We assume that legislators use "deferential" voting strategies, in the sense that when indifferent between a proposed policy and the status quo, a legislator approves. This allows us to focus on no-delay equilibria, in which agenda setters never propose policies that will fail, for a legislator could propose the status quo just as well as a policy that would fail. Then continuation values satisfy

$$v_i(x, \mathcal{L}, \mathcal{P}; \sigma) = \sum_{(\mathcal{L}', \mathcal{P}')} p[\mathcal{L}', \mathcal{P}'|\mathcal{L}, \mathcal{P}] \int_q \int_\theta \sum_j r_j[\mathcal{L}'] U_i(\pi_j(s), \mathcal{L}, \mathcal{P}, \theta_i; \sigma) f(\theta) g(q|x) d\theta dq, \tag{3}$$

for all policies x. As suggested by our notation, this continuation value v_i does not depend on the full state s, but only on the current political state, $(\mathcal{L}, \mathcal{P})$, through the transition probability $p[\cdot]$. As a result, and due to stationarity, the dynamic payoff U_i in (2) only depends on the political state and i's own preference shock.

We can now define a class of stationary Markov perfect equilibria of special interest. Intuitively, we require that legislators always propose optimally, and that legislators and the president only approve policies when it is in their best interest to do so. It is well-known, however, that arbitrary outcomes can be supported in voting games using Nash equilibria in which no voter is pivotal. To address this difficulty, we follow the standard approach of refining the set of Nash equilibria in the approval stage by requiring that legislators and the president eliminate stage-dominated strategies.

Definition 1 *Stationary political equilibrium:* A strategy profile σ is a *stationary political equilibrium* if the following conditions hold:

- for all states s and every legislator i, $\pi_i(s)$ solves

$$\max_y U_i(y, \mathcal{L}, \mathcal{P}, \theta_i; \sigma)$$
$$\text{s.t. } y \in A(s;\sigma),$$

- for all states s, every proposal y, and every player i,

$$\alpha_i(s,y) = \begin{cases} 1 & \text{if } U_i(y, \mathcal{L}, \mathcal{P}, \theta_i; \sigma) \geq U_i(q, \mathcal{L}, \mathcal{P}, \theta_i; \sigma) \\ 0 & \text{else.} \end{cases}$$

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While we impose the restriction that political equilibria be in pure strategies and that voting be deferential, Theorem 2 of Duggan and Kalandrakis (2007) shows that this is without loss of generality: even when players are allowed to mix or disapprove the status quo when indifferent, equilibrium proposal and approval strategies are equivalent to pure strategies with deferential voting for almost every realization of the state.

Before turning to symmetry conditions we impose in our model specification, we first note some properties of stationary political equilibria of fundamental importance. The next result establishes existence of a stationary political equilibrium, and it establishes that in all equilibria, continuation values (and therefore dynamic utilities) are smooth, that legislators have unique optimal proposals at almost all states, and that equilibrium proposal strategies are differentiable almost everywhere. The results follow from the arguments of Duggan and Kalandrakis (2007), though because of differences in our setup here, not as special cases of their theorems. We therefore provide an appendix explaining when and how the proof of Duggan and Kalandrakis is to be modified.

Theorem 1 *There exists a stationary political equilibrium, and every stationary political equilibrium σ is such that:*

1. *Continuation values are smooth: for every legislator i and every political state $(\mathcal{L}, \mathcal{P})$, $v_i(x, \mathcal{L}, \mathcal{P}; \sigma)$ is smooth as a function of x.*

2. *Proposals are almost always strictly best: for almost all states s, every legislator i, and every $y \in A(s; \sigma)$ distinct from the proposal $\pi_i(s)$, we have $U_i(\pi_i(s), \mathcal{L}, \mathcal{P}, \theta_i; \sigma) > U_i(y, \mathcal{L}, \mathcal{P}, \theta_i; \sigma)$.*

3. *Proposal strategies are almost always continuously differentiable: for almost all states s and every legislator i such that $\pi_i(s) \neq q$, $\pi_i(\mathcal{L}, \mathcal{P}, q, \theta, j)$ is continuously differentiable in an open set around (q, θ).*

The properties given in Theorem 1 are conducive to numerical analysis of stationary political equilibria. The second property, in particular, implies that we do not need to consider the possibility that politicians use mixed strategies in equilibrium, and it implies that on a set of states of full measure, equilibrium proposal strategies are independent of the preference shocks of inactive legislators.

Symmetric Policy Making In our computational analysis, we specialize the model to a symmetric setting and focus on a symmetric form of political equilibrium. Toward that end we impose two additional symmetry assumptions on the transition probabilities of the political state. First, we assume *within-party symmetry,* i.e., for all $(\mathcal{L}_t, \mathcal{P}_t)$, all $(\mathcal{L}_{t+1}, \mathcal{P}_{t+1})$, all $\mathcal{L}'_t$, all $\mathcal{L}'_{t+1}$, if

$$d_t = |\mathcal{L}_t \cap \mathcal{D}| = |\mathcal{L}'_t \cap \mathcal{D}| = d'_t \quad \text{and} \quad d_{t+1} = |\mathcal{L}_{t+1} \cap \mathcal{D}| = |\mathcal{L}'_{t+1} \cap \mathcal{D}| = d'_{t+1},$$

then

$$p[\mathcal{L}_{t+1}, \mathcal{P}_{t+1} | \mathcal{L}_t, \mathcal{P}_t] = p[\mathcal{L}'_{t+1}, \mathcal{P}_{t+1} | \mathcal{L}'_t, \mathcal{P}_t].$$

In other words, the probability of transitions between political states depends only on the number of elected Democrats, $d = |\mathcal{L} \cap \mathcal{D}|$, (or Republicans $n - d$) in the legislature $\mathcal{L}$, and not on the identities of individual members of the legislature. Thus, we write the transition probability as $p[d_{t+1}, \mathcal{P}_{t+1}|d_t, \mathcal{P}_t]$, and when appropriate we may refer to a political state by the summary statistic $(d, \mathcal{P})$. Second, we assume that transitions between political states satisfy *across-party symmetry*, i.e., for all $(d_t, \mathcal{P}_t)$ and all $(d_{t+1}, \mathcal{P}_{t+1})$,

$$ p[d_{t+1}, \mathcal{P}_{t+1}|d_t, \mathcal{P}_t] \;\; = \;\; p[n - d_{t+1}, -\mathcal{P}_{t+1}|n - d_t, -\mathcal{P}_t]. \tag{4} $$

Note that in political state $(n - d_t, -\mathcal{P}_t)$, Republicans are in exactly the same situation as Democrats in $(d_t, \mathcal{P}_t)$, and similarly for political states $(n - d_{t+1}, -\mathcal{P}_{t+1})$ and $(d_{t+1}, \mathcal{P}_{t+1})$. Across-party symmetry implies that the transition from one state depends only on the size of the majority party and whether government is unified (the president is a member of the same party as the majority) or divided (the president belongs to the minority party), and not on which party controls the legislature.

Given these assumptions, we can now focus on stationary political equilibria in which players' strategies reflect the symmetry available in the model. Our first refinement of equilibrium requires, informally, that legislators belonging to the same party condition their decisions on preferences in the same way.

Definition 2 *Within-party symmetry:* Given a profile of preference shocks θ and legislators h, i of the same party, let $\rho^{h,i}(\theta) \in \Theta$ be the result of permuting θ_h and θ_i in θ. Given any potential legislators h, i belonging to the same party, $hi > 0$, and any states $s = (\mathcal{L}, \mathcal{P}, q, \theta, j)$ and $s' = (\mathcal{L}', \mathcal{P}', q', \theta', j')$, we say s and s' are symmetric within party with respect to h and i if (a) the president and the status quo are the same in both states, $(\mathcal{P}, q) = (\mathcal{P}', q')$, (b) the set difference of the two legislatures, if $\mathcal{L} \neq \mathcal{L}'$, is equal to $\{h, i\}$, i.e., $\mathcal{L} \, \Delta \, \mathcal{L}' \subseteq \{h, i\}$, and (c) the preference shocks of the two legislators are interchanged, $\theta = \rho^{h,i}(\theta')$. Then we say the stationary strategy profile σ satisfies *within-party symmetry* if whenever s and s' are symmetric within party with respect to any h and i, we have (d) $\pi_h(s) = \pi_i(s')$ and (e) for all y, $\alpha_h(s, y) = \alpha_i(s', y)$.

In words, given the strategic choices of a legislator i in one state, legislator h of the same party will make the same choices whenever he faces the same circumstances, i.e., the other legislators and their preference shocks are fixed, and the stage preferences of h are the same as i's in the first state. Within-party symmetry implies that given a preference shock for a politician, the politician's strategic choice depends only on the size of each party's representation in the legislature and the realized preference shocks of these members, and not on the particular identity of the players in the legislature $\mathcal{L}$.[5] Thus, within-party symmetry

---

[5]This follows since whenever two legislators from the same party use symmetric proposal and approval strategies, other legislators must, essentially (at almost all states), treat them symmetrically so that their strategy does not depend on the identity of the players representing each party. Indeed, other legislators' approval strategies are independent of the agenda setter by definition. Furthermore, by part 2 of Theorem 1 the optimal policy proposals of other legislators are unique at almost all states. Our claim then follows from the fact that at such states, the equilibrium proposals of other legislators depend only on the political approval set and the hypothesis that legislators of the same party use symmetric approval strategies.

implies that the continuation values of all Democratic potential legislators are equal to one another, $v_i(x, \mathcal{L}, \mathcal{P}; \sigma) = v_h(x, \mathcal{L}, \mathcal{P}; \sigma)$ for $h, i = 1, \ldots, n$, as are the continuation values of Republican potential legislators, leaving just two more continuation values, one each for the Democratic and Republican presidential politicians. Furthermore, within-party symmetry continuation values depend only on the number of Democrats elected to the legislature, d, so that we can write $v_i(x, d, \mathcal{P}; \sigma)$ for the continuation value $v_i(x, \mathcal{L}, \mathcal{P}; \sigma)$, and we will more generally abuse notation slightly by occasionally writing the political state as $(d, \mathcal{P})$ instead of $(\mathcal{L}, \mathcal{P})$.

Next, we impose a refinement that restricts the choices of legislators across parties. Given a vector $x = (x_1, x_2) \in \mathbb{R}^2$, let $\varphi(x) = (-x_1, x_2)$ denote the reflection of x through the vertical axis.

Definition 3 *Across-party symmetry:* Given a profile of preference shocks $\theta \in \Theta$, we define

$$\varphi^*(\theta) = (\varphi(\theta_{-1}), \ldots, \varphi(\theta_{-(n+1)}), \varphi(\theta_1), \ldots, \varphi(\theta_{n+1}))$$

to be the result of reflecting each potential legislator i's preference shock θ_i through the vertical axis and then assigning it to the corresponding legislator $-i$ of the opposite party. Given any states $s = (\mathcal{L}, \mathcal{P}, q, \theta, j)$ and $s' = (\mathcal{L}', \mathcal{P}', q', \theta', j')$, with d and d' Democrats in the legislature, respectively, we say s and s' are symmetric across party if (a) $d = n - d'$, (b) $\mathcal{P} = -\mathcal{P}'$, (c) $q = \varphi(q')$, and (d) $\theta' = \varphi^*(\theta)$. Then we say σ satisfies *across-party symmetry* if whenever s and s' are symmetric across party, we have for all $i \in N$, (f) for all y, $\alpha_i(s, y) = \alpha_{-i}(s', \varphi(y))$, and (g) if $-n \leq i \leq n$, then $\pi_i(s) = \varphi(\pi_{-i}(s'))$.

The operator φ^* essentially reverses the situations of politicians in the two parties, e.g., if a Democratic legislator i has extreme partisan preferences (meaning a perturbed ideal point far to the left) in θ, then the corresponding Republican legislator, $-i$, has extreme preferences (now meaning an ideal point far to the right) in $\varphi^*(\theta)$. Thus, across-party symmetry means that if the situation of one party, say the Democrats, in state s is analogous to that of the other party, say the Republicans, in state s', then the strategic decisions of the Democratic politicians in s should be analogous to those of their Republican counterparts in state s'. A consequence of across-party symmetry is that we can compute $v_i(x, d, \mathcal{P}; \sigma)$ from $v_{-i}(\varphi(x), n - d, -\mathcal{P}; \sigma)$, further reducing to two the number of continuation value functions we must compute: it is sufficient to compute the legislative and presidential continuation values for one of the two parties.

In our numerical analysis, we focus on an equilibrium refinement in which the strategic behavior of the players reflects the within- and across-party symmetry of the model primitives.

Definition 4 *Symmetric stationary political equilibrium (SSPE):* A stationary strategy profile σ is a *SSPE* if it is a stationary political equilibrium and satisfies within-party symmetry and across-party symmetry.

The next result establishes existence of a SSPE and collects the implications of our symmetry refinement. Clearly, any SSPE is a stationary political equilibrium and so must

satisfy properties 1–3 of Theorem 1. That any SSPE, if one exists, satisfies properties 4 and 5 of the theorem follows from the above discussion. The argument for existence of SSPE follows that of Duggan and Kalandrakis (2007), but because the argument utilizes structure of the current model not available in that paper, we provide the outline of the proof in the appendix.

Theorem 2 *There exists a SSPE. Every SSPE σ satisfies properties 1–3 of Theorem 1, and in addition, equilibrium continuation values satisfy*

4. for all $i, h \in \{1, ..., n\}$, $v_i(x, d, \mathcal{P}; \sigma) = v_h(x, d, \mathcal{P}; \sigma)$,

5. for all $i \in N$, $v_i(x, d, \mathcal{P}; \sigma) = v_{-i}(\varphi(x), n - d, -\mathcal{P}; \sigma)$.

3   Computation Details

In the first part of this section, we detail the numerical specification of various model parameters required for the numerical analysis of the model of Section 2. In the second part, we describe the algorithm we use to compute SSPE of this model and discuss implementation issues. The results from the implementation of this algorithm are analyzed in the subsequent two sections.

Parameter Specification and Calibration We will numerically analyze a legislature of size $n = 5$ and restrict the possible size of the Democratic representation in that legislature, d, to lie in $\{1, 2, 3, 4\}$. Thus, while the set of all potential political states is given by $\{0, \ldots, n\} \times \{D, R\}$,[6] we obtain a set of eight possible political states that is given by

$$\hat{S} = \{(1, D), (2, D), (3, D), (4, D), (1, R), (2, R), (3, R), (4, R)\}, \tag{5}$$

where, e.g., $(2, R)$ indicates that two Democrats are elected to the legislature and the president is Republican. Note that our this restriction of the state space economically represents the politically relevant configurations of partisan control of the legislature and the presidency, allowing us to distinguish between situations of unified government, i.e., $\hat{s} \in \{(3, D), (4, D), (1, R), (2, R)\}$ or divided government, i.e., $\hat{s} \in \{(1, D), (2, D), (3, R), (4, R)\}$, as well as between cases when the majority party in the legislature controls a simple majority, i.e., $\hat{s} \in \{(2, D), (3, D), (2, R), (3, R)\}$, versus a supermajority that can override a presidential veto, i.e., $\hat{s} \in \{(1, D), (4, D), (1, R), (4, R)\}$.

We have assumed that the political system transitions from a political state $\hat{s} \in \hat{S}$ to $\hat{s}' \in \hat{S}$ with probability $p[\hat{s}'|\hat{s}]$, and given the set of political states in (5), this transition process $p[\cdot|\cdot]$ can be effectively summarized by an 8×8 Markov transition matrix, each row of which represents the probability distribution $p[\cdot|\hat{s}]$ over the eight states in $\hat{S}$, conditional on a state $\hat{s}$ in the previous period. Instead of imputing the entries of this transition matrix in an ad hoc fashion, we follow a calibration procedure using historical US electoral results.

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[6]Recall our convention to write the Presidential politicians of the two parties as $D = n + 1$ and $R = -(n + 1)$, respectively.

	(1,D)	(2,D)	(3,D)	(4,D)	(1,R)	(2,R)	(3,R)	(4,R)		(1,D)	(2,D)	(3,D)	(4,D)	(1,R)	(2,R)	(3,R)	(4,R)
(1,D)	-	-	-	-	-	-	-	-	**(1,D)**	0.1	0.43	0.01	0	0.25	0.2	0	0
(2,D)	0	0	0	0	0	1	0	0	**(2,D)**	0.02	0.44	0.11	0	0.08	0.33	0.02	0
(3,D)	0	1	1	1	0	1	0	0	**(3,D)**	0	0.14	0.33	0.02	0.01	0.24	0.24	0
(4,D)	0	0	0	0	0	0	2	0	**(4,D)**	0	0.01	0.35	0.12	0	0.06	0.43	0.03
(1,R)	-	-	-	-	-	-	-	-	**(1,R)**	0.03	0.43	0.06	0	0.12	0.35	0.01	0
(2,R)	0	0	0	0	0	1	1	0	**(2,R)**	0	0.24	0.24	0.01	0.02	0.33	0.14	0
(3,R)	0	0	2	1	0	0	3	0	**(3,R)**	0	0.02	0.33	0.08	0	0.11	0.44	0.02
(4,R)	-	-	-	-	-	-	-	-	**(4,R)**	0	0	0.2	0.25	0	0.01	0.43	0.1

Figure 1: Historical Transitions since 1944 and Calibrated Transition Matrix.

Specifically, we use data on the partisan control of the US House of Representatives and the presidency, which we may classify into one of the eight political states $\hat{s} \in \hat{S}$ as specified in (5). The first matrix displayed in Figure 1 summarizes these data by reporting a simple count of each of the fifteen observed transitions between political states, starting with the one that occurred in the 1948 elections and ending with the transition that occurred in the 2004 elections, i.e., using a data series from 1944 to 2004.[7] With only fifteen observed transitions, these data are insufficient for us to impute all 64 transition probabilities $p[\hat{s}'|\hat{s}]$ using a frequency estimator. This is true even after we exploit the symmetry restriction that we have imposed in (4), since as is evident by the fact that the first and last row of the first matrix in Figure 1 contains no entries, there is no incidence of a political state with divided government and supermajority control of the legislature. As a consequence, we use more refined data in order to pursue a parametric approach.

To be concrete, let $NDEM_t$ be the number of Democrats elected in the House of representatives in presidential election year t, and let $PRES_t$ be a dummy variable indicating whether the President elected that same year is a Democrat or not, where $t = 1944, 1948, \ldots, 2004$. We use these observations to estimate a Seemingly Unrelated Regressions model (Zellner (1962)) that takes the form

$$
\begin{aligned}
NDEM_t &= \gamma_0^1 + \gamma_1^1 \times NDEM_{t-1} + \gamma_2^1 \times PRES_{t-1} + \varepsilon_t^1 \\
PRES_t &= \gamma_0^2 + \gamma_1^2 \times NDEM_{t-1} + \gamma_2^2 \times PRES_{t-1} + \varepsilon_t^2,
\end{aligned}
$$

where the error terms are independently and identically distributed in each year t according to the bivariate normal distribution

$$
\begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{pmatrix} \sim N\left(0, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right).
$$

---

[7]These data ignore midterm elections that may result in a change of control of the House but not the Presidency, since we do not keep track of such restricted transitions between political states in the model. Similarly, since we have specified a model with a unicameral legislature, we ignore the US Senate in classifying these observations into political states of divided or unified government, or in order to distinguish between simple or supermajority control of the legislature, etc. Both of these additional institutional dimensions can be included in the model and analyzed at an additional computational cost.

Political State	$(1, D)$	$(2, D)$	$(3, D)$	$(4, D)$	$(1, R)$	$(2, R)$	$(3, R)$	$(4, R)$
Probability	0.01	0.21	0.25	0.03	0.03	0.25	0.21	0.01

Table 1: Invariant Distribution over Political States Induced by Calibrated Transition Matrix.

We use the parameter estimates of $\gamma^1, \gamma^2, \sigma_1, \sigma_2$, and ρ, in order to iteratively simulate a long chain of data $(\widehat{NDEM}_\tau, \widehat{PRES}_\tau)_{\tau=1}^k$. At each stage τ in the simulation process, we truncate the simulated continuous data to the nearest integer so that $\widehat{NDEM}_\tau \in \{0, \ldots, 435\}$, and $\widehat{PRES}_\tau \in \{0, 1\}$, before we simulate $\widehat{NDEM}_{\tau+1}, \widehat{PRES}_{\tau+1}$. After discarding the initial k' observations, we classify the simulated data $(\widehat{NDEM}_\tau, \widehat{PRES}_\tau)_{\tau=k'}^k$ into one of the eight political states in $\hat{S}$, and use the resultant $k - k' - 1$ transitions in these simulated data in order to obtain estimates of the probabilities $p[\hat{s}'|\hat{s}]$ using a frequency estimator. By effectively counting each transition in the simulated data twice (once for any simulated transition from state (d, P) to (d', P') and another for a symmetric transition from $(-d, -P)$ to $(-d', -P')$), we arrive at a transition matrix that respects the symmetry restrictions (4) we imposed in the second part of the previous section. This matrix with entries rounded to two decimal points, is displayed in Figure 1. We observe that the end result of the calibration procedure we outlined is a transition matrix that credibly emulates the modal transitions in the historical data also displayed in Figure 1. Table 1 displays the invariant distribution over political states that is induced by the calibrated transition matrix. Note that according to Table 1 the political state is some form of divided government a significant fraction of the time (44%).

With the transition process $p[\hat{s}'|\hat{s}]$ specified, we proceed to determine numerical values for the remaining parameters of the model. We set $\bar{x} = 2$, so that the policy space is given by $X = [-2, 2]^2$. We position the typical Republican legislator's ideal policy on the left-right dimension at $x_L = 1$, so that the typical Democrat legislator is located at -1. We also set $\bar{\theta} = 2$, so that the distribution of preference shocks of each θ_i is uniform in $[-2, 2]^2$. Given our choice for a left-right ideal point location for the legislators at $x_L = 1$ and the discussion in footnote 3, these specifications amount to the assumption that in each period the Democratic legislators' stage ideal points are distributed uniformly in $[-2, 0] \times [-1, 1]$, while the Republican legislators' ideal points are distributed uniformly in $[0, 2] \times [-1, 1]$. Thus, the most moderate Democrat legislator possible on the left-right dimension has an ideal policy in that dimension that coincides with that of the corresponding most moderate Republican. We assume that the electoral process for the selection of the president at the national level is more likely to lead to elected presidents with more moderate preferences compared to the preferences of representatives elected from smaller local districts. Thus, we assume that presidents are on average more moderate than the typical legislators and we position the Republican presidential politician's ideal point at $x_P = 0.5$, and the corresponding Democrat's at $-x_P = -0.5$. We set the discount factor at $\delta = 0.6$, which is a value consistent with the fact that a period in the model corresponds to four calendar

years in the US case. Finally, we specify the support of the density $g(\cdot|x)$ by setting $a = \frac{1}{20}$ and $b = \frac{39}{40}$. These parameter values imply a maximum shock to the policy decision x equal to 0.05 in each coordinate, so that the assumed level of noise on the status quo ensures the equilibrium properties of Theorem 1 but is relatively small. We now proceed to describe the numerical methods we used in order to analyzed the model with the above parameter specifications.

Computational Algorithm and Implementation The point of departure for our numerical analysis is the fact that players' equilibrium strategies are essentially uniquely determined by their equilibrium continuation value functions. Indeed, given such continuation values, voting strategies are pinned down by the assumption of deferential voting, while proposal strategies are essentially unique in equilibrium by property 2 of Theorem 1. As a consequence, even though we define an equilibrium as a profile of strategies, σ, the focus of our numerical analysis is the numerical approximation of equilibrium continuation values. This task is further simplified by the symmetry restrictions we impose on parties in the definition of SSPE, which yield properties 4 and 5 of Theorem 2 and allow us to compute only two continuation value functions, one for a typical Democratic legislator, say v_1, and one for the Democrat presidential politician, $v_{D=n+1}$. Given the $\hat{S}$ subset of possible political states specified in (5), we decompose these continuation values by political state as $(v_{\hat{s},i}(x))_{\hat{s} \in \hat{S}}$. Then the computation of an equilibrium σ reduces to the computation of $2|\hat{S}|$ functions $v_{\hat{s},i}$, where $|\hat{S}|$ of these belong to the representative Democratic legislator, $i = 1$, and $|\hat{S}|$ belong to the Democratic presidential politician, $D = n + 1$.

Let a vector of such SSPE continuation value functions be given by $v = (v_{\hat{s},1}, v_{\hat{s},n+1})_{\hat{s} \in S}$. We numerically approximate these equilibrium continuation values by functions $\hat{v}$. The algorithm, a form of value iteration, is illustrated in Figure 2. Given a vector of approximate continuation values, say $\hat{v}^{k-1}$, at the beginning of iteration k, we obtain an updated vector of values $\hat{v}^k$ as follows. First, for each political state $\hat{s}$ and proposer i, we compute optimal proposals, $\pi_{\hat{s},i}^k(q, \theta)$, at a finite number of points (q, θ) to be specified shortly, assuming players' continuation values are given by $\hat{v}^{k-1}$. For this optimization step we use the *downhill simplex* (or *Nelder-Mead*) optimization algorithm (Press et al. (1992)). This optimization method requires a set of initial feasible points which we locate using a coarse grid search, but does not rely on evaluations of derivatives, and can handle the irregular shape of the set of acceptable policies available to the proposer. Second, we evaluate each continuation value function $\hat{v}_{\hat{s},i}$ using numerical integration at a finite number α of interpolation nodes x_m. Given that we are approximating the high-dimensional integral (3), we opt for a quasi-Monte Carlo integration method, drawing a number β of pseudo-random points $(q_{m,\ell}, \theta_{m,\ell})$ for each x_m. We fix the points $(q_{m,\ell}, \theta_{m,\ell})$ at the beginning of the algorithm using Sobol sequences of quasi-random numbers (Press et al. (1992)), as implemented by Burkardt (2007), and we

INPUT: Termination criterion ϵ.

INPUT: Initial continuation values $\hat{v}^0$.

INPUT: Interpolation nodes $x_m \in X$, for $m = 1, \ldots, \alpha$ and degree $\hat{\alpha}$.

INPUT: Pseudo-random draws $(q_{m,\ell}, \theta_{m,\ell}) \in X \times \Theta$, for $\ell = 1, \ldots, \beta$, $m = 1, \ldots, \alpha$.

OUTPUT: Approximate Equilibrium Continuation values $\hat{v}$.

STEP 1 Optimal proposals.
  – For each $m = 1, \ldots, \alpha$:
  – For each $l = 1, \ldots, \beta$:
  – For each state $\hat{s} = (d, P) \in \hat{S}$:
  – For each possible proposer j at political state $\hat{s}$:
  ∗ Compute $\pi_{\hat{s},j}^k(q_{ml}, \theta_{ml})$ assuming continuation values $\hat{v}^{k-1}$.

STEP 2 Interpolation data.
  – For each $m = 1, \ldots, \alpha$:
  – For each state $\hat{s} \in \hat{S}$:
  ∗ Evaluate $\tilde{v}_{\hat{s}}^k(x_m)$ using (7).

STEP 3 Interpolation/Approximation.
  ∗ Compute $\hat{v}^k$, using interpolation data $(x_m, \tilde{v}_{\hat{s}}^k(x_m))$, $\hat{s} \in \hat{S}$, $m = 1, \ldots, \alpha$.

STEP 4 Convergence check.
  ∗ If $||\hat{v}^k - \hat{v}^{k-1}|| < \epsilon$, set $\hat{v} = \hat{v}^k$ and stop, else proceed to STEP 1 with $\hat{v}^{k-1} = \hat{v}^k$.

Figure 2: Computational Algorithm

compute the integrals

$$
\begin{aligned}
\tilde{v}_{\hat{s},i}^{k}(x_m) \;=\; & \sum_{\ell=1}^{\beta} \beta^{-1} \sum_{\hat{s}'} p[\hat{s}'|\hat{s}] \sum_{j} r_j[\hat{s}'] \Big((1-\delta)[u_i(\hat{\pi}_{\hat{s}',j}^{k}(q_{m,\ell},\theta_{m,\ell})) \\
& + \frac{d}{n}\theta_{m,\ell,i} \cdot \hat{\pi}_{\hat{s}',j}^{k}(q_{m,\ell},\theta_{m,\ell})] + \delta \hat{v}_{\hat{s}',i}^{k-1}(\hat{\pi}_{\hat{s}',j}^{k}(q_{m,\ell},\theta_{m,\ell})) \Big)\,.
\end{aligned}
\tag{6}
$$

At the third step of the k-th iteration, we compute $\hat{v}_{\hat{s},i}^{k}$ for each $\hat{s}$ and i using an $\hat{\alpha}$-th degree Chebyshev polynomial approximation in two dimensions. In particular, after step two of iteration k we have obtained interpolation points $(x_m, \tilde{v}_{\hat{s},i}^{k}(x_m))$, $m = 1, \ldots, \alpha$, where the α interpolation nodes x_m are appropriate (interval adjusted) roots of the Chebyshev polynomials and are fixed throughout the procedure. The details of this regression method to approximate $\hat{v}_{\hat{s},i}^{k}$ from such interpolation data can be found in Press et al. (1992) and Judd (1998). At the conclusion of the third step of the k-th iteration we have generated a sequence $\hat{v}^0, \hat{v}^1, \ldots, \hat{v}^k$ of continuation value functions, and we continue iterating in the above fashion until additional iterations have no appreciable effect on the computed $\hat{v}$.

We implemented this algorithm in order to numerically approximate an equilibrium of the model with its parameters set to the values we described in the first part of this section. All computations reported in this paper were obtained with the following choices of parameters for the implementation of the algorithm. We used $\alpha = 70 \times 70 = 4,900$ interpolation points[8] and an $\hat{\alpha} = 10$-th degree Chebyshev approximation, which amounts to the evaluation of $10 \times 10 = 100$ distinct coefficients for the corresponding Chebyshev polynomials. We set $\beta = 400$ for the numerical integration in STEP 2, and used a *sup* norm to evaluate convergence of the algorithm, requiring a maximum distance no larger than $\epsilon = 10^{-5}$ between any pair of successive Chebyshev coefficients. We preceded each final computation with the above specifications by a quick initial run of the algorithm with less ambitious numerical precision standards, and then used the output from this initial run in order to initiate the algorithm a second time. We encountered no problem with convergence at these levels of precision. We also ran additional checks for the numerical accuracy of the approximate equilibrium continuation values obtained upon convergence. One such check probed the reliability of the proposer optimization method we used, by running an additional iteration using the converged continuation values as input, this time using an expensive but more reliable grid search in order to obtain optimal proposals in step 1 of the iteration. In other checks, we increased the degree of the Chebyshev approximation used in step 3 of the algorithm, and the number of (q, θ) points used for the integration in step 2. The resulting continuation values from these additional iterations did not differ appreciably from the ones obtained upon convergence of the algorithm we described, thus adding confidence to the validity of the approximation. We report the results of these computations in the next two sections.

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[8]We can economize on computations significantly while maintaining comparable levels of precision with a much smaller number of interpolation points.

4 Policy Implications of Presidential Veto

In this section, we explore the properties of the SSPE computed for the Baseline Veto institution, in which a policy proposal passes if and only if either (1) it meets with the approval of a majority of legislators and the president, or (2) it meets with the approval of at least four legislators. The equilibrium of our model generates a rich set of data that bears on the dynamics of the veto in a number of respects. First, we examine the impact of strategic considerations on the preferences of political actors. While the stage utilities of each politician have a simple quadratic form, we see that the anticipation of future play of the game generates more complex preferences; in particular, dynamic considerations about the future location of the status quo have a moderating effect on preferences, inducing legislative agenda setters to compromise more than they otherwise would. Second, we consider the implications of the equilibrium analysis for social welfare. Third, we analyze the nature of winning coalitions that form in equilibrium, and the extent of policy gridlock. We provide a direct evaluation of the contribution of the presidential veto on gridlock, by considering the effect of removing the veto on policies, holding all else constant.

Policy Compromise As is to be expected in an infinite-horizon dynamic game with an infinite state space, the play of the computed SSPE is complex. A concise summary is given, however, by the continuation values and dynamic utilities of the political actors: these pin down equilibrium strategies and enfold all payoff-relevant aspects of the play of the game. In Figure 3, we provide the contour plots for the continuation value function and dynamic utility of the representative Democratic legislator $i = 1$ in the political state $(3, D)$ with preference shock $\theta_1 = 0$. Relative to the legislator's stage preferences, the continuation value function is substantially shifted to the right, with maximizer at roughly $(0, 0)$, the center of the policy space. This is reflected in the legislator's dynamic utility by a widening of contours along the horizontal axis and a (smaller) rightward shift of his ideal point. (The shift in dynamic utility is smaller because the continuation value function is discounted and relatively flat.) Instead of having an ideal point near the stage ideal point $(-1, 0)$, the legislator's dynamic utility is maximized at a policy located in the center/left of the policy space. While no longer having a simple quadratic form, it is noteworthy that dynamic utilities appear to preserve the quasi-concavity of stage utilities. These plots are similar to those in all Democratic minority states $(1, D)$, $(1, R)$, $(2, D)$, and $(2, R)$, and we omit figures for those states.

The moderation exhibited in the legislator's continuation value and dynamic utility are driven by considerations of the location of the status quo in the next period. In all of these states, the probability that the Republicans control the legislature in the following period is rather high; it is .39 in $(3, D)$ and much higher in the other states. If policy outcomes in the current period were moderate, this would limit the ability of a possible future Republican majority in the legislature to move policy even further to the right (in the general direction of their ex ante preference). A Republican agenda setter may still be able to form a majority to move policy rightward, but the status quo will generally be better for his Republican colleagues (improving their default payoffs and restricting the set of policies they would approve), and it will be more difficult to assemble a winning coalition that
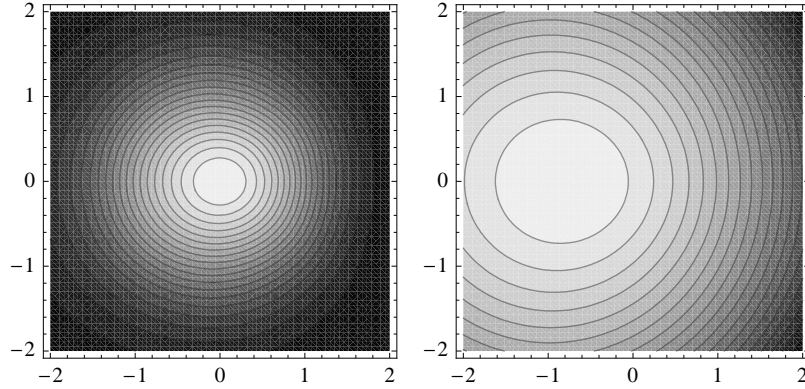
Figure 3: Continuation Value, $v_1(x, 3, D)$, & Expected Utility, $U_1(x, 3, D, \theta_1 = 0)$.

crosses party lines. If the policy in the current period is located near the ex ante Democratic stage ideal point, $(-1, 0)$, in contrast, then a future Republican agenda setter would have greater latitude to move policy further to the right, generating significantly worse payoffs for Democratic politicians.

In Figure 4, we provide the contour plots for the continuation value function and dynamic utility of the representative Democratic legislator $i = 1$ in the political state $(3, R)$ with preference shock $\theta_1 = 0$. Here, there is a marked difference in the continuation value of the legislator relative to state $(3, D)$. At work is the influence of the presidential veto: in moving from $(3, D)$ to $(3, R)$, the probability of a Republican president in the next period increases from .49 to .57. This creates a "dent" in the center/right of the graph of the legislator's continuation value function, for if the current period's policy (and therefore next period's status quo) lies in this region, then the veto of a Republican president would likely keep future policy nearby—preventing possible future Democratic majorities from imposing center/left policies. This is reflected in the legislator's dynamic utility over the center/right region by a widening of contours along the vertical dimension, indicating that the legislator is now more willing to compromise along the non-partisan vertical dimension. These plots are similar to those in the Democratic majority states $(4, D)$ and $(4, R)$, so we omit figures for those states. In sum, we find that a higher probability of opposition control of the legislature next period leads to greater policy compromise, while higher probability of opposition control of the presidency dampens this effect.

The Democratic presidential politician's continuation value and dynamic utility are depicted in Figure 5 for state $(3, R)$ and preference shock $\theta_D = 0$. As in the case of the Democratic legislator, we find significant moderation in the presidential politician's preferences. In contrast to the Democratic legislator, the contours in Figure 5 are fairly similar to those in all other political states; the difference between $(3, D)$ and $(3, R)$ is less pronounced than for the legislator: the president's ex ante stage ideal point is located at $(-.5, 0)$, closer to the center than the legislator, so that (owing to concavity of stage utilities) the drop in utility from center/right outcomes is less significant.

In Table 2, we provide a more systematic summary of the patterns of strategic mod-
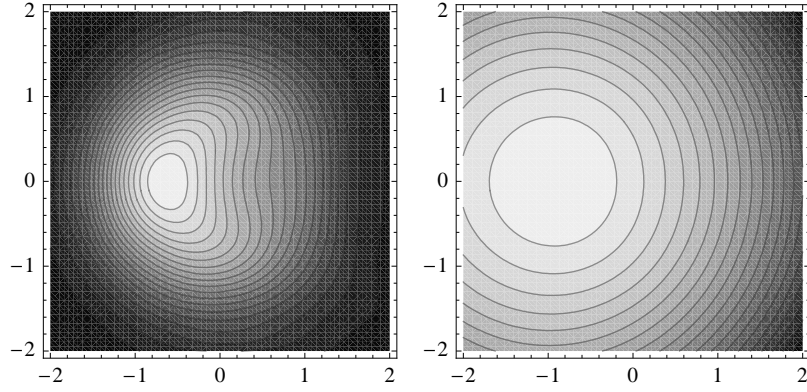
Figure 4: Continuation Value, $v_1(x, 3, R)$, & Expected Utility, $U_1(x, 3, R, \theta_1 = 0)$.
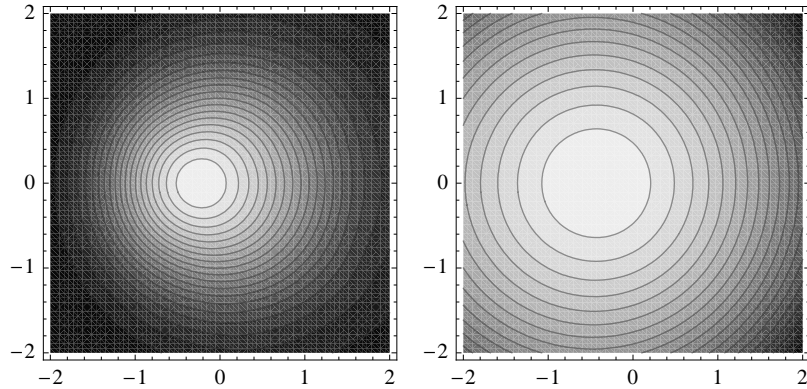

Figure 5: Continuation Value, $v_D(x, 3, R)$, & Expected Utility, $U_D(x, 3, R, \theta_D = 0)$.

eration by legislators and presidential politicians as a function of current period's political state, by reporting the strategic ideal policies of these players on the left-right dimension for each political state, fixing their respective preference shocks at $\theta_i = 0$. The range of variation in these strategic ideal points is significant, with Democratic legislators moving all the way to –0.55 for state (1,D), compared to a stage ideal point of $-x_L = -1$, and with Democratic presidential politicians moving to –0.22 compared to a stage ideal point of $-x_P = -0.5$. Furthermore, we observe identical patterns of compromise for both types of politicians (Democratic legislators and potential presidents) as a function of the political state: both become more moderate as the Republicans' share of seats in the legislature increases, yet both prefer more extreme policies in states in which the Republicans control the presidency.

While these patterns suggest a strong correlation between strategic ideal point locations and the political state in the current period, the underlying cause for this correlation is the

$\hat{s}_t$	Legislator Ideal Point (Left-Right)	President Ideal Point (Left-Right)	$p[d_{t+1} \geq 3|\hat{s}_t]$	$p[\mathcal{P}_{t+1} = D|\hat{s}_t]$
(1,D)	−0.55	−0.22	0.01	0.55
(2,D)	−0.57	−0.25	0.13	0.57
(3,D)	−0.81	−0.36	0.60	0.50
(4,D)	−0.95	−0.46	0.93	0.48
(1,R)	−0.66	−0.29	0.07	0.52
(2,R)	−0.78	−0.35	0.40	0.50
(3,R)	−0.92	−0.43	0.87	0.43
(4,R)	−0.97	−0.49	0.99	0.45

Table 2: Democrats' Equilibrium Ideal Points by Political State ($\theta = (0,0)$).

	Dem. Legislator Ideal Point	Dem. President Ideal Point
$p[d_{t+1} \geq 3|\hat{s}_t]$	−0.31	−0.21
	(0.06)	(0.04)
$p[P_{t+1} = D|\hat{s}_t]$	0.94	0.31
	(0.56)	(0.36)
Intercept	−1.09	−0.41
	(0.31)	(0.20)
R^2	0.97	0.97

Table 3: Strategic Effect of Anticipated Partisan Control on Players' Ideal Policies.

fact that legislators are forward looking, anticipating particular political states occurring with different probabilities in the future, depending on current period's state. To substantiate this claim, in Table 2 we report the probabilities that the legislative majority and the presidency are Democratic in the next period for each political state in the present period. These probabilities appear in the fourth and fifth columns of that table and provide additional evidence in support of the claims we made in discussing Figures 3 to 5, i.e., that future control of the presidency by the Republican party makes Democrats adopt more extreme positions, while an increase in the probability of control of a legislative majority by the Republicans makes them more moderate. Since it may be hard to discern the degree of correlation of these variables with the level of compromise by legislators and presidential politicians, we further summarize these data in Table 3, by providing the results of a regression of Democrats' strategic ideal points as reported in Table 2 on the probabilities that the Democrats control the legislature or the presidency in the ensuing period. According to these regression estimates, the typical Democratic legislator's ideal point moves to the left by 0.31 as the probability of Democratic control of the legislature goes from zero to one. The corresponding effect for the president is somewhat smaller at 0.21, and both effects are

highly statistically significant at any conventional level. On the other hand, if the probability that the president in the next period is a Democrat goes from zero to one, then the typical Democratic legislator becomes more moderate by almost an entire unit (coefficient of 0.94) and the presidential politician by a much smaller magnitude of 0.31. The estimated effect of the probability of control of the presidency, though, is not statistically significant at conventional levels. Looking at the reported R^2s at the bottom row of Table 3, these two variables account for roughly 97% of the variation in equilibrium strategic ideal points, providing convincing support primarily for the hypothesized role of future control of agenda setting and some support for the effect of the presidential veto on the strategic moderation of ideal policies in equilibrium.[9]

Social Welfare Having developed an understanding of the strategic incentives facing politicians in SSPE, we now examine a number of salient aspects of the equilibrium. We first consider long-run welfare from the perspective of a centrist voter with quadratic utility and ideal point at the origin. To calculate the voter's long-run expected payoff, we compute the invariant distribution over policies and perform a mean-variance analysis. The results of this analysis are presented in the first four rows of Table 4. In the computed SSPE, play of the game is symmetric not just across the vertical axis but also across the horizontal, and the unconditional mean of the invariant distribution on the left-right dimension (reported in the first column, second row of Table 4) is at the centrist voter's ideal point, $(0,0)$. Thus, the long-run welfare (aggregated across political states) of the voter is simply negative one times the variance of the invariant distribution, which we depict in Figure 6. Of note, the density of the invariant distribution is bimodal, reflecting the fact that when the Democrats have a majority in the legislature, they generally implement policy to the center/left, while Republicans generally pass policy to the center/right. The modes of the density are closer to the center relative to the underlying ideal points of the parties, reflecting the incentives to compromise discussed above.

Looking at the first row of Table 4, we find that policy variance is higher (and social welfare is lower) when the political state is characterized by unified government than when government is divided, as one would expect: when government is unified, the president's preferences are more likely to align with the majority in the legislature; the veto is generally less restrictive, and the scope for an agenda setter to move policy to his liking is increased. For similar reasons, variance is higher when the party in control of the legislature has a supermajority, as it becomes more likely for the agenda setter to obtain the approval of two (with the president) or three more legislators to push through changes in policy. When we compare the centrist voter's welfare conditional on the political state, we find that the political state (2,D), i.e., divided government without supermajority for the majority party in the legislature, is unambiguously best. At this political state, the voter expects a policy that has the smallest variance, 0.24, and a policy that is on average at $(0.49, 0.01)$, the closest to that voter's ideal policy at $(0,0)$ compared to any other state.

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[9]We also ran regressions adding a third 'interaction' variable that measures the joint probability of unified Democratic government, $p[d_{t+1} \geq 3, P_{t+1} = D|\hat{s}_t]$, but this added variable does no significantly improve over the fit of the simpler linear models we report in Table 3.
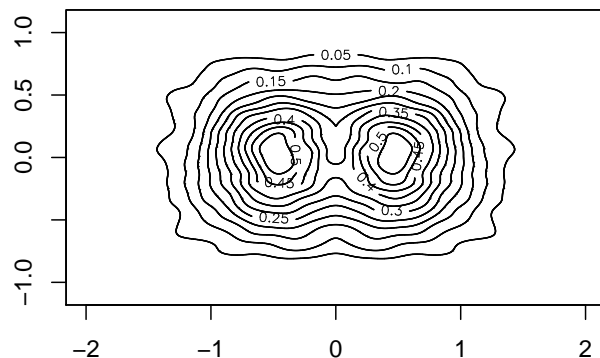
Figure 6: Invariant Distribution of Policy Outcomes

Winning Coalitions and Bipartisanship Next, we consider the nature of winning coalitions formed in the equilibrium invariant distribution. Doing so gives us a first concrete look at the importance of the presidential veto for the long-run policy outcomes of our model. Note that we do not model the explicit exercise of the veto because, as in reality, the veto can affect policy even if it is not exercised. We are ultimately concerned with policy outcomes, and from that perspective it is immaterial whether the agenda setter anticipates the threat of a veto and acts preemptively by compromising his policy proposal, or the president antici-pates a veto override and approves legislation with the support of a legislative supermajority, or the veto is exercised, the bill is sent back to the legislature, with compromise legislation then being produced. The frequency with which different winning coalitions form gives us a preliminary indication of the frequency with which the veto has bite. For instance, when policy passes with the approval of a bare majority in the legislature and the approval of the president, we conclude that the agenda setter was unable (or unwilling) to override the veto and had to satisfy the president.[10] In Table 4, we see that such cases occur with frequency .61. The agenda setter was able to override the veto (or the threat thereof) when a policy passes with the approval of an override majority of four votes in the legislature but without the approval of the president. Such cases occur with frequency .28 (= .35 × .78).

We see that bipartisan winning coalitions form with a frequency of .62 when aggregated across political states. A closer look reveals some patterns of interest. We find that when the agenda setter forms a minimal winning coalition (with the president), the winning coalition is more likely to be bipartisan if the party in control of the legislature has a supermajority. When the agenda setter's party, say the Republicans, has only a bare majority, as in po-

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[10]These cases include situations in which the veto is unrestrictive because the agenda setter was able to pass his dynamic ideal point, receiving approval of exactly two other legislators and the president.

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	All States	Divided Government		Unified Government	
		$(1, D)$	$(2, D)$	$(3, D)$	$(4, D)$
$Var[x]$	0.59	0.30	0.24	0.27	0.30
$E[x_1]$	0.00	0.78	0.49	-0.59	-0.83
s.d.	(0.67)	(0.38)	(0.32)	(0.38)	(0.39)
$E[x_2]$	0.00	0.05	0.01	0.00	0.00
s.d.	(0.37)	(0.40)	(0.37)	(0.36)	(0.39)
welfare	-0.59	-0.91	-0.47	-0.62	-0.99
UNANIMOUS	4%	2%	4%	4%	3%
% President Objects	18%	0%	14%	23%	17%
OVERRIDE	35%	59%	41%	29%	48%
% Bipartisan	92%	20%	100%	100%	28%
% President Objects	78%	92%	82%	72%	76%
MWC	61%	39%	55%	67%	49%
% Bipartisan	42%	65%	60%	31%	37%
$E[||q - x||]$	0.71	0.71	0.61	0.79	0.72
s.d.	(0.53)	(0.46)	(0.47)	(0.57)	(0.49)
$E[|q_1 - x_1|]$	0.49	0.47	0.35	0.60	0.49
s.d.	(0.54)	(0.44)	(0.44)	(0.59)	(0.47)
$E[|q_2 - x_2|]$	0.40	0.43	0.41	0.38	0.41
s.d.	(0.32)	(0.33)	(0.32)	(0.31)	(0.34)

Table 4: Welfare, Winning Coalitions, and Gridlock for the Baseline Institution.

litical state $(2, D)$, he may form a minimal winning coalition either because his preferences are centrist (and therefore close to the president) or simply because there are not enough Republican legislators to override the veto. The first incentive is relatively stronger and the second incentive is weaker when we move to state $(1, D)$, as now there is a supermajority of four Republicans are present in the legislature. Thus, an agenda setter who forms a minimal winning coalition when his party has an override majority is more likely to do so because his preferences are moderate, and he is therefore more likely to form a bipartisan winning coalition. We also find that when the agenda setter forms a minimal winning coalition, it is more likely to be bipartisan if government is divided. In political state $(1, D)$, for example, it is less costly for a Republican agenda setter to form a minimal winning coalition (with the president) if that legislator's preferences are centrist (and therefore close to the president), amplifying the first incentive identified above; in political state $(4, D)$, the president belongs to the same party as a Democratic agenda setter, and so this amplification is absent. Last, we find that when the agenda setter forms an override coalition, it is more likely to be bipartisan if government is unified, reversing the finding when conditioning on formation of a minimal winning coalition. If a legislator proposes a bill that passes despite the objection of a president of the same party, it is more likely that that legislator's preferences are extremist, and less likely that the resultant coalition includes a member of the opposition party.

Gridlock and the Policy Effect of the Veto We now take up the issue of political

gridlock, by which we generally mean the inability of legislative majorities to move policy from the status quo. In Table 4, we measure the extent of political gridlock by (negative one times) the mean distance of equilibrium policy outcomes from the status quo, $E[||q - x||]$, integrating with respect to the equilibrium invariant distribution. This definition of gridlock is similar in spirit to that of Krehbiel (1998), who uses the term in a static model to mean that the status quo remains in place despite the presence of a majority who would overturn it. Our operationalization reflects the fact that the policy outcome will typically be (perhaps only slightly) different from the status quo, as a result our assumption that the policy space is multidimensional and the noise on individual preferences; and when the status quo moves only a small amount, the term "gridlock" should still apply. Thus, it is imperative for us to employ a cardinal measure of gridlock, and we do so simply by focusing on the expected distance by which the status quo is moved.

As can be seen in the last three rows of Table 4, we find as expected that gridlock is higher (i.e., $E[||q - x||]$ is lower) when conditioning on divided government (0.71 and 0.61 under divided government, versus 0.79 and 0.72 under unified government). Perhaps less intuitively, though, under unified government we find that gridlock is lower when the party in control of the legislature has a bare majority than when it has a supermajority. The explanation for the higher gridlock under unified government and supermajority in the legislature has to do with the location of the status quo policy when such political states are reached. In political states with unified government and a bare majority in the legislature it is more likely, relative to a supermajority in the legislature, that the opposition party controlled the legislature in the previous period. Thus, if the current political state is $(3, D)$, for example, then it is more likely, compared to $(4, D)$, that in the previous period the Republicans implemented a policy to the right of the policy space, leading to an undesirable status quo for the Democratic party in the current period and greater scope for moving policy. On the other hand, when Democrats obtain supermajority control of the legislature at state $(4, D)$, it is likely that the status quo policy is already located to the left of the policy spectrum, so that observed policy change is smaller and gridlock higher. This "status quo" mechanism does not operate under divided government, because in those cases the veto power of the president from the opposite party prevents the new majority in the legislature from moving the status quo substantially. In fact, the comparison of the extent of gridlock between supermajority and bare majority control under divided government is reversed: moving from a bare majority to supermajority control of the legislature facilitates the formation of an override majority coalition and significant movement in policy.

An interesting aspect of this apparently counterintuitive comparison of the degree of gridlock under the two types of unified government concerns the policy dimension in which most policy change occurs when the majority party controls a supermajority in the legislature. As seen from the last two rows of Table 4, unified governments without supermajority produce more policy change (less gridlock) on the left-right policy dimension compared to supermajority unified government (0.60 versus 0.49). Yet this comparison is reversed when we look at the second policy dimension (0.38 versus 0.41). Once more, a unified government with supermajority control of the legislature is likely to legislate with a status quo that is already satisfactory on the left-right dimension. Thus, the increased ability of this majority

to produce legislation translates to more policy change in the secondary dimension of political conflict. On the other hand, a unified government at state (3,D) is likely to legislate with priority primarily along with primary left-right policy dimension.

In order to evaluate more concretely the impact of the presidential veto on gridlock and equilibrium policies, we measure the change in policy if we remove the veto in the current period, holding the rules of the game and equilibrium strategies constant in future periods. To formalize this counterfactual comparison, let $\mathcal{W}^{BV}(s)$ denote the winning coalitions in state s under the Baseline Veto institution, and let $\mathcal{W}^{NV}(s)$ denote the winning coalitions in state s under the No Veto institution. Given a stationary political equilibrium σ, let $A^{BV}(s;\sigma)$ be the approval set generated by $\mathcal{W}^{BV}(s)$ as in (1), and let $A^{NV}(s;\sigma)$ be the approval set generated by $\mathcal{W}^{NV}(s)$ as in (1). We use the notation x^{BV} to denote the equilibrium policy outcomes generated by σ, which solve

$$\max_{y} U_i(y, \mathcal{L}, \mathcal{P}, \theta_i; \sigma)$$
$$\text{s.t.}\, y \in A^{BV}(s;\sigma),$$

and we write $\tilde{x}^{NV}$ for the solution to

$$\max_{y} U_i(y, \mathcal{L}, \mathcal{P}, \theta_i; \sigma)$$
$$\text{s.t.}\, y \in A^{NV}(s;\sigma).$$

By Lemma 1 of Duggan and Kalandrakis (2007), the solution of the latter maximization problem is generically unique, so that this random variable is well-defined. We then measure the veto effect by the mean distance, $E[||x^{BV} - \tilde{x}^{NV}||]$, between the equilibrium policy outcomes and the counterfactual policies resulting from removal of the veto, where we integrate with respect to the invariant distribution. We report these calculations in Table 5.

	All States	Divided Government		Unified Government	
		$(1, D)$	$(2, D)$	$(3, D)$	$(4, D)$
$E[||x^{BV} - \tilde{x}^{NV}||]$	0.23	0.21	0.32	0.16	0.16
s.d.	(0.33)	(0.27)	(0.38)	(0.28)	(0.23)
$E[|x_1^{BV} - \tilde{x}_1^{NV}|]$	0.19	0.15	0.28	0.13	0.11
s.d.	(0.31)	(0.22)	(0.36)	(0.25)	(0.18)
$E[|x_2^{BV} - \tilde{x}_2^{NV}|]$	0.09	0.12	0.12	0.07	0.09
s.d.	(0.15)	(0.19)	(0.16)	(0.13)	(0.15)

Table 5: Effect of Veto on Equilibrium Policies

Unsurprisingly, the veto effect is higher under divided government than under unified, as in that case the president is more likely to constrain an agenda setter from the opposing party. Conditional on divided government, the veto effect is notably higher when the party in control of the legislature has a bare majority. In that case, the majority is less likely to

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be able to override the veto, so an agenda setter must propose policies relatively far from his party's ideal point in order to obtain the president's approval. When the veto is removed, this entails a larger move in the policy outcome. On the other hand, the overall impact of removing the veto under bare majority and supermajority is virtually identical conditional on unified government. This apparent tie on the impact of the veto under unified government masks the differential effect of the veto across the two policy dimensions. The president has a bigger effect in constraining proposers from the same party along the left-right dimension when there is no supermajority (0.13 versus 0.11) in the legislature, but has a bigger effect on the secondary policy dimension under supermajority (0.09 versus 0.07). This is despite the fact that we found more gridlock on the left-right dimension under unified government with supermajority control in Table 4, as a result of the typical location of the status quo further to the left (right for state (1,R)) in those political states.

5   Constitutional Experiments

In this section, we conduct a pair of constitutional experiments to examine the consequences of strengthening the veto by removing the veto override provision (the Absolute Veto institution) or weakening the veto by deleting it altogether (the No Veto institution). We compute an equilibrium in each case assuming the corresponding change in the voting rule, and we denote by x^{NV} and x^{AV} the random variables representing SSPE policy outcomes under the No Veto and Absolute Veto institutions, respectively. As above, our interest is in the long run, but in contrast to our earlier measure of the effect of removing the veto reported in Table 5, the comparisons in this section fully capture the impact of these institutional changes in the future expectations of the players and consequently on their equilibrium strategies. We focus on a comparison of social welfare and gridlock in equilibrium, and we find systematic differences across institutions with potentially important consequences for institutional design.

Social Welfare   As in the previous section, we examine long-run social welfare by conducting mean variance analysis for a voter located at the center of the policy space using the invariant distribution over policies induced by each equilibrium. We depict the densities of the invariant distributions corresponding to the No Veto and Absolute Veto institutions in Figure 7. The density under Absolute Veto remains bimodal, while that invariant distribution under No Veto appears to be multi-modal. Importantly, in comparison with the Baseline Veto institution, long-run policies are more concentrated under Absolute Veto and more dispersed under No Veto. This observation is amplified by the evidence in the first row concerning each institution in Table 6, where we see that variance in long-run policy outcomes, in all political states, increases when we delete the veto and decreases when we strengthen the veto by removing the override provision. When we strengthen the veto, it becomes more difficult for an agenda setter to move policy to the extremes of the policy space and further from the ex ante preference of the president. As a result the most preferred institution from the perspective of the voter is the Absolute Veto institution, while removal of the veto leads to a significant drop in welfare. This welfare comparison is unambiguous
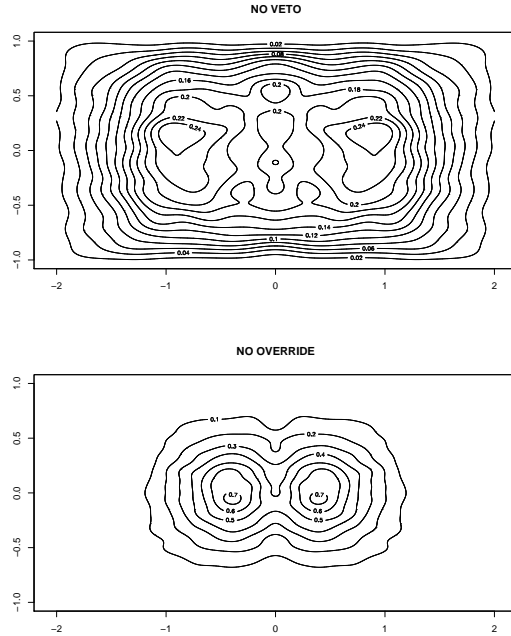
Figure 7: Invariant Distributions under No Veto and Absolute Veto

but not obvious. For example, it would not be unreasonable to expect that more extreme policies would prevail under unified government when the veto is strengthened. A stronger veto could act as an insurance policy for the majority party in the legislature, allowing it to impose extreme policies knowing that a president of the same party would veto moves to the opposite end of the policy space. In order for this incentive to operate in equilibrium, however, it must be that the probability of a president being reelected is high, conditional on unified government, yet for our calibrated transition matrix on political states in Figure 1 that probability is less than one half.

When we compare variation in long term policies by political state, we find that in all three institutional environments, variance of policy outcomes is (weakly) higher under unified government than under divided government, holding supermajority status the same. Furthermore, in the No Veto and Baseline Veto models, variance increases when the party in control of the legislature has a supermajority compared to a bare majority. The Absolute Veto institution is the exception to the latter rule, since variance increases monotonically with an increase of the Democrats' representation in the legislature (with a Democratic president). At work is the effect of the status quo, for when government is divided and the party in control of the legislature has a supermajority, it is more likely, relative to bare majority control, that the party controlled the legislature in the previous period. Thus, if the current state is (1,D), for example, it is more likely, compared to (2,D), that the status quo lies to the right of the policy space. Policy outcomes in this region are likely to be maintained in (1,D) (and indeed, the mean policy outcome on the left-right dimension is

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0.48 in (1,D), compared to 0.41 in (2,D)), but it is less likely that an agenda setter with a realized ideal point to the far right can gain approval for extreme policies, leading to lower variance.

	All States	Divided Government		Unified Government	
		$(1, D)$	$(2, D)$	$(3, D)$	$(4, D)$
$Var[x^{BV}]$	0.59	0.30	0.24	0.27	0.30
$E[x_1^{BV}]$	0.00	0.78	0.49	-0.59	-0.83
Welfare	-0.59	-0.91	-0.47	-0.62	-0.99
$Var[x^{NV}]$	1.09	0.46	0.45	0.47	0.45
$E[x_1^{NV}]$	0.00	0.93	0.80	-0.77	-0.92
Welfare	-1.09	-1.33	-1.09	-1.07	-1.30
$Var[x^{AV}]$	0.53	0.19	0.24	0.26	0.26
$E[x_1^{AV}]$	0.00	0.48	0.41	-0.60	-0.74
Welfare	-0.53	-0.42	-0.40	-0.61	-0.81

Table 6: Welfare under the Three Institutions.

Table 6 reports the welfare of the centrist voter for each political state and institution. As was the case for the Baseline Veto institution, the representative voter's preferred political state remains divided government with a bare majority in control of the legislature in the Absolute Veto institution. A small difference arises in the ranking of political states under the No Veto institution, for which unified government without supermajority control is preferred by the representative voter. Two factors explain this outcome. First, under the Baseline Veto institution, the veto had the greatest bite in political state (2,D), where a bare majority of legislators face a president from the opposing party; in the No Veto institution, the veto constraint on legislative policy proposals no longer binds. Second, as noted in the previous section (and elaborated in Tables 2 and 3), the probability of the opposing party taking control of the legislature in the next period is higher in political state (3,D) than (2,D), creating greater incentives for compromise in the former. Thus, political state (3,D) offers a slight increase in welfare due to more moderate expected policy outcomes under the No Veto institution.

Gridlock We now revisit the issue of political gridlock, and summarize the incidence of the phenomenon under the three institutions in Table 7. As is evident across states in Table 7, when we strengthen the veto it becomes more difficult for an agenda setter to construct a winning coalition to overturn the status quo and gridlock increases (i.e., $E[||q-x||]$ decreases). Furthermore, gridlock is lower under unified government than divided government. We also observe that the expected policy change is higher when the party in control of the legislature has a bare majority than when it has a supermajority. As we discussed in context of the Baseline Veto model, this is the consequence of the fact that on average the status quo is more distant from the preferences of the majority party under unified government in state (3,D) compared to the supermajority state (4,D), since in the latter case it is more likely that the majority party was in power in the previous period so that the status quo was already

positioned on its side of the policy space. In contrast to the Baseline Veto case, however, this effect of the status quo is now also apparent under divided government for both the No Veto and Absolute Veto institutions. For the No Veto case, a move to a political state with supermajority control of the legislature and divided government accrues no gain to the majority party in being able to override the veto of the president (because there is no veto). Thus, the negative effect on gridlock from the more comfortable majority is outweighed by the status quo effect under the No Veto institution: the net effect is that gridlock increases when we move to supermajority and divided government. The same observations hold for the Absolute Veto institution, since a move to a political state with supermajority and divided government no longer has a negative impact on gridlock under this institution, exactly because there is no veto override.

	All States	Divided Government		Unified Government	
		$(1, D)$	$(2, D)$	$(3, D)$	$(4, D)$
$E[\|\|q - x^V\|\|]$	0.71	0.71	0.61	0.79	0.72
s.d.	(0.53)	(0.46)	(0.47)	(0.57)	(0.49)
$E[\|\|q - x^{NV}\|\|]$	1.03	0.94	0.96	1.11	0.90
s.d.	(0.67)	(0.50)	(0.63)	(0.71)	(0.54)
$E[\|\|q - x^{AV}\|\|]$	0.63	0.43	0.50	0.74	0.60
s.d.	(0.54)	(0.36)	(0.45)	(0.60)	(0.50)

Table 7: Gridlock under the Three Institutions.

6   Conclusion

We have computed the equilibrium of a dynamic, multidimensional model of a unicameral legislature and an executive with veto power. We explored the strategic incentives of politicians in light of expectations of future policies, and we examined the implications of different institutional settings. We found that when a veto override provision is in place, dynamic considerations create a moderating effect on preferences: to insure against extreme policy outcomes in the event that the majority party loses control of the legislature, legislators prefer policies toward the center of the policy space. This moderating effect is strengthened when the probability of control of the legislature by the opposition increases, and it is lessened when the opposition is more likely to control the presidency. We compared the welfare of a representative voter across political states and we found that welfare is maximized at the state where policy gridlock is also maximized, i.e., under divided government without supermajority control of the legislature. Gridlock increases when we move from divided government to unified government without supermajority control but then decreases again under unified government with supermajority control of the legislature due to the favorable (endogenous) location of the status quo in the latter political states. We also identified qualitatively different types of policy change from the status quo policy in such political states,

with typical legislation reflecting more change on the secondary policy dimension compared to the main ideological dimension along which parties compete.

We performed counterfactual experiments in order to evaluate the welfare and policy consequences of removing or strengthening the veto. Overall, the veto lowers the variance of the long-run distribution of policies and increases social welfare, and this effect is stronger when the override provision is removed, giving the president absolute veto power. In the pure majority-rule institutional setting, where we remove the veto, these comparisons are reversed. These constitutional experiments are based on an electoral model calibrated from US data, and therefore our analysis holds voting behavior constant and abstracts the effects of institutional variation from electoral effects. With this caveat, our analysis reveals a clear welfare ranking: strengthening the veto decreases variance and increases welfare. Not surprisingly, "gridlock," measured as the expected movement of policy from the status quo, increases with the strength of the veto.

Methodologically, our approach is generalizable from the specific topic of the veto and the specific institutional setting we consider. We build on a general model with the theoretical qualifications (in particular, existence of pure strategy equilibria) needed to extend the analysis to more realistic, and more complex, institutional environments and myriad political phenomena. By incorporating rich institutional detail, we expand the set of political regularities we can explore, and we take a step in ensuring robustness of our findings. Of course, our computations rely on other parametric assumptions (on preferences, discount factors, etc.); this is inherent in the approach, but it can in theory be addressed by estimating a subset of these parameters from data. We in fact employ a simple calibration method in order to specify the transition probability on political states, but we leave the challenges of endogenizing voting behavior and estimating the model to future work.

A   Proofs of Theorems

Theorem 1 *There exists a stationary political equilibrium, and every stationary political equilibrium σ is such that:*

1. *Continuation values are smooth: for every legislator i and every political state $(\mathcal{L}, \mathcal{P})$, $v_i(x, \mathcal{L}, \mathcal{P}; \sigma)$ is smooth as a function of x.*

2. *Proposals are almost always strictly best: for almost all states s, every legislator i, and every $y \in A(s; \sigma)$ distinct from the proposal $\pi_i(s)$, we have $U_i(\pi_i(s), \mathcal{L}, \mathcal{P}, \theta_i; \sigma) > U_i(y, \mathcal{L}, \mathcal{P}, \theta_i; \sigma)$.*

3. *Proposal strategies are almost always continuously differentiable: for almost all states s and every legislator i such that $\pi_i(s) \neq q$, $\pi_i(\mathcal{L}, \mathcal{P}, q, \theta, j)$ is continuously differentiable in an open set around (q, θ).*

Proof Smoothness of continuation values follows from inspection of (3). In Duggan and Kalandrakis (2007), this property follows immediately from the fact that x enters the

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righthand side only through $g(q|x)$, which is smooth in x in that paper. (In fact, this is a special case of the assumptions in that paper.) Here, we assume that $g(\cdot|x)$ is uniform and therefore not differentiable (or even continuous). But we can obtain smoothness by exploiting the special structure of the uniform distribution. Simplify the expression in (3) by writing $v_i(x;\sigma)$ as

$$\int_q h(q)g(q|x)dq \;\; = \;\; \frac{1}{c(x)} \int_{a_2(x)}^{b_2(x)} \left(\int_{a_1(x)}^{b_1(x)} h(q_1,q_2)dq_1 \right) dq_2, \tag{7}$$

where $c(x)$ is equal to $(b_1(x) - a_1(x))(b_2(x) - a_2(x))$, and

$$h(q) \;\; = \;\; \sum_{(\mathcal{L}',\mathcal{P}')} p[\mathcal{L}',\mathcal{P}'|\mathcal{L},\mathcal{P}] \int_\theta \sum_j r_j[\mathcal{L}']U_i(\pi_j(\mathcal{L},\mathcal{P},q,\theta,j),\mathcal{L},\mathcal{P},\theta_i;\sigma)f(\theta)d\theta$$

is a bounded, measurable function of q. Clearly, c is smooth. That $\int_{a_1(x)}^{b_1(x)} h(q_1,q_2)dq_1$ is differentiable in x follows from Leibnitz's rule. In fact, the form of the derivative given by Leibnitz's rule shows that this function is smooth in x. Thus, the righthand side of (7) can be written $\int_{a_2(x)}^{b_2(x)} \hat{h}(q_2,x)dq_2$ divided by $c(x)$, where $\hat{h}$ is smooth in x. This is again differentiable, in fact smooth, by Leibnitz's rule. This establishes smoothness of $v_i(x;\sigma)$ in x for an arbitrary profile σ of stationary Markov perfect strategies.

In addition to smoothness of $v_i(x;\sigma)$, Duggan and Kalandrakis also assume that the derivatives of $g(q|x)$ with respect to x of all orders are uniformly bounded, and they use this to restrict continuation values, a priori, to a space $\mathcal{V}$ that is convex and compact in the C^∞-topology of uniform convergence on compacta. Uniform boundedness of derivatives follows in the present context by the form of the derivative provided by Leibnitz's rule and the boundedness of $\hat{h}$. Thus, we can also place continuation values in a convex, compact space $\mathcal{V}$.

With this result in hand, the existence argument in Duggan and Kalandrakis (2007) goes through unchanged, except for the fact that in the current paper we let the distribution of the proposer and the collection of winning coalitions vary with the state through the political state $(\mathcal{L},\mathcal{P})$. This extends the framework of Duggan and Kalandrakis by adding a finite set of states that evolve according to an exogenous Markov process, and it is easily covered by the existence arguments in that paper. Given a vector of continuation value functions, $v \in \mathcal{V}$, those authors analyze the properties of best response strategies in their Lemmas 1–4. They then define a mapping $\hat{v} = \psi(v) \in \mathcal{V}$ that takes continuation value functions, computes best response strategies, and returns the continuation values generated by them, and their Lemma 5 establishes continuity of ψ. Those lemmas fix continuation values and a status quo q and prove that best responses possess certain useful properties outside measure zero sets $\Theta_k(q;v)$, $k = 1,2,3,4$, and therefore outside the measure zero set $\bigcup_k \Theta_k(q;v)$. Those arguments go through without change if we also fix a political state $(\mathcal{L},\mathcal{P})$, giving us measure zero sets $\Theta_k(\mathcal{L},\mathcal{P},q;v)$. Since the number of political states is finite, $\bigcup_{(\mathcal{L},\mathcal{P})} \bigcup_k \Theta_k(\mathcal{L},\mathcal{P},q;v)$ is still measure zero. The continuity arguments of Duggan and Kalandrakis then apply,

and existence of a fixed point of ψ, which corresponds to a stationary political equilibrium, follows.

Part 1 of the theorem holds for an arbitrary stationary strategy profile and therefore for any stationary political equilibrium. Generic uniqueness of best responses and differentiability of proposal strategies comes out of Lemmas 1 and 3 of Duggan and Kalandrakis (2007), and since those lemmas apply to any best responses, it follows that those properties are exhibited by all stationary political equilibria. QED

Theorem 2 *There exists a SSPE. Every SSPE σ satisfies properties 1–3 of Theorem 1, and in addition, equilibrium continuation values satisfy*

4. for all $i, h \in \{1, \ldots, n\}$, $v_i(x, d, \mathcal{P}; \sigma) = v_h(x, d, \mathcal{P}; \sigma)$,

5. for all $i \in N$, $v_i(x, d, \mathcal{P}; \sigma) = v_{-i}(\varphi(x), n - d, -\mathcal{P}; \sigma)$.

Proof We prove only existence of an SSPE here. This can be obtained by modifying the arguments of Duggan and Kalandrakis (2007) to exploit the symmetry of our model. Once again, we can, a priori, restrict continuation values to a convex, compact space, but we now need to keep track of only two continuation values, v_1 and v_{n+1}, and we construct a symmetrized best response mapping, $(\hat{v}_1, \hat{v}_{n+1}) = \hat{\psi}(v_1, v_{n+1})$. To do so, given (v_1, v_{n+1}), we compute the corresponding dynamic payoffs to each legislator $i = 1, \ldots, n$ as $U_i(x, d, \mathcal{P}, \theta_i) = U_1(x, d, \mathcal{P}, \theta_i)$, where we substitute i's preference shock into legislator 1's dynamic utility, evaluated as in (2). The dynamic utility of presidential politician $i = n + 1$ is evaluated as in (2). For each $i = -1, \ldots, -n$, we define $v_i(x, d, \mathcal{P}) = v_1(\varphi(x), n - d, -\mathcal{P})$, we define $v_{-(n+1)}(x, d, \mathcal{P}) = v_{n+1}(\varphi(x), n - d, -\mathcal{P})$, and we define dynamic utilities analogously. We compute best response approval strategies for each player $i \in N$ as $\hat{\alpha}_i(d, \mathcal{P}, q, \theta) = 1$ if $U_i(y, d, \mathcal{P}, \theta_i) \geq U_i(q, d, \mathcal{P}, \theta_i)$, and $\hat{\alpha}_i(d, \mathcal{P}, q, \theta) = 0$ otherwise. This yields a best response political approval set, $\hat{A}(d, \mathcal{P}, q, \theta, y)$, and we can compute best response proposals for each potential legislator i, $\hat{\pi}_i(d, \mathcal{P}, q, \theta)$, as the solution to

$$\max_y U_i(y, d, \mathcal{P}, \theta_i)$$
$$\text{s.t.} \, y \in \hat{A}(d, \mathcal{P}, q, \theta).$$

Finally, we substitute best response proposals into (3) to obtain best response continuation values, $(\hat{v}_1, \hat{v}_{n+1})$. The argument in Lemma 1 of Duggan and Kalandrakis (2007) directly implies that the best response proposal strategies, $\hat{\pi}_1(d, \mathcal{P}, q, \theta)$, are uniquely defined at almost all states, so $(\hat{v}_1, \hat{v}_{n+1})$ is well-defined. Furthermore, as argued in Section 2, v_1 is in fact the best response continuation value of every Democratic potential legislator, and the best response continuation values of Republican players are obtained by $\hat{v}_i(x, d, \mathcal{P}) = \hat{v}_{-i}(\varphi(x), n - d, -\mathcal{P})$. By the arguments in their Lemmas 2, 3, and 5, the mapping $\hat{\psi}$ is continuous and, by Glicksberg's theorem, possesses a fixed point v^*, and the best response approval and proposal strategies generated by v^* form a SSPE. QED

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