A Theory of Minority and Majority Governments

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Abstract

We develop a theory of the emergence of minority and majority governments in multiparty parliamentary systems using a canonical non-cooperative bargaining model and assuming a policy space of arbitrary finite dimension, any number of political parties, and a general class of preferences over the government agreement space. Only majority governments form in the absence of significant political disagreement. Generically, minority governments form with positive probability when parties represented in parliament are ideologically polarized (or when utility from holding cabinet office is small *relative* to partisan political disagreement). Rather than being paradoxical, minority governments are a regular equilibrium phenomenon.

1 Introduction

The formation of minority governments in parliamentary systems constitutes one of the most intriguing paradoxes in the study of coalition building. Parliamentary systems operate on the principle that the executive's survival in office hinges on the (tacit) support of a majority in parliament. Yet, by definition, minority governments obtain majority support by allocating cabinet positions to a set of parties with only a minority of seats in parliament. Furthermore, a large fraction of coalition governments during the post WWII era (over one third in Western Europe) were minority governments, while in certain countries such as Denmark or Norway, minority governments have been the default cabinet type. Why do political parties support (or tolerate) minority

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governments without receiving cabinet portfolios? Our goal in this study is to develop a parsimonious, yet general theory that accounts for variation in the incidence of minority governments across parliamentary systems. This theory is distinguished from much of the existing literature due to the simultaneous incorporation of three requirements we deem essential for a theory of minority governments. In the remainder of this first section, we first motivate these requirements, then describe the main findings and relate them to the literature on government formation.

Any attempt to understand minority governments brings us squarely in the realm of coalition theories and the literature spawned by William Riker, 1962, and his 'size principle,' the proposition that observed coalitions should be the smallest necessary to win and no larger. An important consideration in applying this dictum is that care should be taken when it comes to the criterion used for measuring the size of winning coalitions. While in abstract theories of coalition formation we may define the winning coalition as the set of voters or supporters that back a particular agreement, that criterion has no bite in the case of government formation: as long as we maintain the assumption that cabinets must enjoy the (tacit) support of a majority in parliament, then all parliamentary governments are majority governments according to that measure of coalition size. Instead, for the purposes of government formation, coalition size has to be measured on the basis of the observed agreement, in particular by summing the parliamentary representation of parties that receive cabinet portfolios. Thus, an essential aspect of any model of minority governments is that (a) the portfolio allocation can be inferred from the government formation agreement.

Government formation agreements determine both the allocation of cabinet posts among parties as well as the policy to be pursued by the new cabinet. While the electorate may not have preferences over the portfolio allocation per se, parties and individuals within parties certainly compete with each other over the allocation of cabinet posts. In particular, it is natural to assume that political parties desire larger fractions of cabinet portfolios, all else equal. This is an essential assumption in a theory of minority governments as such governments are paradoxical only if cabinet positions are desirable *per se*. We thus impose a second requirement that (b) a political party's utility increases with larger share of cabinet portfolios for any given public policy pursued by the cabinet. We emphasize that this assumption does not preclude the possibility that parties' utilities from cabinet office may vary across countries, or over time, nor does this assumption speak in any way to the relative significance of parties' office and policy aspirations.

The third requirement we impose is to (c) avoid dimensionality or other a priori restrictions on the agreement space over which political parties bargain. First, this requirement ensures that our conclusions do not rely on the common but special assumption of a one dimensional policy space, which typically entails equilibrium properties that do not obtain in higher dimensions. We also avoid ad hoc a priori restrictions on the types of agreements that can be attained by particular coalitions, modeling the set of feasible proposals as a continuum. In the present analysis, a formateur that would barely lose an investiture vote due to the objection of one of the intended coalition partners, can achieve the formation of this cabinet by granting an extra concession (in the form of cabinet portfolios or policies) to the objecting party.

Can we obtain equilibrium minority governments if we impose requirements (a), (b), and (c) above? We show that the answer to this question is *almost always* in the affirmative: minority governments emerge with positive probability when political disagreement or policy polarization among bargaining parties is marked relative to the importance of utility obtained by holding cabinet office. On the other hand, when policy disagreement is limited, only majority governments form. Note that we qualify the statement of the result by 'almost always' because it is possible to construct otherwise unspectacular examples in which the stated comparative static does not hold. Yet, in Proposition 2 we show that these examples constitute singularities that obtain only for knife-edge configurations of parameters. Otherwise, the conclusion holds independent of the number of political parties represented in parliament or the number of dimensions of the underlying policy space. In the discussion following Proposition 2 we review systematic empirical evidence and stylized facts about the incidence of minority governments that corroborate these findings.

The mechanism that links policy disagreement with minority governments can be best understood by considering the trade-off faced by the intended coalition partners of a formateur party. When political parties are ideologically polarized, any party rejecting a proposed government agreement faces the risk that a coalition excluding this party will form instead, implementing a distant policy at the wrong side of the ideological spectrum. Thus, holding the value of cabinet posts fixed, policy disagreement reduces parties' bargaining power and makes it more likely for a formateur to extract the consent of other parties without offering them any cabinet posts. More generally, minority governments can be thought to emerge when a formateur's party is strong vis a vis its coalition partners, in equilibrium. In that spirit and under special assumptions, in Result 1 we show that such governments emerge for a larger range of the model's parameters when parties are more impatient. Thus, although these governments are often associated with impaired support for the government's legislative program in parliament, in the present analysis minority governments indicate a relatively strong formateur party capable of mustering support from parties outside the cabinet in order to implement the associated policy agreements. In that regard, the present study adds to the arguments of a number of scholars, e.g., Strom, 1990, Sened, 1995, Tsebelis, 1995, etc., who similarly conclude that minority governments can be stable and viable governing solutions.

Before we proceed with the analysis, we review related theoretical contributions with an emphasis on aspects in which they differ from the present study. Minority governments have been associated with policy polarization in one of the earliest accounts of the phenomenon to appear in the comparative politics literature by Dodd, 1976. In his account, though, the connection between policy polarization and minority governments is almost assumed. It amounts to an inability of polarized parties to participate in the same cabinet. Furthermore, minority governments of that flavor are expected to be of short duration (*e.g.*, Powell, 1982, page 142). In a seminal contribution for the study of minority governments Kaare Strom, 1984, 1990, provided an explanation for these cabinets based on the inter-temporal trade-offs parties face when considering their options for government participation. A key assumption in Strom's argument is that gaining office immediately may not be optimal for parties which, by "deferring gratification" of their office aspirations, may avoid costly electoral consequences associated with holding cabinet portfolios. Thus, it may be rational to allow a minority government to form – particularly if parties are patient, have opportunities to influence policy in the legislature even if not present in the cabinet, and face competitive elections.

Perhaps the earliest formal theory result associating policy polarization with minority governments is provided by Itai Sened, 1995, 1996. He considers a model with both policies and cabinet portfolio allocations and shows that equilibrium minority governments emerge when led by a large, centrally located party and when other parties are significantly ideologically polarized (Proposition 2, page 292 in Sened, 1995, and Proposition 3, page 361, in Sened, 1996). The main differences between Sened's theory and the present study are, first, that he uses a cooperative solution concept while the present analysis is non-cooperative; second, that he assumes that parties incur a policy related payoff (a cost in Sened's terminology) only when participating in government while policy payoffs from the implemented government agreement obtain in the present analysis whether the party receives cabinet portfolios or not; and, finally, that in the present study minority governments may emerge even if there exists no party in a central dominant location – as is the case in Example 1.

The bargaining process of government formation we assume was introduced in political science by Baron and Ferejohn, 1989. They study a divide-the-dollar game, which we may interpret as a game for the division of cabinet portfolios, and obtain only minimum winning coalitions in equilibrium. Thus their model produces no minority governments, a finding that is consistent with the present analysis since there is no policy disagreement in their model. In a similar bargaining space, Baron, 1998, considers a dynamic model with an exogenous random status quo in which minority governments are preferred by the formateur but do not form in equilibrium. In contrast, Kalandrakis, 2004, 2007, obtains minority allocations in a divide-the-dollar game when the status quo is endogenous. Baron, 1991, considers bargaining over a two-dimensional ideological space, but his analysis is silent on the emergence of equilibrium minority/majority cabinets, as he does not make portfolio allocations an explicit choice among bargaining parties.

The assumption that parties only care about policies and cabinet portfolios accrue no office-holding payoff is made by Laver and Schofield, 1990, Laver and Shepsle, 1990, and Austen-Smith and Banks, 1990, who propose theories of government formation premised on cooperative solution concepts. All three contributions reach the conclusion that minority governments may emerge under conditions that ensure the policies pursued by these cabinets are invulnerable or *core* policies.[1] While Laver and Schofield assume that bargaining parties may consider the entire range of possible policy agreements, Laver and Shepsle and Austen-Smith and Banks restrict possible policy and portfolio allocations to a finite number of what they deem credible policy alternatives for each coalition. Thus existence of a core or stable government is more likely under their assumptions, but such core points, with or without minority cabinets, are not guaranteed to exist in the absence of exogenous restrictions on feasible policies.[2]

A number of authors consider noncooperative government formation models with assumptions related to those in the present study. Austen-Smith and Banks, 1988, consider a model of both elections and post-election bargaining that takes place among three parties that must both split cabinet portfolios and determine a policy drawn from a one dimensional space. In their model, minority governments are not obtained in equilibrium because the authors assume at the outset that policy disagreement is small relative to the spoils of office, although minority governments emerge in bargaining subgames that are not reached in equilibrium. Related three party, one dimensional models are analyzed by Crombez, 1996, who associates minority governments with the size of the median party, by Kalandrakis, 2000, and by Cho, 2005, the latter model being dynamic with an endogenous status quo and elections. Also, part of the results in Morelli, 1999, concern a similar model without equilibrium minority governments (Proposition 4, page 816).

[1] Schofield, 1993, 1995, considers a generalized cooperative solution concept, the *heart*.

[2] Conditions for their existence are discussed by, e.g., McKelvey and Schofield, 1986, Banks, 1995, and Austen-Smith and Banks, 1990.

Badyopadhyay and Oak, 2004, consider a single period coalition formation model, assuming that formateurs propose one among a finite number of coalitions, with the agreement to be implemented by any coalition restricted to an exogenously fixed compromise. They derive conditions that induce or preclude minority governments, as do Diermeier and Merlo, 2000, in a model that also addresses the stability of these governments. They work with three parties and a two-dimensional policy space, identical to that of Example 1 in the present study, but assume that utility is transferable. These utility transfers (negative or positive) are not construed as the division of cabinet positions among parties under their assumptions. Diermeier and Merlo define the coalition of parties receiving cabinet portfolios as the 'proto-coalition' that eventually offers the final government proposal. In order for minority governments to form, extra-cabinet parties (those excluded from the 'proto-coalition') must receive compensation in the form of non-policy transfers[3] in order to support the minority cabinet. Baron and Diermeier, 2001, use a similar formal definition of a government in a related model but do not obtain equilibrium minority governments, as they restrict transfers only among parties in the proto-coalition.

We shall now proceed to the main part of the analysis. We start in the following section by presenting the model. Next, we show that this model produces majority governments with probability one in the absence of significant policy disagreement. In the penultimate section we establish the advertised result concerning minority governments, and further discuss its interpretation and robustness questions. We conclude in the last section. All proofs have been relegated to an Appendix.

2 Government Formation Bargaining

We assume a parliament consisting of $n \geq 3$ parties and denote the set of these parties by $N = \{1, ..., n\}$. Each party i has a positive share of seats in parliament equal to $s_i > 0$ and no single party controls a parliamentary majority, i.e., we have $\sum_{i=1}^{n} s_i = 1$ and $s_i \leq \frac{1}{2}$ for every party i. A government must receive the support of some coalition, $C \subseteq N$, that controls a majority of seats ($\sum_{i \in C} s_i > \frac{1}{2}$) in order to be invested. Parties bargaining over the formation of a government must agree on a policy $\mathbf{x} \in X$. The policy space X is a subset of the d-dimensional Eucledian space $\mathbb{R}^d$, $d \geq 1$, that encompasses all the public policies that can be pursued by any cabinet. In addition to the policy to be pursued by the government, bargaining parties must also agree on the allocation

[3]These transfers to parties outside the government have to take the form of private goods other than cabinet office: if these 'bribes' were cabinet office, then these parties should be considered part of the cabinet.

of portfolios, which we represent as a vector $\mathbf{g} = (g_1, g_2, ..., g_n) \in \mathbb{R}^n$ that satisfies $g_i \geq 0$ for each party i and $\sum_{i=1}^n g_i = G > 0$. Hence, we define a *government* as follows:

Definition 1 *A* government *is a pair* $(\mathbf{x}, \mathbf{g})$ *consisting of a policy* $\mathbf{x}$*, and an allocation of cabinet portfolios* $\mathbf{g}$*.*

We distinguish minority governments from majority governments using the default empirical criterion, i.e., whether cabinet portfolios are allocated only among parties that control a minority of seats in parliament:

Definition 2 *A government* $(\mathbf{x}, \mathbf{g})$ *is a* minority government *if the set of parties that receive positive share of cabinets is not a winning coalition, i.e., if* $\sum_{i \in C} s_i \leq \frac{1}{2}$ *where* C *is the coalition of all the parties with positive portfolio allocation,* $g_i > 0$*.*

Of course, if a government $(\mathbf{x}, \mathbf{g})$ is not a minority government, then it is a majority government. We emphasize that a minority government must still be approved (or tolerated) by a winning coalition.

In each period $t = 1, 2, ...$ before the attainment of an agreement party i becomes the *formateur* with probability π_i, where $\sum_{i=1}^n \pi_i = 1$. When party i is the formateur in period t, it proposes a government. If this proposal is accepted by a winning coalition, the game ends with the formation of that government. Otherwise the game moves to the next period and continues as above until an agreement is reached. We shall assume that parties have preferences over governments given by a utility function U_i that takes the form

$$U_i(\mathbf{x}, \mathbf{g}; c_i) = u_i(\mathbf{x}, g_i) + c_i g_i. \tag{1}$$

Party i discounts the future with a discount factor $\delta_i \in (0, 1]$. Thus, if a government $(\mathbf{x}, \mathbf{g})$ is invested in period t, the payoff of party i is given by $\delta_i^{t-1} U_i(\mathbf{x}, \mathbf{g}; c_i)$.

We now impose a number of assumptions on the policy space, partisan preferences, and the bargaining protocol. First, we assume that the policy space X is convex and compact and can be cut out by a finite number of concave functions. Also, the function $u_i : X \times \mathbb{R}_+ \to \mathbb{R}$ is smooth and concave and satisfies $u_i(\mathbf{x}, g_i) > 0$, for all parties i and all $\mathbf{x}, g_i$. We strengthen concavity over the policy component of u_i's arguments by requiring that for any porfolio allocation g_i, the function u_i has negative definite second derivative, $D^2_{\mathbf{x}} u_i(\mathbf{x}, g_i)$. This assumption implies that, for each g_i, party i has a unique ideal policy that maximizes u_i over policies in X. We denote this ideal policy by $\hat{\mathbf{x}}^i(g_i)$ and we require some political disagreement when parties' cabinet portfolio allocations are zero, so that $\hat{\mathbf{x}}^i(0) \neq \hat{\mathbf{x}}^j(0)$ for all distinct parties j and i. When it comes to preferences over cabinets we assume that party i's utility is strictly increasing with its portfolio allocation, so that u_i satisfies $\frac{\partial u_i(\mathbf{x},g_i)}{\partial g_i} > 0$ for all $\mathbf{x} \in X$, and that $c_i \in (0, \overline{c})$ for some $\overline{c} > 0$. We mildly restrict the form of interaction between policies and portfolio allocations implied by u_i by requiring that the marginal utility from cabinet office is bounded and independent of policies at zero portfolio allocation, i.e., that $\frac{\partial u_i(\mathbf{x},0)}{\partial g_i} = m_i < +\infty$ for all $\mathbf{x} \in X$. We require that for every winning coalition C and all parameters $(c_i)_{i \in N} \in (0, \overline{c})^n$ there exists a policy $\mathbf{x}$ that solves

$$\sum_{j \in C} (m_j + c_j)^{-1} D_{\mathbf{x}} u_j(\mathbf{x}, 0) = \mathbf{0}, \tag{2}$$

and $\mathbf{x}$ is in the interior of X. We assume that $\pi_i = 0$ if party i is a dummy party[4] and that $\pi_i > 0$ for every party i that is not a dummy party. Lastly, we require that if party i is not a dummy party, then for every winning coalition C such that coalition $C \setminus \{i\}$ is also a winning coalition, there exists $j \in C$, $j \neq i$, such that $C \setminus \{j\}$ is also a winning coalition. The last two assumptions ensure that a formateur is not redundant in any winning coalition in which no other coalition partner is redundant.

We have now specified the model and this is a good point to pause in order to comment on some of the assumptions we have imposed and their interpretation. A central focus of the analysis to follow will be on the effect of changes in cabinet parameter G on the types of governments that form in equilibrium. As we vary this parameter, holding everything else constant, we vary the significance of the office component in parties' utility. This interpretation is most obvious if we recast the model in the following equivalent way: we may represent the division of cabinet portfolios by a vector $\sigma \in [0,1]^n$ with $\sum_{i=1}^n \sigma_i = 1$, so that σ_i represents party i's share of cabinet posts, and reexpress party i's utility as $U_i(\mathbf{x}, G\sigma; c_i)$. An application of the chain rule of differentiation then reveals that increases in G simultaneously increase the positive utility effect of a given portfolio allocation for all recipient parties. Of course, cabinet office can be significant only *relative* to other sources of utility so that, in the model, lower G can be interpreted either as an aggregate decrease of the significance of cabinet office as a source of utility compared to policy, or as an increase in policy disagreement or polarization. We have also introduced parameters c_i, that regulate *individual* parties' marginal preference for cabinets. These parameters will serve to make precise the notion that minority governments occur generically, by allowing us to perturb parties' preferences away

[4] A party i is a dummy party if coalition $C \setminus \{i\}$ is a winning coalition whenever coalition C is winning.

from singular specifications of the model. The reader may wonder whether there is any redundancy by the inclusion of both G and c_i as parameters of the model, which is not generally the case since u_i need not be linear in g_i.[5]

We shall restrict the analysis to the study of no delay stationary subgame perfect (SSP) Nash equilibria, in pure or mixed strategies. By no delay we mean that parties becoming formateurs propose a government that is invested with probability one. We also impose a standard restriction on voting strategies that ensures that parties vote on proposed agreements as if they are pivotal, i.e., they approve governments that they strictly prefer over their expected utility if the game continues in the next round and reject governments when they have the reverse strict preference. Since the class of SSP equilibria we study forms a subset of the set of subgame perfect equilibria, minority governments can certainly emerge in a subgame perfect equilibrium if they can emerge in a stationary equilibrium. Thus, the restriction to stationary equilibria makes our task harder in what follows. We can readily check that this government formation model satisfies the assumptions of Banks and Duggan, 2000. As a consequence, there exists at least one equilibrium. We emphasize that the game may (and in general does) admit multiple equilibria.

3 Majority Governments

The main goal of this section is to show that only majority governments can form in equilibrium if the cabinet parameter G is too high, i.e., if the impact of an increase in parties' cabinet shares is high relative to the effect of a change in policies. In the second part of this section, we show that the policy agreements reached by majority governments satisfy certain necessary conditions. We will use these necessary conditions in order to show that minority governments occur generically for low G in the next section. Accordingly, the first result is:

Proposition 1 *Fix utility functions u_i, preference parameters c_i, seat shares s_i, recognition probabilities π_i, and discount factors δ_i. There exists $\overline{G}$ such that for every $G > \overline{G}$, all equilibrium governments are majority governments in all equilibria of the corresponding government formation game.*

Proposition 1 ensures that in parliamentary systems with small policy polarization or parties for which cabinet portfolios are a (relatively) significant source of utility, majority governments are

[5] In any case, both Propositions 1 and 2 in the sequel state properties of the equilibrium correspondence as we vary G, for fixed (generic) values of c_i.

guaranteed to occur with probability one. Loosely speaking, the argument relies on the fact that if, by increasing G, we increase the size of the pie to be distributed among parties in the form of spoils from the control of cabinet office, parties' expected utility if they prolong the negotiations by one more period increases. This is because proposing parties are guaranteed to be able to get a share of that augmented pie in the event they become the formateur party in the next period. Since parties expect more in the next period if negotiations are prolonged, they must receive higher compensation in the present period in order to approve a government and terminate the negotiations. But there is an upper bound on the utility that parties can extract from public policies, without receiving any cabinets. As a result, there exists some level of G above which parties must receive cabinets in order to approve any government, even if this government implements their ideal policy. For such high G, the only possible governments are majority governments.

Continuing the study of majority governments, we now characterize the types of policy compromises that prevail whenever majority governments form in equilibrium.

Lemma 1 *If a majority government* $(\mathbf{x}, \mathbf{g})$ *proposed by party* i *forms in equilibrium and the set of parties receiving cabinet portfolios is given by* C*, then* $i \in C$*, all parties* $j \in C, j \neq i$*, are indifferent between accepting or rejecting* $(\mathbf{x}, \mathbf{g})$*, and the policy* $\mathbf{x}$ *maximizes*

$$\sum_{j \in C} w_j u_j(\mathbf{y}, g_j), \tag{3}$$

over $\mathbf{y} \in X$*, holding the portfolio allocation* $\mathbf{g}$ *and weights* $w_j = \left(\frac{\partial u_j(\mathbf{x}, g_j)}{\partial g_j} + c_j\right)^{-1}$ *fixed.*

Thus, the policy compromises pursued by majority governments maximize a weighted average of political parties' utilities. Note that these policies are not independent of the attained portfolio allocation, $\mathbf{g}$, since the maximizer of (3) varies with that portfolio allocation compromise. As is evident from the proof of Lemma 1, the result follows simply from the optimization considerations of formateurs. In the next section, we use this necessary condition in order to show that minority governments must occur when G is low, for almost all parameters of the model.

4 Minority Governments

In the previous section we showed that there exists some level of cabinet parameter $\overline{G} > 0$ such that only majority governments form in all equilibria of the associated game for all $G > \overline{G}$. In this section we wish to show a partial converse, i.e., that there exists some $\underline{G} > 0$ with $\underline{G} \leq \overline{G}$ such that for all $G < \underline{G}$ minority governments form with positive probability in all equilibria of the game. Figure 1 gives a graphic rendition of the comparative static we wish to establish. Note that we allow the cutoff point $\underline{G}$ to be strictly smaller than $\overline{G}$, a consequence of the fact that the model typically admits multiple equilibria: it is possible that for G in a range $\underline{G} < G < \overline{G}$, minority governments form with positive probability in some equilibria but not in others. Yet, despite allowing for this range of indeterminacy, the above formalization, if true, establishes the main substantive conclusion of the analysis.

[insert Figure 1 about here]

In order to allow the reader to develop intuition for the reasons why we might expect such a result to hold, we now analyze an example. The configuration of party policy preferences in this example is a focal case in Baron, 1991, and is assumed by Diermeier and Merlo, 2000, and Baron and Diermeier, 2001. The example differs from the former study in that in the present model we explicitly introduce portfolio allocations, in addition to policies, as part of the government agreement, and from the latter two studies because we do not assume transferable utility.

[insert Figure 2 about here]

Example 1 *Let the space of policies be of dimension $d = 2$. Assume $n = 3$ parties with quadratic policy preferences given by*[6]

$$U_i(\mathbf{x}, \mathbf{g}) = \tilde{u}_i(\mathbf{x}) + g_i = p^2 - (x_1 - \hat{x}_1^i)^2 - (x_2 - \hat{x}_2^i)^2 + g_i,\ i = 1, 2, 3.$$

Parties' ideal policy points lie at the corners of an equilateral triangle as shown in Figure 2(a), and are given by $\hat{\mathbf{x}}^1 = (\frac{p}{2}, \frac{\sqrt{3}p}{2})$, $\hat{\mathbf{x}}^2 = (0, 0)$, and $\hat{\mathbf{x}}^3 = (0, p)$, where $p > 0$. Probabilities of recognition and discount factors are given by $\pi_i = \frac{1}{3}$ and $\delta_i = \delta$ for every party i, respectively. There exists a pure strategy equilibrium in this game,[7] *which is displayed in Figure 2(a), such that proposing party 1 coalesces with party 2, party 2 with party 3, and party 3 with party 1. The equilibrium policy proposal $\mathbf{y}^i$ offered by party i coalescing with party j is given by*

$$\mathbf{x}^{\{i,j\}} = \mathbf{y}^i = \tfrac{1}{2}\hat{\mathbf{x}}^i + \tfrac{1}{2}\hat{\mathbf{x}}^j,$$

[6] The linear term $c_i g_i$ is implicitly incorporated in the term $.... + g_i$, while we can easily determine the feasible set X so that solutions to (2) lie in its interior.

[7] There exists a second pure strategy equilibrium that is identical to the above except parties coalesce in reverse order (party 1 with 3, 3 with 2, and 2 with 1); and there also exists a continuum of mixed strategy equilibria in which proposals are appropriate mixtures of proposals in the two pure strategy equilibria.

when $G \geq \tilde{G} = \frac{p^2(9-7\delta)}{4\delta}$*, while it is a policy closer to the ideal point of party* i *when* $G < \tilde{G}$*. Minority governments such that proposing party* i *sets* $g_i = G$*, form with probability one when* $G \leq \tilde{G}$*. Majority governments form with probability one when* $G > \tilde{G}$*.*

Example 1 constitutes a sharp illustration of the comparative statics we outlined in Figure 1. For high policy polarization or low cabinet parameter $G \leq \tilde{G}$, we get minority governments (in fact with probability one), while we only get majority governments when G is above that value. Figure 2(b) displays the change in equilibrium policy compromises as a function of the parameter G. In order to understand the dependence of the equilibrium outcome on parameters, we now consider (without loss of generality) the calculus of parties 1 and 2, with party 1 being the proposing party. Party 1 wishes to propose a government that maximizes its utility and obtains the support of party 2. Keeping with the notation in the example, we denote by $\mathbf{y}^1$, $\mathbf{y}^2$, and $\mathbf{y}^3$ the policies that prevail in equilibrium. Then, by rejecting a proposal from party 1, party 2 expects to receive

$$\tfrac{1}{3}\tilde{u}_2(\mathbf{y}^1) + \tfrac{1}{3}\tilde{u}_2(\mathbf{y}^2) + \tfrac{1}{3}\tilde{u}_2(\mathbf{y}^3) + \tfrac{G}{3}, \tag{4}$$

one period later. Here we make use of the symmetry of the equilibrium in order to infer that, in expectation, party 2 receives one third of the cabinets, $\frac{G}{3}$, in the next period.

Now suppose that we have an equilibrium in which majority governments prevail with probability one. From Lemma 1 in the previous section, we must have $\mathbf{y}^i = \mathbf{x}^{\{i,j\}}$, where policies $\mathbf{x}^{\{i,j\}}$ maximize (3) for a majority government coalition by parties i and j, and are depicted graphically in Figure 2(a).[8] Using the expression in (4) we deduce a contradiction to our hypothesis that majority governments form when

$$\tilde{u}_2(\mathbf{x}^{\{1,2\}}) \geq \delta(\tfrac{1}{3}\tilde{u}_2(\mathbf{x}^{\{1,2\}}) + \tfrac{1}{3}\tilde{u}_2(\mathbf{x}^{\{2,3\}}) + \tfrac{1}{3}\tilde{u}_2(\mathbf{x}^{\{3,1\}}) + \tfrac{G}{3}), \tag{5}$$

since in this case party 2 is willing to approve a policy at $\mathbf{x}^{\{1,2\}}$ without receiving any cabinets. A substantive interpretation of condition (5) is most clearly obtained if we set $\delta = 1$, whence inequality (5) is equivalent to

$$\tilde{u}_2(\mathbf{x}^{\{1,2\}}) - \tilde{u}_2(\mathbf{x}^{\{3,1\}}) \geq G. \tag{6}$$

[8] These compromises are independent of the exact portfolio allocation, $\mathbf{g}$, due to the fact that party preferences in Example 1 are quasi-linear.

If we fix the location of political parties (hence the location of compromises $\mathbf{x}^{\{i,j\}}$), then the above condition is met when G is small. But another interpretation is that, for given G, condition (6) is satisfied when the policy of a majority government in which party 2 does not participate, $\mathbf{x}^{\{3,1\}}$, is close (in utility terms) to the corresponding policy in a majority government this party participates, $\mathbf{x}^{\{1,2\}}$: in that case, party 2 is willing to accept a minority government with a policy at $\mathbf{x}^{\{1,2\}}$ without receiving any cabinets, because otherwise it faces the risk of a policy at $\mathbf{x}^{\{1,3\}}$ implemented by a coalition of parties 1 and 3 in the next period. In order to further highlight the latter interpretation of condition (6) – that minority governments emerge in the presence of significant policy disagreement – consider what happens in Example 1 when parties' ideal points all coincide (when $p = 0$). Then compromises $\mathbf{x}^{\{i,j\}}$ also coincide, and condition (6) fails for all $G > 0$, so that all equilibrium governments are majority governments in the absence of political disagreement.

Assuming some political disagreement, as we do in the model, our task is to show that the above mechanism operates in more general bargaining environments. It turns out, though, that we cannot rule out the possibility of knife-edge configurations of model parameters for which the desired comparative static is not valid, because an inequality analogous to (5) fails for all G.[9] Yet, by combining certain continuity results established by Banks and Duggan, 2000, and Kalandrakis, 2006, we are able to establish that the mechanism illustrated by Example 1 operates generically. We state this result in the next Proposition:

Proposition 2 *Fix utility functions u_i, seat shares s_i, and recognition probabilities, π_i. For almost all discount factors δ_i, and for almost all preference parameters c_i, there exists $\underline{G}$ with $\underline{G} > 0$ such that for all $G < \underline{G}$, minority governments form with positive probability in all equilibria of the corresponding government formation game.*

Before we conclude this section, we take some time to further clarify the implications of Proposition 2, discuss empirical evidence and additional results under stronger assumptions, and explore possible extensions and generalizations. A first remark is that Proposition 2 only ensures that in every equilibrium, minority governments occur with positive probability, not necessarily with probability one. This is fortunate, as it allows for the possibility of political systems in which both minority and majority governments occur with positive probability over time in the same

[9] An example of such a singularity is available in the Appendix.

equilibrium. Minority governments *may* occur with probability one in an equilibrium, as in the equilibrium of Example 1, but this is not always the case.

In the introduction, we argued that minority governments are paradoxical only if political parties prefer more cabinet office to less, all else constant, and we insisted in imposing this requirement in the analysis (requirement (b) in the introduction). It may then appear upon a first reading, that Propositions 1 and 2 run counter to that requirement, as they jointly yield minority governments when the cabinet parameter G is small. But there is no real inconsistency between Propositions 1 and 2 and requirement (b). In particular, requirement (b) is always imposed in the model: parties prefer larger share of cabinet portfolios to less, all else equal, for all values of $G > 0$. As we have already emphasized, requirement (b) has no implications for the relative importance of office utility compared to policy disagreement, and it is such *relative* changes captured by changes in cabinet parameter G that lead to the different equilibrium outcomes in Propositions 1 and 2.

A related objection to Propositions 1 and 2 is the following: if we imprecisely read these results to mean that minority governments occur only when cabinet office is not significant, then it is tempting to discount the importance of the results on the grounds that they are too intuitive. If "intuitive" in this context means a result that is obviously true, then this objection has already been addressed: we have already cautioned the reader that a naive statement of Proposition 2 has a counterexample, and statements that have counterexamples cannot be obviously true.

More to the point, it is not the case that Propositions 1 and 2 jointly produce minority governments only when cabinet posts are insignificant. Minority governments can emerge in the model even if we fix the *absolute* value of holding cabinet posts to an arbitrarily high level. For that high level, we can increase the ideological distance between the policy compromises pursued by different coalitions in order to ensure that at least some equilibrium governments are minority governments. This becomes obvious by increasing the difference $\tilde{u}_2(\mathbf{x}^{\{1,2\}}) - \tilde{u}_2(\mathbf{x}^{\{3,1\}})$ in inequality (6) of Example 1. Thus, this critique of Proposition 2 can have merit only if the model yields minority governments when cabinet posts have *unrealistically* small value relative to policy. To dispel the latter possibility, consider Example 1, set $p = 1$, and note that for discount factor $\delta = \frac{9}{11}$, we get $\underline{G} = \tilde{G} = 1$. That means party 1 values cabinets so much that it is willing to implement the ideal policy of another party in order to get all cabinets, instead of implementing its own ideal policy without cabinets. For a smaller discount factor $\delta = \frac{3}{5}$, we get $\underline{G} = 2$ so that parties value holding cabinet office twice as much as moving policy from the ideal of their worst opponent to their own ideal point. Although such trades may appear desirable for some individual politicians within parties, they seem highly inconsistent with partisan preferences and a pair of parties that are ideologically far apart. Yet, in Example 1 we obtain minority governments with probability one for any $G < \tilde{G}$ under either pair of parameter specifications.

Intuitive or not, Propositions 1 and 2 are statements that are testable, and the pertinent question is whether the theory put forth by Propositions 1 and 2 is corroborated by the data or not. Do we observe more minority governments as parties get more ideologically polarized or as cabinet office becomes less important as a source of party payoffs? A number of scholars have provided evidence that is consistent with these predictions. For example, minority governments are more likely in Scandinavian countries where political disagreement is marked, and where strong norms of meritocracy in the public sector and a long tradition of scrutiny of the government by independent bodies limit the spoils that political parties can extract from the tenure of cabinet posts. Indridason, 2005, attributes the higher incidence of majority governments in Iceland compared to other Scandinavian countries to the higher importance of cabinet office due to the prevalence of clientelism practices. Furthermore, in 'large n' studies, Warwick, 1998, and Indridason, 2004, show that minority governments are more likely to form when policy polarization increases or when cabinet office becomes more important, respectively. Sened, 1996, using data from Israel shows how the emergence of minority governments there is related to the polarization of the party system. Additional evidence involving more countries, and extensive review of the evidence can be found in Sened and Schofield, 2006, whose monograph theoretically and empirically addresses both electoral and coalition formation politics.

A final comment is that Proposition 2 asserts the existence of threshold, $\underline{G}$, below which minority governments are guaranteed to occur with positive probability, but it does not explicitly determine the magnitude of that threshold. As the statement of the Proposition implies, the exact value of this threshold varies in general with the remaining aspects of the model (policy preferences, recognition probabilities, etc.). If we are willing to impose stronger assumptions than those maintained in the analysis so far, we can get a handle on such additional forces that may determine that threshold, $\underline{G}$. Specifically, by simple differentiation we get:

Result 1 *Assume party payoffs, recognition probabilities, and seat shares as given in Example 1. Then the threshold* $\underline{G}$ *below which minority governments occur decreases with parties' common discount factor,* δ.

The comparative static in Result 1 is straightforward. The range of values of cabinet parameter G for which minority governments occur increases with parties' impatience. As parties get more impatient, their bargaining power decreases so that they are more willing to accept government proposals that do not allocate them any cabinet portfolios, all else held constant.

The observation that minority governments emerge when formateurs deal with coalition partners that have low bargaining power offers a unifying interpretation of both Proposition 2 and Result 1. In the case of Proposition 2, the weakness of the formateur's coalition partners arises from the polarization of the political system: if the formateur fails, then a government at the wrong side of the political spectrum may emerge implementing very undesirable policies, thus generating an incentive to accept minority governments instead. Result 1 identifies another potential source of weakness for the formateur's coalition partners, i.e., their impatience. In either case, rather than being weak and feeble governing solutions, minority governments arise as a consequence of the strength of the formateur's party vis a vis its coalition partners. This strength supports theoretically what we already know empirically, i.e., that minority governments can be both stable and viable governing solutions.

Extensions and Generalizations

Before we conclude, we briefly discuss the possibility of extending the main results of the analysis under different assumptions. We have allowed interaction effects between payoffs from policy and positive portfolio allocations. These interaction effects may represent, for example, ministerial corruption such that cabinet members extract private benefits by altering a public policy (a contract, law, public office appointment, etc.), or the possibility that cabinet appointments have policy implications as is assumed in a much stronger form by Laver and Shepsle (1990). We thus generalized significantly over the assumption of additive separability that is standard in the literature.[10] We barred interaction effects between parties' policy payoff and the portfolio allocation of other parties, a restriction that allowed a more transparent implementation of the assumption that parties prefer larger share of portfolios to less. But we can admit a more general class of preferences, by adding the entire vector of portfolio allocations to the arguments of utility function $u_i(\mathbf{x}, g_i)$. The externalities represented by such a generalized function $u_i(\mathbf{x}, \mathbf{g})$ can, for example, capture the incentives that lead to the occurrence of surplus coalitions. Proposition 2 then still

[10]The even stronger assumption of quasi-linearity is imposed between policies and cabinets by, e.g., Austen-Smith and Banks, 1988, Crombez, 1996, Morelli, 1999, and between policies and transfers by Diermeier and Merlo, 2000, and Diermeier and Baron, 2001.

holds if we mildly restrict the extent of these interactions at $\mathbf{g} = \mathbf{0}$ portfolio allocations, as we do in the present study.

The essence of the main argument admits further generalization. In particular, the main findings extend directly to alternative government formation bargaining models, as long as they satisfy certain continuity properties. One such generalization involves the related model of Banks and Duggan, 2006, who relax the assumption that agreements are desirable by adding a status quo policy that is implemented each period coalition negotiations fail.[11] Focusing on alternative bargaining protocols, Baron and Diermeier, 2001, and Diermeier and Merlo, 2000, propose a bargaining game that allows formateur parties to select proto-coalitions which negotiate over agreements prior to the resultant government being presented for an overt or tacit investiture vote. The analysis we pursued is applicable directly in the case of such alternative bargaining protocols, but by employing the definition of government used in the present study, without assuming transferable utility as these authors do.

5 Conclusions

We have derived a general, yet parsimonious theory for the emergence of minority and majority governments in multiparty parliamentary systems using a sequential bargaining model of coalition formation in the tradition of Baron and Ferejohn, 1989. We established a comparative static to the effect that minority governments are (for almost all parameters) guaranteed to emerge with positive probability when policy disagreement or polarization is significant, or when utility from cabinet posts is small relative to partisan policy disagreement. On the other hand, only majority governments form when these conditions are reversed. Throughout the analysis we maintained that cabinet office is valuable per se, and we avoided dimensionality or other ad priori restrictions on the government agreements space. These findings are corroborated by a number of independent empirical studies.

[11] Note that the two models coincide as discount factors $\delta_i \to 1$, for all $i \in N$.

APPENDIX

In this appendix we prove the two Propositions and the Lemma stated in the main body of the paper. Before we start, we introduce some necessary notation. We denote the set of portfolio allocations given parameter G by $\mathbf{G}_G = \left\{\mathbf{g} \in \mathbb{R}^n_+ : \sum_{i \in N} g_i = G\right\}$. We also use v_i to indicate the continuation value of party i in an equilibrium, i.e., the expected utility of this party if the proposal in the current round is rejected.

Proof of Proposition 1 Define $\overline{u}_i = \max\{u_i(\mathbf{x}, 0) : \mathbf{x} \in X\}$. We first claim that:

($\mathbf{1}$)*Consider an equilibrium with discounted continuation values that satisfy $\delta_i v_i > \overline{u}_i$ for all parties $i \in N$ with $\pi_i > 0$. All governments that form in that equilibrium are majority governments.* Suppose not. Then there exists equilibrium proposal $(\mathbf{y}, \mathbf{g}) \in X \times \mathbf{G}_G$, a winning coalition C that approve government $(\mathbf{y}, \mathbf{g})$, and a non-dummy party $j \in C$ such that $g_j = 0$. Then $U_j(\mathbf{y}, \mathbf{g}; c_j) = u_j(\mathbf{y}, 0) \geq \delta_j v_j > \overline{u}_j$, a contradiction.

We shall next show that:

($\mathbf{2}$) *For each party $i \in N$ with $\pi_i > 0$, there exists $\overline{G}_i$ such that $G > \overline{G}_i \Rightarrow \delta_i v_i > \overline{u}_i$ in every equilibrium.* Fix some player i and let $\underline{u}_i = \min\{u_i(\mathbf{x}, 0) : \mathbf{x} \in X\}$. If $(\overline{\mathbf{y}}, \overline{\mathbf{g}}) \in X \times \mathbf{G}_G$ is the expected value of proposals in an SSP equilibrium, we have $\delta_j v_j \leq U_j(\overline{\mathbf{y}}, \overline{\mathbf{g}}; c_j)$, for all $j \in N$, due to the concavity of u_j. For government $(\overline{\mathbf{y}}, \overline{\mathbf{g}})$, let $h \neq i$ be such that $\overline{g}_h \geq \overline{g}_j$ for all $j \in N \backslash \{i\}$, i.e., h is the party with the highest expected portfolio allocation among parties other than i. Since $\overline{g}_h \geq \frac{G - \overline{g}_i}{n-1}$ we have $\overline{g}_i + \overline{g}_h \geq \frac{G}{n-1}$. Thus, proposal $(\overline{\mathbf{y}}, \mathbf{g}') \in X \times \mathbf{G}_G$ with $g'_j = \overline{g}_j$ if $j \neq i, h$, $g'_h = 0$, and $g'_i = \overline{g}_i + \overline{g}_h \geq \frac{1}{n-1} G$, is approved by all parties except party h. Hence, party i can guarantee her party a utility level $U_i(\overline{\mathbf{y}}, \mathbf{g}'; c_i)$ when proposing. Thus, i's continuation value must satisfy $v_i \geq (1 - \pi_i)\underline{u}_i + \pi_i \left(u_i\left(\overline{\mathbf{y}}, \frac{G}{n-1}\right) + c_i \frac{G}{n-1}\right)$. Note that $u_i(\overline{\mathbf{y}}, 0) \geq \underline{u}_i$ for all $\overline{\mathbf{y}} \in X$. Since $\pi_i > 0$, $\delta_i \in (0, 1]$, $c_i > 0$, and $\frac{\partial u_i(\overline{\mathbf{y}}, g_i)}{\partial g_i} > 0$ for all $\overline{\mathbf{y}} \in X$, there exists $\overline{G}_i > 0$ such that $\delta_i \left((1 - \pi_i)\underline{u}_i + \pi_i \left(u_i\left(\overline{\mathbf{y}}, \frac{G}{n-1}\right) + c_i \frac{G}{n-1}\right)\right) > \overline{u}_i$ for all $G > \overline{G}_i$ and all $\overline{\mathbf{y}} \in X$. As a result, $G > \overline{G}_i \Rightarrow \delta_i v_i > \overline{u}_i$ and we have completed the proof of step (2).

Set $\overline{G} = \max\left\{\overline{G}_i \mid i \in N\right\}$. We now have $G > \overline{G} \Rightarrow \delta_i v_i > \overline{u}_i$ for all $i \in N$ with $\pi_i > 0$, in every equilibrium by step (2). But then only majority governments form for $G > \overline{G}$ in every equilibrium, by step (1), and the proof of the Proposition is complete. *QED*

Next, we prove Lemma 1.

Proof of Lemma 1 Let majority government $(\mathbf{x}, \mathbf{g})$ be proposed by party i in an equilibrium with continuation values $v_i, i \in N$. We first observe via standard arguments that

$$U_j(\mathbf{x}, \mathbf{g}; c_j) = \delta_j v_j, \text{ for all } j \in C \setminus \{i\}, \tag{7}$$

which follows from the fact that $\frac{\partial u_h(\mathbf{x}, g_h)}{\partial g_h} + c_h > 0$, $\frac{\partial u_h(\mathbf{x}, g_h)}{\partial g_l} = 0$ for all $\mathbf{x} \in X$, $l \neq h$, $h, l \in N$. Specifically, if $U_j(\mathbf{x}, \mathbf{g}; c_j) > \delta_j v_j$, $j \in C \setminus \{i\}$, then by the continuity of $U_j(\mathbf{x}, \mathbf{g}; c_j)$ it is possible to reduce g_j and increase g_i (and party i's utility) with the new government still being invested. Similarly, if $U_j(\mathbf{x}, \mathbf{g}; c_j) < \delta_j v_j$, $j \in C \setminus \{i\}$ then it is possible to set $g_j = 0$ and increase g_i (and party i's utility) with the new government still being invested. Furthermore, the same arguments and the fact that the proposing party i cannot be redundant in any winning coalition in which no other party is redundant ensure that $i \in C$, i.e., the proposing party is included among the parties receiving cabinets.

Now suppose $\mathbf{x}$ does not maximize the objective in (3). First we show that $\mathbf{x}$ must be at least a local maximizer of (3). If not, then we can change $\mathbf{x}$ in a feasible direction $\mathbf{v}$ such that

$$\sum_{j \in C} \left(\frac{\partial u_j(\mathbf{x}, g_j)}{\partial g_j} + c_j \right)^{-1} D_{\mathbf{x}} u_j(\mathbf{x}, g_j) \cdot \mathbf{v} = d > 0.$$

For $n' = |C|$, set

$$d_j = \frac{d}{n'} - \left(\frac{\partial u_j(\mathbf{x}, g_j)}{\partial g_j} + c_j \right)^{-1} D_{\mathbf{x}} u_j(\mathbf{x}, g_j) \cdot \mathbf{v},$$

for all $j \in C$, set $d_j = 0$ for all $j \notin C$ and note that $\sum_{j \in N} d_j = 0$. Thus, set a direction of change for $\mathbf{g}$ given by $\mathbf{d} = (d_1, ..., d_n)$, and consider the effect that a change of $(\mathbf{x}, \mathbf{g})$ in the feasible direction $(\mathbf{v}, \mathbf{d})$ has on $U_j(\mathbf{x}, \mathbf{g}; c_j)$:

$$D_{(\mathbf{x},\mathbf{g})} U_j(\mathbf{x}, \mathbf{g}; c_j) \cdot (\mathbf{v}, \mathbf{d}) = D_{\mathbf{x}} u_j(\mathbf{x}, g_j) \cdot \mathbf{v} + \left(\frac{\partial u_j(\mathbf{x}, g_j)}{\partial g_j} + c_j \right) d_j = \left(\frac{\partial u_j(\mathbf{x}, g_j)}{\partial g_j} + c_j \right) \frac{d}{n'} > 0,$$

for all $j \in C$, which is a contradiction: government $(\mathbf{x}, \mathbf{g})$ cannot be optimal for formateur $i \in C$ since there exists a feasible direction that improves all coalition partners' payoff. Thus, $\mathbf{x}$ is a local maximizer of (3). Since the latter is strictly concave as the sum of strictly concave functions, $\mathbf{x}$ is also a global maximizer over alternatives in X. *QED*

Before we prove Proposition 2 we prove a second Lemma. The gist of that Lemma is illustrated graphically in Figure 3. This Figure displays a two-dimensional policy space and the ideal policy points of members of two winning coalitions with party i belonging in both coalitions. The two highlighted policy points display the policies that satisfy equations (2) for these two coalitions. The Lemma, which is illustrated graphically in Figure Figure A1, establishes that for almost all parameters $(c_i)_{i\in N}\in(0,\overline{c})^n$ these policies cannot lie on the same indifference contour of players in the intersection of C and C'.

Lemma 2 *Consider distinct majority coalitions C, C'. If distinct policies $\mathbf{y},\mathbf{y}'$ satisfy (2) for coalitions C and C', respectively, then outside a measure zero set $\mathcal{C}(C,C')\subset(0,\overline{c})^n$ of parameters $(c_i)_{i\in N}$, equations*

$$U_j(\mathbf{y},\mathbf{0};c_j)=U_j(\mathbf{y}',\mathbf{0};c_j),\ \textit{for all } j\in C\cap C',$$

are inconsistent.

Proof. Let $\nu=|C\cup C'|$. By assumption $\mathbf{y},\mathbf{y}'\in int(X)$. Furthermore, since these policies are distinct, we have $(\mathbf{y},\mathbf{y}')\in(int(X)\times int(X)-\Delta)$, where Δ is the diagonal of $int(X)\times int(X)$. Set $S=(int(X)\times int(X)-\Delta)\times(0,\overline{c})^\nu$ and define $|C\cap C'|$ functions $F_j:S\to\mathbb{R}^{2d+1}$, $j\in C\cap C'$, to be the left-hand side of the following $2d+1$ equations:

$$\begin{aligned}\sum_{h\in C}(m_h+c_h)^{-1}D_{\mathbf{y}}u_h(\mathbf{y},0)&=\mathbf{0}\\ \sum_{h\in C'}(m_h+c_h)^{-1}D_{\mathbf{y}'}u_h\left(\mathbf{y}',0\right)&=\mathbf{0}\\ u_j(\mathbf{y},0)-u_j\left(\mathbf{y}',0\right)&=0.\end{aligned}$$

The first $2d$ equations represent equations (2) for coalitions C and C', respectively. The domain of each F_j is the space of policies $\mathbf{y}$, $\mathbf{y}'$ and that of the $|C\cup C'|$ parameters c_h, $h\in C\cup C'$, i.e., $(0,\overline{c})^\nu$. Recall that $\mathbf{z}\in S$ is a critical point of mapping F_j if the Jacobian of F_j evaluated at $\mathbf{z}$, $D_{\mathbf{z}}F_j(\mathbf{z})$, does not have full rank. Consider the set

$$K_j=\left\{\mathbf{z}\in F_j^{-1}(\mathbf{0}):\mathbf{z}\text{ is a critical point of }F_j\right\}.$$

We will first show:

$$\bigcap_{j\in C\cap C'}K_j=\emptyset. \qquad (\star)$$

Assume there exists $\mathbf{z}\in\cap_{j\in C\cap C'}K_j$, instead, to get a contradiction. In what follows we index parties using the convention $q\in C\setminus C'$, $q'\in C'\setminus C$, and $l\in(C\cup C'-\{j\})$. Calculating the

Jacobian $D_{\mathbf{z}}F_j(\mathbf{z})$ using the order of variables implied by $\mathbf{z} = \left(\mathbf{y}, \mathbf{y}', c_j, c_l, ..., c_q, ..., c_{q'}\right)$, we get:

$$D_{\mathbf{z}}F_j(\mathbf{z}) = \begin{bmatrix} A & \mathbf{0} & -w_j^{-2}\mathbf{a}_j & -w_l^{-2}\mathbf{a}_l & ... & -w_q^{-2}\mathbf{a}_q & ... & \mathbf{0} & ... \\ \mathbf{0} & B & -w_j^{-2}\mathbf{b}_j & -w_l^{-2}\mathbf{b}_l & ... & \mathbf{0} & ... & -w_{q'}^{-2}\mathbf{b}_{q'} & ... \\ \mathbf{a}_j^T & -\mathbf{b}_j^T & 0 & 0 & ... & 0 & ... & 0 & ... \end{bmatrix},$$

where $w_h = m_h + c_h$, $h \in C \cup C'$, $A = \sum_{h\in C} w_h^{-1} D_{\mathbf{y}}^2 u_h(\mathbf{y}, 0)$, $B = \sum_{h\in C'} w_h^{-1} D_{\mathbf{y}'}^2 u_h(\mathbf{y}', 0)$, $\mathbf{a}_h = D_{\mathbf{y}} u_h(\mathbf{y}, 0)$, and $\mathbf{b}_h = D_{\mathbf{y}'} u_h(\mathbf{y}', \mathbf{0})$. Note that by assumption $w_h > 0$, $h \in C \cup C'$. Performing a few equivalence operations on $D_{\mathbf{z}}F_j(\mathbf{z})$ we get a new matrix with the same rank:

$$\begin{bmatrix} A & \mathbf{0} & w_j^{-1}\mathbf{a}_j & w_l^{-1}\mathbf{a}_l & ... & w_q^{-1}\mathbf{a}_q & ... & \mathbf{0} & ... \\ \mathbf{0} & B & w_j^{-1}\mathbf{b}_j & w_l^{-1}\mathbf{b}_l & ... & \mathbf{0} & ... & w_{q'}^{-1}\mathbf{b}_{q'} & ... \\ w_j^{-1}\mathbf{a}_j^T & -w_j^{-1}\mathbf{b}_j^T & 0 & 0 & ... & 0 & ... & 0 & ... \end{bmatrix}.$$

Since A, B are negative definite, the above matrix has full rank if and only if the $1 \times m$ matrix

$$M_j(\mathbf{z}) = \mathbf{0} - \begin{bmatrix} w_j^{-1}\mathbf{a}_j \\ -w_j^{-1}\mathbf{b}_j \end{bmatrix}^T \begin{bmatrix} A^{-1} & \mathbf{0} \\ \mathbf{0} & B^{-1} \end{bmatrix} \begin{bmatrix} w_j^{-1}\mathbf{a}_j & w_l^{-1}\mathbf{b}_l & ... & w_q^{-1}\mathbf{a}_q & ... & \mathbf{0} & ... \\ w_j^{-1}\mathbf{b}_j & w_l^{-1}\mathbf{b}_l & ... & \mathbf{0} & ... & w_{q'}^{-1}\mathbf{b}_{q'} & ... \end{bmatrix}.$$

has rank 1. Since $\mathbf{z} \in \cap_{j\in C\cap C'} K_j$, we have $\mathbf{z} \in F_j^{-1}(\mathbf{0})$ for all j, so that $\mathbf{y}$ and $\mathbf{y}'$ satisfy equations (2) and we obtain:

$$\sum_{h\in C} w_h^{-1}\mathbf{a}_h = \mathbf{0} \Rightarrow \sum_{j\in C\cap C'} w_j^{-1}\mathbf{a}_j = -\sum_{q\in C\setminus C'} w_q^{-1}\mathbf{a}_q, \text{ and} \tag{A}$$

$$\sum_{k\in C'} w_h^{-1}\mathbf{b}_h = \mathbf{0} \Rightarrow \sum_{j\in C\cap C'} w_j^{-1}\mathbf{b}_j = -\sum_{q'\in C'\setminus C} w_{q'}^{-1}\mathbf{b}_{q'}. \tag{B}$$

Furthermore, since $\mathbf{z} \in \cap_{j\in C\cap C'} K_j$, $\mathbf{z}$ is a critical point of each F_j, so we must have $M_j(\mathbf{z}) = \mathbf{0}$ for all $j \in C \cap C'$. $M_j(\mathbf{z}) = \mathbf{0}$ means $w_j^{-1}\mathbf{a}_j^T A^{-1} w_q^{-1}\mathbf{a}_q = 0$ for all $q \in C \setminus C'$ and all $j \in C \cap C'$, and we can obtain (by summing equations $w_j^{-1}\mathbf{a}_j^T A^{-1} w_q^{-1}\mathbf{a}_q = 0$ first over all $q \in C \setminus C'$ for each $j \in C \cap C'$, and then over all j):

$$\left(\sum_{j\in C\cap C'} w_j^{-1}\mathbf{a}_j^T\right) A^{-1} \left(\sum_{q\in C\setminus C'} w_q^{-1}\mathbf{a}_q\right) = 0.$$

Using (A) and the fact that A^{-1} is negative definite, we deduce

$$\sum_{j\in C\cap C'} w_j^{-1}\mathbf{a}_j = \sum_{j\in C\cap C'} (m_j + c_j)^{-1} D_{\mathbf{y}} u_j\left(\mathbf{y},0\right) = \mathbf{0}. \tag{C}$$

An identical argument using (B) gives us

$$\sum_{j\in C\cap C'} w_j^{-1}\mathbf{b}_j = \sum_{j\in C\cap C'} (m_j + c_j)^{-1} D_{\mathbf{y}'} u_j\left(\mathbf{y}',0\right) = \mathbf{0}. \tag{D}$$

Since (C) and (D) imply that $\mathbf{y}$, $\mathbf{y}'$ both maximize the strictly concave function $\sum_{j\in C\cap C'}(m_j+c_j)^{-1}u_j(\mathbf{x},0)$, we must have $\mathbf{y}=\mathbf{y}'$, which is impossible since $\mathbf{z}\in\cap_{j\in C\cap C'}K_j\subset S$. This completes the proof of ($\star$).

Now for each $j\in C\cap C'$ define the map $\hat{F}_j : S\setminus K_j \to \mathbb{R}^{2d+1}$ to be the restriction of F_j on $S\setminus K_j$. Recall that $\mathbf{0}\in\mathbb{R}^{2d+1}$ is a regular value of $\hat{F}_j$ if $D_{\mathbf{z}}\hat{F}_j(\mathbf{z})$ has full rank for every $\mathbf{z}\in\hat{F}_j^{-1}(\mathbf{0})$. Thus, since $\hat{F}_j^{-1}(\mathbf{0})\cap K_j=\emptyset$ by construction, $\mathbf{0}\in\mathbb{R}^{2d+1}$ is a regular value of $\hat{F}_j$ for all $j\in C\cap C'$. Note that S is open and K_j is a (relatively) closed set for all $j\in C\cap C'$, so that $S\setminus K_j$ is an open set for all $j\in C\cap C'$. Thus, each $\hat{F}_j$ is a smooth mapping between smooth manifolds since $S\setminus K_j$ is open and u_h is smooth. As a result, the Preimage theorem (Guilemin and Pollack, 1974, page 21) ensures that $R_j=\hat{F}_j^{-1}(\mathbf{0})$ is a $(2d+\nu)-(2d+1)=(\nu-1)$-dimensional manifold.

We are now ready to conclude the proof of the Lemma. In order for $\mathbf{z}\in S$ to satisfy $F_j(\mathbf{z})=\mathbf{0}$ for some $j\in C\cap C'$ we must have $\mathbf{z}\in R_j\cup K_j$. It follows that in order for $\mathbf{z}\in S$ to satisfy $F_j(\mathbf{z})=\mathbf{0}$ for all $j\in C\cap C'$, we must have $\mathbf{z}\in\bigcap_{j\in C\cap C'}(R_j\cup K_j)$. Since $\bigcap_{j\in C\cap C'}K_j=\emptyset$ by ($\star$), the set $\bigcap_{j\in C\cap C'}(R_j\cup K_j)$ becomes the finite union of sets that are themselves the intersection of $|C\cap C'|$ sets, at least one of which is a $(\nu-1)$-dimensional manifold. Thus, $\bigcap_{j\in C\cap C'}(R_j\cup K_j)$ is itself at most $(\nu-1)$-dimensional. This is one dimension smaller than the space of parameters $(c_i)_{i\in C\cup C'}\in(0,\bar{c})^{\nu}$ and, as a consequence, the equations

$$\begin{aligned}
\sum_{h\in C}(m_h+c_h)^{-1}D_{\mathbf{y}}u_h\left(\mathbf{y},0\right) &= \mathbf{0}\\
\sum_{h\in C'}(m_h+c_h)^{-1}D_{\mathbf{y}'}u_h\left(\mathbf{y}',0\right) &= \mathbf{0}\\
u_j\left(\mathbf{y},0\right)-u_j\left(\mathbf{y}',0\right) &= 0, j\in C\cap C',
\end{aligned}$$

are consistent only for a set of measure zero $\mathcal{C}(C,C')$ of parameters c_h, $h\in N$. Since $U_j(\mathbf{x},\mathbf{0};c_j)=u_j(\mathbf{x},0)$, the proof is complete. ■

We can now prove Proposition 2.

Proof of Proposition 2 Fix any π_i, s_i, u_i, $i \in N$, consistent with our assumptions. We break the proof into four steps:

(**1**) *For all discount factors $(\delta_i)_{i\in N}$ outside a measure zero set $\mathcal{D} \subset (0,1]^n$, there is a finite number of pure strategy equilibria in the game with $G = 0$. These equilibria are independent of $(c_i)_{i\in N} \in (0,\overline{c})^n$.* The game with $G = 0$ satisfies condition $(A1)$ of Lemma 2, page 318, of Kalandrakis, 2006. Thus, the number of pure strategy equilibria of this game is finite by Theorems 3(i), page 323, and Theorem 5, page 325, of Kalandrakis, 2006, for almost all discount factors. Clearly, parameters $(c_i)_{i\in N} \in (0,\overline{c})^n$ do not affect the equilibrium set of the game with $G = 0$. Thus, step 1 is proved.

In what follows, we will say that a sequence of equilibria e_k, one for each version of the game with $G = G_k > 0$, is *minority barring* if all governments proposed in each equilibrium e_k are majority governments. We first conclude:

(**2**) *Consider a sequence $G_k \to 0$ and an associated minority barring sequence of equilibria $e_k \to e$. e is an equilibrium of the game with $G = 0$, and for every party i with $\pi_i > 0$, if i proposes policy $\mathbf{y} \in X$ in equilibrium e, then $\mathbf{y}$ satisfies equations (2) for some majority coalition C with $i \in C$.* The fact that e is an equilibrium follows from the upper-hemicontinuity of the equilibrium correspondence (Banks and Duggan, 2000, Theorem 3, page 81). Suppose party i proposes policy $\mathbf{y} \in X$ in equilibrium e. Since only majority governments form in every equilibrium e_k, there exists a subsequence (still indexed by k), $e_k \to e$, a corresponding sequence of majority governments $(\mathbf{y}_k, \mathbf{g}_k) \to (\mathbf{y}, \mathbf{0})$ that are in the support of party i's proposal strategy in equilibrium e_k, and a majority coalition C with $i \in C$, such that $g_{h,k} > 0$ for all $h \in C$ and $g_{h,k} = 0$ for all $h \notin C$. By Lemma 1 and the Theorem of the Maximum we deduce that $\mathbf{y}$ maximizes $\sum_{j\in C} (m_j + c_j)^{-1} u_j(\mathbf{y}, 0)$, which is a strictly concave function. By assumption, there exists $\mathbf{x}$ in the interior of X such that

$$\sum_{j\in C} (m_j + c_j)^{-1} D_{\mathbf{x}} u_j(\mathbf{x}, 0) = \mathbf{0}.$$

We deduce that $\mathbf{x} = \mathbf{y}$ is the unique maximizer satisfying equations (2) which coincide with the first order conditions for a maximum of (3) at $\mathbf{g} = \mathbf{0}$.

Lemma 2 allows us to show:

(**3**) *Fix any $c_i, i \in N$, outside a measure zero set $\mathcal{C}^* \subset (0,\overline{c})^n$ of parameters $(c_i)_{i\in N}$. Consider a sequence $G_k \to 0$ and an associated minority barring sequence of equilibria $e_k \to e$. Equilibrium*

e is in pure strategies. Suppose equilibrium e is in mixed strategies so that two distinct policies $\mathbf{y}, \mathbf{y}' \in X$ lie in the support of some party i's proposal strategy with $\pi_i > 0$. Then, by the same argument used in the previous step, there exists a subsequence (still indexed by k), $e_k \to e$, corresponding sequences of majority governments $(\mathbf{y}_k, \mathbf{g}_k) \to (\mathbf{y}, \mathbf{0}), (\mathbf{y}'_k, \mathbf{g}'_k) \to (\mathbf{y}', \mathbf{0})$, that are in the support of party i's proposal strategy in equilibrium e_k, and distinct winning coalitions C and C', $i \in C \cap C'$, such that $g_{h,k} > 0$ and $g'_{q,k} > 0$ if and only if $h \in C$, $q \in C'$, and

$$U_j(\mathbf{y}_k, \mathbf{g}_k; c_j) \quad = \quad U_j(\mathbf{y}'_k, \mathbf{g}'_k; c_j), \text{ for all } j \in C \cap C',$$

for all k. The indifference of all players $j \in C \cap C'$ follows from the fact the the proposer i mixes between proposals $(\mathbf{y}_k, \mathbf{g}_k), (\mathbf{y}'_k, \mathbf{g}'_k)$ and leaves all cabinet recipient parties indifferent between its proposal and their continuation value in equilibrium e_k (equations (7) in the proof of Lemma 2). By continuity and Step 2, we deduce that proposals $\mathbf{y}, \mathbf{y}'$ offered in the limit equilibrium e must satisfy

$$\begin{aligned}
\sum_{j \in C} (m_j + c_j)^{-1} D_{\mathbf{y}} u_j(\mathbf{y}, 0) \quad &= \quad \mathbf{0} \\
\sum_{j \in C'} (m_j + c_j)^{-1} D_{\mathbf{y}} u_j(\mathbf{y}', 0) \quad &= \quad \mathbf{0} \\
U_j(\mathbf{y}, \mathbf{0}; c_j) \quad &= \quad U_j(\mathbf{y}', \mathbf{0}; c_j), \text{ for all } j \in C \cap C'.
\end{aligned}$$

Lemma 2 guarantees this is impossible outside a measure zero set $\mathcal{C}(C, C')$. Since there exists a finite number of possible pairs of distinct winning coalitions C, C', and finite unions of sets of measure zero have measure zero, outside a measure zero set of parameters $\mathcal{C}^* \subset (0, \bar{c})^n$, equilibrium e must be in pure strategies. This completes the proof of step 3.

The last step is:

(4) *Fix any policy $\mathbf{x} \in X$. For all $(c_i)_{i \in N}$ outside a measure zero set $\mathcal{C}(\mathbf{x}) \subset (0, \bar{c})^n$, $\mathbf{x}$ does not satisfy (2) for any winning coalition C.* Since $|C| \geq 2$ for any winning coalition C, and parties' ideal points do not coincide, the set of parameters $(c_i)_{i \in N}$ that solve

$$\sum_{j \in C} (m_j + c_j)^{-1} D_{\mathbf{x}} u_j(\mathbf{x}, 0) = \mathbf{0},$$

is a lower dimensional set (since $D_{\mathbf{x}} u_j(\mathbf{x}, 0) \neq \mathbf{0}$ for at least $|C| - 1$ parties $j \in C$). So the claim follows since there are only a finite number of possible majority coalitions C, and finite unions of sets of measure zero have measure zero.

We are now ready to conclude the proof of the Proposition. Fix any discount factors $(\delta_h)_{h \in N} \notin \mathcal{D}$. By Step 1, party i, $\pi_i > 0$, may propose at most a finite number of policies $\{\mathbf{x}_1, ..., \mathbf{x}_\tau\}$, which are independent of $(c_h)_{h \in N}$, in any of the $\tau \geq 0$ pure strategy equilibria of the game with $G = 0$. Consider any parameter $(c_h)_{h \in N} \in (0, \overline{c})^n$, outside the measure zero set

$$\mathcal{C} = \mathcal{C}^* \cup \mathcal{C}(\mathbf{x}_1) \cup ... \cup \mathcal{C}(\mathbf{x}_\tau).$$

Suppose that there is no $\underline{G}> 0$ such that minority governments form with positive probability in all equilibria of the game when $0 \leq G <\underline{G}$. In other words, the working hypothesis is that for each $G > 0$ there exists some G' with $G > G' > 0$ for which an equilibrium exists with all proposed governments being majority governments. Then there is a sequence $G_k \to 0$ and an associated minority barring sequence of equilibria e_k. By going to a subsequence if necessary, we have $e_k \to e$, and the limit e is an equilibrium by Step 2. Since $(c_h)_{h \in N} \notin \mathcal{C}^*$, e is a pure strategy equilibrium by Step 3, so that party i proposes one of policies $\mathbf{x} \in \{\mathbf{x}_1, ..., \mathbf{x}_\tau\}$. Step 2 ensures that this policy solves (2) for some winning coalition C with $i \in C$. But this is impossible by Step 4, since we have assumed that $(c_h)_{h \in N} \notin \mathcal{C}$. Thus, for almost all discount factors, and almost all parameters $(c_h)_{h \in N}$, there exists $\underline{G} > 0$ such that minority governments form with positive probability in every equilibrium of the game with $G < \underline{G}$. *QED*

In the last part of this Appendix we shall show that it is possible to specify instances of the model that satisfy all maintained assumptions and admit equilibria without minority governments for all positive levels of cabinet parameter, $G > 0$.

Counter-Example: Let the space of policies be of dimension $d = 2$ and set X to be the square defined by points $(0, b)$, $(b, 0)$, $(0, -b)$, and $(-b, 0)$. Assume $n = 4$ parties with equal share of seats in parliament $s_i = \frac{1}{4}$, $i = 1, ..., 4$ and preferences given by

$$U_i(\mathbf{x}, \mathbf{g}) = p - \left(x_1 - \hat{x}_1^i\right)^2 - a\left(x_2 - \hat{x}_2^i\right)^2 + g_i, \ i = 1, 3, \text{ and}$$

$$U_i(\mathbf{x}, \mathbf{g}) = p - a\left(x_1 - \hat{x}_1^i\right)^2 - \left(x_2 - \hat{x}_2^i\right)^2 + g_i, \ i = 2, 4,$$

where $\hat{\mathbf{x}}^1 = (0,1)$, $\hat{\mathbf{x}}^2 = (1,0)$, $\hat{\mathbf{x}}^3 = (0,-1)$, and $\hat{\mathbf{x}}^4 = (-1,0)$. Probabilities of recognition and discount factors are identical and given by $\pi_i = \frac{1}{4}$ and $\delta_i = \delta \in (\frac{6}{7}, 1)$ for all $i \in N$, respectively. Set the weight $a \in (0, \frac{7\delta-6}{6-5\delta})$, the constant p at

$$p = \frac{2a(1+a)(4+a) - \delta a(8+9a+3a^2)}{2(2+a)^2(1-\delta)},$$

and $b \in (\frac{a}{2+a}, \frac{\sqrt{p}-\sqrt{a}}{\sqrt{a}})$. For every level of $G > 0$ there exists an equilibrium such that party 1 proposes $\mathbf{x} = \left(0, \frac{a}{2+a}\right)$ and $\mathbf{g} = \left(\frac{(2-\delta)G}{2}, \frac{\delta G}{4}, 0, \frac{\delta G}{4}\right)$, party 2 proposes $\mathbf{x} = \left(\frac{a}{2+a}, 0\right)$ and $\mathbf{g} = \left(\frac{\delta G}{4}, \frac{(2-\delta)G}{2}, \frac{\delta G}{4}, 0\right)$, party 3 proposes $\mathbf{x} = \left(0, -\frac{a}{2+a}\right)$ and $\mathbf{g} = \left(0, \frac{\delta G}{4}, \frac{(2-\delta)G}{2}, \frac{\delta G}{4}\right)$, and party 4 proposes $\mathbf{x} = \left(-\frac{a}{2+a}, 0\right)$ and $\mathbf{g} = \left(\frac{\delta G}{4}, 0, \frac{\delta G}{4}, \frac{(2-\delta)G}{2}\right)$.

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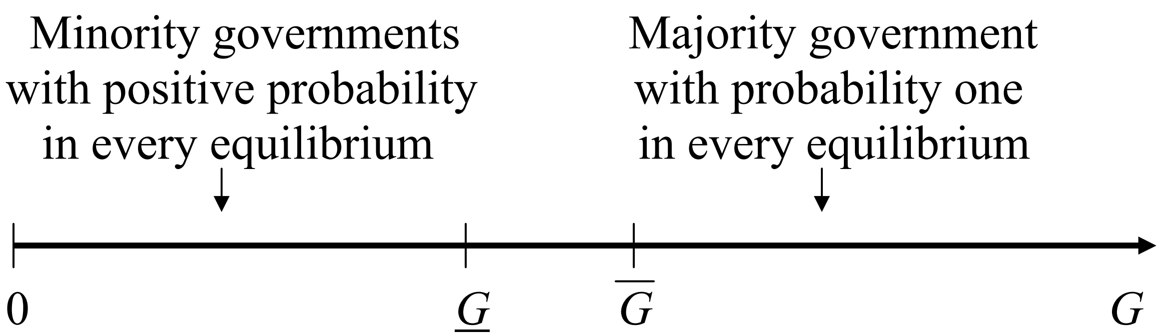
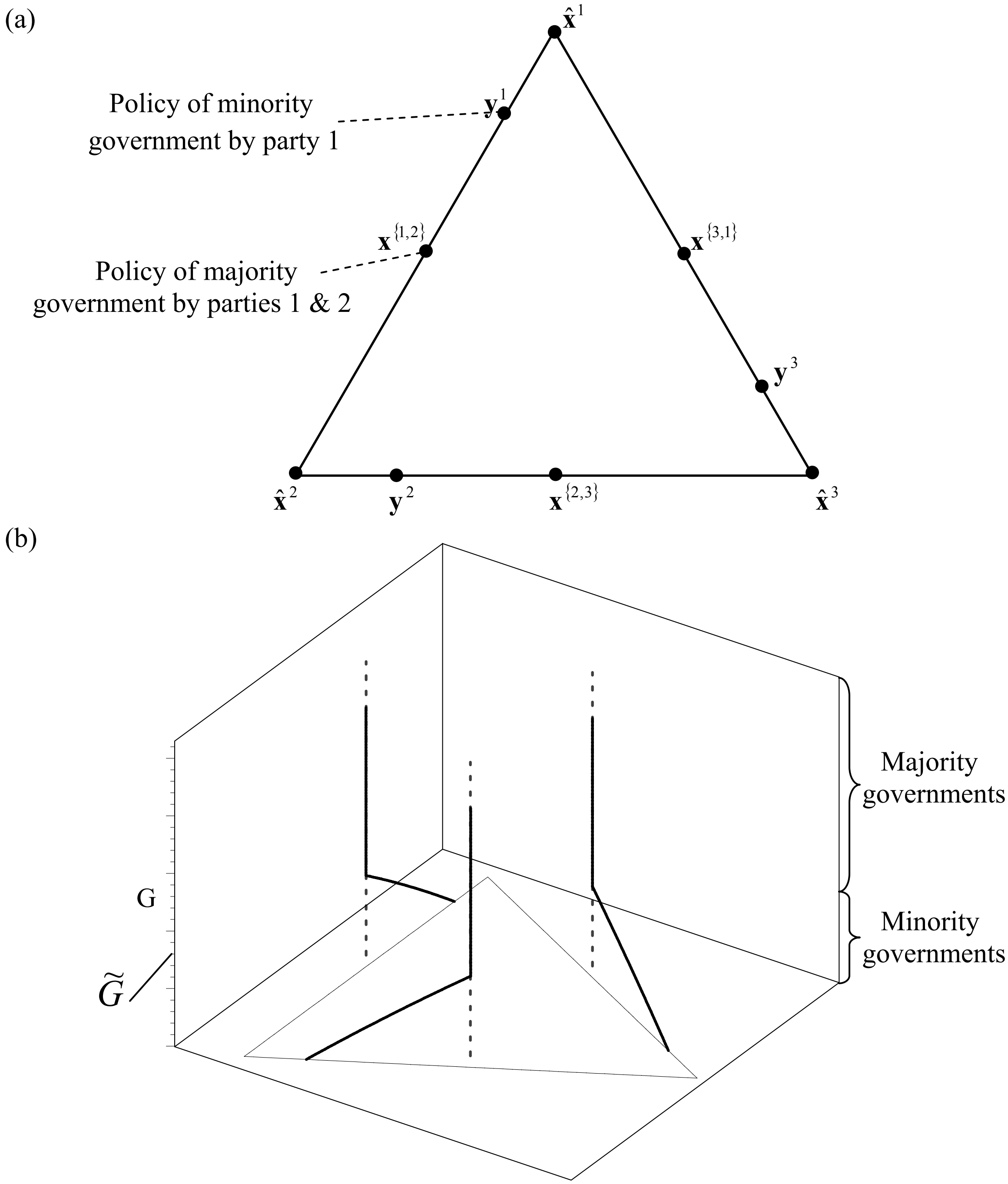
Figure 1: Comparative Statics



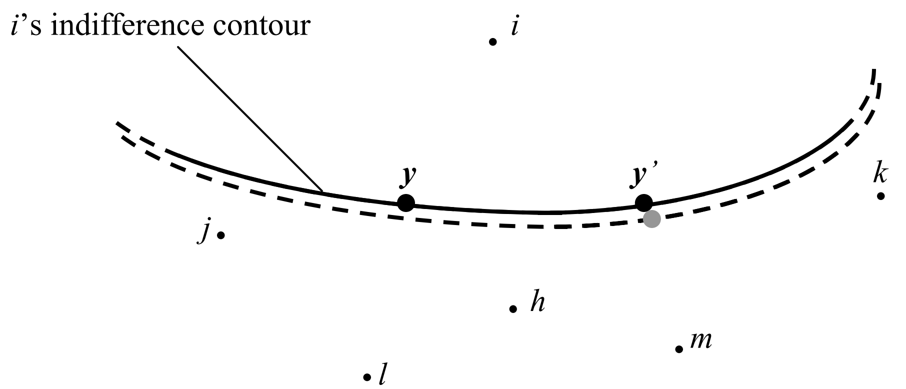
Key: Since the bargaining game may admit multiple equilibria, it is possible that in some intermediate range of cabinet spoils ($\underline{G}$, $\overline{G}$) a subset of equilibria involve minority governments and the rest do not.

Figure 2: Minority and Majority Governments in Example 1



Key: *(a) The (unconstrained) ideal policies of the three parties are located at the vertices of the equilateral triangle. The highlighted points are different policies proposed in equilibrium for different values of parameter G.*
(b) For G below $\tilde{G}$, minority governments form and policies are leveraged toward the ideal point of the formateur. For G above that level, only majority governments form with policies at the midpoint between the ideal points of the two parties in government.

Figure A1: Graphic Illustration of Lemma 2



Key: *Policy **y** represents the solution to equations (2) for coalition C = {i, j, l, h} , while policy **y'** represents the respective policy for coalition C = {i, m, k}. There exists a perturbation of parties' preference parameters c, that ensures that the two policies do not fall on the same indifference contour of party* $i \in C \cap C'$.