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A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-sectional Heterogeneity?*

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Abstract

In this paper we study the asset pricing implications of a parsimonious two-agent macroeconomic model with two key features: limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution. The parameter values for the model are taken from the real business cycle literature and are not calibrated to match any financial statistic. Yet, with a risk aversion of two, the model is able to explain a large number of asset pricing phenomena including all the facts matched by the external habit model of Campbell and Cochrane (1999). Examples in this list include a high equity premium and a low risk-free rate; a counter-cyclical risk premium, volatility and Sharpe ratio; predictable stock returns with coefficients and R^2 values of long-horizon regressions matching their empirical counterparts, among others. In addition the model generates a risk-free rate with low volatility (5.7 percent annually) and with high persistence. We also show that the similarity of our results to those from an external habit model is not a coincidence: the model has a reduced form representation which is extremely similar to Campbell and Cochrane's framework for *asset pricing*. However, the *macroeconomic implications* of the two models are very different, favoring the limited participation model. Moreover, we show that policy analysis yields dramatically different conclusions in each framework.

Keywords: Limited stock market participation, asset pricing, the equity premium puzzle, incomplete markets, habit formation, elasticity of intertemporal substitution.

JEL classification: E32, E44, G12

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1 Introduction

No other puzzle in the last two decades has probably generated as much interest and subsequent research as the equity premium puzzle of Mehra and Prescott (1985). These authors essentially showed that the historical equity premium (i.e., the excess return of stocks over bonds) was puzzling in the context of the canonical general equilibrium portfolio-choice model, which is also the foundation of most neoclassical macroeconomic analysis. The long quest for a resolution has uncovered further puzzling facts such as the risk-free rate puzzle of Weil (1989), adding to the list of challenges. After many extensions and enrichments of the basic framework, a satisfactory explanation still seemed elusive.[1]

Building on the earlier insight of Abel (1990) and Constantinides (1990) a number of recent papers have introduced models featuring preferences with endogenous habit formation (Jermann 1998; Boldrin, Christiano and Fisher 1999) or "catching-up with Joneses" (Campbell and Cochrane 1999) which can explain both the equity premium and the risk-free rate puzzles. In fact, Campbell and Cochrane (1999) go even further and are able to match a wide variety of financial statistics including the time series and cyclical patterns of asset prices as well as the predictability of excess returns.

Although habit persistence could be an appealing description of behavior at an introspective level, it is fair to say that it is probably not the best understood aspect of individual preferences. At a more concrete level, the empirical evidence seems mixed. Studies using aggregate data usually find evidence in favor of habit[2] with quite large persistence (Ferson and Constantinides 1991; Heaton 1995; Daniel and Marshall 1998) whereas individual-level data does not seem to reveal even the slightest hint of such behavior (Naik and Moore 1996; Dynan 2000; McKenzie 2002). Furthermore, Otrok, Ravikumar and Whiteman (2002) show that both the equity premium and the risk-free rate implied by these models are extremely sensitive to the specification of the shock process. For example, even small changes in the persistence of consumption growth—much more modest than those observed in the U.S. historical data—imply movements in *average* returns in the order of 15 to 25 percentage points. Thus it is still useful to study asset prices in a model with standard preferences (for example, CRRA) but by allowing for other elements, such as sources of heterogeneity which are observable or empirically well-established.

In this paper we follow this second route and study asset prices in a parsimonious two-agent macroeconomic model with two key features: limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution. In other respects the framework is a standard real business cycle model. More specifically, we consider an economy with a neoclassical production technology and competitive markets. There are two types of agents. The majority of households (first type) do not participate in the stock market where claims to the firm's future dividends are traded. However, a risk-free bond is available to all households, so non-stockholders can also accumulate wealth and smooth consumption intertemporally. We

[1] For excellent surveys of the literature on these puzzles, see Kocherlakota (1996) and Campbell (1999).

[2] A number of papers have argued that time aggregation might strongly bias results in favor of habit persistence even when there is none. See for example Porter and Wheatley (1999) and the references therein.

also model the capital adjustment costs faced by firms (as in Lucas and Prescott 1971; Jermann 1998; and Boldrin, et. al 1999). Finally, consistent with empirical evidence which we document in Section 3, we assume that stockholders have a higher elasticity of substitution than non-stockholders. With CRRA utility this also implies heterogeneity in risk aversions, but as we show in the robustness analysis with Epstein-Zin (1989) preferences the results of the paper remain essentially the same when both agents' risk aversions are set equal to two, while keeping the elasticities fixed at their baseline values.

We find that the model is surprisingly successful in explaining a large number of asset pricing phenomena, many of them considered to be puzzles. To provide a benchmark, the model can match all the facts studied in Campbell and Cochrane (1999) *as well as* a number of others on which that model is silent. This is true even though the model has no free parameters, and all the structural ones are calibrated to standard values from the business cycle literature. This approach is in contrast to some recent papers on asset pricing where a number of parameters are chosen specifically to match certain moments of financial variables.

Here is an overview of our results. With a risk aversion of two for both agents, the Sharpe ratio is 23 percent (and 29 percent with a risk aversion of three) compared to 25 to 30 percent in the U.S. data. The model also matches the levels of returns as well as their volatilities. Especially the low variability of the interest rate has proved to be a particularly tough challenge for habit persistence models—a risk-free rate volatility above 20 percent is common (c.f., Boldrin et. al. 1999). In contrast, the annual volatility of the interest rate in our model is 5.7 percent compared to 5.4 percent in the U.S. data. Moreover, the model generates a countercyclical equity premium, conditional volatility, and Sharpe ratio, all of which are documented features of asset prices. Furthermore, stock returns are predictable and both the coefficients and the R^2 values of the long-horizon regressions of future stock returns on the price-dividend ratio match those obtained in the finance literature (Campbell and Shiller 1988; Fama and French 1989). Finally, the model explains a number of other features of asset prices, such as their autocorrelation patterns, their cross-correlations with each other, and so on.

These results do not rely on high risk aversion, idiosyncratic shocks or binding borrowing constraints. Limited participation is the critical element generating all of these results. On the other hand, preference heterogeneity has a big impact on the first two moments of returns, but is not as critical for other findings. The driving force behind the time-series results is the evolution of the wealth distribution over the business cycle. More specifically, a positive persistent technology shock increases the value of equity significantly benefiting the stockholders immediately. On the other hand non-stockholders gain only gradually as their wages grow with increasing capital and they respond by accumulating more wealth. Thus, initially the share of wealth held by stockholders go up coming back down slowly. As a result, each group's influence over the determination of asset prices change endogenously through the business cycle giving rise to interesting time-series behavior.

Many of these results can also be found in Campbell and Cochrane (1999). They study a representative-agent exchange economy with a slow-moving external habit term in the utility function. They reverse-engineer the parameters of this habit process to match a number of financial moments and find that the model performs impressively in other dimensions as well. The remarkable similarities between our results suggests an interesting possibility: Even though the two models look very different on the surface, could there be a deeper connection between them? We find that this is indeed the case. *The limited participation model has a reduced form which is extremely similar to Campbell and Cochrane's framework in terms of asset pricing implications.* In particular, the complicated process they recover (and call external habit) turns out to be just the consumption process of non-stockholders.

A simple way to see this point is by considering the Euler equation for the stockholder in our model. Let X be non-stockholders' consumption and C^A be aggregate consumption. So, stockholders' consumption is: $C^h = C^A - X$, and we have:

$$E_t\left[\beta\left(\frac{C_{t+1}^A - X_{t+1}}{C_t^A - X_t}\right)^{-\alpha}\left(R_{t+1}^S - R_t^f\right)\right] = 0.$$

where α and β are the risk aversion and time discount parameters respectively, and R_{t+1}^S and R_t^f are the risky and risk-free rates. Note that a similar Euler equation does not hold for aggregate consumption. Now, if in fact asset prices are determined by this condition, an observer who ignores limited participation and takes C^A as the right measure of consumption would still have to subtract X to successfully explain asset prices and then would have to somehow reinterpret this X. In the case of Campbell and Cochrane (hereafter, C-C), they call it a habit process. In Section 5 we make this point rigorously and show that the process X assumed in C-C has virtually the same statistical properties as non-stockholders' consumption in our model. In a way this finding verifies their conjecture that the representative agent preferences they recover "could result from aggregation of heterogenous consumers with quite different preferences" (p. 241).

What do we learn from these results for studying macroeconomic problems? This question is critical since an important reason for macroeconomists' interest in financial anomalies is because these puzzles challenge the foundations of the very framework used for policy analysis. It would be hard to place a lot of confidence in the *quantity* implications of a framework which has notoriously poor *pricing* implications. Unfortunately, the extension of the external habit model to study macroeconomic issues has not been all that successful. For example, Lettau and Uhlig (2000) show that incorporating external habit results in a number of macroeconomic anomalies in an otherwise standard business cycle model. One has to introduce a number of frictions in production and more habits in leisure and so on, in order to partly counteract this effect.

Fortunately, this is where the similarities between our model and the habit framework ends. The macroeconomic implications of the limited participation model are as good as a standard business cycle model without any adjustment costs and are still in the right ball park with the frictions imposed. The main source of difference between the two models leading to these results is the extremely high risk aversion (around 80) assumed in C-C compared to a risk aversion of only two used in our framework.[3] Finally, the model generates substantial wealth inequality together with a more even consumption distribution both of which match the empirical statistics from the U.S. data. We study the macroeconomic implications of this model in more detail in a companion paper (Guvenen 2002) and show that it can also help resolve some *macroeconomic* puzzles.

In section 7, we conduct a simple policy experiment (a capital income taxation problem) and find that the two models yield drastically different policy conclusions. This point suggests extreme caution as the success of habit persistence models encouraged many researchers to address policy questions in that framework (among others, McCallum and Nelson 1998; Fuhrer 2000; Ljungqvist and Uhlig 2000; Christiano, Eichenbaum and Evans 2001; and Abel 2001).

The paper is organized as follows. Section 2 introduces the model and the parametrization is discussed in Section 3. We then analyze asset prices in Section 4 and establish the connection between the two models in Section 5. Starting with Section 6 we look at macroeconomic behavior where the similarities end. Section 6 analyzes the positive implication and Section 7 presents a normative perspective in the context of a capital income taxation problem. Section 8 contains sensitivity analysis and relates the paper to the existing literature, and Section 9 concludes.

2 The Model

The model is an extension of the framework studied in Guvenen (2000). For transparency of results, our modeling goal is to stay as close to the standard real business cycle model (e.g., Hansen 1985) as possible and only introduce two key features: limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution. We consider an economy populated by two types of agents who live forever. The population is constant and is normalized to unity. Let λ $(0 < \lambda < 1)$ denote the measure of the first type of agents (who will be called "stockholders" later) in total population.

PREFERENCES

Both agents have time separable expected utility functions defined over future consumption streams:

$$E_t \left(\sum_{j=1}^{\infty} \beta^{t+j} u^i \left(c_{t+j} \right) \right) \tag{1}$$

for $i = h, n,$ where the superscripts h and n denote stock*h*olders and *n*on-stockholders respectively, and β is the subjective discount rate. As for the parameterization of the momentary utility function, we have two considerations in mind. On the one hand, we want to keep preferences standard to highlight the effect of limited participation and other endogenous features

[3] Although endogenous habit models imply a low risk aversion they still result in a very low EIS which is the driving force for macroeconomic models. In our model, capital is controlled by stockholders who have a high EIS (at least 0.5) with our calibration.

of the model. This suggests a standard CRRA utility function: $u(C) = \frac{C^{1-\alpha}}{1-\alpha}$ and we adopt it throughout much of the paper. On the other hand, it is well-known that with this specification the parameter α controls both the relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS) which are different aspects of individuals' tastes. To clarify the intuition of the results in Section 8 we will generalize the preferences to the recursive utility function of Epstein and Zin (1989). This exercise will also allow us to discuss which features of the results are driven by the risk aversion versus the elasticity of intertemporal substitution.

The Firm

There is a single aggregate firm producing the consumption good using capital (K_t) and labor (L_t) inputs according to a Cobb-Douglas technology:

$$Y_t = Z_t K_t^{\theta} L_t^{1-\theta},$$

where $\theta \in (0,1)$ is the factor share parameter. The logarithm of the stochastic technology level evolves as an AR(1) process:

$$\begin{aligned} \log(Z_{t+1}) &= \rho \log(Z_t) + \varepsilon_{t+1} \\ & \varepsilon \sim N\left(0, \sigma_{\varepsilon}^2\right) \end{aligned}$$

The goal of the firm's managers (hopefully) is to maximize the value of the firm to owners:[4]

$$P_t^S = \underset{\{I_{t+j}, L_{t+j}\}}{Max} E_t \left[\sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} \left(Z_{t+j} K_{t+j}^{\theta} L_{t+j}^{1-\theta} - W_{t+j} L_{t+j} - I_{t+j} \right) \right]$$

subject to the technology constraint which features adjustment costs in investment

$$K_{t+1} = (1-\delta) K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t,$$

where P_t^S is the ex-dividend value of the firm in a given period, and $\beta^j \frac{\Lambda_{t+j}}{\Lambda_t}$ is the discount rate (i.e., the marginal rate of substitution between periods t and $t+j$). The adjustment cost function $\Phi(\cdot)$ is concave in investment which captures the difficulty of quickly changing the level of capital installed in the firm. Consequently, the prices of capital and consumption goods are not equal and Tobin's q is not necessarily equal to unity. This specification is the same as the one used in Jermann (1998) and Boldrin, et. al (1999) to make comparison easier.

The firm is 100 percent equity financed as commonly assumed in the real business cycle literature. It is relatively straightforward to introduce leverage into this framework (say, as a fixed proportion of capital) although we will not pursue that approach here. A share in this

[4] To save on notation (but at the expense of slightly abusing notation) we use K_t to denote the firm's capital choice and we will also use the same variable to denote aggregate capital which the firm takes as given when making its choice. In equilibrium, of course, the two are the same.

firm entitles its owner to the entire stream of future dividends given by

$$D_t = Z_t K_t^{\theta} L_t^{1-\theta} - W_t L_t - I_{t.}$$

The firm does not issue new shares and finances investment through retained earnings. For convenience we normalize the number of shares outstanding to unity so that P_t^S is also the stock price. In this environment, the basic asset valuation condition holds:

$$P_t^S = E_t \left[\beta \frac{\Lambda_{t+1}}{\Lambda_t} \left(D_{t+1} + P_{t+1}^S \right) \right]. \tag{2}$$

Finally, the firm's first order conditions together with competitive labor markets imply that workers are paid their marginal products: $W_t = (1-\theta) Z_t (K_t/L_t)^{\theta}$.

Stockholders and Non-stockholders

Both agents have one unit of time endowment in each period, which they supply inelastically to the firm. Besides the productive capital asset there is also a one-period riskless household bond (in zero net supply) that is traded in this economy. The crucial difference between the two groups is in their investment opportunity sets: "Non-stockholders" can freely trade in the bond, but as their name suggests, they are restricted from participating in the capital market. "Stockholders," on the other hand, have access to both markets and hence are the sole capital owners in the economy. Following the incomplete markets literature we impose portfolio constraints as a convenient way to prevent Ponzi schemes. As we discuss in the quantitative analysis these constraints can be quite loose and actually they never bind in our simulations.

The timing of events is as follows: each period starts with production; agents are paid their wages and asset returns are realized after production takes place. Then consumption and portfolio choice decisions are made and asset trading is carried out. Finally, consumption takes place and the period ends. Before we move to agents' problem a final remark is in order.

Remark: It is possible to think of the participation structure assumed here as an endogenous outcome of a model where there is a one-time fixed cost of participating. With a cost of appropriate magnitude, type 1 agents (with low risk aversion) will enter the stock market whereas the other group will stay out. The resulting equilibrium is identical to the one studied here; see Guvenen (2002) for a further discussion about endogenizing participation. We feel that for the purposes of the current paper this is a reasonable assumption and our main conclusion is not likely to be overturned by this extension.

Individuals' Dynamic Problem and the Equilibrium

In order to state the individual's problem recursively, we need to specify the aggregate state-space for this economy. The Markov characteristic of the exogenous driving force naturally suggests concentrating on equilibria which are dynamically simple. Thus, the portfolio holdings of each group together with the exogenous technology shock constitute a sufficient state space which summarizes all the relevant information for the equilibrium functions.

In a given period, the portfolios of each group can be expressed as functions of the *beginning-of-period* capital stock, K, the aggregate bond holdings of non-stockholders *after* production,

B, and the technology level, Z. Let us denote the financial wealth of an agent in the current period by ω where we suppress superscripts for clarity of notation. The dynamic programming problem of a stockholder can be expressed as follows:

$$\begin{aligned}
V^h(\omega; K, B, Z) &= \max_{b',\ s'} \left\{ U(c) + \beta E\left[V^h(\omega'; K', B', Z') \mid \Omega \right] \right\} \\
& \quad s.t \\
c + P^B(K, B, Z) * b' + P^S(K, B, Z) * s' &\leq \omega + W(K, Z) \\
\omega' &= b' + s' * \left(P^S(K', B', Z') + D(K', B', Z') \right) \\
K' &= \Gamma_K(K, B, Z) \\
B' &= \Gamma_B(K, B, Z) \\
b' &\geq \underline{B}^h,
\end{aligned}$$

where the expectation is conditional on the set Ω containing all the information at the time of decision, and b' and s' denote bond and stock holdings respectively. The endogenous functions Γ_K and Γ_B denote the laws of motion for aggregate wealth distribution which are determined in equilibrium; P^B denotes the equilibrium bond pricing function. Note that each agent is facing a constraint on bond holdings with possibly different (and negative) lower bounds. The problem of the non-stockholder can be written as above with $s' \equiv 0$.

A stationary recursive competitive equilibrium for this economy is given by a pair of value functions $V^i(\omega^i; K, B, Z)$, $(i = h, n)$, bond holding decision rules for each agent $b^i(\omega^i; K, B, Z)$, stockholding decision for the stockholder, $s(\omega^h; K, B, Z)$, stock and bond pricing functions, $P^S(K, B, Z)$ and $P^B(K, B, Z)$, a competitive wage function, $W(K, Z)$, an investment function for the firm, $I(K, B, Z)$, and laws of motion for aggregate capital and aggregate bond holdings of non-stockholders, $\Gamma_K(K, B, Z)$, $\Gamma_B(K, B, Z)$, such that:

1) Given the pricing functions and the laws of motion, the value functions and decision rules of each agent solve that agent's dynamic problem

2) Given the equilibrium discount rate process $\frac{\Lambda_{t+j}}{\Lambda_t}$ and $W(K, Z)$, the investment function $I(K, B, Z)$ and labor choice are optimal.

3) Bond market clears: $\lambda b^h(\varpi^h; K, B, Z) + (1 - \lambda) b^n(\varpi^n; K, B, Z) = 0$, where ϖ^i denote the aggregate wealth of a given group; and the labor market clears: $L = \lambda * 1 + (1 - \lambda) * 1 = 1$.

4) Aggregates result from individual behavior:

$$K_{t+1} = (1 - \delta) K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t, \tag{3}$$

$$B_{t+1} = (1 - \lambda) b^n(\varpi^n, K_t, B_t, Z_t). \tag{4}$$

5) There exists an invariant probability measure $\mathbf{P}$ defined over the ergodic set of equilibrium distributions.

3 Quantitative Analysis

Since an analytical solution is not possible, we use numerical methods to solve for the equilibrium. The task is quite challenging requiring solutions to three dynamic programs (one for each agent plus one for the firm) where each program depends on the solution of other problems in quite a nonlinear way. For example, the firm takes the discount factor (stockholders' MRS) as given where in fact that MRS is obtained from the solution to the stockholder's problem who in turn takes the investment decision and stock prices as given. The crucial step is the first one: to get good initial guesses for equilibrium laws of motion and especially for pricing functions. We relegate the discussion of these and other computational issues (as well as the accuracy of the solution) to the Appendix.

Baseline Parameterization

A currently common method for calibrating asset pricing models is to choose a number of parameters to match certain *financial statistics*, such as the risk-free rate and the equity premium (as in Jermann 1998, and Boldrin, et. al. 1999), the persistence of the price-dividend ratio (Campbell and Cochrane 1999) and so on. Then additional moments of the data serve as overidentifying restrictions to be examined. In contrast, we follow the real business cycle tradition and calibrate the parameters to replicate *the long-run macroeconomic facts* of the U.S. economy such as the average capital-output ratio, the persistence of the Solow residuals and so on. In particular, no parameter is chosen to replicate a financial statistic.

The time period in the model corresponds to 3 months of calendar time. The capital share of output, θ, is set equal to 0.4 following Cooley and Prescott (1995). As for the technology shock, we match the persistence of the Solow residual, $\rho = 0.95$, and set the standard deviation of the innovation equal to 2 percent. Although this latter number is larger than the one reported in Cooley and Prescott (1995), it is consistent with the estimates obtained by Christiano and Eichenbaum (1992), and the values used in Danthine and Donaldson (2001), and Storesletten, Telmer and Yaron (1999) among others. Moreover, we will compare the model to data extending back to 1890, and given that output and consumption were substantially more volatile prior to World War II, this seems like a sensible choice. We use a 12-state Markov chain to discretize the AR(1) process for Z_t following Tauchen and Hussey's (1991) method. As Table 11 in the Appendix shows, the autocorrelation structure (from lag 1 to 5) of the approximation tracks the AR(1) process quite closely, and the standard deviation is equal to the true value.

To complete the description of technology, we choose the adjustment cost parameter, $\xi = 0.23$, which is the value used in a number of recent papers (Jermann 1998, Boldrin et. al 1999, Francis and Ramey 2002). This value is near the low end of the empirical estimates for this variable (the elasticity of investment with respect to Tobin's q), so we will also conduct sensitivity analysis with respect to this parameter.

Participation rates: The stock market participation rate has gradually increased from around 5 percent in the 1950s to approximately 19 percent in 1982 (Survey of Consumer Finances). During 1990s, for many reasons ranging from the emergence of mutual funds and reduced costs of (on-line) trading to the retirement saving by baby-boomers, this trend has accelerated in the U.S. as in the rest of the world. As a result, in 1999 the stockholding rate has reached almost 50 percent.[5] Since in this paper we are studying a stationary economy, we want to focus on the pre-1990s U.S. economy. Moreover, a significant fraction of households are holding very small amounts of stocks. For example, in the 1984 PSID data, 24 percent of households declare themselves as stock owners whereas the fraction holding more than $10,000 worth of stocks is less than 10 percent (Mankiw and Zeldes 1991). A different way to put this is that more than 95 percent of all stocks is held by the top 10 percent stock owners. (This is true even in 1998 SCF data, after the participation boom of the '90s). Thus, in the baseline case the fraction of stockholders, λ, is set equal to 20 percent. We will also report results for $\lambda = 30\%$, which is close to the average raw participation rate in the 1980s. The main conclusion we will draw is that it does not affect the main message of the paper (Section 8).

Borrowing constraints are harder to measure and calibrate. We want to choose these bounds to reflect the fact that stockholders can potentially accumulate capital which can then be used as collateral for borrowing in the risk-free asset, whereas non-stockholders have to pay all their debt through future wages. For the baseline case, we allow stockholders to borrow in bonds up to twelve quarters of expected labor income ($\underline{B}^h = 12 * E(W)$). As for non-stockholders, we calibrate their borrowing limit to two quarters of expected income, which is twice the credit limit most short-term creditors, such as credit card companies, impose. Again, these constraints do not bind in our simulations and can be relaxed without affecting any of the main results.

Preference Parameters

The subjective discount factor, β, is set equal to 0.99 in order to match the U.S. capital-output ratio of 3.3 reported by Cooley and Prescott (1995). We calibrate the curvature parameter α based on the implied elasticity of intertemporal substitution. There is a large body of evidence documenting significant heterogeneity in the EIS across population (Blundell, Browning and Meghir 1994; Attanasio and Browning 1995; Atkeson and Ogaki 1996; Barsky, Juster, Kimball and Shapiro 1997; Attanasio, Banks and Tanner 2002; Vissing-Jorgensen 2002).[6] Furthermore, using individual level data, these studies find that stockholders (or the wealthy in general) have significantly higher elasticity of substitution than the poor. To capture these differences in a parsimonious way we set $\alpha^h = 2$ and $\alpha^n = 10$ which is close to values obtained by Attanasio et. al. Note that although with the CRRA utility adopted in our baseline model these values also give rise to heterogeneity in the risk aversion, for *none* of our results this heterogeneity plays an essential role. In Section 8, we disentangle the two parameters using Epstein-Zin preferences and show that with the same EIS values used here, if we set both agents' risk aversion parameters to two, the results of the paper remain intact. What matters for the results is mainly the risk aversion of stockholders and the EIS of non-stockholders. Table 1 summarizes our baseline parameterization.

[5] In terms of wealth-weighted participation rates, participation boom is less pronounced because old stockholders still own most of all the equity outstanding (the "Equity Ownership in America" report by the Investment Company Institute 1999).

[6] See Browning, Hansen and Heckman (1999) and Guvenen (2002) for detailed descriptions of this evidence and discussions of the importance of heterogeneity in the EIS in various contexts.

Table 1: Baseline Parametrization

Quarterly Model		
Parameter		Value
β	Time discount rate	0.99
α^h	Risk aversion of stockholders	2
α^n	Risk aversion of non-stockholders	10
λ	Participation rate	0.2
ρ	Persistence of aggregate shock	0.95
σ_ε	Standard deviation of shock	0.02
θ	Capital share	0.4
ξ	Adjustment cost coefficient	0.23
δ	Depreciation rate	0.02
$\underline{B}^h$	Borrowing limit of stockholders	$12\overline{W}$
$\underline{B}^n$	Borrowing limit non-stockholders	$2\overline{W}$

Note: The baseline model assumes CRRA utility function for both agents implying that the respective elasticities of intertemporal substitution are 0.5 and 0.1 for stockholders and non-stockholders respectively. The borrowing limits are indexed to the average wage rate, $\overline{W}$.

4 Results: Asset Prices

In this section we study the asset pricing implications of our model. We will look at a large number of pricing phenomena that have received attention in the literature.

We start with the first two moments of the stock and bond return. Table 2 displays the statistics from the simulated model along with their empirical counterparts from the U.S. historical return data taken from Campbell (1999). The "long sample" corresponds to the period $1890-1991$, and the "post-war period" covers $1947-1991$. When calculating the stock return in the post-war data there is one point worth mentioning. In a recent paper McGrattan and Prescott (2001) convincingly argue that approximately 1.8 percent of yearly stock returns from 1960 onward can be attributed entirely to favorable changes in the tax code (and specifically to the reduction of dividend and corporate income taxes). Since the model here abstracts from all taxes we subtract 1.8 percent from stock returns each year after 1960. With this adjustment the equity premium is roughly 5 percent per year in both sample periods.

Comparing the model to the data, we first see that the average risk-free rate exactly matches empirical values. Second, the stock return is 5.3 percent compared to 7 percent in the data and consequently the equity premium in the model is around 3.3 percent, which is approximately 2/3 of the historical value. Although, by slightly increasing the risk aversion of stockholders (from two to three), the equity premium rises to 4.9 percent and the Sharpe ratio to 0.29 without any pronounced side effects (column 4), we will not pursue this strategy here. The reason is that the risky return in the model corresponds to the stock of a firm which is entirely equity financed (hence unlevered) unlike the empirical counterpart which measures the return

Table 2: Moments of Asset Prices and Returns

	US Data		Model		RBC
	Long Sample	Post-War	$\alpha^h = 2$	$\alpha^h = 3$	
	Panel A: The Risky Rate and Risk-free Rate				
$E\left(R^S\right)$	6.89	7.04	5.30	6.53	4.16
$\sigma\left(R^S\right)$	18.2	16.7	14.1	16.7	0.37
$E(R^f)$	1.91	1.68	1.98	1.63	4.16
$\sigma(R^f)$	5.44	2.23	5.73	6.51	0.18
$E(R^S - R^f)$	**4.82**	**5.36**	**3.32**	**4.90**	**.004**
$\sigma(R^S - R^f)$	19.1	16.8	14.7	17.1	0.27
$\frac{E(R^S - R^f)}{\sigma(R^S - R^f)}$	**0.25**	**0.32**	**0.23**	**0.29**	**.014**
$\rho(R^S, R^f)$	−0.04	0.02	−0.01	0.02	0.97
	Panel B: The Stock Price and Dividend				
$\sigma\,(d)$	13.3	13.6	11.2	10.6	–
$p - d$	21.1	24.7	23.2	23.8	–
$\sigma(p - d)$	27.0	26.2	15.6	19.6	–

Notes: The mean and standard deviation of each variable is annualized and reported in percentages. The data is from Campbell (1999) and covers 1890-1991. Long sample covers the entire period and post-war data is from 1947 to 1991. Following McGrattan and Prescott (2001) we subtract 1.8 percent per year from stock returns after 1960 to adjust for tax changes as described in the text.

to U.S. firms which have significant debt in their capital structures.[7] The effects of leverage in production economies are well understood: it raises both the level and the volatility of the stock return (see Benninga and Protopapadakis 1991, and Cecchetti, Lam and Mark 1993). Thus, it does not seem reasonable to try hard to match the risky rate in the current framework otherwise introducing debt into the capital structure would cause us to overshoot the risk premium.[8] Of course one interesting question is: How does the model generate a large equity premium with standard preferences and low risk aversion? The mechanism here is quite different compared to a representative agent model and we defer that discussion to the end of this subsection.

Second, the volatility of the risky rate is around 14 percent which is slightly lower than in the data ($17 - 18$ percent). This small discrepancy is again consistent with the fact that there is no leverage in the model. Also in the data the risky rate and the equity premium have very similar variabilities (18.2 versus 19.1) which is captured well by the model (14.1 versus 14.7). As can be anticipated from these numbers, stock and bond returns are almost uncorrelated: $\rho(R_t^S, R_t^f) \approx 0.0$ in the long sample as well as in our simulations. For comparison the two returns have a correlation of 0.97 in the standard RBC model. We also report the corresponding statistics for the standard RBC model in the last column, but since the shortcomings of that

[7] Jermann (1998) reports from Masulis (1988) that the debt-to-value ratio varies between 0.13 to 0.44 for market values and between 0.53 to 0.75 for book values.

[8] In principal it is very easy to incorporate leverage into our model. However, the computational demand of the current version is already extremely high. So we postpone that exercise for now.

Table 3: VOLATILITY OF THE RISK-FREE RATE IN VARIOUS MODELS

		US Data Long Sample	This Paper	Boldrin et. al.	Jermann	Campbell-Cochrane
β		–	0.99	0.99999	0.99	0.97
$E(R^f)$	%	1.91	1.98	1.20*	0.82*	0.94*
$\sigma(R^f)$	%	5.44	5.73	24.6	11.46	0.0*

Notes: A "*" indicates that some parameters in that particular model were chosen to match the statistic. The volatility statistic from Boldrin et. al is the result in their baseline model (preferred two-sector). In Jermann (1998) the figure is from the baseline case with stationary shocks; the volatility is 11.98 percent with random walk shocks.

framework are quite well-known we do not comment on them here (see Rouwenhurst, 1995).

Now we return to the risk-free rate. As noted above, the risk-free rate is low with a reasonable (and quite standard) discount rate of 0.99. But arguably the more interesting feature is the low volatility of the interest rate that has proved quite a challenge even for recent models which have been quite successful otherwise (Table 3). For example, in Boldrin et. al (1999) the variability ranges from 17.4 percent all the way up to 25.4 percent and in fact the risk-free asset is more volatile than the stock in all specifications they consider (except when they introduce leverage in which case the risky rate goes up to 25 percent). Cognizant of this fact C-C chose the functional form and the free parameters for the law of motion of the exogenous habit process judiciously to generate a constant risk-free rate.

In contrast, and to our pleasant surprise, the limited participation model yields the same volatility for the bond return as in the U.S. data without any degrees of freedom to choose. This outcome will appear even more surprising later when we further document many similarities between the asset pricing implications of this model and the habit persistence framework. So, what explains the difference?

To clearly see the difference consider the bond market diagram in Figure 1. The left panel depicts the case of a representative agent with habit formation in consumption. Even though external and endogenous habit differ in the risk aversion they imply, both specifications result in very low EIS and hence in a very inelastic bond demand curve. The interaction of this steep demand curve with the bond supply which is also *perfectly inelastic* at zero (due to the representative agent assumption) means that even small shocks to demand will generate large movements in the bond price and hence a substantial volatility of the risk-free rate.[9] On the other hand, in our framework the explanation is quite simple (right panel) . First, note that just like the representative agent above, the majority of the population (non-stockholders) have very inelastic bond demand. The crucial difference is that the bond supply is not inelastic at all. In fact, stockholders' supply schedule is quite flat both because of their preferences ($EIS \geq 0.5$)

[9] Campbell and Cochrane observe this problem and ingeniously (but also in an ad-hoc manner) specify the habit process such that shocks to this demand function exactly cancel out and the bond price never moves.

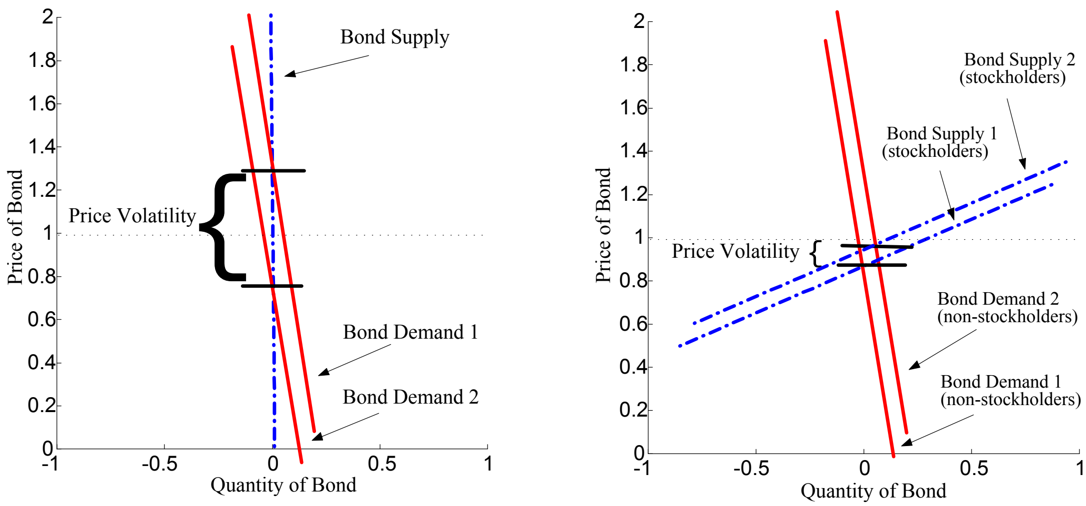

Figure 1: Determination of Bond Price Volatility in A Representative Agent Model (Left) versus in the Limited Participation Model (Right)

and also because they have another asset to substitute. A movement in the bond demand of similar magnitude now results only in small changes in the interest rate and the rest will be reflected in the variability of trade volume in the bond market.[10] The existence of shocks to stockholders does not overturn this result. This explanation also captures the fact that unlike in representative agent models people do actually trade in bond markets even though the total supply may be zero and it seems that heterogeneity among participants is an important factor determining asset prices.

We now move from returns to prices. The lower panel in Table 2 displays the moments of the stock price and the dividend process. First, the volatility of the dividend process is around 11 percent in the model which is quite close to what is in the data. Second, the average price-dividend ratio in the model is around 23, in line with the empirical evidence. The P/D ratio however seems a little too smooth (but higher when $\alpha^h = 3$) which could increase once leverage is introduced into the capital structure.

How Does the Model Generate a Large Risk Premium?

Saito (1995) studied a model with limited participation where both agents consume out of wealth. He showed that the equity premium is linearly increasing in the share of wealth owned by non-stockholders. A large premium is then possible if stockholders borrow a significant fraction of the capital invested from non-stockholders. This is basically a leverage story: high borrowing makes stockholders' portfolio risky and they demand a high premium to hold stocks. Given that there is no labor income and non-stockholders constitute a large fraction of the population this

[10] The terms demand and supply are rather arbitrary here since both groups do demand bonds depending on the interest rate. We denote $-b^h$ as the bond supply of stockholders for convenience and because they are actually net borrowers in equilibrium.

outcome happens naturally, potentially yielding a high equity premium. However, in the long-run non-stockholders' share of wealth goes to zero (in a growing economy without labor income) and so does the premium. Furthermore, in reality even though U.S. firms have significant leverage ratios, a large fraction of this debt is in fact held by other stockholders (either as corporate bonds or other non-equity financial assets). An arguably better measure is to look at the fraction of productive capital owned by stockholders (capital in publicly traded companies plus private businesses) as a fraction of the total. By this measure stockholders own more than 90 percent of all financial assets and private capital which means that total borrowing cannot be more than 10 percent of aggregate capital (Poterba and Samwick 1995). This amount is too small to yield any significant risk premium.

As we demonstrate in Section 5 when we analyze the macroeconomic implications, stockholders in fact hold 90 percent of wealth in our economy too. So there must be another mechanism generating the risk premium. What happens here is that both agents face significant risks in their labor income which is also persistent given the AR(1) nature of the shock. This feature is absent in Saito (1995) and creates more demand for precautionary wealth in our model. Non-stockholders can do that only through holding bonds which is possible when stockholders borrow. But note that stockholders can also smooth shocks ex-post by saving and dissaving the capital. This is an option not available to non-stockholders. Further exacerbating the situation for this poor group is the fact that they have very low EIS and thus they desperately want to insure (a feature absent in Saito 1995, and Basak and Cuoco 1998). This asymmetry in market incompleteness coupled with heterogeneity in preferences results in stockholders only trading if non-stockholders pay a high premium. The high premium in turn depresses the risk-free rate which means accumulating wealth does not pay off for non-stockholders and they end up with low wealth in equilibrium. In other words, the fact that the observed borrowing of stockholders is not high does not mean that non-stockholders do not demand wealth. They desperately do but their very demand lowers the return they face in turn lowering their wealth holdings.

This explanation also highlights an important difference of this framework from a model with hand-to-mouth consumers, advocated for example by Campbell and Mankiw (1989). Since non-stockholders hold only a tiny fraction of capital in the U.S. economy (as well as in our model), it is sometimes suggested that these households behave as if they consume their income period by period. However, such a modeling would miss the important interaction in the bond market described above and eliminate the main mechanism which gives rise to the equity premium.

The Effect of Adjustment Costs

The mechanism just described for the equity premium more precisely determines the price of risk, or in other words, the premium for each unit of the risk. Thus the level of the premium is also determined by the volatility of the return. In a frictionless production economy this volatility is very low with the small size of the aggregate shock. Introducing adjustment costs is one way to reduce the elasticity of capital supply and increase the volatility of returns and as a result the *level* of the equity premium. However, adjustment costs have little effect on the Sharpe ratio which is displayed in Table 4. The Sharpe ratio goes from 24 percent down to 22 percent when we move essentially from an exchange economy ($\xi = 0.01$) to the frictionless limit

Table 4: Sharpe Ratio for Different Values of the Adjustment Cost Parameter

	Adjustment Cost Parameter (ξ)				
	0.01	0.10	0.23	0.5	∞
Sharpe Ratio	0.238	0.234	0.229	0.223	0.216

The reported Sharpe ratios are from a version of the baseline model with a two point approximation to the AR(1) process (due to computation time) where the standard deviation is adjusted to keep the same unconditional standard deviation of Z.

when the cost is completely eliminated. Furthermore, the other asset pricing results studied below are not sensitive to the adjustment cost: even setting $\xi = 2$, which is quite higher than empirical estimates has no appreciable effect.

Finally, we should also stress that we use adjustment costs as a simple way to introduce a friction into the technology and also to make our results comparable to recent papers which employed the same specification. However, this particular type of friction is not an essential part of the model. Alternative ways to increase the volatility of return such as stochastic depreciation (as in Storesletten et. al 1999) could also generate a high *level* of risk premium.

4.1 Cyclical Properties of Returns

Even a few years ago explaining the first two moments of returns would probably be enough to declare success. However, the hugely successful paper by Campbell and Cochrane raised the bar higher. We will thus examine the implications of the model in a number of dimensions that have received attention both in the finance and macroeconomic literatures.

A large literature in finance has documented the predictability of stock returns, and in particular, the fact that many of the variables predicting the returns also predict the business cycle (Campbell and Shiller, 1988; Poterba and Summers, 1988; Fama and French, 1989, among others). Most of these studies focus on the predictive power of such variables as the dividend-price ratio, the default spread or the term premium which also closely track business cycle fluctuations. Furthermore, Fama and French (1989) documented the *long term co-movement* between the D/P ratio and business conditions that extend well beyond the duration of a typical business cycle. They state:

> The major movements in these variables (D/P ratio and default premium) and in the expected return components they track, seem to be related to longer term business episodes that span several measured business cycles. The dividend yield and the default spread forecast high returns when conditions are persistently weak and low returns when conditions are persistently strong.

Figure 2 plots the D/P ratio together with the NBER business cycle dates. The long-run co-movement is striking. The D/P ratio is high during the 1930s and the early years of World War II, and is persistently low from 1955 to 1973, a period of stronger economic conditions that

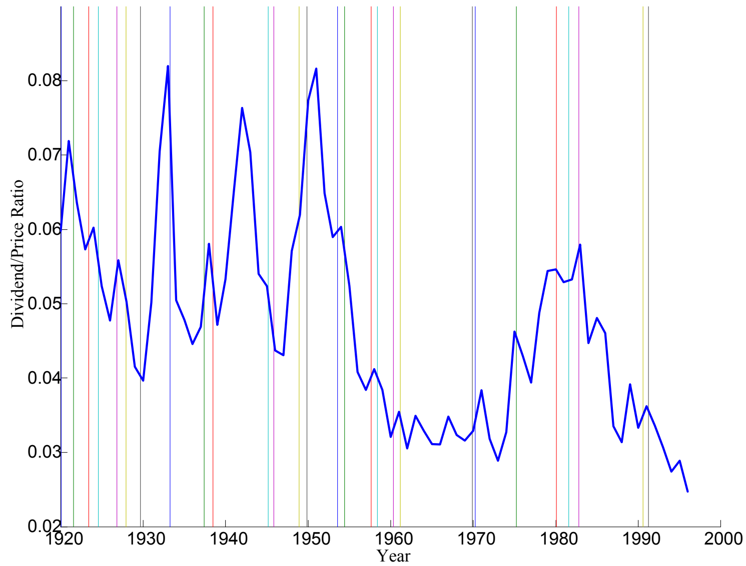

Figure 2: Co-movement Between the Dividend/Price Ratio and Business Conditions

nevertheless includes four recessions, and increases after the oil shock of 1973 and remains high until mid-80s.

To further substantiate this claim Fama and French (1989) regress future risky returns on the D/P ratio *and* the term premium where each variable captures the long- and short-term co-movement with output respectively and find *both of them* to be significant. Moreover each variable increases the R^2 of the regression when added separately.

In order to test whether the model can capture the twists and turns of the relationship between financial variables and business conditions we conduct the following exercises. We first investigate if the model can account for the cyclical movement of asset prices including the countercyclicality of the expected equity premium, the conditional volatility and the Sharpe ratio documented in the literature (see Schwert 1989, Nelson 1991, and Chou, Engle and Kane 1992). Moreover, in the next subsection we also replicate the long-horizon regressions of expected returns on the D/P ratio of Campbell and Shiller (1988).

One advantage of having a fully specified model is that we can compute $E_t\left(R^S_{t+1}\right)$ and $\sigma_t\left(R^S_{t+1}\right)$ from the equilibrium functions exactly (rather than having to regress the return data on variables predicting business conditions) and examine their co-movements with aggregate output directly. First, in order to separate high frequency fluctuations typically associated with business cycles from longer term movements we follow the RBC tradition and filter output using a Hodrick-Prescott filter. We then document the co-movement both in the short-run and in the long-run as stressed by Fama and French. Table 5 presents the relevant statistics.

Column 1 shows that besides the long-term positive co-movement with output depicted in Figure 2, the price-dividend ratio is also pro-cyclical at business cycle frequencies. Both of these correlations are captured by the model. Second, an interesting observation is that the realized

Table 5: Cross-Correlation of Financial Variables with Output

	US Data		Model	
Variable	Short	Long	Short	Long
$p_t - d_t$	0.13	p-cycl*	0.53	0.94
$R^S_{t+1} - R^f_t$	0.31	p-cycl	0.30	0.33
$E_t\left(R^S_{t+1}\right)$	c-cycl*	c-cycl	−0.64	−0.92
$E_t(R^S_{t+1} - R^f_t)$	c-cycl	c-cycl	−0.31	−0.51
$\sigma_t(R^S_{t+1} - R^f_t)$	c-cycl	c-cycl	−0.28	−0.47
$\frac{E_t(R^S_{t+1}-R^f_t)}{\sigma_t(R^S_{t+1}-R^f_t)}$	c-cycl	c-cycl	−0.34	−0.51

Notes: All statistics are annual from the long sample covering 1890-1991. The columns with headings Short and Long report the correlation with output at business cycle frequencies and in the long-run respectively. The notation p-cycl and c-cycl denote pro- and counter-cyclical.

equity premium is *procyclical* even though ex ante excess returns are strongly *countercyclical* suggesting that innovations to returns and output move in the same direction. This feature is also true in the model: the correlation of realized excess returns from the model is equal to its empirical counterpart (0.30), even though the expected return is countercyclical (−0.31).

As for the conditional volatility of returns, there are two well-documented facts. One, conditional volatility is countercyclical in the U.S., which is also true in our model both in the long-run (−0.47) and at business cycle frequencies (−0.28). Two, conditional volatility is highly persistent and thus clusters over time. This feature is clearly seen in our model as well: the quarterly first order autocorrelation of $\sigma_t\left(R^S_{t+1} - R^f_t\right)$ is 0.85. Moreover, peeking ahead to Table 8 (bottom of Panel A) we see that absolute stock returns are positively autocorrelated just like in the data, which again is another indication of persistence in variability.

The countercyclical variation in volatility would seem to provide a justification for the countercyclical movement in the risk premium. After all, the premium can be expressed as the product of the price of risk $(\frac{E_t(R^S-R^f)}{\sigma_t(R^S-R^f)})$ and the quantity of risk $\left(\sigma_t(R^S - R^f)\right)$, and the movement in the latter will give rise to a change in the risk premium even if the former is constant. However, empirical evidence does not seem to back up this argument. Expected returns move a lot more than conditional volatility, and hence the price of risk (Sharpe ratio) is also countercyclical (Chou, Engle and Kane 1992). The same pattern emerges in simulated data both in the long-run (−0.51) and in the short-run (−0.34).

This result is due to the significant countercyclical redistribution of wealth away from stockholders which makes them effectively more risk averse in recessions because of the concavity of the value function (as a result of precautionary motives). On the other hand, the change in non-stockholders' wealth is much more modest. As a result, the bond price jointly determined by both groups is not affected as much creating countercyclical movement in the price of risk.

Table 6: Predictability of Stock Returns: Long-Run Regression on Price/Dividend Ratio

	US. Data					Model		
	Long Sample			Post-War				
Δd included?	No	No	Yes	No	No	No	No	Yes
Horizon	Coeff.	R^2	R^2	Coeff.	R^2	Coeff.	R^2	R^2
1	−0.08	.06	.06	−0.10	.11	−0.12	.12	.12
3	−0.16	.12	.13	−0.18	.31	−0.20	.33	.33
5	−0.25	.21	.22	−0.33	.51	−0.28	.41	.41
7	−0.34	.27	.28	−0.44	.57	−0.36	.48	.48
10	−0.36	.39	.39	−0.64	.73	−0.41	.53	.53

Notes: The data is from Campbell (1999) and covers 1890-1991. The coefficients for the regression when Δd are very similar to those with dividends left out and hence are not reported.

4.2 Predictability of Returns: Long Horizon Regressions

The previous section documented the fact that expected returns are correlated with the business conditions and hence are predictable in our model. Although these cyclical patterns are interesting they do not tell us directly *the size of the predictable component* in stocks. The long-horizon regressions of Campbell-Shiller, and Fama-French provide a more direct answer. Furthermore, because many of the conditional moments cannot be easily calculated from data, these regressions form the basis of most empirical evidence. Thus we replicate these regressions in our framework which also serve as a more direct comparison of the model to existing evidence.

We first regress stock returns on the price/dividend ratio for the long sample and post-war U.S. data (Table 6). The classic pattern documented in the literature can seen here: the coefficients are negative indicating that a high P/D ratio forecasts lower returns in the future. Moreover both the coefficients and the R^2 values are increasing with horizon and reach impressive levels.

The model counterpart is reported in the last three columns. Both the coefficient estimates and the R^2 values are strikingly similar to the empirical results: predictability is modest at one year horizon but increases steadily and reaches 50 percent at 10 year horizon. The coefficients also increase quickly first and then grow more slowly. Another interesting finding by Campbell and Shiller is also presented in column 3. Basically, lagged dividend growth has almost no predictive power once the P/D ratio is included in the regression. The R^2 values remain virtually unchanged and the coefficients (not reported) are not significant. The same result is replicated in simulated data where the R^2 values do not change up to the third digit and the coefficients are mostly insignificant even with a simulated sample size of 15,000 years.

An alternative manifestation of return predictability is the excess volatility of stock prices. A simple way to see this is by first decomposing the variance of the log P/D ratio following

Table 7: Variance of Price-Dividend Ratio Explained by Future Covariances

Variance explained by	U.S. Data		Model
	Long Sample	Post-War	
r^S_{t+j}	101	137	124
Δd_{t+j}	−10	−31	−27

Notes: Each cell reports the percentage of variance explained by the corresponding variable (that is, $\left(\sum_{j=1}^{\infty} \gamma^j cov\left(p_t - d_t, x_{t+j}\right)\right) / var(p_t - d_t)$ where x is Δd and $-r^S$ in each case). The formula is calculated using 15 lags (years) both in the data and in the model.

Cochrane (1992). Defining $\gamma = \left(\overline{P}^S / \overline{D}\right) / \left(1 + \left(\overline{P}^S / \overline{D}\right)\right)$ at the steady state, we have:

$$var(p_t - d_t) \approx \sum_{j=1}^{\infty} \gamma^j cov\left(p_t - d_t, \Delta d_{t+j}\right) - \sum_{j=1}^{\infty} \gamma^j cov\left(p_t - d_t, r^S_{t+j}\right)$$

In the U.S. data (Table 7) a substantial fraction of total volatility is accounted for by the covariance of the log P/D ratio with future returns and only a small component is explained by varying expectations of future dividend growth. Moreover, both autocovariances are negative, consistent with the idea that a high P/D ratio signals low dividend growth which in turn means low returns in the future. The model captures both aspects of the data, and the covariances fall between the values observed in the post-war data and in the long-sample.

4.3 Time Series Properties: Autocorrelation and Cross-correlation

The autocorrelation structure of financial variables display some very interesting patterns. For example, although the risk-free rate is highly persistent, the risky rate (as well as the equity premium) has no significant persistence and displays mild mean reversion. On the other hand, the absolute value of the risky rate is positively autocorrelated both at short and long-horizon indicating clustering of volatility. Finally, as noted earlier the dividend yield (D/P) is also extremely persistent and its fluctuations extend well beyond the duration of typical business cycle episodes.

In the standard RBC model many variables have time-series behavior very similar to that of the exogenous technology shock. This is not totally surprising given that the assumed functional forms with little curvature together with a single source of uncertainty ties most variables tightly to the properties of the shock. In this sense the variety of autocorrelation patterns observed in financial data might seem too rich to be explained with our model which is based on the same framework.

The model turns out to be surprisingly successful in explaining the time series of financial data. For example, as reported in Table 8, the autocorrelation structure of the P/D ratio matches that in the data all the way from 1-year to 10-year horizon. C-C view this persistence as

Table 8: Autocorrelation Structure of Key Financial Variables

		LAG (years)					
		1	2	3	5	7	10
		Autocorrelation					
$p-d$							
US Data		.79	.59	.52	.35	.32	.23
Model		.80	.64	.52	.35	.23	.18
$r^s - r^f$							
US Data		.03	$-.22$	.08	$-.14$	.10	.12
Model		$-.02$	$-.04$	.01	$-.01$	$-.01$	$-.02$
r^s							
US Data		$-.02$	$-.17$	.10	$-.11$	.08	.09
Model		$-.12$	$-.07$	$-.05$	$-.03$	$-.02$	$-.03$
r^f							
US Data	(R^*)	.53	.36	.23	.14	.15	.10
US Data	(N^*)	.83	.73	.69	.60	.57	.43
Model		.84	.66	.52	.34	.22	.11
$\|r^s\|$							
US Data		.13	.09	.06	.14	.15	.07
Model		.08	.06	.04	.04	.04	.03
		Cross-Correlation					
$p_t - d_t, \|r^s_{t+j}\|$							
US Data		$-.12$	.02	$-.06$	$-.10$	$-.05$	$-.04$
Model		$-.16$	$-.13$	$-.10$	$-.07$	$-.04$	$-.03$

Notes: The empirical statistics are calculated for the period 1890-1991. The rows denoted by R* and N* report the correlation for real and nominal interest rates respectively.

key for their results and they calibrate one of the free parameters to match this autocorrelation structure. In contrast, here the model delivers this result as a natural outcome.

Second, the model generates weak mean reversion in the equity premium similar to that in the data. The autocorrelation is very small and tends to revert to its mean. The sum of the first k autocorrelations of excess returns (for $k = 1, .., 10$) is also small and negative in the U.S. data and is also tracked quite well by the model (not reported). For comparison, in the RBC model the first autocorrelations of the risky rate and excess return are 0.63 and 0.55 respectively and are still above 0.30 at 10 year horizon

Third, since the risk-free rate is time-varying in our model we can examine its time series behavior, a feature absent from C-C's framework. The interest rate is highly persistent in the model. Measuring the empirical counterpart is a little tricky because in reality bonds are only nominally riskless since they have inflation risk.[11] Using the ex-post real rate is one possible

[11] One could potentially look at index bonds to get a measure of the real interest rate. Index bonds however are very new in the US, and they have been traded in the UK since 1982. Also these bonds have an indexation lag

approach (the row denoted by R^*) but due to unanticipated inflation, the autocorrelation structure calculated this way is downward biased. An alternative option is to use the ex-ante nominal return which is probably a better indicator of the risk-free rate investors anticipate (the row denoted with an N^*). This series is significantly more persistent. The truth probably lies somewhere in between and so does the risk-free rate in the model. The persistence is the same as that of the nominal rate for short horizons and falls to levels closer to the autocorrelation of real interest rates at longer horizons (for 5 to 10 years).

Finally, the model also generates the correct pattern of volatility clustering and predictability: absolute returns are positively autocorrelated consistent with the U.S. data implying that high volatility is usually followed by more volatility. This evidence complements the high persistence revealed in the conditional standard deviation ($\rho(\sigma_t, \sigma_{t+1}) = 0.85$) reported earlier. Furthermore, as shown in the lower panel, a low P/D ratio predicts higher volatility in the future and both the sign and size of the autocorrelation structure are similar to empirical values. This is the leverage effect documented by Schwert (1989) and Nelson (1991) among others.

Correlation of Consumption Growth with Asset Returns

One interesting observation about the linkage between asset prices and quantities is that the market return seems to do a better job in explaining asset returns than does consumption growth. An alternative statement of this fact is that the static CAPM outperforms consumption-based asset pricing models. C-C show that when consumption is aggregated over time before calculating the growth rate, the same relation holds in their model as well which is surprising since the true data generating process for asset prices is the consumption-based model.

At this point, may be not too surprisingly, this result holds in our model too. The correlation of the true discount factor (M) with the growth rate of time-aggregated consumption falls very quickly (Table 9). On the other hand the risky rate has a substantially higher correlation of 0.96 with M even when aggregated over 12 periods. For example, if we simulated our model at a monthly frequency, aggregating variables to yearly data would make it look as if the risky rate is a much better proxy for the true discount factor compared to consumption growth. Moreover, although Campbell and Cochrane attribute this outcome to the existence of habit, here the same result obtains even when $\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}$ is correctly constructed from stockholders' consumption and by construction there is no habit, which suggests that this outcome is a result of time aggregation.

Overall, we believe that these results demonstrate the potential of the limited participation model as a parsimonious framework for conducting serious asset pricing analysis.

5 Habit versus Limited Participation: What is the Connection?

In the previous section we showed that a macroeconomic model with limited participation and heterogeneity in the EIS is able to explain a large number of asset pricing phenomena

of 8 months which means they are essentially nominal at shorter horizons. Thus with few data points at annual frequency it is not clear how useful that data will be for our purposes and we do not pursue that approach here.

Table 9: Cross-Correlation of the True Discount Factor with Variables

	Correlation with:	
Aggregated over (periods)	$\left(\frac{C^h_{t+1}}{C^h_t}\right)^{-\alpha}$	R^S
1	1.000	0.992
4	0.713	0.975
8	0.572	0.963
12	0.534	0.959

Notes: Consumption growth is aggregated up by adding each period's consumption as it is done with actual data.

including all the results presented in C-C.[12] The similarities between the two models' results are remarkable. Although the current framework arguably has further advantages (such as the time-varying and realistic interest rate) the similarities are still many.

One curious question that comes to mind is whether the similarities are just a coincidence or whether there might be a deeper connection between the two frameworks. On the surface though the two models look quite different (e.g., representative- versus heterogeneous-agents; external habit versus standard CRRA utility; exchange versus production economy; random walk versus autoregressive shocks, to name a few). Nevertheless, we will show that our model has a reduced form representation which looks strikingly similar to the external habit formulation of C-C including their parameterization of free parameters!

To develop the argument, first recall that the Euler equations for the stockholder's portfolio choice imply:

$$E_t\left[\beta\left(\frac{C^h_{t+1}}{C^h_t}\right)^{-\alpha^h}\left(R^S_{t+1}-R^f_t\right)\right]=0$$

Note that this condition only holds for stockholders, and not for aggregate consumption. Making the substitution, $C^h = C^A - X$, where C^A denotes aggregate consumption, and letting $X \equiv C^n$ for reasons that will become clear in a moment, we get:

$$E_t\left[\beta\left(\frac{C^A_{t+1}-X_{t+1}}{C^A_t-X_t}\right)^{-\alpha^h}\left(R^S_{t+1}-R^f_t\right)\right]=0$$

This is the same Euler equation that C-C derive once X_{t+1} is reinterpreted as an external

[12] There are a few results discussed in Campbell-Cochrane (such as the correlation of consumption growth with returns, the actual time series of stock price data generated by the model, etc.) that we have not reported here to save space. They are all as similar to C-C's findings as the ones reported. Available upon request.

habit term. In other words, because some households do not participate in the stock market and satisfy their Euler equations (with respect to stocks), representative agent models ignoring this fact would still have to subtract their consumption (X_{t+1}) from the aggregate (C_{t+1}^A) to successfully explain asset prices. Thus ignoring limited participation would make it look as if the representative agent was displaying habit persistence. Of course for this claim to have any validity, we have to establish that the properties of X_t are very much the same as the neglected term, that is, non-stockholder's consumption. To that end, we follow C-C and define $S_t \equiv \frac{C_t^A - X_t}{C_t^A}$, which is the key figure in their model and the properties of this ratio drives all the action. In that framework S_t is the consumption above habit as a percentage of total, and is called "the surplus consumption ratio." In our model, $S_t \equiv \frac{C_t^h}{C_t^A}$, is simply the share of stockholder's consumption in the aggregate. Now manipulating this last equation we obtain

$$E_t \left[\beta \left(\frac{C_{t+1}^A - X_{t+1}}{C_{t+1}^A} \right)^{-\alpha^h} \left(\frac{C_t^A}{C_t^A - X_t} \right)^{-\alpha^h} \left(\frac{C_{t+1}^A}{C_t^A} \right)^{-\alpha^h} \left(R_{t+1}^S - R_t^f \right) \right] = 0$$

$$\Longrightarrow E_t \left[\beta \left(\frac{S_{t+1}}{S_t} \right)^{-\alpha^h} \left(\frac{C_{t+1}^A}{C_t^A} \right)^{-\alpha^h} \left(R_{t+1}^S - R_t^f \right) \right] = 0.$$

This alternative expression also holds in both frameworks, and can be viewed as adding an extra state variable, S_t, to the standard Euler equation of Mehra and Prescott (1985). For either model to have any hope of success S_t must be doing something magical. Stressing this critical role C-C introduce a fairly elaborate exogenous process for $s_t \equiv \log(S_t)$:[13]

$$s_{t+1} = (1 - \phi)\, \overline{s} + \phi s_t + \lambda\,(s_t) \left(c_{t+1}^A - c_t^A \right)$$

and calibrate it to match a number of financial moments. Apart from the persistence term ϕ, the nonlinear sensitivity function, $\lambda\,(s_t)$, is chosen to satisfy three conditions viewed desirable for matching asset prices (more on this later). As C-C acknowledge, this is a reverse engineering exercise as it is difficult to justify the specific functional form and the parameter values chosen for $\lambda\,(s_t)$ on economic grounds.

So how similar are the two models really? In the analysis below we look at the statistical properties of S_t as well as the features of $\lambda\,(s_t)$ assumed in C-C and the one implied by our model. The first obvious step is to check whether S_t in our model is procyclical which is, a priori, not at all clear. In the habit model the surplus ratio increases in good times because habit evolves very slowly and lags behind actual consumption, so this ratio has a strong positive correlation with output. Similarly, in our model S_t (stockholders' share of consumption) has a correlation of 0.92 with output. This correlation is driven by two separate effects. First, stockholders have a higher

[13] We set the growth rate of consumption (g) in their formulation equal to zero since we are looking at a stationary model.

EIS so their consumption rise more in response to a positive shock. And second, a technology shock increases the stock price and thus stockholders' wealth substantially on impact, whereas the immediate effect for non-stockholders is much weaker because their savings increase only gradually as their wage grow slowly with capital.

After passing this first test of sensibility we can now proceed to take a closer look. First, we compare the densities of S_t which are displayed in Figure 3. The one in the right panel is obtained by simulating the AR(1) process above with the parameter choices in C-C. As can be seen, its mode is at its upper bound and the distribution has significant negative skewness. In the left panel we plot the unconditional density of $\left(C^h/C^A\right)$ in our model. To us, the similarity looks just remarkable, especially considering that $\left(C^h/C^A\right)$ is an endogenous outcome and does not have any of the above specifications or free parameters.[14] The two distributions almost have the same shape including the location of the mode, the sharp upper bound and the left tail.[15] Moreover, the left skew in the ratio is despite the fact that neither C^h nor C^A have significant skewness[16] and the two are highly correlated at business cycle frequencies.

Second, turning to conditional information one can get an even clearer picture. C-C calibrate the persistence parameter ϕ above to match the autocorrelation structure of the price/dividend ratio. Their parameter choice implies a quarterly persistence of 0.96 for S_t which is virtually the same as the autocorrelation of $\left(C^h/C^A\right)$ in our model: 0.954! This could be expected at some level since our model was able to match the autocorrelation of the P/D ratio without calibrating any extra parameters.

Third, as C-C emphasize the functional form of $\lambda(s_t)$ is at the heart of their model. Especially the fact that the sensitivity is decreasing in S_t so that habit is more sensitive to consumption shocks when it is already low is credited for generating many of the results. By substituting ϕ (0.96) and the model generated sequences of s_t and c_t^A ($\equiv log\left(C_t^A\right)$) into the AR(1) process above we can back out the implied sensitivity function from our model which we call $\widehat{\lambda}(s_t)$.

Figure 4 plots both λ and $\widehat{\lambda}$ on the same graph. The cloud of points representing $\widehat{\lambda}$ are downward sloping and almost parallel to λ as if it was shifted down. However, recall that the two functions could not possibly be identical since one of the three requirements C-C had in mind when choosing λ was to get a constant risk-free rate. One can relax this assumption following the approach in the working paper version of Campbell and Cochrane (1999) and allow for a variable risk-free rate:

$$r_t^f = r_0^f + B\left(s_t - \overline{s}\right), \tag{5}$$

[14] The small dip on the right end of the distribution of $\left(C^h/C^A\right)$ is due to the Markov approximation. Even though the 12-state process is able to match the autocorrelation and standard deviation of the AR(1), it is still truncated at both ends. When we reduce the persistence of the AR(1) process we get very similar distributions with slightly less dispersion but without the dip in $\left(C^h/C^A\right)$.

[15] Note that in C-C's benchmark the interest rate is assumed to be constant. If this restriction is relaxed (as in equation 5 in the main text) and the interest rate volatility is calibrated to 6 percent per year the density for λ becomes slightly less skewed and looks even more similar to that in the limited participation model.

[16] The skewness coefficients are -0.07 and -0.01 for C^h and C^A respectively, but it is -0.43 for C^h/C^A.

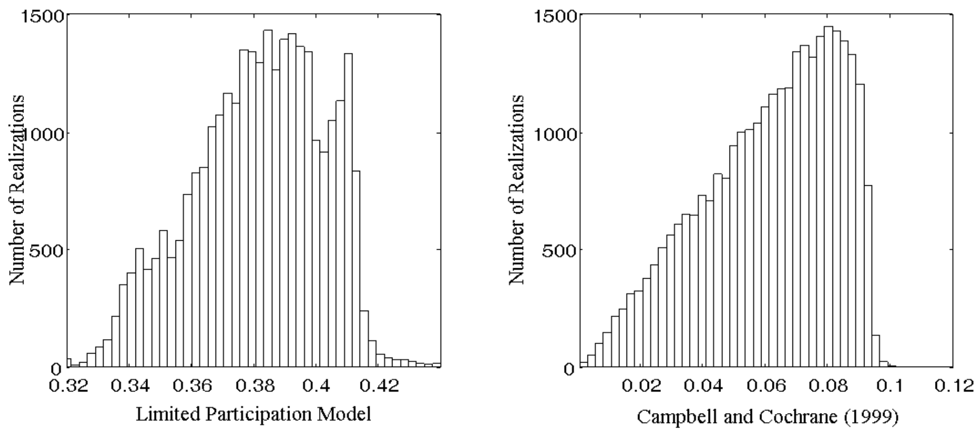

Figure 3: Comparing Empirical Distributions: Stockholders' share in Aggregate Consumption (Left) Versus the Surplus Consumption Ratio in Campbell-Cochrane (Right)

where B is again a free parameter. This modifies some of the equations in C-C and in particular implies a higher steady state surplus ratio which at the baseline case was lower than the average value in our model. This modification scales λ down by $\sqrt{\frac{1-\phi}{1-\phi-B/2}}$ with C-C's calibration. By choosing B such that the risk-free rate has roughly 1.5 percent quarterly standard deviation as in our model, the implied $B = 0.039$ and the scaling factor is 1.78. The adjusted λ function now falls almost on top of the one estimated in our model!

Finally, we can compare the models in yet another dimension by looking at the implied habit process X_t itself. When choosing the λ function, C-C discuss three desirable properties that X_t should satisfy. First, it is a smoothly evolving process and in particular $\sigma_C^2 \gg \sigma_X^2$. Second, $C > X$ everywhere so that the power utility function is well-defined. Third, around the steady state X_t is predetermined and moves positively with consumption elsewhere. This essentially implies that locally C and X are uncorrelated and globally they have a positive correlation.

All three of these conditions are naturally satisfied by non-stockholders' consumption (X_t) in our framework. First, aggregate consumption is much more variable than that of non-stockholders' ($\frac{\sigma_C^2}{\sigma_X^2} \approx 4 - 5$) even though 80 percent of the population are non-stockholders (more on this in the next section). Second, $C^A > X$ is satisfied naturally since stockholder's consumption has to be positive. And third, the two group's consumption has a relatively low correlation (0.72) implying that C^A and X have a positive but less than perfect correlation.

Given that asset prices in both models are determined by the Euler equation above, the striking similarities between S_t (and X_t) largely explains how the two models' implications are virtually identical confirming our conjecture that the external habit specification can be viewed as a reduced form representation of the limited participation model. The endogenous redistribution of wealth between the two groups with different utility curvatures is reflected in a time-varying risk aversion in the representative agent framework with external habit. However,

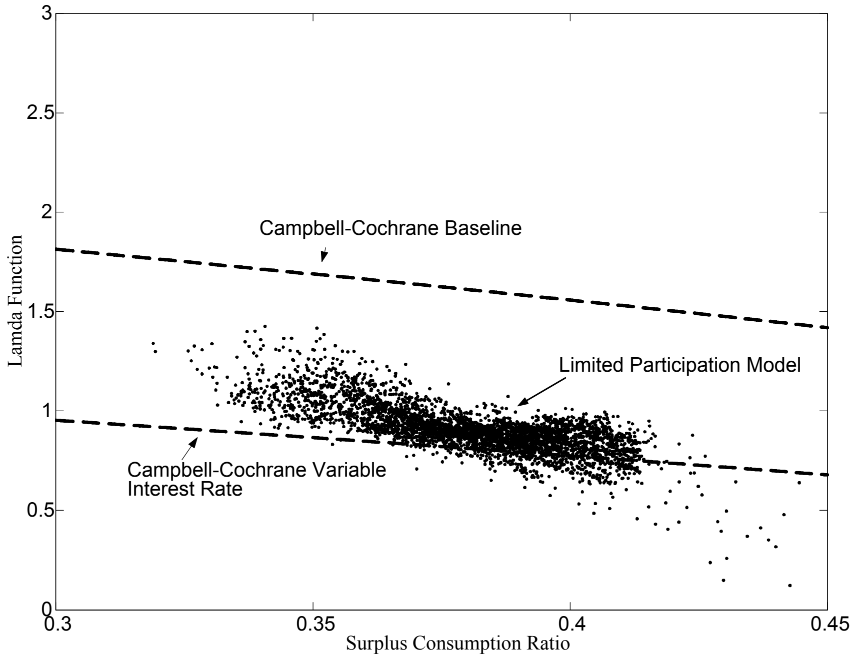

Figure 4: Comparison of the sensitivity function $\lambda(s_t)$ assumed in Campbell-Cochrane to the one implied by the Limited Participation model

as we demonstrate in the next two sections these similarities do not extend to macroeconomic implications or to policy analysis conducted in the two frameworks.

Before concluding this section we want to show how our reinterpretation of external habit preferences eliminates an unattractive feature of C-C's model: an incredibly high risk aversion. To see this note that the utility function used by C-C is $u = \frac{(C-X)^{1-\alpha}}{1-\alpha}$ and with the representative agent assumption they interpret C as the actual consumption of the agent and X as an externality. When calculating risk aversion one would then differentiate all expressions with respect to C_t and get

$$
\begin{aligned}
RRA_t &= -\omega \frac{V_{\omega\omega}}{V_\omega} = \psi_t \frac{\partial \ln C_t}{\partial \ln \omega_t}, \\
\psi_t &\equiv C^A \frac{U_{cc}}{U_c} = \frac{\alpha C_t}{C_t - X_t} = \frac{\alpha}{S_t}
\end{aligned}
$$

where the second equality is obtained by using the envelope condition, $u_c = V_\omega$, and then substituting for u_c. With C-C's parameter choice $\frac{\alpha}{S_t}$ is approximately 40 at steady state and grows without bound as C_t falls near X_t during a recession. Moreover, $\frac{\partial \ln C_t}{\partial \ln W_t}$ is greater than 1 resulting in an average risk aversion of 80, and in hundreds during recessions.

In the context of our model, however, if one correctly interprets X_t as non-stockholders' consumption, then the agent who prices stocks consumes not the aggregate consumption, C_t^A, but only $C_t^h = C_t^A - X_t$. Consequently, differentiating u correctly with respect to $(C^A - X)$ we obtain

$$\psi_t = -C^h \frac{U_{cc}}{U_c} = \alpha \left(C^A - X\right) \frac{\left(C^A - X\right)^{-\alpha}}{\left(C^A - X\right)^{-\alpha-1}} = \alpha!$$

Moreover, $\frac{\partial \ln C_t}{\partial \ln \omega_t}$ is just 1 with CRRA utility in the absence of borrowing constraints (and is only slightly above 1 in our model since stockholders are far from their constraints) which implies $RRA_t \approx \alpha = 2$ with C-C's calibration independent of X_t! However, this is not to say that the difference between the two models is just a matter of interpretation as will become clear in the macroeconomic analysis below.

6 Results: Macroeconomics

The interest of macroeconomists in the equity premium puzzle (despite being primarily an asset pricing phenomenon) is not all that surprising. Mehra and Prescott (1985) essentially showed that the historical excess return was puzzling in the context of a standard general equilibrium (consumption-portfolio choice) model which is also the foundation of most neoclassical macroeconomic analysis.[17] Naturally, one would wonder, how much confidence can be placed in a framework which has grossly counterfactual implications for most pricing problems? Thus it is compelling, from macroeconomists perspective, to examine if a model proposed to resolve these asset pricing puzzles also displays plausible macroeconomic behavior.

Despite the enormous success of the Campbell and Cochrane model in explaining asset prices, its extension to study macroeconomic problems has not been as impressive. Lettau and Uhlig (2000) embedded the external habit specification used by C-C in a standard real business cycle model and found that it creates a number of macroeconomic anomalies. For example, consumption turns out to be extremely smooth: the quarterly standard deviation is a mere 0.02 percent which is 70 times smaller than observed in the U.S. data. Introducing another habit formation, one in leisure, does not help much but instead gives rise to other puzzles of its own: labor hours become too smooth and countercyclical. Overall, they concluded that "introducing habit formation in consumption and leisure yields counterfactual cyclical behavior in an otherwise standard real business cycle model."[18]

It is then natural to ask if the successful asset pricing results of the limited participation framework also come at the cost of unrealistic macroeconomic behavior. In Guvenen (2002) we analyze the quantity implications of this model (without adjustment costs) in detail and find that it not only delivers plausible statistics, but it also helps us understand some facts which

[17] Although Mehra and Prescott used an exchange economy version of the canonical model, further extensions to include production, more general preferences, as well as many other elements to enrich the model did not alter the conclusion appreciably until the introduction of habit persistence models discussed here.

[18] Boldrin et. al (1998) tackle this problem by imposing restrictions on the production side of the economy such as immobility of labor and/or capital between sectors, another assumption which forces agents to make savings decisions before they observe the aggregate shocks and so on. Also, the EIS is not as low in their framework as in C-C.

Table 10: Macroeconomic Statistics – Time Series and Cross-Section

	US Data Post-War	Model $\xi = \infty$	$\xi = 0.23$	$\xi = 0.5$
		Panel A: Time Series		
$\sigma(Y)$	1.9	2.3	2.4	2.5
$\sigma(C^A)$	1.4	1.3	2.2	1.6
$\sigma(C^h)$	–	1.8	4.4	3.6
$\sigma(C^n)$	–	1.0	1.1	1.0
$\sigma(I)$	8.2	8.6	3.0	5.3
		Panel B: Cross-Sectional		
ω^h/ω^A	88%	85%	89%	86%
C^h/C^A	32%	34%	37%	35%

Notes: The statistics reported in panel A are from *quarterly* U.S. data covering 1947.1-1991.4. Standard deviations are calculated from data after the trend is removed with a Hodrick-Prescott filter from the log of raw data with a bandwidth parameter of 1600. The investment figure includes changes in inventories; if excluded the volatility is 5.8 percent. The source for the distribution of financial wealth is Wolff (2000), and for consumption data is Guvenen (2002).

appear puzzling in a standard RBC model.[19] Thus in this section we present a brief overview of the macroeconomic results of the model both with and without adjustment costs imposed. Table 10 displays a summary of both aggregate business cycle and cross-sectional statistics.

First, we start with the case when $\xi = \infty$ (no adjustment costs) which is more directly comparable to Lettau and Uhlig's analysis. In contrast to their findings, here the volatilities of output, aggregate consumption and investment are all quite close to their empirical values. Note that this is true even though most households have a very low elasticity of substitution. If these households were holding a significant fraction of wealth (which is not the case; see panel B), their preference for very smooth consumption would induce them to save and dissave vigorously in response to shocks causing aggregate investment and thus output to be overly volatile. Instead, virtually all the wealth is held by households with a high EIS, so both output and investment have more plausible variabilities.

Second, when adjustment costs are imposed, with our baseline calibration ($\xi = 0.23$) output remains fairly unchanged, but the volatility of aggregate consumption goes up. When we decompose consumption across the two groups, it becomes clear that the increase is mainly due to stockholders. One possible way to reconcile this finding with empirical evidence is to observe that stockholding is very unequal even among stockholders. For example, in 1998, almost 95 percent of all equity (including pensions) was held by the top 10 percent of the population, but

[19] An example of such a puzzle is the apparent inconsistency between econometric studies using aggregate data which estimate a very low EIS parameter (Hall 1988, Campbell and Mankiw 1989) and calibrated models which find a high EIS to be more consistent *with the same* aggregate data. The limited participation model is consistent with the findings of both literatures, a results largely due to the realistic heterogeneity created by the model.

even more remarkably, the top 1 percent of households owned 48 percent of all equity (Survey of Consumer Finances, 1998). Since consumption is much more evenly distributed than wealth in the U.S. (panel B), even if these wealthy stockholders have very volatile consumption paths, their effect in aggregate consumption is likely to be quite modest.[20] In other words, the two-agent framework attributes a lot of variability to the consumption of all stockholders whereas in reality most of the aggregate risk is borne by a much smaller fraction within this group.

Nevertheless, the model still predicts the *average* consumption volatility of stockholders to be higher than that of non-stockholders. There is considerable evidence that this is indeed the case. For example, Mankiw and Zeldes (1991) report that the variance of stockholders' consumption growth is *more than twice* that of non-stockholders. This is true even though their consumption measure consists of only food expenditures from PSID. Furthermore, Attanasio, Banks and Tanner (2002) use non-durables and services as their consumption measure from the Family Expenditure Survey on British households and calculate stockholders' consumption growth variance to be *four times* larger than non-stockholders'. In a recent paper, Aït-Sahalia, Parker and Yogo (2001) focus on the sales of luxury goods as a measure of the wealthy's consumption.[21] They find for example that the annual standard deviation of the sales of luxury retailers is between 17 to 23 percent, and the volatility of charitable contributions is 21 percent. Although there is certainly some durability in these expenditures, these numbers are even much larger than the standard deviation of durable consumption expenditures which is around 8 percent annually in the post-war NIPA data. Again, comparing between durable goods, the sales of Porsche cars have an annual volatility of 34.3 percent compared to 8.5 percent for overall luxury cars (defined as having a price higher than \$24,000). Hence, although indirect, this evidence also seems to suggest that the wealthy (and similarly the stockholders) have more volatile consumption paths than non-stockholders consistent with the model.

Finally, the baseline parameter choice for adjustment costs ($\xi = 0.23$) which we take from previous studies (Jermann 1998 and Boldrin et. al 1999) is on the low side of empirical estimates. If one sets $\xi = 0.5$ then, as shown in column 4, both consumption and investment come closer to empirical values. The only noticeable change in financial statistics is a small reduction in the *level* of the premium without affecting the Sharpe ratio (Table 4) or any other asset pricing questions investigated before. As we discussed before the level of the premium can be increased by leverage or a slightly higher risk aversion. Alternatively, adjustment costs may not be the most appealing and realistic friction to consider. Irreversibilities in the investment decision, stochastic depreciation of capital or frictions in inter-sectoral movements of capital are other realistic features of the production process which can also generate high return volatility.

Turning to the cross-section, the model reveals an interesting pattern of heterogeneity: in the baseline case, stockholders consume only 37 percent of aggregate consumption despite holding 89 percent of aggregate wealth. Interestingly, this is exactly the same kind of distribution observed in the U.S. data. This result is intuitive since consumption is proportional to income

[20] See Poterba (2000) for an elaboration of this point.

[21] Of course the link from sales to the consumption of a certain group is clearly not a direct one, but the evidence should still be informative.

in the long-run which is more evenly distributed than wealth. If one interprets the relevant empirical measure as financial wealth (since we have no housing or durables in the model) then the share of wealth held by the top 20 percent is approximately 90 percent (Poterba 2000). This result is also robust to adjustment costs and holds even in the limit when capital is perfectly elastic (Panel A). In this sense, limited participation does not only have successful asset pricing implications but also gives rise to plausible macroeconomic aggregates and heterogeneity.

Putting all this evidence together it is clear that the similarities between the two models do not extend to macroeconomic analysis. Both models have implications not captured by the Euler equation for portfolio choice. In particular, in the limited participation model, although non-stockholders do not hold a lot of wealth they contribute substantially to aggregate consumption and affect its properties.

7 How Different Are The Two Models?: A Policy Experiment

The recent success of habit formation models (both endogenous and external variety) has encouraged researchers to perform policy analysis in these frameworks. For example, Fuhrer (2000) documents this success and proceeds to conduct monetary policy analysis in an endogenous habit model. In a similar framework, Christiano, Eichenbaum and Evans (2001) analyze optimal monetary and fiscal policy. On the fiscal policy side, Ljungqvist and Uhlig (2000) and Abel (2001) study optimal taxation in environments with external habit (For other examples, see also McCallum and Nelson 1998, D'Amato and Laubach 2002, Levin and Williams 2002). In this section we conduct a simple policy experiment to demonstrate that one can reach dramatically different conclusions in the limited participation model compared to the habit persistence model.

The experiment we have in mind is the capital income taxation problem which has been widely studied in the literature. It has long been recognized that the welfare effects of this tax critically depend on the degree of intertemporal substitution (Summers 1981, King and Rebelo 1990). Indeed, Hall (1988) concludes that his estimate of a small average EIS also imply a weak response of savings to changes in interest rates. To the contrary, we argue that the effect of taxation on savings will be determined by the wealth-weighted elasticity measure, which is, given the enormous wealth inequality, very close to that of the stockholders.

In order to demonstrate our point, we study a simple tax reform problem similar to the one studied by Lucas (1990). We imagine that initially the government imposes a flat-rate tax on capital income and returns the proceeds to households in a lump-sum fashion. Suppose that, at a certain date, capital income tax is completely eliminated and agents have not previously anticipated it. We set the initial tax rate $\tau^k = 36$ percent which roughly corresponds to the average rate in the U.S. All aspects of the baseline model remain intact. Also, in order to make our results comparable to the previous literature, we first consider the welfare gain from this reform in a representative agent framework. If the agent has $\rho = 0.5$, the welfare benefit of this policy is 0.75 percent of consumption per period taking the transition path into account. Although it may not seem much, as Lucas argues, this is an order of magnitude larger than the gain from eliminating the business cycle fluctuations, and almost twice the gain from eliminating a 10 percent inflation. If, on the other hand, we assume $\rho = 0.02$, (which is higher than the EIS implied by C-C's model) the welfare gain is reduced to 0.2 percent of consumption instead, mainly because now the transition takes approximately 250 years compared to 20 years in the former case.

Now we subject our two-agent economy to the same tax experiment. The welfare gain is 0.71 percent of total consumption.[22] In effect, this economy behaves as if it was populated only by agents with unit elasticity and non-stockholders' preferences virtually vanished from the problem.

There is an even more interesting side to this problem that transpires from explicitly modeling heterogeneity: based on policy experiments like the one above, many economists have argued in favor of eliminating capital income taxes. But, a representative agent framework is silent on the question of "who is actually gaining from this reform?" since all agents are assumed to be identical. In reality, all agents are not identical, and as we have shown so far, in some dimensions, they differ substantially. So, we would like to take this question seriously and break down the gains from this reform. *It turns out that, in consumption terms, stockholders gain by 4.7 percent, whereas non-stockholders, who constitute 70 percent of the population, actually lose 1.9 percent of their consumption!* Clearly, this is a different conclusion than what comes out from the representative agent economy. Moreover, if there is an exogenous government expenditure stream that needs to be financed eliminating capital income tax can be viewed as imposing a consumption tax instead. Hence, it is possible to interpret this result as suggesting that consumption taxes must be progressive from an equity point of view.

In sum, both kinds of habit persistence imply a low EIS for the representative agent, when in fact almost all productive capital is owned by households with a high elasticity. Thus, any policy question which depend on intertemporal substitution is likely to give substantially different answers in each framework.

8 Robustness and Relation to the Literature

[**To be Written**] Summary:

1. What happens when borrowing constraints are relaxed? The equity premium, its volatility and the Sharpe ratio remain virtually unchanged. Other statistics studied in this paper are not affected either qualitatively and quantitatively. The only curious finding is that the countercyclicality results are slightly weaker when borrowing constraints are tighter.

2. What happens when risk aversion and EIS are disentangled? Equity premium is determined mainly by the risk aversion of stockholders and the elasticity of the non-stockholders. More precisely, when the two are disentangled using Epstein-Zin preferences and the risk

[22] This calculation is based on the assumption that there is a utilitarian government which tries to attain the same social welfare index as without taxes and makes transfers to agents in such a way to minimize the total amount of transfers.

aversion of both agents are kept at 2, but non-stockholders' EIS is set equal to 0.1, the results are virtually identical to what is presented in the paper.

3. What happens when participation rate, λ, is assumed to be 30 percent? The equity premium falls to 2.96 percent (from the baseline of 3.3 percent) and the standard deviation falls to 13.8 percent. So the change in the Sharpe ratio is quite modest: instead of 23 percent, it falls to 21.5 percent. All other results essentially remain qualitatively the same with only minor changes quantitatively.

4. When preference heterogeneity is eliminated (both agents have elasticities of 0.5 and risk aversions of 2), the equity premium falls to 0.8 percent and the volatility falls to 8.9% reducing the Sharpe ratio also to 9 percent. Most of the other results are not affected: for example, the conditional excess return, volatility and Sharpe ratio continue to be countercyclical, stock returns are still predictable (although the R^2's fall by approximately 40 percent). The autocorrelation patterns of variables (Table 8) remain largely unaffected. Thus, preference heterogeneity seems to affect the first two moments of the returns (Table 2) with the rest of the results mainly driven by limited participation. However, note that in this case non-stockholders' consumption becomes more volatile than stockholders' and the share of stockholder's consumption ceases to be procyclical, breaking the close link between this model and the external habit model. In that sense preference heterogeneity is essential for the results overall.

Literature Review

There is a vast and growing literature on the asset pricing puzzles addressed in this paper. For excellent surveys of this literature see Kocherlakota (1996), and Campbell (1999). This paper is more closely related to the strand of literature which emphasizes the role of limited participation starting with Mankiw and Zeldes (1991). Saito (1995), and Basak and Cuoco (1998) studied general equilibrium models with limited participation in the stock market. Our environment differs in that we emphasize the role of preference heterogeneity as well as labor income risk. As discussed in Section 4.1, both of these elements play essential roles in generating the results in this paper. In a recent paper, Guo (2002) studies an exchange economy model with limited participation and is able to match some of the financial statistics mentioned above. However, as he also emphasizes, his results also rely on large income shocks to each group (36 percent per year) as well as frequently binding borrowing constraints (40 percent of the time). In a related line of research, Abel (2001) and Diamond and Geanakoplos (2001) address policy questions in economies with limited participation whereas Heaton and Lucas (1999) investigate the consequence of increasing participation for stock prices. This paper complements this literature by showing that limited participation together with preference heterogeneity of plausible form can provide a compelling explanation for a large number of asset pricing phenomena.

9 Conclusions

In this paper we introduced a macroeconomic model with limited stock market participation and heterogeneity in the EIS parameter. This particular two-agent representation is parsimonious compared to the traditional (fully) heterogenous-agent frameworks of Aiyagari (1994), Krusell and Smith (1998), and Storesletten et. al (1999), yet it captures a lot of interesting heterogeneity absent in representative agent models, which turns out to be crucial for understanding asset prices. We calibrate the parameters of the model to standard values from the real business cycle literature and in particular we do not choose them to match any financial statistic.

The model is surprisingly successful in accounting for the behavior of financial variables. This includes the level and the volatility of the stock return and the interest rate as well as the cyclical movement of the price-dividend ratio, the equity premium, its volatility and the Sharpe ratio. Moreover, stock prices have a large predictable component in the model as documented by many papers in the literature.

Many of the results explained here are also features of the external habit model of Campbell and Cochrane (1999). We find that this is not a coincidence. The limited participation model has a reduced form which is extremely similar to the external habit model. By abstracting from limited participation representative agent models still has to subtract non-stockholders' share from aggregate consumption to successfully explain asset prices and then reinterprets it as an externality faced by the representative agent. One major difference is that the risk aversion in our model is just 2 compared to an average value of 80 in the external habit model.

The model also generates plausible macro statistics even when there are no frictions in production whereas the external habit model is hard to reconcile with aggregate macro data (Lettau and Uhlig 2000) largely due to the extremely low EIS implied by that model. Furthermore, policy analysis in the two frameworks also yield substantially different conclusions. This point suggests extreme caution as the success of habit persistence models encouraged many researchers to address policy questions in that framework.

Another lesson that we draw from these results is about the potential dangers of reverse of engineering. Although reverse engineering can do an impressive job in matching a certain dimension of data (such as asset prices) this success is in no way a guarantee that the model is a satisfactory description of any other dimension of interest (such as for policy analysis).

We believe that these results would encourage further research on the reasons behind limited participation which is not addressed in this paper. Furthermore, given the central role played by non-participation, another natural question concerns the consequences of the recent trends in participation observed in most countries for asset prices as well for wealth inequality and welfare.

A Appendix [*Incomplete*]

The discretization method understates the true persistence of the AR(1) process. So, in order to get a first order autocorrelation of 0.95 we simulate an AR(1) with $\rho = 0.96$ for 30,000 periods.

Table 11: Autocorrelation Structure of the Markov Chain Approximation

	$\sigma\,(\varepsilon)$	Autocorrelation at Lag				
		1	2	3	4	5
AR(1)	2.000%	.950	.903	.857	.815	.774
Markov Approximation	2.009%	.950	.902	.856	.814	.772

Applying the Tauchen and Hussey's (1991) method with 12 state points we obtain the desired transition matrix. Table 11 shows that the generated Markov process behaves almost exactly like an AR(1) process with $\rho = 0.95$.

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