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Signaling Quality by Delaying Sales

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Abstract

This paper studies the problem of a monopolist privately informed about its product quality, who can sell its product in advance, and faces forward-looking buyers who learn about quality over time. We show that if the monopolist prefers to sell sooner than later, the unique equilibrium satisfying a standard refinement criterion will be such that high-quality monopolists will postpone sales so as to separate themselves from low-quality ones. An application of the analysis is the allocation of sales among season tickets and event tickets for sport or musical events. Several testable implications are derived in the comparative static analysis. A somewhat unexpected result is that an increase in the precision of monopolist's information has a *negative* effect on economic efficiency.

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1 Introduction

In this simple paper, I study the problem of a capacity-constrained monopolist privately informed about its product quality, who can sell its product in advance, and faces forward-looking buyers who are disclosed public information about quality over time. I show that if the monopolist prefers to sell sooner than later, the unique equilibrium satisfying Universal Divinity (Banks and Sobel 1987) will be such that high-quality monopolists will postpone sales so as to separate themselves from low-quality ones, and induce consumers to buy at a higher price. As well as deriving testable implications in the comparative static analysis, the paper presents some implications in terms of welfare analysis. An unexpected result is that an increase in the precision of monopolist's information has a *negative* effect on economic efficiency.

As a straightforward application, consider a sport team where the management must decide how to allocate the tickets among season and game tickets, and is privately informed about the quality of the team. The buyers value quality and make inferences based both on the quantity of season tickets put on sale, and, as the season goes on, on the performance of the team. If the management prefers to sell sooner than later, I show that in equilibrium, high-quality teams sell out season tickets at a high price, and lower-quality teams offer larger quantity of tickets in the season market, for a lower price. Beyond this example, the model applies to environments or industries characterized by imperfect competition and the coexistence of forward and spot markets. With respect to financial markets, for instance, this paper's results are consistent with the common perception that a large volume of sales discloses inside information about a company's poor performance, and that in such markets the timings of sales are what matters.[1] Along these lines, the paper can be related to the work of Kyle (1985). In that paper however, information is separate from market power.

[1] While in this paper we consider the case in which all forward markets are open only at period 0, the essence of the results would still hold in a model that allows also for forward markets open at different periods. For the case of slow disclosure of public information, the equilibrium in that game would involve an intertemporal mixed strategy structure symilar to the one introduced by Noldeke and Van Damme (1990).

Specifically, the market makers set prices efficiently and make on average zero profit, and a different price-taking player is privately informed about the liquidation value of a risky asset. Since the market is assumed to be efficient, welfare analysis is precluded.

Applied to the mathematically equivalent problem of dynamic monopoly, my analysis helps explaining the marketing practice of launching innovations with small quantity offers and a high price, so as to induce the perception of high quality. In that respect the contribution resembles Bagwell and Riordan (1991) in that firms signal their quality in equilibrium, and sell for a higher price (see also Bagwell 1992, Albaek and Overgaard 1992). However, unlike Bagwell and Riordan (1991), our model endogeneizes the learning component, and thus the separating equilibrium presented here is sustained without restricting the analysis to industries where high quality translates into high variable costs. Because of that feature, this paper's result does not suffer a well-known critique of Bagwell and Riordan 1991 (see for example Judd and Riordan 1994, Shieh 1993): if high prices signal high costs that translate into high quality, then there is no incentive for cost reduction and technological innovation.

Finally, this paper's results are of relevance for the general issue of the timing of trade, a question on which clear consensus has not been reached, despite extensive study in the field of economic theory. In a repeated bargaining model, Gul and Sonnenschein (1988) point out that, while "with incomplete information during the bargaining process, agents might be expected to signal their valuations with their offers, and this takes time,"[2] a delay in bargaining can only be imputed to be a delay between the times of the offers. Even though Ausubel and Deneckere (1992) show that if the informed party is granted the right to make irrelevant offers, agreement will be delayed in real time regardless of the offers' frequency, the stated conjecture is only partially vindicated because in their paper all the types of informed party pool, avoiding to make initial serious offers.[3]

[2] Gul and Sonneschein (1992), pag. 602.

[3] In a related contribution, Merlo and Wilson (1995) point out that in the case of stochastic bargaining with common information, if the gains from trade are initially small with respect to possible future realizations, the parties will wait to settle. See also Admati and Perry (1987).

In a durable-good dynamic monopoly model, Coase (1972) formulated the opposite conjecture, verified by Gul Sonnenschein and Wilson (1986), that prices will immediately converge to marginal cost, because the monopolist cannot credibly commit not to serve residual consumers after selling those willing to pay the monopoly price. The Coasian conjecture does not extend to the case of two-sided private information: Ausubel and Deneckere (1992) show that trade may be delayed for very long periods of time as neither party is willing to signal its weakness by settling for unfavorable terms. Moreover, Ausubel and Deneckere (1989) study the case with one-sided information, and establish a folk-theorem result in which the possibility of reputation allows for any outcome in between monopolistic and competitive prices. This paper elicits a different source of trade delays. If the informational advantage of the seller deteriorates over time, in the *unique* equilibrium, high-quality sellers will find it advantageous to partially postpone trade, until quality is disclosed.

Technically, this paper's model belongs to the class of signaling games that follow Spence (1973). As is well known, incomplete information in these games prevents efficiency, and a second-best separating equilibrium can be attained if and only if a single-crossing condition is realized between each pair of types' indifference curves (see also Athey 1997 for the supermodular approach). In this paper's game, the single-crossing condition is guaranteed by the fact that the informed party prefers -ceteris paribus- to sell sooner than later, and by the fact that public information is exogenously disclosed over time.[4]

I study two different cases with respect of the timing of information disclosure. In the simplest case, the monopolist is initially fully-informed and her information is subsequently disclosed to the buyers all at once. In the second case, information is slowly released to the buyers according to a Beta-Bernoulli updating model, and the monopolist's initial

[4] This paper assumes that the disclosure of information about quality over time is independent of the amount of previous sales. The opposite polar case is studied in the models on word-of-mouth communication, where potential buyers may learn about quality only from current costumers, and thus high-quality companies may try to increase initial sales in order to have more buyers informed about their quality (see for example Rogerson 1983, and Vettas 1995).

information is represented as an extraction from a binomial experiment. That allows us to analyze equilibrium behavior for different monopolist's information precision, by varying the size of the experiment. Also, the infinite-period model allows us to describe a stochastic sequence of spot-market prices and study their dynamics. By restricting attention to the case of almost infinitely-patient individuals, by stipulating that forward markets are open only at period zero, and that the monopolist operates on forward markets by offering perpetuities, the players' incentives are mathematically equivalent to the simpler two-period case.

A counterintuitive result is that forward sales depend on the monopolist's extraction, and not on the quality estimate consisting of the ratio between the extraction and the size of the experiment. This result is understood by noticing that the loss of reducing sale is borne by a high quality-monopolist only because she needs to differentiate herself from all lower-quality monopolists. Thus the relevant statistic is the order of the extraction, regardless of the size of the experiment. The result that the monopolist signals high quality by delaying sales is established by showing a negative relationship between forward sales and the experiment extraction.

As a consequence of the last two results, it turns out that the relationship between economic efficiency and the precision of monopolist's information is also *negative.* In fact, fixing the underlying unknown quality, an increase in the size of the binomial experiment, shifts mass onto higher extractions, and thus results on average in less forward sales. A larger postponement of sales reduces welfare and increases the informational loss. By the same token, it can also be shown that, fixing the underlying quality, bad news, in the form of a smaller monopolist's extraction, increases economic efficiency. The result is related with the distinction between social and private information. While an increment in the quality of social information is surely beneficial, an increase in private information, and thus in the informational gap between the players, may have adverse effect on welfare, as it tightens the constraints associated with the revelation of information.

It will also be shown that forward sales increase with the monopolists' impatience. The equilibrium is supported by the requirement for low-quality monopolists not to copy high-quality sales choices. When the loss induced by postponing sales decreases, separation may be supported only if the high-quality monopolists delay a more sales. In the limit case in which the monopolist is almost indifferent between selling sooner and later, she will sell a negligible amount in the forward markets, unless she holds the worst-possible quality observation. While an increase in the probability of high quality increases ex-ante utility, it also increases the informational loss. In fact, the low-quality monopolist's behavior does not impose any direct social effect, as she does not postpone sales. When the monopolist becomes less impatient, it will be shown that welfare increases, and the informational loss decreases, because the loss induced by fewer forward sales is offset by reduced impatience.

Since prices are the Bayesian estimates of quality, they consist of a convex combination of the prior mean and the quality observations. Thus an increase in the monopolist's quality extraction translates into higher prices, but with a multiplier smaller than 1. If the prices are above the prior mean, the price increment is positively related to the variance. Intuitively, if the prior is more flexible, then the price gives more weight to the observed values than on the prior beliefs. In the case of an almost perfectly informed monopolist, it will be shown that, upon observing the quantity offered in the forward markets, the buyers will be able to make an almost surely a correct estimate of quality. As a consequence, price variation in the spot markets uniformly vanishes almost surely, so that the spot market prices are almost constant over time. In that sense, a continuity result is established with the case of perfectly informed monopolists.

This paper is presented as follows. The second section studies the simple case in which the monopolist is perfectly informed and quality is disclosed to the buyers all at once. The third section deals with the case where the monopolist is not necessarily perfectly informed, and information is slowly released over time. Calculations and proofs omitted from the main body are in the Appendix.

2 Two-Period Case

This section studies the case where the monopolist is initially perfectly informed about quality, and her information is subsequently disclosed to the buyers all at once. As well as having interest of its own, this simple case serves as an introduction to the issues that we will encounter in the following section.

In this section, the product's quality is a random variable $\theta \in \{\theta_H, \theta_L\}$, $0 < \theta_L < \theta_H < 1$, with $\Pr(\theta = \theta_L) = \lambda \in (0, 1)$. At time $t = 0$, the quality is known by the monopolist only, it is disclosed to the buyers at time $t = 1$. There is a continuum of buyers identified with the interval $[0, 1]$. The monopolist chooses the quantity q to be sold in advance at time 0. This quantity is observed by the buyers who use it to update their expectations of quality. I require that the monopolist prefers to sell sooner than later. This assumption implies that the buyer's discount factor for money is higher than the monopolist's one. To simplify the analysis, I assume that the buyers do not discount the utility of a forward purchase, and that demand is infinitely elastic. At time 1, each buyer is willing to pay a price equal to θ, and at time $t = 0$, she is willing to pay $E(\theta|q)$. The monopolist discounts future sales with factor δ. Thus the type-θ monopolist's profit for selling quantity q at time 0 is:

$$u_\theta(q) = qE[\theta|q] + \delta(1 - q)\theta. \tag{1}$$

I begin the analysis by considering the separating equilibrium profiles $\mathbf{q} = (q_H, q_L)$. Notice first that in any separating equilibrium, the buyers' beliefs must be correct on path, and hence $E(\theta|q_H) = \theta_H$ and $E(\theta|q_L) = \theta_L$. Thus, when choosing q_L, the low-quality monopolist enjoys the expected utility $q_L\theta_L + \delta q_L\theta_L$, and so sets $q_L = 1$ whenever $\delta < 1$, and picks any $q_L \in [0, 1]$ when $\delta = 1$. As is well known, in signaling models such as this one there are multiple separating equilibria since off-path beliefs are free. Any such equilibrium must satisfy the incentive compatibility constraint for the low quality monopolist:

$$q_L\theta_L + \delta(1 - q_L)\theta_L \geq q_H\theta_H + \delta(1 - q_H)\theta_L. \tag{2}$$

These equilibria are supported by off-path beliefs such as those satisfying $E(\theta|q) = \theta_L$ for all $q < q_H$ and $E(\theta|q) = \theta_H$ for all $q \geq q_H$. At the optimal separating equilibrium, incentive compatibility is satisfied without slack, so that:[5]

$$q_H = \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}.$$

In the Appendix, I show that incentive compatibility is also satisfied for the high quality monopolist, and thus that any profile $\mathbf{q} = (1, q_H)$, with $q_H \in \left[0, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right]$ is a separating equilibrium. In the Appendix, it is also shown that *the only separating equilibrium that satisfies the Intuitive Criterion* (see Cho and Kreps 1987) *is the second-best PBE* $\mathbf{q}^* = \left(1, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)$.[6]

As well as separating equilibria, the model allows also for pooling equilibria in which $q_H = q_L = q$, supported by off-path beliefs such as $E(\theta|q') = \theta_L$, for all $q' \neq q$. However, while separating Perfect Bayesian Equilibrium profiles exist given any parameter specification, it turns out that any equilibrium pooling behavior is incompatible with monopolists being too impatient to sell. Specifically, it is shown in the Appendix that *there do not exist any pooling Perfect Bayesian Equilibrium for* $\delta \in (\lambda + (1-\lambda)\theta_L/\theta_H, 1)$. At the same time, for any $\delta < \lambda + (1-\lambda)\theta_L/\theta_H$, the pooling equilibrium profiles are $q \in [\frac{(1-\delta)\theta_L}{\lambda(\theta_H - \theta_L) + (1-\delta)\theta_L}, 1]$, and *each of them fails to conform with the Intuitive Criterion.*

A special case is that of a monopolist indifferent between selling sooner or later. For $\delta = 1$, there exists a pooling equilibrium where $q_H = q_L = 0$, and separating equilibria where $q_H = 0, q_L \in (0, 1]$. All these equilibria satisfy the Intuitive Criterion. While formally, the equilibrium $q_H = q_L = 0$ is a pooling equilibrium, it must be noted that all transactions are conducted at period 1, when the monopolist's private information has been revealed. In fact, the pooling equilibrium yields exactly the same payoff outcomes as the separating equilibrium, for the case of $\delta = 1$.

[5]It is easy to check that under this paper's assumptions, $\frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L} \in [0, 1)$.

[6]A Perfect Bayesian Equilibrium fails the Intuitive Criterion if any type is willing to unilaterally deviate, once the buyers' beliefs off-path are adjusted so as not to lay any positive probability to any type θ taking actions that are equilibrium dominated.

As the second-best PBE $\mathbf{q}^* = \left(1, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)$ is the unique intuitive equilibrium (up to payoff equivalence for $\delta = 1$), I conclude this section by exploring its welfare and comparative statics properties.

Since $\partial q_H / \partial \delta = -\theta_L \frac{\theta_H - \theta_L}{(\theta_H - \delta\theta_L)^2}$, *the high-quality forward sales are decreasing in* δ. As the monopolist becomes less impatient to sell, the requirement for the low-quality monopolist not to copy the high-quality one becomes more constrictive, because the loss of postponing sales becomes smaller. In order to avoid pooling, the high-quality monopolist must postpone more sales to period 1.

By the same token, since $\partial q_H / \partial(\theta_H - \theta_L) = -\frac{\theta_H(1-\delta)}{(\theta_H - \delta\theta_L)^2} < 0$, if the gap in quality increases, the high-quality monopolist makes less sales at period 0. When the gap in quality is higher the low-quality monopolist has a stronger incentive to pool with the high-quality one.

Ex-ante social welfare is the discounted equilibrium value of trade:

$$W = \lambda \left[\frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L} + \delta \left(1 - \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L} \right) \right] \theta_H + (1-\lambda)\theta_L.$$

It is straightforward to see that an increase in the probability of high quality increases ex-ante welfare. Less obvious is the fact that, *when the monopolist becomes more patient, ex-ante welfare increases,* in fact $\partial W / \partial \delta = \frac{\lambda\theta_H(\theta_H - \theta_L)^2}{(\theta_H - \delta\theta_L)^2} > 0$. In order to appreciate this result, notice that the discount factor enters the loss function in two separate ways. It enters indirectly by reducing the equilibrium period-0 sales of the high-quality monopolist, and thus increasing the undiscounted loss, but it also enters directly by discounting (and thus reducing) the loss. It turns out that the direct effect dominate the indirect one, so that the net effect is a loss reduction.

It is also possible to show that the parameters θ_H and θ_L have a positive impact on the social welfare. In fact, $\partial W / \partial \theta_H = \lambda\delta \frac{(\theta_H - \delta\theta_L)^2 - (1-\delta)^2\theta_L^2}{(\theta_H - \delta\theta_L)^2}$, which is strictly positive for $\delta \leq 1$, and $\partial W / \partial \theta_L = \frac{(\theta_H - \delta\theta_L)^2 - \lambda\delta(\theta_H - \theta_L)[(1-\delta)\theta_H + (\theta_H - \delta\theta_L)]}{(\theta_H - \delta\theta_L)^2}$ which is positive for $\delta \in [0,1]$, and $\lambda \leq 1$.

The efficiency loss with respect to the perfect information case equals the value of high quality transactions that are postponed to period 1, times the deterioration rate, multiplied by the probability of a high quality product:

$$L = \lambda(1-\delta)\left[1 - \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right]\theta_H = \lambda\frac{\theta_H\left(\theta_H - \theta_L\right)(1-\delta)}{\theta_H - \delta\theta_L}.$$

It is straightforward to see that the loss increases in the probability of a high-quality product. In fact, the low-quality monopolist does not impose any direct social cost, as she does not postpone sales. When the monopolist becomes less impatient to sell, the loss decreases, in fact $\partial L/\partial\delta = -\theta_H\frac{(\theta_H-\theta_L)^2}{(\delta\theta_L-\theta_H)^2} < 0$. While an improvement of the low-quality product reduces the efficiency loss ($\partial L/\partial\theta_L = -\frac{\lambda(1-\delta)^2\theta_H^2}{(\theta_H-\delta\theta_L)^2} < 0$), *an increase of the high quality product makes the efficiency loss larger:* $\partial L/\partial\theta_H = \lambda\left(1-\delta\right)\frac{(\theta_H-\delta\theta_L)^2+\delta(1-\delta)\theta_L^2}{(\theta_H-\delta\theta_L)^2} > 0$. When the high-quality product is more valuable, in fact, the low-quality monopolist has a stronger incentive to copy the high-quality one. This results in a larger quantity of high-quality product withdrawn from sale at period 0, and thus more trade inefficiently delayed.

3 The Infinite-Period Case

3.1 The Model

In this section public information about quality is slowly disclosed over time. I denote the information disclosed at time $t \in \mathcal{N}$ by x_t, an extraction from a Bernoulli distribution of unknown parameter θ. The parameter θ is extracted from the Beta distribution[7] with parameters (α, β), that are common knowledge. At time $t = 0$, the monopolist is privately informed of the extraction y from an experiment of size m, where the probability of success of each trial is θ, thus y is distributed according to a binomial distribution of parameters (θ, m). The statistic y/m is an estimate of the quality, as $E(y/m) = \theta$. The size m is common knowledge and represents the precision of the monopolist's extraction: as $V(y/m) = \theta(1-$

[7] The Beta distribution is the standard prior used to model updating of Bernoulli extractions. It is a distribution on $[0, 1]$ that allows great flexibility with respect to the first two moments. See for example Mood Graybill and Boes (1988).

$\theta)/m$, the variance of the monopolist's information is inversely proportional to the size of the experiment.

I assume that for each time $t > 0$, there is a market for the forward sale of the product at time t, that all forward markets are open at time 0, and that at any other time only the spot market t is open. Also, the monopolist operates on forward markets only by offering perpetuities. Specifically, after observing the extraction y, she must decide the constant quantity $q_y \in [0, 1]$ to offer for sale in all the forward markets, and the residual quantity will be offered in the spot markets. The monopolist discounts profits with factor δ.

Buyers are indexed by i on the continuum $[0, 1]$. Given time τ information Ω_τ, the i-th buyer's utility from a time-t purchase $(t > \tau)$ is $u_i(t|\Omega_\tau) = \gamma^{t-\tau}[E[x_t|\Omega_\tau] + a(1-i)]$, where γ is her discount factor, and $a > 0$. At any time τ, Ω_τ includes q_y and the history of past Bernoulli extractions. The sum of past realizations $\sum_{s=1}^{\tau-1} x_s$ is denoted by $\mathbf{x}_{\tau-1}$, and $\mathbf{x}_0$ is set equal to zero. The buyers use the quantity q_y to infer the monopolist's extraction y. That, together with the sum of past realizations $\mathbf{x}_{\tau-1}$, allows them to form the expectation $E[x_t|\Omega_\tau]$, according the Beta-binomial updating model introduced above.

As in the previous section, to simplify the analysis, I only consider the case for small heterogeneity among buyers, and so I take $a \to 0$. I require buyers to be impatient to make forward purchases and the monopolist to be impatient to make forward sales, that assumption translates in the implication that $\gamma > \delta$. I restrict attention to the case of infinitely patient players, and take $\delta \to 1$. For simplicity, the results are derived by holding $\frac{1-\delta}{1-\gamma}$ fixed and equal to k. Since $\gamma > \delta$, it follows that $k > 1$. The results are derived by taking first the limit $a \to 0$, and then the limit $\delta \to 1$.

3.2 Results

The following Lemma shows that at any period $t > 0$, the price equilibrium p_t of each spot market t will be close to $E[\theta|q, \mathbf{x}_{t-1}]$, for a close to 0, and that the monopolist will exhaust buyers' demands. Since heterogeneity is very small, it is in fact intuitive that the monopolist does not have any incentive withholding sales in the spot markets.

Lemma 1 *For any given $k > 1$, and forward sales q, at each time $t > 0$, the monopolist will sell quantity $Q_t = 1 - q$. For a close to 0, the buyers will pay approximately the price:*

$$p_t = E[\theta|q, \mathbf{x}_{t-1}].$$

The second Lemma derives the equilibrium price of a perpetuity given the quantity offered q. Since buyers have rational expectations, they include the information contained in q in the calculation of their willingness to pay. Since they discount future payoffs, they will pay a finite price for a perpetuity.

Lemma 2 *In equilibrium, given the monopolist's forward sales offer q, the buyers are willing to pay $P_0 = E[\theta|q]\gamma/(1-\gamma)$ for a perpetuity.*

In light of the above Lemmata, for a close to zero, the monopolist's problem after observing y consists of picking q such that:

$$\max_{q\in[0,1]} u_y(q) = q\frac{1-\delta}{1-\gamma}\gamma E[\theta|q] + [1-q](1-\delta)\delta E\left[\sum_{t=0}^{\infty}\delta^t E[\theta|q,\mathbf{x}_t]\middle| y\right]. \tag{3}$$

By carrying out calculations with respect to the Bayesian updating, and by taking the limit for δ close to 1, the monopolist problem can be represented in a functional form equivalent to Equation (1) in the second section.

Lemma 3 *Given any k>1, and any m, taking a close to 0, and then take δ close to 1, for any type $y \in \{0, 1, ..., m\}$, the expected net present value of choice $q \in [0, 1]$ is approximately:*

$$u_y(q) = qkE[\theta|q] + [1-q]E[\theta|y] = qkE[\theta|q] + [1-q]\frac{\alpha+y}{\alpha+\beta+m}. \tag{4}$$

While comparing Equations (1) and (4), one realizes that the monopolist's incentives are mathematically equivalent to those studied in the second section, the introduction of a Beta-binomial model of the monopolist's information requires analyzing the problem for any experiment size m, and thus requires extending the analysis from the case of two types to any number $m + 1$ of types.

As in the second section, the model allows for a plethora of Perfect Bayesian Equilibria. In the Appendix, I show that, as in the Spence model (cf. Banks and Sobel 1987, Fudenberg and Tirole 1991) with more than two types, the Intuitive Criterion may be ineffective in selecting the second-best separating equilibrium. Following Banks and Sobel (1987), I restrict attention to Perfect Bayesian Equilibria satisfying Universal Divinity, a refinement of the Intuitive Criterion. Intuitively, consider any action off the equilibrium path. For each type of monopolist determine the set of buyers' strategies that improve the monopolist's payoff with respect to the equilibrium. Whenever the set associated with one type strictly contains the set associated with another, the equilibrium beliefs are required to lay at most infinitesimal mass on the second type.

The following Proposition characterizes the unique Perfect Bayesian Equilibrium satisfying Universal Divinity. It is a fully-separating equilibrium where the monopolist reveals all her information by means of the forward-sale offer.

Proposition 1 *Given any k>1, and any m, taking a close to* 0, *and then* δ *close to* 1, *there exists a unique PBE satisfying Universal Divinity. In this equilibrium, upon observing signal* y, *the monopolist offers for sale in the forward markets approximately the quantity* q_y, *where* $q_0 = 1$, *and,* $\forall y \in \{1, ..., m\}$,

$$q_y = \prod_{s=1}^{y} \frac{(\alpha + s - 1)(k - 1)}{(\alpha + s - 1)(k - 1) + k}.$$

The forward price P_0 *is such that:*

$$(1 - \gamma)P_0 = \gamma \frac{\alpha + y}{\alpha + \beta + m},$$

and each time $t \geq 0$, *the spot-market price is approximately:*

$$p_{t+1} = \frac{\alpha + y + \mathbf{x}_t}{\alpha + \beta + m + t}.$$

The key insight of Proposition 1 is that q_y is strictly decreasing in y, so that the monopolist chooses to postpone sales in order to signal higher quality. Specifically, for any $y \geq 1$,

$q_y = z_y q_{y-1}$, where $z_y = \frac{(\alpha+y-1)(k-1)}{(\alpha+y-1)(k-1)+k}$. Since for any y, z_y is in $(0,1)$, it follows that q_y is decreasing in y. The coefficient z_y may be interpreted as the loss incurred by the $(y-1)$-th lowest quality type on the y-th lowest quality type. In order to avoid the $(y-1)$-th type to copy her, the y-th type must put on sale at most a fraction z_y of the amount put on sale by the $(y-1)$-th type. Since the $(y-1)$-th type avoids being copied by the $(y-2)$-th type, though, the latter type imposes an indirect loss on the y-th type too. Iteratively, all the types $s = 0, 1, ..., y-1$ impose an informational loss on type y, as $q_y = \prod_{s=1}^{y} z_s$.

Since the equilibrium is perfectly separating, all information contained in y is signalled in the forward-sale offer q_y. Because of that, the forward price P_0 consists of the discounted value of a stream of purchases with expected quality equal to the Bayesian estimate of θ given the draw y from a binomial experiment of size m. The spot prices also incorporate public information disclosed over time. Thus the period $t+1$ spot price p_{t+1} consist of the Bayesian estimate of θ given the extraction $y + \mathbf{x}_t$ from a binomial experiment of size $m+t$.

3.3 Forward-Sale Comparative Statics

It has already been shown in the previous section that q_y is strictly decreasing in y. This subsection presents further testable implications with respect to forward sales. As is customary working out, these implications involve exercises in comparative statics.

A somewhat counterintuitive result is that while the monopolist's estimate of quality coincide with the statistic y/m, the forward-sale offer q_y depends only by y, the (inverse) order of quality of the monopolist, regardless of m, the size of the experiment. The result is understood by noticing that the informational constraint of postponing sales is bore by a high quality monopolist only because she needs to differentiate herself from all lower quality monopolists, regardless of her actual quality.

It is also interesting to note that while q_y is decreasing in y, the absolute value of the decrement $q_{y+1} - q_y$ is decreasing in y. Notice in fact that $|q_{y+1} - q_y| = |(z_{y+1} - 1)q_y| = \frac{k}{(\alpha+y-1)(k-1)} q_y$, and both these quantities are decreasing in y. This means that the informational loss imposed by the y-th lowest type on the $(y+1)$-th type becomes smaller and smaller as y increases. The bigger share of sale postponement suffered by a monopolist with a high quality y/m is thus imposed by the lowest-quality types, rather than by the types whose quality is closer to y/m.

Let us focus now on the effect of a change in the ratio of k on the forward-sale quantity q_y. I will show by induction that for any $y \geq 1$, $\partial q_y/\partial k > 0$. Notice first that $\partial q_1/\partial k = \partial z_1/\partial k = \frac{\alpha}{(\alpha k - \alpha + k)^2} > 0$. For any $y \geq 1$, $\partial q_y/\partial k = q_{y-1}\partial z_y/\partial k + z_y \partial q_{y-1}/\partial k$. By the induction hypothesis, $\partial q_{y-1}/\partial k > 0$. Since, $q_{y-1} > 0$, and $z_y > 0$, it suffices to show that $\partial z_y/\partial k > 0$, for any y. In fact, $\partial z_y/\partial k = \frac{\alpha+(y-1)}{((\alpha+y)(k-1)+1)^2} > 0$. Thus, *when k increases, forward sales increase.* High-quality monopolists are required to postpone sales to avoid being copied by lower-quality types. Since a larger impatience to sell increases the disincentive for sale postponements, it allows separation with the delay of a smaller amount of sales.

Considering the limit cases, we first see that for $k \to 1^+$, even the monopolist with the second-lowest experiment extraction will offer a negligible quantity in the forward markets: $q_1 \to 0^+$. This does not imply that the separating equilibrium unravels however. Regardless of how large m is, each $y \in \{1, ..., m\}$ will sell smaller and smaller quantities q_y, and these apparently negligible differences will be sufficient to separate out the types. When $k \to \infty^+$, the monopolist is infinitely impatient to sell. For any given m, even the best quality monopolist nearly exhaust sales in the forward markets: in fact $q_m \to 1^-$. Separation occurs with very little constraint on forward sales, because the immediate revenue from selling a perpetuity is much higher than the present value of the stream of income obtained by sales on the spot markets.

Finally, I consider the effect of a change in the mean or variance of the prior distribution over quality. Since $\theta \sim Beta(\alpha, \beta)$, its mean is $\mu = \frac{\alpha}{\alpha+\beta}$, and standard deviation is $s = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$. Solving out for α as a function of μ, and s, we obtain $\alpha = -\mu\frac{\mu^2+s^2-\mu}{s^2}$. Since $\partial z_y/\partial \alpha = \frac{k(k-1)}{((\alpha+y)(k-1)+1)^2} > 0$, analogously as the case for k, it can be shown by induction that $\partial q_y/\partial \alpha > 0$, for all $y \geq 1$. Since $\partial \alpha/\partial \mu = -\frac{(\mu-s)^2+2\mu^2}{s^2} < 0$, and $\partial \alpha/\partial s = -\mu^2\frac{1-\mu}{s^2} < 0$, it turns out that *an increase in either the mean or in the variance of the distribution over* *ex-ante quality results in less forward sales.* In order to understand the result, note that when expected quality is higher, copying higher-quality types yields a higher payoff, that translates in requiring more sales from higher types in order to separate themselves from lower types.

3.4 Welfare Implications

The welfare and informational loss it the equilibrium derived in Proposition 1 display the same functional forms as those derived for the 2-period case.

Proposition 2 *In the equilibrium derived in Proposition 1, for given θ and y, the welfare and loss are approximately:*

$$W(\theta, y) = \theta[q_y k + (1 - q_y)] \qquad L(\theta, y) = \theta k(1 - q_y).$$

For the case of a close to 0 and δ close to 1, welfare and informational loss are controlled by the quality θ (and thus in an ex-ante sense by α and β, the parameters of the prior distribution over quality), by k, the monopolist impatience to sell, and by the monopolist forward sales q_y (and thus indirectly by y, and so, in an interim sense, by m and θ). Note however that welfare and loss do not depend directly on the quality estimate y/m, but only on the order extraction y.

The monopolist's impatience to sell k enters the welfare function $W(\theta, y)$ directly, as a multiplier of $q_y\theta$, and indirectly in the formula for q_y. Since $\partial q_y/\partial k > 0$, it is straightforward to conclude that *the effect of a change of k on the welfare function $W(\theta, y)$ is positive,* and that the result extends to interim and ex-ante welfare. The effect of k on the loss function is in general ambiguous: consider, for instance, $\partial L(\theta, 1)/\partial k = \theta(1 - q_1 - k\partial q_1/\partial k) = \theta k \frac{\alpha(k-1)+k-\alpha}{(\alpha k-\alpha+k)^2}$, that quantity is positive for $k > \frac{2\alpha}{\alpha+1}$, and negative otherwise.

We now show that the quality parameter θ, and the mean of its distribution have the same sign of impact on economic efficiency as in the two-period case. It is straightforward to show that *an increase in θ increases the informational loss $L(\theta, y) = \theta k(1 - q_y)$.* It also increases the interim loss $E(L(\theta, y)|\theta)$: since y is an extraction from a binomial distribution of parameters θ and m, an increase in θ shifts mass onto higher values of y, and thus it results in fewer forward sales q_y. Since $E(L(\theta, y)|\alpha, \beta) = E(E[L(\theta, y)|\theta]|\alpha, \beta)$, an increase in the mean of the distribution of θ increases the ex-ante loss $E(L(\theta, y)|\alpha, \beta)$.

The effect of a change in θ on interim welfare for fixed m is seen by analyzing the relation:

$$E(W(\theta, y)|\theta) = \theta \sum_{y=0}^{m} [q_y k + (1 - q_y)] \binom{m}{y} \theta^y (1 - \theta)^{m-y}.$$

The parameter θ enters the interim expected welfare function both as a positive multiplier and as the control for $E([q_y k + (1 - q_y)]|\theta)$, where it has a negative effect because it shifts mass onto lower q_y. Since we know that the quantity $q_y - q_{y+1}$ is decreasing in y, and that all the quantities q_y are increasing in k, we can conclude that the effect of θ on interim welfare is always positive if that is so for the worst-case scenario, where $k \to 1^+$. In fact, since $\lim_{k=1^+} q_y = 0$ for $y > 0$, and $q_0 = 1$, $\lim_{k=1^+} E(W(\theta, y)|\theta) = \theta(1 - \theta)^m + \theta \sum_{y=1}^{m} \binom{m}{y} \theta^y (1 - \theta)^{m-y} = \theta > 0$. Again, the result is the same in the ex-ante sense: an increase in the mean of the random variable θ increases expected welfare.

Unlike in the previous section, the introduction of a Beta-binomial model of updating allows us to determine the impact of the monopolist's information on economic efficiency, independently of the actual quality of the good. A straightforward yet unexpected result is that, *fixing the unknown parameter θ, and the size of the experiment m, bad news improves efficiency.* In fact, it has been previously shown that q_y is decreasing in y, and since $k > 1$, welfare $W(\theta, y)$ is increasing in q_y, and loss $L(\theta, y)$ is decreasing in q_y.

The most surprising result of the section is that the effect of the monopolist's information precision m on economic efficiency is also negative. This result holds both in the interim case, when the quality θ is fixed and unknown, and in the ex-ante case, in which θ is a random extraction of a Beta distribution parametrized by α and β.

Proposition 3 *In the equilibrium derived in Proposition 1, the interim welfare $E(W(\theta, y)|\theta, m)$ and the ex-ante welfare $E(W(\theta, y)|\alpha, \beta, m)$ are strictly decreasing in m;*

likewise, the interim loss $E(L(\theta,y)|\theta,m)$ and the ex-ante loss $E(L(\theta,y)|\alpha,\beta)$ are strictly increasing in m.

Proof. Since y is an extraction from a binomial distribution with parameters θ and m, $Pr(y|\theta,m) = \binom{m}{y}\theta^y(1-\theta)^{m-y}$. Thus, for a fixed θ, the function

$$\sum_{s=0}^{y} Pr(s|\theta,m) = \sum_{s=0}^{y} \binom{m}{s}\theta^s(1-\theta)^{m-s}$$

is strictly decreasing in m. That is to say, if $m' > m$, then $y|\theta,m'$ first-order stochastically dominates $y|\theta,m$. Since q_y is a decreasing function of y, it follows that $E(q_y|\theta,m') < E(q_y|\theta,m)$. Then, as

$$E(W(\theta,y)|\theta,m) = \theta[kE(q_y|\theta,m) + (1 - E(q_y|\theta,m))],$$

and $k > 1$, it follows that $E(W(\theta,y)|\theta,m') < E(W(\theta,y)|\theta,m)$. Therefore the interim expected welfare $E(W(\theta,y)|\theta)$ is strictly decreasing in m, and an analogous argument shows that the interim expected loss $E(L(\theta,y)|\theta)$ is strictly increasing in m.

The result extends to the ex-ante case in which θ is a random extraction of a Beta distribution parametrized by α and β: since $E(W(\theta,y)|\alpha,\beta) = E(E[W(\theta,y)|\theta,m]|\alpha,\beta)$, an increase in m reduces $E(W(\theta,y)|\alpha,\beta)$, and increases $E(L(\theta,y)|\alpha,\beta)$. ■

3.5 Spot-Market Prices

This sub-section is devoted to giving predictions regarding the spot-market price paths implied by the equilibrium derived in Proposition 1. Since the prices $p_{t+1} = \frac{\alpha+y+\mathbf{x}_t}{\alpha+\beta+m+t}$ consist of the Bayesian estimates of θ given y and $\mathbf{x}_t$, we can rewrite

$$p_{t+1} = \mu\frac{\alpha+\beta}{\alpha+\beta+m+t} + (y/m)\frac{m}{\alpha+\beta+m+t} + (\mathbf{x}_t/t)\frac{t}{\alpha+\beta+m+t},$$

and note that the price at time t is the convex combination of the prior mean of θ, of the monopolist's private information y/m, and of the average observed quality $\mathbf{x}_t/t$. Therefore, fixing m, an increase in y implies a higher spot price; and, fixing t, a higher x_t implies higher a price. Conversely, an increase in m, for a fixed y, reduces the spot price p_t. For a fixed β, an increase in α yields a higher price, and in general, if the distribution of θ stochastically dominates the distribution of θ', the price associated with θ dominates the price associated with θ'. However, in the mean/variance space, the effect of an increase of the prior mean of θ is in general ambiguous.[8] The effect of prior variance in the mean/variance space is simpler to see, because:

$$\frac{dp_{t+1}}{ds^2} = \frac{\beta + (m-y) + (t-x_t)}{(\alpha+\beta+m+t)^2} \frac{\alpha^2(\frac{\alpha}{\alpha+\beta}-1)}{(\alpha+\beta)^2 s^2} + \frac{\alpha+y+x_t}{(\alpha+\beta+m+t)^2} \frac{\alpha\left(\frac{\alpha}{\alpha+\beta}-1\right)^2}{(\alpha+\beta)s^2},$$

and the above quantity is positive if and only if $\alpha/(\alpha+\beta) < (y+x_t)/(m+t)$. So that *if the observed mean is above the prior mean, then an increase in variance increases* p_t, *and vice-versa.* Intuitively, if the prior is more flexible, then the price gives more weight to observed values than to prior beliefs.

Now, consider price variations:

$$\Delta p_{t+1} = \frac{\alpha+y+\mathbf{x}_{t-1}+x_t}{\alpha+\beta+m+t} - \frac{\alpha+y+\mathbf{x}_{t-1}}{\alpha+\beta+m+t-1} = \frac{x_t(\alpha+\beta+m+t-1)-(\mathbf{x}_{t-1}+\alpha+y)}{(\alpha+\beta+m+t)(\alpha+\beta+m+t-1)}.$$

Obviously, if $x_{t+1}=0$, the price variation is negative and, if $x_{t+1}=1$, the price variation is positive. Intuitively, a larger m and a larger t yield lower price variation, in the sense that $\partial|\Delta p_{t+1}|/\partial(m+t) < 0$. Also, a larger x_t or a larger y yield a lower increase in price, in the sense that $\partial\Delta p_{t+1}/\partial x_t < 0$, and $\partial\Delta p_{t+1}/\partial y < 0$. The effect of prior parameters are generally ambiguous.

In terms of expected prices and price variation, we distinguish between the standpoint of an external observer who knows the realization of θ, and that of the players. Given θ,

$$E(p_{t+1}|\theta) = \frac{\alpha+E(y|\theta)+E(\mathbf{x}_t|\theta)}{\alpha+\beta+m+t} = \frac{\alpha+(m+t)\theta}{\alpha+\beta+m+t} = \mu\frac{\alpha+\beta}{\alpha+\beta+m+t} + \theta\frac{m+t}{\alpha+\beta+m+t},$$

[8]Notice that $\partial p_{t+1}/\partial\alpha = \frac{\beta+(m-y)+(t-x_t)}{(\alpha+\beta+m+t)^2} > 0$, $\partial p_{t+1}/\partial\beta = -\frac{\alpha+y+x_t}{(\alpha+\beta+m+t)^2} < 0$, $\partial\alpha/\partial\mu = -\frac{s^2+3\mu^2-2\mu}{s^2} > 0$ whenever $1-\sqrt{(1-3s^2)} < 3\mu < 1+\sqrt{(1-3s^2)}$, and $\partial\beta/\partial\mu = \frac{3\mu^2-4\mu+1+s^2}{s^2} > 0$ whenever $3\mu < 2-\sqrt{(1-3s^2)}$ or $3\mu > 2+\sqrt{(1-3s^2)}$. Thus, while for $0 < s^2 < 1/4$, $\partial p_{t+1}/\partial\mu > 0$, when $2-\sqrt{(1-3s^2)} < 3\mu < 1+\sqrt{(1-3s^2)}$, it is also the case that for higher variance $s^2 > 1/3$, $dp_{t+1}/d\mu < 0$.

so that an external observer expects the prices to move over time from the prior mean $\alpha/(\alpha+\beta)$ to the realization of θ: the quantity

$$E(\Delta p_{t+1}|\theta) = \frac{(\theta-\mu)(\alpha+\beta)}{(\alpha+\beta+m+t)\,(\alpha+\beta+m+t-1)},$$

is positive or negative depending whether μ underestimates the realized θ, or vice-versa. By the same token, given the realized information y, and $\mathbf{x}_t$, the external observer expects the price variation Δp_{t+1} to adjust the price p_t closer to the realized θ, specifically,

$$E(\Delta p_{t+1}|\theta, y, \mathbf{x}_t) = \frac{(\theta - p_t)}{(\alpha+\beta+m+t)}.$$

On the other hand, the players do not know θ while playing the game. Since $E(\theta|y, x_{t-1}) = p_t$, they take the price sequence as a martingale: $E(\Delta p_{t+1}|p_t) = 0$, for any $t \geq 0$.

It is a standard result from Bayesian statistics (see for example Feller 1950, Chapter 8) that for t approaching infinity, p_t, the Beta-binomial estimate of θ converges almost surely to the realized θ. By the same token, Δp_{t+1} converges almost surely to 0.

3.6 An Almost Perfectly-Informed Monopolist

I conclude the paper by considering the case of an (almost) perfectly-informed monopolist and the slow disclosure of public information. From standard Bayesian statistics, it can be shown that the information contained in the forward-sale offer immediately yields almost surely a correct estimate of quality. As a consequence of that, the price variation Δp_{t+1} uniformly vanishes almost surely as m goes off to infinity. Thus spot prices are almost constant over time.

Proposition 4 *For $m \to \infty$, $(1-\beta)P_0$ converges to θ almost surely, all prices p_t uniformly converge almost surely to θ, and all price variations Δp_{t+1} uniformly vanish almost surely.*

4 Appendix

4.1 Calculations Omitted from Section 2

For any profile $\mathbf{q} = (1, q_H)$, with $q_H \in \left[0, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right]$ to be a separating equilibrium, it must be the case that incentive compatibility is satisfied for the high-quality monopolist. This requires that:

$$q_H\theta_H + \delta(1 - q_H)\theta_H \geq q\theta_L + \delta(1 - q)\theta_H, \ \forall q > q_H \tag{5}$$

For $\theta_L < \delta\theta_H$, this translates into: $q_H\theta_H + \delta(1 - q_H)\theta_H \geq \delta\theta_H$, which is satisfied, since $q_H(1 - \delta)\theta_H \geq 0$. For $\theta_L > \delta\theta_H$, it must be that $q_H\theta_H + \delta(1 - q_H)\theta_H \geq \theta_L$, which is satisfied for any $q_H \geq -\frac{\theta_L - \delta\theta_H}{\theta_H(1-\delta)}$. Since the last quantity is negative, I obtain that any $q_H \in \left[0, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right]$ characterizes a separating equilibrium.

Recall that a set of Perfect Bayesian Equilibrium beliefs conforms with equilibrium dominance, whenever they do not lay any positive probability to any type θ taking actions that are equilibrium dominated. Thus a PBE fails to satisfy the Intuitive Criterion if a type is willing to unilaterally deviate, once the buyers' beliefs are adjusted to conform with equilibrium dominance.

Consider any separating equilibrium: $\mathbf{q} = (1, q_H)$, $q_H \in \left[0, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right]$. By incentive compatibility, any strategy $q \in \left(q_H, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)$ is equilibrium dominated for type θ_L. Once the buyers' beliefs are adjusted to follow $E(\theta|q) = \theta_H$ for all $q \in \left(q_H, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)$, Equation 5 implies that all actions $q \in \left(q_H, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)$ are unilateral deviations that make type θ_H better off. Thus the only separating PBE satisfying the Intuitive Criterion is $\mathbf{q}^* = \left(1, \frac{(1-\delta)\theta_L}{\theta_H - \delta\theta_L}\right)$.

All pooling Perfect Bayesian Equilibrium outcomes $q_H = q_L = q$ may be supported by off-path beliefs such as $E(\theta|q') = \theta_L$, for all $q' \neq q$. The necessary conditions for q to be a PBE outcome are that for any $q' \neq q$, neither type of monopolist wants to deviate, i.e:

$$\begin{aligned} q(\lambda\theta_H + (1 - \lambda)\theta_L) + \delta(1 - q)\theta_H &\geq q'\theta_L + (1 - q')\delta\theta_H, \\ q(\lambda\theta_H + (1 - \lambda)\theta_L) + \delta(1 - q)\theta_L &\geq q'\theta_L + (1 - q')\delta\theta_L. \end{aligned}$$

For any δ, the second condition holds only if:

$$q(\lambda\theta_H + (1-\lambda)\theta_L) + \delta(1-q)\theta_L \geq \theta_L,$$

or

$$q \geq \frac{(1-\delta)\theta_L}{\lambda(\theta_H - \theta_L) + (1-\delta)\theta_L}.$$

It is easily checked that $\frac{(1-\delta)\theta_L}{\lambda(\theta_H-\theta_L)+(1-\delta)\theta_L} \in (0,1)$, unless $\delta = 1$. So, in particular, $q > 0$ unless $\delta = 1$.

With respect to the first condition, in the case where $\theta_L < \delta\theta_H$, we just need it to be that:

$$q(\lambda\theta_H + (1-\lambda)\theta_L) + \delta(1-q)\theta_H \geq \delta\theta_H,$$

or $q((1-\lambda)\theta_L + (\lambda-\delta)\theta_H) \geq 0$.

So, when $\lambda(\theta_H - \theta_L) < \delta\theta_H - \theta_L$, we should have $q = 0$, but this is ruled out by the low-quality monopolist's equilibrium condition, unless $\delta = 1$.

When $\theta_L/\theta_H < \delta < \lambda + (1-\lambda)\theta_L/\theta_H$, instead any $q \in [\frac{(1-\delta)\theta_L}{\lambda(\theta_H-\theta_L)+(1-\delta)\theta_L}, 1]$ is a pooling PBE outcome.

In the case where $\theta_L > \delta\theta_H$, we need it to be that:

$$q(\lambda\theta_H + (1-\lambda)\theta_L) + \delta(1-q)\theta_H \geq \theta_L,$$

or $q((1-\lambda)\theta_L + (\lambda-\delta)\theta_H) \geq \theta_L - \delta\theta_H$. Since $\theta_L/\theta_H < \lambda + (1-\lambda)\theta_L/\theta_H$, the necessary condition is:

$$q \geq \max\{\frac{\theta_L - \delta\theta_H}{\theta_L - \delta\theta_H + \lambda(\theta_H - \theta_L)}, \frac{(1-\delta)\theta_L}{\lambda(\theta_H - \theta_L) + (1-\delta)\theta_L}\} = \frac{(1-\delta)\theta_L}{\lambda(\theta_H - \theta_L) + (1-\delta)\theta_L}.$$

To see that, when $\delta \leq \lambda + (1-\lambda)\theta_L/\theta_H$, all pooling PBE profiles q fail to satisfy the Intuitive Criterion, note that any action $q' \in [0, q'')$ is equilibrium dominated for the low-quality monopolist, when $q'' = q\frac{\lambda(\theta_L-\theta_H)+\theta_H-\delta\theta_L}{\theta_H-\delta\theta_L}$. In fact, the best that the low-quality monopolist can achieve by taking q' is $q'\theta_H + (1-q')\delta\theta_L$, which is less than the equilibrium payoff $q(\lambda\theta_L+(1-\lambda)\theta_H)+(1-q)\delta\theta_L$ for the specified q'. At the same time, $\frac{\lambda(\theta_L-\theta_H)+\theta_H-\delta\theta_L}{\theta_H-\delta\theta_L} \in$

$(0, 1)$, because $\delta < 1 < \frac{\lambda\theta_L + (1-\lambda)\theta_H}{\theta_L}$, $\theta_H - \delta\theta_L > 0$, and $\lambda(\theta_L - \theta_H) < 0$. Since $q \geq \frac{(1-\delta)\theta_L}{\lambda(\theta_H - \theta_L) + (1-\delta)\theta_L} > 0$, it follows that $q'' > 0$.

Once off-path buyers' beliefs have been adjusted to conform with equilibrium dominance, the PBE profile q fails the Intuitive Criterion test because there exists an ε small enough, for which the high-quality monopolist prefers to play $q' = q'' - \varepsilon$, rather than taking the equilibrium action q. In fact,

$$\begin{aligned} & q''\theta_H + (1-q'')\delta\theta_H = q''\theta_H + (1-q'')\delta\theta_L + (1-q'')\delta(\theta_H - \theta_L) \\ = \; & q(\lambda\theta_L + (1-\lambda)\theta_H) + (1-q)\delta\theta_L + (1-q'')\delta(\theta_H - \theta_L) \\ = \; & q(\lambda\theta_L + (1-\lambda)\theta_H) + (1-q)\delta\theta_H + \delta(\theta_H - \theta_L)(q - q'') \\ > \; & q(\lambda\theta_L + (1-\lambda)\theta_H) + (1-q)\delta\theta_H. \end{aligned}$$

4.2 Proofs Omitted from Section 3

Proof of Lemma 1. Given any firm offer Q_t, by the law of demand and supply, the equilibrium price p_t is determined by the marginal consumer $i = q + Q_t$ such that $U_i(t|\Omega_t) = p_t$.

Thus, $p_t = a(1 - q - Q_t) + E(x_t|\Omega_t)$, and the monopolist profit is $\pi_t = a(1 - q - Q_t) + E(x_t|\Omega_t)Q_t$. The derivative $E(x_t|\Omega_t) + a(1 - q - 2Q_t)$ is strictly positive for $Q_t \in [0, 1]$ for $a \to 0$, so that the firm sets $Q_t = 1$ and $p_t \to E[\theta|q, \mathbf{x}_{t-1}]$. ■

Proof of Lemma 2. Consider the buyers expected value for the ticket conditional on $\Omega_t = (q, \mathbf{x}_t)$, the information held at time t. Since the buyers have rational expectations, for any $t \geq 1$, $E[\theta|q] = E\left[E[\theta|q, \mathbf{x}_{t-1}]|q\right]$. Therefore, each consumer i forecasts a forward purchase to yield utility of $\sum_{t=1}^{\infty} \gamma^t u_i(t|q) = \sum_{t=1}^{\infty} \gamma^t [E[x_t|q] + a(1-i)] = \sum_{t=1}^{\infty} \gamma^t [E[\theta|q] + a(1-i)] = [E[\theta|q] + a(1-i)]\gamma/(1-\gamma)$.

Again, the price is set by the marginal consumer $i = q$ such that $[E[\theta|q] + a(1-i)]\gamma/(1-\gamma) = P_0$, and taking $a \to 0$, $P_0 \to E[\theta|q]/(1-\gamma)$. ■

Proof of Lemma 3. Given that $\theta \sim Beta(\alpha, \beta)$, and that $y \sim bin(\theta, m)$, it follows that $\theta|y \sim Beta(\alpha + y, \beta + m - y)$, so that

$$E(\theta|y) = \frac{\alpha + y}{\alpha + \beta + m}.$$

In particular,

$$E[\theta|q, \mathbf{x}_t] = \frac{\alpha + E(y|q) + \mathbf{x}_t}{\alpha + \beta + m + t}.$$

Since $\mathbf{x}_t|y \sim bin(t, \theta|y)$, it is also the case that:

$$E(\mathbf{x}_t|y) = \frac{\alpha + y}{\alpha + \beta + m}t.$$

so that

$$(1-\delta)\delta E\left[\sum_{t=0}^{\infty} \delta^t E[\theta|q, \mathbf{x}_t] \middle| y\right] = \delta \frac{\alpha + E(y|q) + \frac{\alpha+y}{\alpha+\beta+m}t}{\alpha + \beta + m + t}.$$

Therefore,

$$\text{for } \delta \to 1, \ (1-\delta)\delta E\left[\sum_{t=0}^{\infty} \delta^t E[\theta|q, \mathbf{x}_t] \middle| y\right] \downarrow \frac{\alpha + y}{\alpha + \beta + m},$$

and

$$u_y(q) \downarrow qkE[\theta|q] + [1-q]\frac{\alpha + y}{\alpha + \beta + m}.$$

■

Proof of Proposition 1. The proof consists of two separate Lemmata.

Lemma 4 *There exists a PBE such that for any* $y \in \{1, ..., m\}$, $q_y = \prod_{t=1}^{y} \frac{(\alpha+t-1)(k-1)}{(\alpha+t-1)(k-1)+k}$.

Proof. At any fully separating equilibrium,

$$E[\theta|q_y] = E(\theta|y) = \frac{\alpha + y}{\alpha + \beta + m},$$

and, the Incentive Compatibility constraints

$$u_y(q_y) \geq u_y(q), \ q \in [0, 1], \tag{6}$$

must be satisfied. Thus, since the type 0 monopolist obtains payoff equal to:

$$u_0(q_0) = q_0 k \frac{\alpha}{\alpha + \beta + m} + (1 - q_0)\frac{\alpha}{\alpha + \beta + m},$$

it must also be the case that $q_0 = 1$, as $k > 1$.

I propose an equilibrium where q_y is strictly decreasing in y, $q_0 = 1$, the supporting beliefs off-path are a step function, defined as follows:

$$\Pr[y|q] = 1 \text{ iff } q \in (q_{y+1}, q_y], \text{ where } q_{m+1} = 0, \text{ and } \Pr[m|0] = 1$$

and all the constraints

$$u_y(q_y) \geq u_y(q_{y+1}), \quad y \in \{0, 1, ..., m-1\}, \tag{7}$$

are binding.

Given that $q_0 = 1$, the binding constraints (7) uniquely pin down all the equilibrium quantities q_y, for $y \in \{1, 2, ..., m\}$, in a recursive fashion. To show that the profile that I propose is an equilibrium, I will only be left to show that the q_y pinned down are admissible, i.e. $q_y \in [0, 1]$, and that the remaining Incentive Compatibility requirements are satisfied.

To explicitly calculate q_1, see that the constraint that $u_0(q_0) \geq u_0(q_1)$ translates as:

$$k\frac{\alpha}{\alpha+\beta+m} = q_1 k \frac{\alpha+1}{\alpha+\beta+m} + [1-q_1]\frac{\alpha}{\alpha+\beta+m}$$

that yields solution

$$q_1 = \frac{\alpha(k-1)}{\alpha(k-1)+k} \in (0,1),$$

regardless of m.

For any $y \in \{1, 2, ..., m-1\}$, the constraint $u_y(q_y) \geq u_y(q_{y+1})$, translates as:

$$q_y k \frac{\alpha+y}{\alpha+\beta+m} + [1-q_y]\frac{\alpha+y}{\alpha+\beta+m} = q_{y+1} k \frac{\alpha+y+1}{\alpha+\beta+m} + [1-q_{y+1}]\frac{\alpha+y}{\alpha+\beta+m}$$

that yields solution:

$$q_{y+1} = q_y \frac{(\alpha+y)(k-1)}{(\alpha+y)(k-1)+k} \in (0,1).$$

All q_y can be calculated by recursion to yield:

$$q_y = \prod_{t=1}^{y} \frac{(\alpha+t-1)(k-1)}{(\alpha+t-1)(k-1)+k}, \quad \text{for } y = \{1, ..., m\}.$$

For future reference I define

$$z_{y+1} = \frac{(\alpha + y)(k-1)}{(\alpha + y)(k-1) + k},$$

and rewrite the above solution as $q_{y+1} = z_{y+1} q_y$.

Now I show that for any $y \in \{0, 1, ..., m-1\}$, and any $h \in \{1, 2, ..., m-y\}$, the incentive constraints requiring any type $y+h$ not to adopt q_y are satisfied and not binding. That is, we want to show that:

$$u_{y+h}(q_{y+h}) - u_{y+h}(q_y) > 0, \ \forall y \in \{0, 1, ..., m-1\}, \ \forall h \in \{1, 2, ..., m-y\} \tag{8}$$

First consider $u_{y+1}(q_{y+1}) - u_{y+1}(q_y)$, for arbitrary y.

$$\begin{aligned} & u_{y+1}(q_{y+1}) - u_{y+1}(q_y) \\ = \ & q_y z_{y+1} k \frac{a+y+1}{a+b+m} + [1 - q_y z_{y+1}] \frac{a+y+1}{a+b+m} - q_y k \frac{a+y}{a+b+m} - [1-q_y] \frac{a+y+1}{a+b+m} \\ = \ & \frac{k q_y}{(a+b+m)[(k-1)(a+y)+k]} > 0, \text{ for any } y. \end{aligned}$$

Notice that, for any $y \in \{0, 1, ..., m-1\}$, for any $l \in \{0, 1, ..., m-y-1\}$, and for any $h \in \{l+1, l+2, ..., m\}$,

$$\begin{aligned} & u_{y+h}(q_{y+l+1}) - u_{y+h}(q_{y+l}) \\ = \ & z_{y+l+1} q_{y+l} k \frac{a+y+l+1}{a+b+m} + [1 - z_{y+l+1} q_{y+l}] \frac{a+y+h}{a+b+m} \\ & - q_{y+l} k \frac{a+y+l}{a+b+m} - [1 - q_{y+l}] \frac{a+y+h}{a+b+m} \end{aligned}$$

with the substitution $y' = y + l$,

$$\begin{aligned} = \ & z_{y'+1} q_{y'} k \frac{a+y'+1}{a+b+m} + [1 - z_{y'+1} q_{y'}] \frac{a+y'+h-l}{a+b+m} \\ & - q_{y'} k \frac{a+y'}{a+b+m} - [1 - q_{y'}] \frac{a+y'+h'-l}{a+b+m} \\ = \ & [u_{y'+1}(q_{y'+1}) - u_{y'+1}(q_{y'})] + [1 - z_{y'+1} q_{y'}] \frac{h-l-1}{a+b+m} - [1 - q_{y'}] \frac{h-l-1}{a+b+m} \\ = \ & [u_{y'+1}(q_{y'+1}) - u_{y'+1}(q_{y'})] + (h-l-1) \frac{(1 - z_{y'+1}) q_{y'}}{a+b+m}. \end{aligned}$$

So, for any $y \in \{0, 1, ..., m-1\}$, and any $h \in \{1, 2, ..., m-y\}$,

$$u_{y+h}(q_{y+h}) - u_{y+h}(q_y) = \sum_{l=0}^{h-1} [u_{y+h}(q_{y+h-l}) - u_{y+h}(q_{y+h-l-1})]$$
$$= \sum_{l=0}^{h-1} \left[\left[u_{y+h-l}(q_{y+h-l}) - u_{y+h-l}(q_{(y+h-l)-1}) \right] + l \frac{(1 - z_{y+h-l}) q_{(y+h-l)-1}}{a+b+m} \right] > 0.$$

Finally, I show that the incentive constraints requiring any type $y \in \{0, ..., m-1\}$ not to adopt q_{y+h}, for any $h \in \{1, 2, ..., m-y\}$ are satisfied, and not binding. That is, we want to show that:

$$u_y(q_y) - u_y(q_{y+h}) > 0, \ \forall y \in \{1, ..., m-1\}, \ \forall h \in \{1, 2, ..., m-y\}. \tag{9}$$

By construction, $u_y(q_y) - u_y(q_{y+1}) = 0$, for any arbitrary $y \in \{0, 1, ..., m-1\}$, expanding the expression for further reference, we obtain:

$$\begin{aligned} 0 &= u_y(q_y) - u_y(q_{y+1}) \\ &= q_y k \frac{a+y}{a+b+m} + [1 - q_y] \frac{a+y}{a+b+m} - z_{y+1} q_y k \frac{a+y+1}{a+b+m} - [1 - z_{y+1} q_y] \frac{a+y}{a+b+m}. \end{aligned}$$

Notice that, for any $y \in \{0, 1, ..., m-1\}$, for any $l \in \{0, 1, ..., m-y-1\}$,

$$\begin{aligned} & u_y(q_{y+l}) - u_y(q_{y+l+1}) \\ = \ & q_{y+l} k \frac{a+y+l}{a+b+m} + [1 - q_{y+l}] \frac{a+y}{a+b+m} \\ & - z_{y+l+1} q_{y+l} k \frac{a+y+l+1}{a+b+m} - [1 - z_{y+l+1} q_{y+l}] \frac{a+y}{a+b+m} \end{aligned}$$

with the substitution $y' = y + l$,

$$\begin{aligned} &= q_{y'} k \frac{a+y'}{a+b+m} + [1 - q_{y'}] \frac{a+y'-l}{a+b+m} - z_{y'+1} q_{y'} k \frac{a+y'+1}{a+b+m} - [1 - z_{y'+1} q_{y'}] \frac{a+y'-l}{a+b+m} \\ &= [u_{y'}(q_{y'}) - u_{y'}(q_{y'+1})] + [1 - q_{y'}] \frac{-l}{a+b+m} - [1 - z_{y'+1} q_{y'}] \frac{-l}{a+b+m} \\ &= [u_{y'}(q_{y'}) - u_{y'}(q_{y'+1})] + [1 - q_{y'}] \frac{-l}{a+b+m} - [1 - z_{y'+1} q_{y'}] \frac{-l}{a+b+m} \\ &= l \frac{q_{y'}(1 - z_{y'+1})}{a+b+m} > 0. \end{aligned}$$

So, for any $h \in \{1, 2, ..., m - y\}$,

$$u_y(q_y) - u_y(q_{y+h}) = \sum_{l=0}^{h-1} [u_y(q_{y+l}) - u_y(q_{y+l+1})]$$
$$= \sum_{l=0}^{h-1} l \frac{q_{y+l}(1 - z_{y+l+1})}{a + b + m} > 0.$$

■

Lemma 5 *The unique equilibrium satisfying Universal Divinity is the PBE introduced in Lemma 4.*

Proof. Consider any fully-separating equilibrium $\mathbf{q}' \neq \mathbf{q}$. Since $\mathbf{q}$ satisfies the incentive compatibility constraints in Condition (7) without slack, for $\mathbf{q}'$ to be a separating equilibrium, it must be the case that there exists a type y such that $u_y(q'_y) > u_y(q'_{y+1})$. Suppose that the supporting beliefs are such that: $\Pr[y|q] = 1$ iff $q \in (q'_{y+1}, q'_y]$, where $q'_{m+1} = 0$, and $\Pr[m|0] = 1$.

Consider any quantity $q \in (q'_{y+1}, z_{y+1}q'_y)$. Condition (7) assures that the equilibrium payoff of type y is higher than the payoff obtained when taking q if the buyers would believe that in that case the monopolist is of type $y + 1$. However, q is not necessarily equilibrium dominated because Condition (7) does not imply that $u_y(q'_y)$ is higher than the payoff obtained by the y type when taking q if the buyers believe $\Pr(y + 2|q) = 1$. Consider the following numerical example. Say that $\alpha = 1$, $\beta = 1$, $k = 2$, and $m = 3$. Thus $u_0(1) = 2/5$, $z_1 = 1/3$. Say that $q'_1 = 1/5$, simple calculations show that none of the quantity $q \in (1/3, 1/5)$ is equilibrium dominated. Since the only possible equilibrium dominance refinement of the stipulated supporting beliefs concerns the quantities $q \in (q'_{y+1}, z_{y+1}q'_y)$, it is concluded that the Intuitive Criterion fails to refine the fully separating equilibrium $\mathbf{q}'$.

In order to refine fully-separating equilibrium, we invoke Banks and Sobel's Universal Divinity. Equilibrium beliefs satisfy criterion D1 if, whenever the set of consumer's best-response that makes a type y willing to deviate to q is strictly smaller than the set of responses that makes a type y' willing to deviate to q, then $\Pr(y|q) = o(\Pr(y'|q))$. A PBE satisfies Universal Divinity whenever its supporting beliefs satisfy D1.

Consider now the quantities $q \in (q'_{y+1}, z_{y+1}q'_y)$, the set of buyers' prices that make type y willing to deviate from q'_y is $[(\alpha + y)\frac{q'_y(k-1)+q}{(\alpha+\beta+m)qk}, 1]$, whereas the set of buyers' prices that make type $y+1$ willing to deviate from q'_{y+1} is $[(\alpha + y + 1)\frac{q'_{y+1}(k-1)+q}{(\alpha+\beta+m)qk}, 1]$. So I need to check when the latter set is larger. Let $D = (\alpha + y + 1)\frac{q'_{y+1}(k-1)+q}{(\alpha+\beta+m)qk} - (\alpha + y)\frac{q'_y(k-1)+q}{(\alpha+\beta+m)qk}$. Since $q'_{y+1} < z_{y+1}q'_y$, it is the case that $\partial D/\partial q > z_{y+1}q'_y$. So I need to have q smaller than the threshold q'' solving $D = 0$. Since $q'_{y+1} < z_{y+1}q'_y$, it is the case that $q'' > z_{y+1}q'_y$. Thus, for any $q \in (q'_{y+1}, z_{y+1}q'_y)$, the only beliefs satisfying D1 must be such that $\Pr(y|q) = o(\Pr(y+1|q))$. But when that is the case, type $y+1$ may deviate from q'_{y+1} to $z_{y+1}q'_y - \varepsilon$, so the equilibrium $\mathbf{q}'$ fails Universal Divinity.

I now consider the separating equilibrium $\mathbf{q}$ such that, for any $y \in \{0, 1, ..., m-1\}$, it is the case that $q_{y+1} = \frac{(\alpha+y)(k-1)}{(\alpha+y)(k-1)+k}q_y$, and $\Pr[y|q] = 1$ iff $q \in (q_{y+1}, q_y]$, where $q_{m+1} = 0$, $\Pr[m|0] = 1$. Given any q, the set of buyers prices that make monopolist y willing to deviate to q is $[(\alpha + y)\frac{q_y(k-1)+q}{(\alpha+\beta+m)qk}, 1]$. For any type y, and any $h \in \{-y, -y+1, ..., -2, -1, 1, 2, ..., m-y\}$, I introduce the following function of q:

$$D^{yh}(q) = (\alpha + y + h)(q_{y+h}(k-1) + q) - (\alpha + y)(q_y(k-1) + q).$$

Following the previous derivations, the equilibrium $\mathbf{q}$ satisfies Universal Divinity whenever for any y, and any $q \in (q_{y+1}, q_y)$, $D^{yh}(q) \geq 0$ for any $h \in \{-y, -y+1, ..., -2, -1, 1, 2, ..., m-y\}$.

First note that for any y,

$$\begin{aligned}
(\alpha + y + 1)\, q_{y+1}(k-1) - (\alpha + y)\, q_y(k-1) &= -q_{y+1} \\
(\alpha + y - 1)\, q_{y-1}(k-1) - (\alpha + y)\, q_y(k-1) &= q_y.
\end{aligned}$$

For any $h > 0$, and $q \in (q_{y+1}, q_y)$,

$$\begin{aligned}
D^{yh}(q) &= (\alpha + y + h)(q_{y+h}(k-1) + q) - (\alpha + y)(q_y(k-1) + q) \\
&= (\alpha + y + h)\, q_{y+h}(k-1) - (\alpha + y)\, q_y(k-1) + hq \\
&> (\alpha + y + h)\, q_{y+h}(k-1) - (\alpha + y)\, q_y(k-1) + hq_{y+1}
\end{aligned}$$

$$= \sum_{l=1}^{h}[(\alpha+y+l)\, q_{y+l}(k-1) - (\alpha+y+l-1)\, q_{y+l-1}(k-1)] + hq_{y+1}$$

$$= -\sum_{l=1}^{h} q_{y+l} + hq_{y+1} = \sum_{l=1}^{h}[q_{y+1} - q_{y+l}] \geq 0.$$

And similarly, for any $h < 0$, and $q \in (q_{y+1}, q_y)$,

$$\begin{aligned}
D^{yh}(q) &= (\alpha+y+h)\, q_{y+h}(k-1) - (\alpha+y)\, q_y(k-1) + hq \\
&> (\alpha+y+h)\, q_{y+h}(k-1) - (\alpha+y)\, q_y(k-1) + hq_y \\
&= \sum_{l=0}^{-(h-1)}[(\alpha+y-l-1)\, q_{y-l-1}(k-1) - (\alpha+y-l)\, q_{y-l}(k-1)] + hq_y \\
&= \sum_{l=0}^{-(h-1)} q_{y-l} + hq_y = \sum_{l=0}^{-(h-1)}[q_{y-l} - q_y] \geq 0.
\end{aligned}$$

Let us now consider pooling and semi-pooling equilibrium. Take any equilibrium $\mathbf{q}'$, where more than one type plays the quantity q' with positive probability. A necessary condition for q' to be an equilibrium is that y_-, the smallest type playing q', is unwilling to deviate. Since $k > 1$, that requires at least that $q'\frac{\alpha+E(y|q')}{\alpha+\beta+m}k + (1-q')\frac{\alpha+y_-}{\alpha+\beta+m} = \frac{\alpha+y_-}{\alpha+\beta+m}k$, thus $q' \geq \frac{(k-1)(\alpha+y_-)}{k(\alpha+E(y|q'))-(\alpha+y_-)} =: \hat{q}$, and note that $\hat{q} > 0$.

Consider any quantity $q \in (0, \hat{q})$, while the above condition requires that $u_{y_-}(q')$ dominates the payoff achievable with any q if the buyers hold beliefs yielding $E(y|q) \leq E(y|q')$, still q is not necessarily equilibrium dominated as the buyers may believe for instance that $E(y|q) = m$. As for the case of perfectly separating equilibrium, the Intuitive Criterion fails to refine Perfect Bayesian Equilibrium.

In order to show that any pooling or semi-pooling equilibrium fails to satisfy Universal Divinity, consider the set of types that play q' with positive probability, and denote by y^+ the largest of such types. Note that for any small $\varepsilon > 0$, the set of buyers prices that make y^+ willing to deviate to $q' - \varepsilon$ is strictly larger than the set of responses that makes any other type playing q' deviate. In fact the condition $q'\frac{\alpha+E(y|q')}{\alpha+\beta+m}k + (1-q')\frac{\alpha+y}{\alpha+\beta+m} = (q'-\varepsilon)pk + (1-q'+\varepsilon)\frac{\alpha+y}{\alpha+\beta+m}$ yields the solution $p = \frac{q'k(\alpha+E(y|q'))-\varepsilon(\alpha+y)}{(\alpha+\beta+m)(q'-\varepsilon)k}$ which is decreasing in y.

But once the beliefs have been fixed to conform with criterion D1, it is the case that for any $\varepsilon > 0$, $\Pr(y^+|q' - \varepsilon)$ is arbitrarily close to 1. By deviating to $q' - \varepsilon$, thus, type y^+ will achieve payoff of almost $(q' - \varepsilon)\frac{\alpha+y^+}{\alpha+\beta+m}k + (1 - (q' - \varepsilon))\frac{\alpha+y^+}{\alpha+\beta+m}$ which dominates $q'\frac{\alpha+E(y|q')}{\alpha+\beta+m}k + (1 - q')\frac{\alpha+y^+}{\alpha+\beta+m}$ for ε small enough. ■ ■

Proof of Proposition 2. The welfare is the sum of the seller profit and of the buyers' utility, with appropriate normalizations.

$$\begin{aligned}
W(\theta, y, \delta) &= (1-\delta)\left[\left(q_y P_0 + (1-q_y)\sum_{t=1}^{\infty}\delta^t p_t\right) + \left(\sum_{t=1}^{\infty}\gamma^t\theta - \sum_{t=1}^{\infty}\gamma^t(1-q_y)p_t - q_y P_0\right)\right] \\
&= (1-\delta)\left[(1-q_y)\sum_{t=1}^{\infty}\delta^t p_t + \sum_{t=1}^{\infty}\gamma^t(\theta - (1-q_y)p_t)\right] \\
&= (1-\delta)(1-q_y)\sum_{t=1}^{\infty}\delta^t p_t + \gamma\frac{1-\delta}{1-\gamma}\theta - (1-\delta)(1-q_y)\sum_{t=1}^{\infty}\gamma^t p_t.
\end{aligned}$$

For $\delta \to 1$, thus,

$$\begin{aligned}
W(\theta, y, \delta) \to W(\theta, y) &= (1-q_y)\theta + \frac{1-\delta}{1-\gamma}\theta - \frac{1-\delta}{1-\gamma}(1-q_y)\theta \\
&= (1-q_y)\theta - k(1-q_y)\theta + k\theta \\
&= \theta\left[kq_y + (1-q_y)\right].
\end{aligned}$$

With analogous derivations, the informational loss is approximately $L(\theta, y) = \theta k(1 - q_y)$.

■

Proof of Proposition 4. Henceforth I will denote by y_m the extraction y from experiment of size m. I need to show that $\Pr(\lim_{m\to\infty} p_{t+1} = \theta) = 1$ uniformly in $t \geq 0$. By the first Borel-Cantelli Lemma, letting the event $A^t_{Mm} = \cap_{\nu=M}^{m}\{|\frac{\alpha+y_m+\mathbf{x}_t}{\alpha+\beta+m+t} - \theta| \leq \varepsilon\}$, that corresponds (cf. Feller pag 156) to $\forall\varepsilon > 0$, $\forall\delta > 0$, $\exists M$ s.t. $\forall t \geq 0$, $\Pr(A^t_{Mm}) > 1 - \delta$, for all $m \geq M$.

For any $\varepsilon > 0$, there exists an M such that $|\frac{\alpha+y_m+\mathbf{x}_t}{\alpha+\beta+m+t} - \frac{y_m+x_t}{m+t}| < \varepsilon/2$, for any $m \geq M$, and $t \geq 0$. By the strong law of large numbers, $\forall\varepsilon > 0$, $\forall\delta > 0$, $\exists M'$ s.t. $\Pr(\cap_{\nu=M'}^{m}\{|\frac{y_m}{m} - \theta| \leq \varepsilon/2\}) > 1 - \delta$, for all $m \geq M'$.

Since y_m is an extraction from binomial with parameters θ and m, and for any t, the $\mathbf{x}_t$ are extractions from binomial with parameters θ and t, it also the case that for any

t, $y_m + \mathbf{x}_t$ are extractions from a binomial distribution with parameters θ and $m+t$. Therefore, the above expression of the law of large numbers implies that for any $t \geq 0$, $\Pr(\cap_{\nu=M'}^{m}\{|\frac{y_m+\mathbf{x}_t}{m+t} - \theta| \leq \varepsilon/2\}) > 1-\delta$, for all $m \geq M'$.

By triangular inequality, $|\frac{\alpha+y_m+\mathbf{x}_t}{\alpha+\beta+m+t} - \theta| \leq |\frac{\alpha+y_m+\mathbf{x}_t}{\alpha+\beta+m+t} - \frac{y_m+\mathbf{x}_t}{m+t}| + |\frac{y_m+\mathbf{x}_t}{m+t} - \theta|$. By taking M'' as the largest between M and M', thus, $\forall t \geq 0$, $\Pr(A^t_{M''m}) > 1-\delta$, for all $m \geq M''$.

By the above result, $\forall \varepsilon > 0$, $\forall \delta > 0$, $\exists M$ s.t. $\forall t \geq 0$, $\Pr(\cap_{\nu=M}^{m}\{|\frac{\alpha+y_m+\mathbf{x}_t}{\alpha+\beta+m+t} - \theta| \leq \varepsilon/2\}) > 1-\delta$, for any $m \geq M$, and $\exists M'$ s.t. $\forall t \geq 0$, $\Pr(\cap_{\nu=M}^{m}\{|\frac{\alpha+y_m+\mathbf{x}_{t+1}}{\alpha+\beta+m+t+1} - \theta| \leq \varepsilon/2\}) > 1-\delta$, for any $m \geq M'$. By taking M'' as the largest between M and M', and applying triangular inequality, I obtain that $\forall t \geq 0$, $\Pr(\cap_{\nu=M''}^{m}\{|\frac{\alpha+y_m+\mathbf{x}_{t+1}}{\alpha+\beta+m+t+1} - \frac{\alpha+y_m+\mathbf{x}_t}{\alpha+\beta+m+t}| \leq \varepsilon\}) > 1-\delta$, for all $m \geq M''$. ■

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