

KELLEY SCHOOL OF BUSINESS

INDIANA UNIVERSITY

Department of Business Economics and Public Policy

Price dispersion in the lab and on the Internet: Theory and evidence

Michael R. Baye*

John Morgan**

Price dispersion is ubiquitous in settings that closely approximate textbook Bertrand competition. We show (Propositions 2 and 3) that only a little bounded rationality among sellers is needed to rationalize such dispersion. A variety of statistical tests, based on data sets from two independent laboratory experiments and structural estimates of the parameters of our models, suggest that bounded rationality based theories of price dispersion organize the data remarkably well. Evidence is also presented which suggests that the models are consistent with data from a leading Internet price comparison site.

*Department of Business Economics and Public Policy, Kelley School of Business, Indiana University, Bloomington, IN 47405. Phone: (812) 855-2779. E-mail: mbye@indiana.edu.

** Haas School of Business and Department of Economics, University of California, Berkeley, CA 94720. Phone: (510) 642-2669. E-mail: morgan@haas.berkeley.edu.

We thank the Editor, the referees, and numerous seminar participants for helpful comments on earlier versions of this paper. We are especially grateful to Dilip Abreu, Martin Dufwenberg, Glenn Ellison, Nancy Epling, Ed Glaeser, Uri Gneezy, Paul Klemperer, Dan Kovenock, David Laibson, John Maxwell, Martin Sefton, Dale Stahl, and Abdullah Yavas. The authors thank Nuffield College (University of Oxford), Trinity and Corpus Christi Colleges (University of Cambridge), University of Nottingham, The Hoover Institution, and the University of Bonn for their gracious hospitality and for providing a stimulating environment where some of this work was completed. Morgan gratefully acknowledges the financial support of the National Science Foundation.

1 Introduction

A puzzling empirical regularity has been identified in price setting environments where firms sell identical products: There is considerable price dispersion and transactions prices tend to be above marginal cost. For example, with a few mouse clicks at the Internet price comparison site Shopper.com on 21 August 2001, a consumer could obtain a list of ten firms offering prices for an Epson Expression 1600 Professional Edition scanner. These prices ranged from a low of \$625.85 all the way up to a high of \$903. These prices are also inconsistent with Bertrand-Nash behavior. The range was 44.3% of the lowest price, and the coefficient of variation (defined as the ratio of the standard deviation of prices to the mean price) was 15.2%. There is nothing unique about this date or product. Even today, price dispersion for a variety of products sold in online markets is ubiquitous and persistent.¹

While one might surmise that this casual evidence of price dispersion may be explained by transactions costs, imperfect information, differing seller costs, and so forth, similar patterns of price dispersion occur in laboratory settings where these considerations are absent by design. Plott (1982) surveys the results of numerous complete information homogeneous product pricing experiments, and notes that this institution generally results in “a higher variance in behavior” (p. 1513) than other market institutions, and that even after multiple rounds of experiments with the same subjects the “...adjustment [of price to marginal cost] tends to be from above and either converges [to marginal cost] slowly or does not converge at all.” (p. 1498). He concludes that “...a slight upward bias relative to the competitive equilibrium, even when the number of firms is “large,” appears to be part of the general properties of the posted-price institution” (p. 1514). Davis and Holt (1996) obtain similar findings.²

¹Our website, www.nash-equilibrium.com, provides both current and historical data on price dispersion in online markets. There is also considerable evidence of price dispersion at traditional retail outlets; see Carlson and Pescatrice (1980), Pratt, Wise, and Zeckhauser (1979), Stigler (1961), Villas-Boas (1995), Warner and Barsky (1995), among others.

²This is not to say that all laboratory studies are at variance with theory. Cason and Friedman (forthcoming) find that theory performs well, especially when sellers sell to robot buyers.

This paper shows that introducing just a little bounded rationality among sellers goes a long way in reconciling theory with the price dispersion commonly observed on the Internet as well as in the lab. We begin in Section 2 by more formally documenting the pervasiveness of price dispersion in homogeneous product settings. Section 3 then develops equilibrium implications of bounded rationality models that are relevant for homogeneous product, winner-take-all price competition. Our Propositions 2 and 3 show that, in these settings, models of bounded rationality based on solution concepts proposed by Radner (1980) and McKelvey and Palfrey (1995) generate price dispersion similar to that observed on the Internet and in the lab. Section 4 uses experimental data to structurally estimate the parameters of these pricing distributions. A variety of statistical tests offer considerable support for the bounded rationality hypothesis, and little support for the Nash hypothesis. We conclude in Section 5 by providing some evidence that data from the Internet are also consistent with bounded rationality models of price dispersion. All proofs are relegated to the Appendix.

2 Price Dispersion in Homogeneous Product Markets

As Hal Varian (1980) noted over two decades ago, "...‘the law of one price’ is no law at all" (p.651). Since theoretical explanations of the failure of the law of one price typically stress the role of search frictions (cf. Reinganum, 1979), information asymmetries (cf. Varian, 1980), or both (Burdett and Judd, 1983), one might expect that price dispersion would be trivial at price comparison sites on the Internet and non-existent in laboratory data where sellers compete in a classic Bertrand fashion. Unfortunately, this conclusion is not borne out in a number of empirical studies (cf. Brynjolfsson and Smith (2000); Baye, Morgan, and Scholten (2001a); and Plott (1982)). This section presents some additional evidence that price dispersion is ubiquitous in a variety of homogeneous product settings.

Evidence from the Internet

Price comparison sites have become an increasingly popular way to shop for merchandise on

the web. One such site, Shopper.com, offers comparisons on over 100,000 consumer electronics products. Using a “spider” written in the PERL programming language, we downloaded data for the top 50 products (based on consumer “click-throughs”) from the Shopper.com site on 26 March 2001. This information includes the name of the product, the identity of all firms selling each product, each firm’s reputational ranking, inventory, shipping costs, and list price. Based on the list prices, we constructed two statistics to summarize the level of price dispersion for the different products: the coefficient of variation (the sample standard deviation of prices charged for a product divided by the mean price of the product) and the range (the highest price minus the lowest price charged for a given product).³ Unlike the range, the coefficient of variation is a unitless measure of price dispersion that may be used to compare the relative dispersion of different products that sell for different prices.

Table 1 presents descriptive statistics for these data (in descending order of each product’s coefficient of variation). The table illustrates that, even at a price comparison site like Shopper.com where consumers may almost costlessly observe an entire list of prices that different firms charge for identical electronics products, price dispersion is pervasive. The average product in the sample sells for \$498.25, while the average range in prices is \$140.39. The coefficients of variation range from a high of 28.5% (for a Sony VAIO personal computer) to zero (for a Voodoo5 3dfx graphics card). The “law of one price” holds exactly for only two products, and these are each sold by only a single firm. Expressed differently, for every product where two or more firms competed, the Internet data display coefficients of variation ranging from 1.9% up to 28.5% and price ranges from \$10.95 to \$1,607.00.

Baye, Morgan, and Scholten (2001) document that the levels of price dispersion reported in Table 1 are not an artifact of firms’ listing “bogus prices” for the products or the date on which these data were collected. Indeed, they observe patterns of price dispersion similar to that in Table 1 both across time and for the top 1,000 products at Shopper.com. Baye, Morgan, and Scholten (forthcoming) also provide evidence that the dispersion observed at Shopper.com is not an artifact

³These measures have been used in a variety of other studies, including Carlson and Pescatrice (1980), Sorensen (2000), and Baye, Morgan and Scholten (2001 and forthcoming).

of differences in shipping costs or vendor characteristics (such as inventories or reputation).

Evidence from the lab

Price dispersion is not only observed at Internet sites such as Shopper.com, but in controlled environments designed to exactly match textbook models of Bertrand competition. We now describe two sets of experiments that examine price setting in homogeneous product markets and show that price dispersion is also ubiquitous in controlled laboratory settings. Overviews of the experimental designs are provided below; the interested reader should refer to the cited papers for details.

Dufwenberg and Gneezy (2000) report on hand-run experiments in which twelve Dutch subjects participated in a ten round Bertrand pricing game. In each session, subjects competed in either a duopoly, triopoly, or quadopoly treatment with random rematching of subjects between rounds. Subjects selected integer prices between 2 and 100 (inclusive) in each round. The subject in a match choosing the lowest price was awarded a number of points (convertible into cash) equal to the price she selected. Thus, the monopoly price of 100 is also the highest price that subjects were permitted to select. The unique Nash equilibrium is for each subject to select a price equal to two.

For the duopoly treatments with Dutch subjects, the average posted price was 33.33, with an interquartile range from 19 to 49. Increasing the number of sellers from two to four decreased the average price to 25.75; however the interquartile spread remained fairly substantial, ranging from 5 to 33.5. The average lowest price also exceeded the Nash prediction. Under duopoly, the mean winning price was 27.52, for the triopoly treatments it was 8.19, and for the quadopoly treatments the average winning price was 8.51. While the mean (and median) prices observed declined from round 1 to round 10, the spread in prices actually increased from rounds 1 to round 10. The levels of price dispersion observed in these experiments are, in fact, much higher than what we observed in Table 1. For instance, in the duopoly experiments, the coefficient of variation for prices across all rounds was about 60%. Price dispersion persisted even in the last round—the coefficient of variation here was about 52%. The triopoly and quadopoly treatments yielded even greater dispersion across all rounds and in the last round. In short, recent homogeneous product price setting experiments with Dutch subjects resulted in prices above the Nash prediction and considerable price dispersion.

Similar conclusions may be drawn from experiments reported in Abrams, Sefton, and Yavas (2000), which were computer-run with US subjects. In contrast to the Dutch experiments, Abrams, Sefton and Yavas used live buyers and live sellers. These experiments consisted of eight buyers and eight sellers participating in a 25 round pricing game. In each round a buyer received price quotes from two different sellers selected at random, and could either purchase from one of the sellers or refuse their offers and (possibly) exercise an outside option.⁴ The value of the object to a buyer was 120 points; hence, a buyer’s surplus from accepting a price quote was simply the difference between 120 and the purchase price. A seller whose price quote was accepted by a buyer received points equal to the amount of the quote. The feasible set of price quotes that sellers could offer were integers from 0 to 200 inclusive. Thus, the monopoly price was 120; however, in contrast to the Dutch experiments, sellers were permitted to price above the monopoly price. Despite the more complicated experimental design, seller incentives are identical to those in the classical Bertrand setting: It is a Nash equilibrium for each seller to price at zero.⁵

The data with US subjects are qualitatively similar to those with Dutch subjects. While there is an initial decline in the median, spread, and interquartile range of prices, all three measures stabilize after seven rounds. In particular, the average price stabilizes at 33.87 for periods 8 through 25, well above the Nash equilibrium level of zero. The coefficient of variation for the Abrams, Sefton, and Yavas data exceed those in Table 1 as well as those reported above for the duopoly experiments with Dutch subjects.

⁴In the Abrams, Sefton, and Yavas design, the outside option is such that, in any subgame perfect equilibrium, consumers would never choose to exercise it. For this reason, all tests reported below are based on pricing data from what they call the “Initial Bertrand Stage” of their experiment.

⁵This equilibrium is also subgame perfect. Due to the discreteness of the strategy space, there are two other subgame perfect equilibria: $p_1 = p_2 = 1$ and $p_1 = p_2 = 2$. The tests reported below are based on the conventional Nash hypothesis that $p_1 = p_2 = 0$; however the conclusions are not affected by which of these three Nash equilibria is used for the null hypothesis.

3 Theory

While one may “explain away” price dispersion on the Internet by arguing that it stems from unobserved heterogeneities (cf. Gatti, 2000; Janssen and Moraga, 2000; Rosenthal, 1980; Spulber, 1995; Stahl, 2000; and Varian, 1980) or by the fact that it is costly for firms to post prices (Baye and Morgan, 2001), similar patterns are observed in laboratory settings that explicitly control for these and other frictions. Since significant dispersion is observed in two quite different environments, we offer a theoretical framework that helps explain the price dispersion observed in the field and in the lab.

Preliminaries

Consider an environment in which a set $N = \{1, 2, \dots, n\}$ of $n \geq 2$ risk-neutral players compete to supply some homogeneous product. Let $\pi(p)$ denote the payoff to a monopolist charging a price p . Let c be the initial breakeven price of a monopolist operating in this market. This price satisfies $\pi(c) = 0$ and $\pi(p) < 0$ for all $p < c$. Let $p^M \in \arg \max_{p \in \mathcal{P}} \pi(p)$ and $\pi^M \equiv \pi(p^M) > 0$ denote the monopoly price and monopoly payoff, respectively. For simplicity, we assume that $\pi(p)$ is continuous and strictly increasing on the interval $[c, p^M]$, and restrict the players to choose prices from the set $\mathcal{P} \equiv [c, p^M]$.⁶

Each player simultaneously chooses a price, $p_i \in \mathcal{P}$, and the player charging the lowest price earns the monopoly payoff corresponding to that price. If several players tie for the lowest price, one of them is randomly selected to earn the monopoly payoff at the lowest price. Thus, if (p_1, p_2, \dots, p_n) are the prices chosen by the n players, the (expected) payoff to player i is given by:

$$\pi_i(p_1, p_2, \dots, p_n) = \begin{cases} \pi(p_i) & \text{if } p_i < p_j \ \forall j \neq i \\ \frac{1}{m} \pi(p_i) & \text{if } i \text{ ties } m - 1 \text{ other players for low price} \\ 0 & \text{otherwise} \end{cases}$$

Let Φ be the set of all cumulative distribution functions on \mathcal{P} , so that a strategy for player i is a probability measure $F_i \in \Phi$. Given a vector of strategies $F = (F_1, F_2, \dots, F_n)$, player i 's expected

⁶One can readily extend all of the results below to cases where players are free to choose prices above the monopoly price.

payoff is $E\pi_i(F) = \int_{\mathcal{P}^n} \pi_i(p_1, p_2, \dots, p_n) dF$. If we let $\Pi = (E\pi_1, E\pi_2, \dots, E\pi_n)$, then a homogeneous product pricing game is given by $\Gamma \langle N, \Phi^n, \Pi \rangle$.

Our analysis applies three solution concepts to homogeneous product pricing games: Nash equilibrium, ε -equilibrium, and quantal response equilibrium. The latter two solution concepts incorporate bounded rationality into an equilibrium framework.

Definition 1. A vector of strategies $F^* = (F_1^*, \dots, F_n^*) \in \Phi^n$ comprises a Nash equilibrium of Γ if, for all $i \in N$ and for all $F'_i \in \Phi$,

$$E\pi_i(F'_i, F_{-i}^*) - E\pi_i(F_i^*, F_{-i}^*) \leq 0.$$

Definition 2. A vector of strategies $F^\varepsilon \in \Phi^n$ comprises an ε -equilibrium of Γ if, for all $i \in N$, for all $F'_i \in \Phi$, and a fixed $\varepsilon > 0$:

$$E\pi_i(F'_i, F_{-i}^\varepsilon) - E\pi_i(F_i^\varepsilon, F_{-i}^\varepsilon) \leq \varepsilon.$$

An ε -equilibrium is a set of strategies with the property that no player can obtain more than ε in additional payoffs by deviating from the prescribed strategies. There are economic as well as psychological rationales for this solution concept. An ε -equilibrium may arise because of bounded rationality stemming from cognitive or motivational constraints. For example, if a Weber-Fechner-Stevens law holds with respect to just noticeable differences in perceptions of payoffs, then players may lack the cognitive capacity to rationally choose between two strategies that yield payoffs that are within ε of one another. Alternatively, if it is costly to reformulate pricing strategies, players may lack the motivation to incur these economic or psychic costs when the resulting gain is small (less than ε). In addition, ε -equilibria may also arise from ‘‘satisficing behavior;’’ see March and Simon (1958).

To define a quantal response equilibrium for homogeneous product pricing games, it is useful to let $E\pi_i(p_i, F_{-i}^Q)$ denote player i 's expected payoff from charging a price p_i when the other players

adopt the vector of strategies $F_{-i}^Q \in \Phi^{n-1}$. Let T_i map $E\pi_i(p_i, F_{-i}^Q)$ into the probability that player i charges a price less than or equal to p_i .

Definition 3. A vector of strategies $F^Q \in \Phi^n$ comprises a quantal response equilibrium (QRE) if, for all $i \in N$ and $p_i \in \mathcal{P}$:

$$F_i^Q(p_i) = T_i\left(E\pi_i\left(p_i, F_{-i}^Q\right)\right).$$

The nondegenerate distributions of player actions in the quantal response framework may be thought of as the result of preference shocks (McFadden, 1984), which stem from privately observed (and perhaps non-pecuniary) costs or benefits of charging different prices. For example, an online retailer might suffer a temporary variation in the implicit cost of order-processing as a result of absenteeism by inventory “pickers” on a given day. Alternatively, nondegenerate price distributions might be viewed as thought of as resulting from decision errors (Luce, 1959). These errors might reflect limitations in subjects’ cognitive processing (in laboratory settings) or “bugs” in the yield management and/or pricing algorithms used by Internet retailers.

In a QRE, the likelihood that players choose a particular strategy depends on the expected payoffs from that strategy. Strategies are determined by a decision rule (such as the logistic or power function) that is probabilistic and has the property that actions generating higher expected payoffs are more likely to be selected, although not necessarily with probability one. Each player’s expected payoffs from different actions depend on the probability distributions of other players’ actions. For a given probabilistic decision rule, a QRE requires that all players hold correct beliefs about the probability distributions of other players’ actions. Anderson, Goeree, and Holt (1998) pioneered the techniques used below to solve for a quantal response equilibrium in games where the action space is continuous.⁷

⁷Their theoretical analysis examines QRE in rent-seeking games.

Analysis

This subsection applies the above solution concepts to any homogeneous product pricing game, Γ . We begin with the familiar result that the unique symmetric Nash equilibrium in such a game entails breakeven pricing. The proof of this result is standard and hence is omitted.

Proposition 1. The following comprises a symmetric Nash equilibrium to Γ : For all $p \in [c, p^M]$ and $i \in N$, $F_i^*(p) = 1$.

Next, we examine ε -equilibria in homogeneous product pricing games. For the case of pure-strategies, ε -equilibrium behavior leads to prices and payoffs that do not substantially differ from Nash behavior. To be more precise, the following lemma shows that, in any pure-strategy ε -equilibrium to such a game, each player necessarily earns a payoff that is less than ε .

Lemma 1. In any pure-strategy ε -equilibrium to Γ , players earn no more than ε .

Proof. See the Appendix.

Nonetheless, there does exist a symmetric (mixed-strategy) ε -equilibrium in which each player earns an expected payoff that exceeds ε , as the following proposition shows.

Proposition 2. For any $\varepsilon \in \left(0, \left(\frac{n}{n-1}\right)^{n-1} 2^{-n} \pi^M\right)$, the following comprises a symmetric ε -equilibrium to Γ in which each player earns an expected payoff that exceeds ε : For all $p \in [c, p^M]$ and $i \in N$,

$$F_i^\varepsilon(p) = \begin{cases} 0 & \text{if } p < \pi^{-1}(\theta) \\ 1 - \left[\frac{\theta}{\pi(p)}\right]^{\frac{1}{n-1}} & \text{if } p \in [\pi^{-1}(\theta), p^M) \\ 1 & \text{if } p = p^M \end{cases} \quad (1)$$

where

$$\theta = \left[\varepsilon^{n-1} \left(\frac{n}{n-1}\right)^{n-1} \pi^M \right]^{\frac{1}{n}}. \quad (2)$$

Proof. See the Appendix.

Proposition 2 identifies a dispersed-price ε -equilibrium in which prices range from $\pi^{-1}(\theta)$ to the monopoly price, p^M . Here, θ (the parameter we will estimate in the sequel), is an increasing function of the bounded rationality parameter, ε . Thus, θ is zero when players are fully rational (i.e., $\varepsilon = 0$) and increases as bounded rationality increases. Further, higher values of θ are also associated with a shrinking of the range of prices since, as ε increases, the lowest price charged by firms in this ε -equilibrium moves upward toward the monopoly price. For $\varepsilon > 0$, firms earn strictly positive profits. In fact, it follows from Lemma 1 that this ε -equilibrium Pareto dominates all pure strategy ε -equilibria, as each player earns an expected payoff in excess of ε . Even for small values of ε , the level of profits earned in this ε -equilibrium is quite sizeable. In a duopoly setting where ε is 1% of monopoly profits, industry profits in this ε -equilibrium are 26.4% of monopoly profits.

It is useful to highlight three additional implications of the ε -equilibrium identified in Proposition 2. First, as the number of competing players increases, prices become more concentrated in the tails of the distribution. Second, there is a jump in the distribution of prices at the monopoly price, and the size of this jump at p^M is increasing in the number of players. In contrast, there is no point mass at the lower support of the distribution of prices, and the lower support is decreasing in n . Finally, one can show that, for small ε , each player's expected payoff is increasing in ε and decreasing in n ; thus greater competition or lower levels of bounded rationality lead to more competitive outcomes. In the sequel, we test several of these implications using both field and laboratory data.

Our second main proposition identifies a symmetric quantal response equilibrium in homogeneous product pricing games. Following Lopez-Acevedo (1997), suppose that the probability (more formally the density) with which player i chooses price p is generated by the power function decision rule with (bounded rationality) parameter $\lambda \in [0, \frac{1}{n-1}]$.⁸ In this case,

$$T_i(E\pi_i(p, F_{-i})) = \int_c^p \frac{E\pi_i(q, F_{-i})^\lambda}{\int_c^{p^M} E\pi_i(t, F_{-i})^\lambda dt} dq. \quad (3)$$

⁸We examine behavior for the power function rather than the more usual logistic specification. This choice is motivated purely by analytical tractability; we were able to obtain closed-form representations for equilibrium pricing strategies under the power function specification (see Proposition 3 below) but not under the logistic specification.

Proposition 3. For any $\lambda \in \left[0, \frac{1}{n-1}\right)$, let T_i be defined by equation (3). Then the following comprises a symmetric QRE to Γ : For all $p \in [c, p^M]$ and $i \in N$,

$$F_i^Q(p) = 1 - \left(\frac{g(p^M) - g(p)}{g(p^M) - g(c)} \right)^{\frac{1}{1+\lambda-n\lambda}} \quad (4)$$

where $g(p) \equiv \int \pi(p)^\lambda dp + K$.

Proof. See the Appendix.

Proposition 3 identifies a dispersed-price quantal response equilibrium where prices range from marginal cost to the monopoly price. Thus, firms earn positive expected profits in the QRE. Here, λ is the bounded rationality parameter. Values of λ near zero are associated with minimal levels of rationality. As λ increases, players are more prone to make optimal decisions; hence higher values of λ are associated with greater levels of rationality.

It is useful to contrast the distribution of prices arising in Proposition 2 with that in Proposition 3. First, in contrast to the ε -equilibrium, there is no jump in the distribution of prices at p^M under the QRE. Second, again in contrast to the ε -equilibrium, the lower support of the distribution of prices in the QRE is independent of the bounded rationality parameter (λ). Finally, the limit of both the ε -equilibrium and QRE correspond to the symmetric Nash equilibrium. To see this, note that as ε tends to zero, θ goes to zero and the equilibrium in Proposition 2 converges to that in Proposition 1. Similarly, as $\lambda \rightarrow \frac{1}{n-1}$, increasingly more of the probability mass in the QRE is allocated to prices close to c and the equilibrium in Proposition 3 also converges to that in Proposition 1. At the other extreme, as $\lambda \rightarrow 0$, the decision error component of the QRE strategies overwhelms the weight attached to relative expected profits and one obtains *random behavior* (prices chosen uniformly on $[c, p^M]$) as a symmetric QRE. Clearly, if $\varepsilon \geq \pi^M$, random behavior is also a symmetric ε -equilibrium. Thus, both extreme assumptions of rationality are nested within the ε -equilibrium and QRE frameworks.

4 Data Analysis

In our view, a theory of behavior in homogeneous product pricing games should be broadly applicable in environments that approximate winner-take-all competition. As we noted above, price comparison sites on the Internet as well as the experimental data with US and Dutch subjects share these characteristics. Since the US and Dutch data are derived from controlled experiments that, by design, match the conditions of the model given in Section 3, we use these data to formally analyze the theoretical models presented above. Throughout, we organize the experimental data as follows: For each treatment (duopoly, triopoly, quadopoly) in the experiments with Dutch subjects, we pool both sessions and report separate results for data based on all rounds and the last round (as this most closely corresponds to the one-shot game). Similarly, we pool all duopoly sessions of the experiments with US subjects and analyze data pooled over all rounds as well data from only the last round.

Before proceeding with formal testing, it is useful to highlight key features of the experimental data. Figures 1 and 2 compare the empirical distribution of prices for all rounds of the duopoly and quadopoly treatments with Dutch subjects to that predicted under the two polar rationality assumptions: Nash behavior and random behavior. Notice that the pricing does not appear consistent with the Nash hypothesis (where all firms price at two with probability one), nor with random behavior. In contrast, consistent with both the epsilon and quantal response hypotheses, as one moves from two-seller to four-seller treatments, lower prices become more frequently observed than higher prices. Further, consistent with the ϵ -equilibrium hypothesis, in both figures there is a distinct upward jump at the monopoly price in the empirical distribution of prices.

One might speculate that the observed departures from Nash behavior stem from inexperienced subjects or repeated-play strategies that sustain cooperative outcomes in early rounds. However, as Figure 3 shows, data from the last round of the duopoly treatments with Dutch subjects exhibit features similar to those in Figures 1 and 2. One major difference in the last round duopoly data is that the upward jump at the monopoly price is more pronounced.

The empirical distributions of prices based on the US data and the Dutch three-seller treatments

have features similar to those described above. In short, observed behavior in two independent sets of experiments with subjects from different countries and different experimental procedures suggest that behavior in pricing games is qualitatively consistent across experimental conditions and lies between the Nash prediction and random behavior. The observed distributions of prices in these experiments share features observed in field data. In light of this, we use this experimental data to estimate the bounded rationality parameters associated with the ε -equilibrium and QRE strategies defined in Propositions 2 and 3.

Hypotheses

In the US and Dutch experimental designs, the monopoly profit function is $\pi(p) = p$ for all $p \in \mathcal{P}$. Applying the results from Propositions 1-3 to the strategy space in the Abrams, Sefton, and Yavas experiments leads to the following predictions about the distributions of prices under the three competing equilibrium concepts:

$$\text{Nash Hypothesis: } F_i^*(p) = 1 \quad \text{if } 0 \leq p \leq 120$$

$$\text{QRE Hypothesis: } F_i^Q(p) = 1 - \left(1 - \left(\frac{p}{120}\right)^{1+\lambda}\right)^{\frac{1}{1+\lambda-n\lambda}} \quad \text{if } 0 \leq p \leq 120. \quad (5)$$

$$\varepsilon\text{-equilibrium Hypothesis: } F_i^\varepsilon(p) = \begin{cases} 0 & \text{if } p < \theta \\ 1 - \left(\frac{\theta}{p}\right)^{\frac{1}{n-1}} & \text{if } \theta \leq p < 120 \\ 1 & \text{if } p = 120 \end{cases} \quad (6)$$

We also consider the naive hypothesis that subjects choose strategies at random:

$$\text{Random Behavior Hypothesis: } F_i^R(p) = \frac{p}{120} \quad \text{if } 0 \leq p \leq 120$$

Similar hypotheses obtain for the Dufwenberg and Gneezy data, except that the monopoly price is 100 and the minimum feasible price is 2.

Estimation

The expressions for the QRE and ε -equilibrium solution concepts in equations (5) and (6) depend on the number of players (n), the monopoly price, and the (unobservable) bounded rationality parameters λ and θ . Our approach is to use the experimental data to estimate λ and θ . The hypothesis tests described in the next section are based on parameter estimates for each data set. These parameter estimates were obtained in the following manner.

For each price p in the strategy space of an experiment, let $\widehat{F}(p)$ denote the empirical cumulative distribution function associated with that price; that is, $\widehat{F}(p)$ is the fraction of prices observed in a given data set that is less than or equal to p . Similarly, let $F^Q(p; \lambda)$ and $F^\varepsilon(p; \theta)$ denote the predicted distributions of prices under the QRE and ε -equilibrium as a function of their respective bounded rationality parameters. For the Abrams, Sefton, and Yavas data, these expressions are given by equations (5) and (6). For each data set, we estimate the parameters λ and θ by minimizing the sum of squared errors between the empirical cumulative distribution function and the theoretical cumulative distribution function. To account for the discrete strategy space, we adapt equation (5) for the Abrams, Sefton, and Yavas data as follows: Let \underline{p} denote the largest integer strictly less than lowest price in the strategy space. We estimated using

$$F_i^Q(p) = 1 - \left(1 - \left(\frac{p - \underline{p}}{p^M - \underline{p}} \right)^{1+\lambda} \right)^{\frac{1}{1-\lambda+n\lambda}}.$$

This ensures that both random behavior and Nash remain special cases of QRE with $\lambda = 0$ and $\lambda = \frac{1}{n-1}$, respectively. A similar estimation procedure is employed for the Dufwenberg and Gneezy data.

The parameter estimates for θ and λ are reported in Table 2. Recall that in the QRE, feasible values of λ are bounded from above by $\frac{1}{n-1}$, which declines as the number of players increases. The closer the parameter estimate is to $\frac{1}{n-1}$, the closer is the QRE prediction to the Nash prediction. The estimates of λ reported in Table 2 for both the U.S. and Dutch duopoly experiments are considerably less than 1, thus suggesting a departure from Nash behavior. Specifically, the estimates of λ based on all rounds of U.S. duopoly data are 76.1% of that under Nash behavior. Similarly, for the Dutch duopoly all round data, the estimated value of λ is 78.4% of that under Nash behavior.

Moreover, there is no obvious convergence of the QRE to Nash behavior as the number of players increases from two to four: In the triopoly treatment the estimated value of λ is 93.6% of that under Nash, and for the quadopoly treatment the estimated value is 86.1% of that under Nash behavior. Examining the estimates of last round data, the evidence for convergence to Nash is mixed. In the U.S. duopoly treatment, the estimate of λ increases to 90.7% of Nash, whereas the estimate for the Dutch duopoly treatment decreases to only 45.7% of that under Nash behavior. In contrast, estimates of λ for the last round triopoly and quadopoly treatments are both in excess of 98% of those under Nash behavior.

We now turn to the parameter estimates for the ε -equilibrium reported in Table 2. Recall that in an ε -equilibrium, feasible values of θ are bounded from below by zero, which corresponds to Nash behavior. Unlike the case of the QRE, this lower bound is independent of the number of players. The estimates of θ based on all rounds of data for the duopoly experiments are considerably above zero, and generally decline as we move from duopoly to triopoly to quadopoly. This is consistent with the theory: Equation (2) implies that for a given ε , θ is decreasing in n .

Given estimates of θ , we may invert equation (2) to obtain implied values of ε expressed both in US dollars and as a percentage of the potential monopoly profits. These are also reported in Table 2. As is readily seen, the degree of bounded rationality is quite small. The largest estimate of ε is six cents, which means that a subject will not deviate from a given strategy unless doing so increases her payoff by more than six cents. The median estimate of ε is less than one cent, or less than one-half of one percent of the monopoly profits. We will see below that even these small values of ε are consistent with behavior that differs significantly from the Nash prediction.

Figures 4 and 5 plot the estimated distributions of prices for the ε -equilibrium and QRE, along with the empirical distribution, for all rounds of the duopoly and quadopoly treatments with Dutch subjects. These figures are representative of all of the estimates reported in Table 2, and suggest that our estimates of λ and θ predict theoretical pricing distributions that are fairly close to the observed distributions. However, the estimated distributions for QRE tend to underpredict the frequency of prices near the monopoly price. In Figures 4 and 5, the empirical distribution of

prices jumps up at the monopoly price, and this jump is not consistent with a QRE. In contrast, the estimated ε -equilibrium price distributions correctly predict a jump at the monopoly price, but the predicted size of this jump is larger than that observed in the data.

Plots of the estimated price distributions are similar for the other treatments, regardless of whether one focuses on data from all rounds or only the last round. For instance, Figure 6 displays results based on the last round of the duopoly treatment with Dutch subjects. One difference in plots based on the last round data is that the upward jumps in the empirical distribution of prices near the monopoly price tend to be more pronounced. This is highlighted in Figure 6, which shows that the estimated ε -equilibrium for the quadopoly treatment predicts the observed jump in the empirical distribution of prices, even though the implied value of ε for that case is very close to zero.

It is interesting to note that the distributions of prices under the estimated QRE and ε -equilibrium roughly bracket the empirical distribution of prices for each treatment. The ε -equilibrium estimates tend to form an upper bound on the empirical distribution for low prices, and a lower bound on for higher prices. The QRE estimates, on the other hand, tend to form a lower bound on the empirical distribution for lower prices and an upper bound for higher prices. This suggests that a combination of the two concepts might provide even greater explanatory power. The theoretical foundations of what might be termed an ε -QRE are beyond the scope of the present paper.

Tests

Despite the fact that only small departures from full rationality are implied by the estimates in Table 2, the models of bounded rationality are superior to the Nash (or random behavior) predictions at organizing the data. Next, we employ a variety of hypothesis tests to formally test the data against the hypotheses of QRE, ε -equilibrium, Nash, and random behavior.

In Table 3 we report the results of three different types of hypothesis tests applied to each of the data sets. The first are standard t -tests of the hypothesis that the mean of the empirical distribution of prices is equal to the mean of the theoretical distribution of prices under four null hypotheses (ε -equilibrium, QRE, random behavior, and Nash). The second are sign-tests for equality of medians

between the empirical distribution of prices and that under each of the null hypotheses. The last are chi-squared tests for equality of the empirical and estimated theoretical distributions under the ε -equilibrium, QRE, and random behavior hypotheses.

Overall, the statistical evidence favors the bounded rationality models of pricing behavior over Nash and random behavior. For the duopoly treatments, we fail to reject the null hypothesis of ε -equilibrium behavior in 10 of 12 hypothesis tests. Under QRE, we fail to reject in 8 of 12 tests. In contrast, Nash behavior is rejected in all of the tests and random behavior is rejected in all but two of the tests. In all cases where we fail to reject the QRE hypothesis, we also fail to reject the ε -equilibrium hypothesis; however, in some cases where the QRE solution concept is rejected, we fail to reject the ε -equilibrium solution concept.

The statistical support for the bounded rationality models is weaker in the triopoly and quadopoly data. For these treatments, we fail to reject ε -equilibrium in only 3 of 12 tests. Likewise, we fail to reject QRE behavior in 6 of 12 tests. Still, both of these hypotheses fare better than random behavior and the Nash hypothesis, both of which are rejected in all cases.

Looking in more detail at the test of equality of means in Table 3, we fail to reject the ε -equilibrium hypothesis in all cases but one (Dutch triopoly, all rounds). Thus, the ε -equilibrium hypothesis appears to be quite useful in predicting the mean price, regardless of the experience of subjects or the competitiveness of the pricing environment. For the duopoly treatments, QRE does almost as well in that it is only rejected for the US all rounds data. For the triopoly and quadopoly treatments, we fail to reject QRE using last round data but reject it when we pool all rounds. The mean prices predicted under Nash and random behavior are overwhelmingly rejected in the data; in all cases, the Nash predicts prices that are too low and random behavior predicts prices that are too high.

However, tests of medians in Table 3 reveal that QRE is more successful than the competing models at predicting median prices. In particular, QRE is rejected only twice—once under duopoly (US, all rounds) and once under quadopoly (all rounds). In contrast, the ε -equilibrium is rejected in all triopoly and quadopoly treatments. Once again, the predictions of Nash and random behavior

are overwhelmingly rejected in the data. Random behavior consistently predicts a median price that is too high, while the Nash prediction is too low.

The last set of results in Table 3 are chi-square tests of the equality of the theoretical and observed distributions of prices for the three hypotheses that entail nondegenerate distributions of prices. Specifically, for each estimated theoretical distribution, we constructed bins such that, under the associated theoretical distribution, an equal number of price realizations are expected in each bin. We then constructed the statistic

$$s = \sum_{i=1}^3 \left(\frac{N_i - EN_i}{EN_i} \right)^2$$

where N_i denotes the actual number of price realizations observed in bin i and EN_i denotes the expected number of price observations in bin i under the relevant null hypothesis. Three bins were used to insure that for all i , $EN_i \geq 5$.

We reject the hypothesized ε -equilibrium pricing distribution only once under duopoly, while the hypothesized pricing distribution under QRE is rejected twice (both US and Dutch, all rounds). Neither hypothesis does particularly well for triopoly and quadopoly: the ε -equilibrium solution concept is rejected in all cases and QRE in all cases but one (quadopoly, last round). Nonetheless, the QRE and ε -equilibrium solution concepts still do better than random behavior and Nash, which are rejected in every case.

5 Discussion

The theoretical results presented in Section 3, coupled with the empirical evidence in Section 4, suggests that a little bounded rationality goes a long way towards rationalizing the price dispersion observed in homogeneous product settings. Based on two different experimental datasets with significant differences in treatments (duopoly, triopoly, and quadopoly), differences in the nationality of subjects (US vs. Dutch), and differences in experimental designs (live versus robot buyers, outside options vs. no outside options, hand-run vs. computerized environments), we find considerable statistical support for the hypotheses that subjects are pricing in a manner consistent with models

of bounded rationality.

In light of these findings, one might wonder whether these models are also a plausible explanation for the price dispersion observed on the Internet and documented in Table 1. While the absence of data on firms' costs and other variables precludes us from providing structural estimates and tests with these data, we conclude by providing some evidence that models of bounded rationality are also consistent with Internet data.

Proposition 1 predicts that a disproportionate number of firms will post the highest (monopoly) price for a given product. For the Shopper.com data summarized in Table 1, two or more firms were tied for listing the highest price in 14.6% of the products for which there were multiple price listings. In contrast, for only 8.3% of products two or more firms were tied for the lowest price. This mass observed in the data at the highest price for each product is consistent with the theory, which predicts a discrete jump in the cumulative distribution function at the monopoly price.

Models of bounded rationality also shed light on an empirical regularity documented in Baye, Morgan, and Scholten (2001). Based on a data set consisting of four million observations at Shopper.com for the top 1,000 electronic products sold between 2 August 2000 and 1 March 2001, they find that the coefficient of variation in prices tend to increase as the number of firms listing prices rises from 2 to 5, and then decreases as the number of firms increases from 5 to 30. A qualitatively similar pattern holds for the theoretical coefficient of variation in an ε -equilibrium based on payoffs in the Dufwenberg and Gneezy experiments. In particular, for small levels of bounded rationality (one cent), the theoretical coefficient of variation increases up to four sellers and then decreases thereafter. While the levels of price dispersion observed in lab are higher than those observed on the Internet, only a small degree of bounded rationality is needed to rationalize the patterns of price dispersion observed in both environments.

A Appendix

This Appendix contains proofs of Lemma 1 and Propositions 2 and 3 in the paper.

Proof of Lemma 1

Without loss of generality, consider an ε -equilibrium in which players $1, 2, \dots, k$ charge a price of p^ε and the remaining $n - k$ players charge a strictly greater price. Players $i \leq k$ thus earn a payoff of $E\pi_i = \pi(p^\varepsilon)/k$, while players $j > k$ earn payoffs of zero.

Case 1: $k > 1$.

If player $i \leq k$ charges p^ε she earns $\pi(p^\varepsilon)/k$, and if she deviates by charging a slightly lower price she earns no more than $\pi(p^\varepsilon)$. By hypothesis, the initial constellation of prices comprises an ε -equilibrium, so it follows that

$$\pi(p^\varepsilon) \leq \varepsilon \left(\frac{k}{k-1} \right).$$

Hence, each player $i \leq k$ earns a payoff of $E\pi_i \leq \varepsilon$ and each player $j > k$ earns a payoff of $E\pi_j = 0$.

Case 2: $k = 1$.

If player $j > k$ conforms she earns a payoff of 0. If she deviates by undercutting p^ε she earns no more than $\pi(p^\varepsilon)$. By hypothesis, the initial constellation of prices comprises an ε -equilibrium, so it follows that

$$\pi(p^\varepsilon) \leq \varepsilon.$$

This implies that no player earns more than ε . *Q.E.D.*

Proof of Proposition 2

Since $\pi(\cdot)$ is continuous and strictly increasing on $[c, p^M]$, $\pi^{-1}(\cdot)$ exists. For a given $\varepsilon \in \left(0, \left(\frac{n}{n-1} \right)^{n-1} 2^{-n} \pi^M \right)$, we will show that the strategies in equation (1) comprise a symmetric ε -equilibrium, and furthermore, that $E\pi_i(F^\varepsilon) > \varepsilon$ for all $i \in N$.

First, observe that by charging $p \in [\pi^{-1}(\theta), p^M]$ when the other $n - 1$ players adopt strategies F_{-i}^ε , player i 's expected payoff is

$$E\pi_i(p, F_{-i}^\varepsilon) = \pi(p) (1 - F_i^\varepsilon(p))^{n-1}$$

$$\begin{aligned}
&= \pi(p) \left[\frac{\theta}{\pi(p)} \right] \\
&= \theta.
\end{aligned}$$

Hence, player i 's expected payoff is constant on this interval. If player i prices below $\pi^{-1}(\theta)$, she earns $\pi(p) < \theta$. When player i prices at p^M , she earns $\pi(p^M)$ only if all other $n-1$ players also charge p^M . Thus,

$$\begin{aligned}
E\pi_i(p^M, F_{-i}^\varepsilon) &= \left[\Pr(p_i = p^M) \right]^{n-1} \frac{\pi^M}{n} \\
&= \left[\frac{\theta}{\pi^M} \right] \frac{\pi^M}{n} \\
&= \frac{\theta}{n}.
\end{aligned}$$

Thus, if player i plays according to F_i^ε when the other $n-1$ players also use F_i^ε , her expected payoff is:

$$\begin{aligned}
E\pi_i(F^\varepsilon) &= \Pr(p_i < p^M) \theta + \Pr(p_i = p^M) \frac{\theta}{n} \\
&= \theta \left[1 - \left(\frac{\theta}{\pi^M} \right)^{\frac{1}{n-1}} \left(\frac{n-1}{n} \right) \right].
\end{aligned}$$

The best deviation available to player i is to charge a price $p \in [\pi^{-1}(\theta), p^M)$. In this case, she earns expected payoffs of θ . The expected gain from such a deviation, however, is only

$$\left[\frac{\theta}{\pi^M} \right]^{\frac{1}{n-1}} \theta \left(\frac{n-1}{n} \right) \leq \varepsilon$$

which implies that F^ε constitutes a symmetric ε -equilibrium.

Since $\varepsilon < \left(\frac{n}{n-1} \right)^{n-1} 2^{-n} \pi^M$, it is a routine matter to verify that

$$E\pi_i(F^\varepsilon) = \theta \left[1 - \left(\frac{\theta}{\pi^M} \right)^{\frac{1}{n-1}} \left(\frac{n-1}{n} \right) \right] > \varepsilon. \quad Q.E.D.$$

Proof of Proposition 3

Recall that a symmetric QRE satisfies

$$F_i^Q(p) = T_i \left(E\pi_i(p, F_{-i}^Q) \right)$$

for all $i \in N$. Using the definition of T_i , differentiating, and imposing symmetry yields:

$$f_i^Q(p) = \frac{[1 - F_i^Q(p)]^{(n-1)\lambda} \pi(p)^\lambda}{\mu}$$

where $\mu \equiv \int_{-\infty}^{\infty} \left(\pi(t) (1 - F_i^Q(t))^{n-1} \right)^\lambda dt$.

We may rewrite this expression as

$$\int [1 - F_i^Q(p)]^{-(n-1)\lambda} dF_i^Q = \int \frac{1}{\mu} \pi(p)^\lambda dp.$$

Integrating yields

$$-\frac{1}{1 + \lambda - n\lambda} (1 - F_i^Q(p))^{1+\lambda-n\lambda} = \frac{1}{\mu} (g(p) - K)$$

where $-K$ is a constant of integration. Solving for F_i^Q we obtain

$$F_i^Q(p) = 1 - \left(-\frac{1 + \lambda - n\lambda}{\mu} (g(p) - K) \right)^{\frac{1}{1+\lambda-n\lambda}}.$$

Using the fact that $F_i^Q(p^M) = 1$ yields

$$F_i^Q(p) = 1 - \left(\frac{1 + \lambda - n\lambda}{\mu} (g(p^M) - g(p)) \right)^{\frac{1}{1+\lambda-n\lambda}}. \quad (A1)$$

Finally, using the fact that $F_i^Q(c) = 0$ gives us

$$\mu = (1 + \lambda - n\lambda) (g(p^M) - g(c)).$$

Substituting μ into equation (A1) yields equation (4). Finally, it is straightforward to verify that

F_i^Q is a well-defined cdf. *Q.E.D.*

References

- Abrams, E., M. Sefton, and A. Yavas. "An Experimental Comparison of Two Search Models." *Economic Theory*, Vol. 16 (2000), pp. 735-749.
- Anderson, S., J. Goeree, and C.A. Holt. "Rent-seeking with Bounded Rationality: An Analysis of the All-Pay Auction." *Journal of Political Economy*, Vol. 106 (1998), pp. 828-853.
- Baye, M.R. and J. Morgan. "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets." *American Economic Review*, Vol. 91 (2001), pp. 454-74.
- Baye, M.R., J. Morgan, and P. Scholten. "Price Dispersion in the Small and in the Large: Evidence from an Internet Price Comparison Site." Mimeo, Indiana University, 2001.
- Baye, M.R., J. Morgan, and P. Scholten. "Persistent Price Dispersion in Online Markets." In D. Jansen, ed., *The New Economy*. Chicago: University of Chicago Press, forthcoming.
- Brynjolfsson, E. and M.D. Smith. "Frictionless Commerce? A Comparison of Internet and Conventional Retailers." *Management Science*, Vol. 46 (2000), pp. 563-585.
- Burdett, K. and K. Judd. "Equilibrium Price Dispersion," *Econometrica*, Vol. 51 (1983), pp. 955-969.
- Carlson, J.A. and D.R. Pescatrice. "Persistent Price Distributions." *Journal of Economics and Business*, Vol. 33 (1980), pp. 21-27.
- Cason, T. and D. Friedman. "Buyer Search and Price Dispersion: A Laboratory Study." *Journal of Economic Theory*, forthcoming.
- Davis, D. and C. Holt. "Consumer Search Costs and Market Performance." *Economic Inquiry*, Vol. 34 (1996), pp. 133-151.
- Dufwenberg, M. and U. Gneezy. "Price Competition and Market Concentration: An Experimental Study." *International Journal of Industrial Organization*, Vol. 18 (2000), pp. 7-22.
- Gatti, J.R.J. "Equilibrium Price Dispersion with Sequential Search." Mimeo, Cambridge University, November 2000.
- Janssen, M. and J.L. Moraga, "Pricing, Consumer Search and the Size of Internet Markets." Mimeo, Tinbergen Institute Rotterdam, June 2000.
- Lopez-Acevedo, G. "Quantal Response Equilibria for Posted Offer Markets." *Estudios Económicos*, Vol. 12 (1997), pp. 95-131.
- Luce, R. *Individual Choice Behavior: A Theoretical Analysis*. New York: John Wiley & Sons, 1959.
- March, J. G. and H. A. Simon, *Organizations*, New York: John Wiley & Sons, 1958.
- McFadden, D. "Econometric Analysis of Qualitative Response Models." In Z. Griliches and M. Intriligator, eds., *Handbook of Econometrics*, Amsterdam: Elsevier, 1984.
- McKelvey, R. and T. Palfrey. "Quantal Response Equilibria for Normal Form Games." *Games and Economic Behavior*, Vol. 10 (1995), pp. 6-38.
- Plott, C. "Industrial Organization Theory and Experimental Economics." *Journal of Economic Literature*, Vol. 20 (1982), pp. 1485-1527.
- Pratt, J., D. Wise, and R. Zeckhauser. "Price Differences in Almost Competitive Markets." *Quarterly Journal of Economics*, Vol. 93 (1979), pp. 189-211.
- Radner, R. "Collusive Behavior in Oligopolies with Long But Finite Lives." *Journal of Economic Theory*, Vol. 22 (1980), pp. 136-156.

- Reinganum, J.F. "A Simple Model of Equilibrium Price Dispersion." *Journal of Political Economy*, Vol. 87 (1979), pp. 851-858.
- Rosenthal, R.W. "A Model in Which an Increase in the Number of Sellers Leads to a Higher Price." *Econometrica*, Vol. 48 (1980), pp. 1575-1580.
- Sorensen, A. "Equilibrium Price Dispersion in Retail Markets for Prescription Drugs." *Journal Political Economy*, Vol 108 (2000), pp. 833-50.
- Spulber, D.F. "Bertrand Competition when Rivals' Costs are Unknown." *Journal of Industrial Economics*, Vol. 43 (1995), pp. 1-11.
- Stahl, D.O.II. "Strategic Advertising and Pricing in E-Commerce." *Advances in Applied Microeconomics*, Vol. 9 (2000), pp. 69-100.
- Stigler, G. "The Economics of Information." *Journal of Political Economy*, Vol. 69 (1961), pp. 213-225.
- Varian, H. "A Model of Sales." *American Economic Review*, Vol. 70 (1980) pp. 651-659.
- Villas-Boas, J.M. "Models of Competitive Price Promotions: Some Empirical Evidence from the Coffee and Saltine Crackers Markets." *Journal of Economics and Management Strategy*, Vol. 6 (1995), pp. 85-107.
- Warner, E. and R. Barsky. "The Timing and Magnitude of Retail Store Markdowns with Reference to Weekends and Holidays." *Quarterly Journal of Economics*, Vol. 110 (1995), pp. 321-52.

Table 1. Price Dispersion on the Internet

<i>Product</i>	<i>Coefficient of Variation</i>	<i>Price Range</i>	<i>Average Price</i>	<i>Lowest Price</i>	<i>Number of Sellers</i>	<i>Product Rank</i>
Sony VAIO PCG_XG28	28.5%	\$1,607.00	\$2,150.70	\$1,592.00	10	43
Palm Ille	23.9%	\$80.99	\$162.24	\$139.00	4	19
HP Pavilion 6736c Multimedia PC	22.9%	\$164.00	\$507.00	\$425.00	2	33
Kodak DC280 Zoom	22.5%	\$279.01	\$433.65	\$319.99	8	10
Abit KT7 (Socket A)	15.3%	\$51.53	\$143.91	\$128.00	5	34
256MB PC 100 SDRAM	15.0%	\$29.12	\$78.75	\$65.88	10	24
Olympus C_3030 Zoom	14.4%	\$376.00	\$774.12	\$623.00	50	6
Palm V	11.7%	\$81.24	\$355.54	\$318.75	5	7
Palm IlIxe	11.3%	\$74.01	\$190.16	\$164.99	27	11
Olympus D_490 Zoom	11.2%	\$165.01	\$416.37	\$333.99	45	21
128MB 16X64 SDRAM PC133 8NS	10.9%	\$13.11	\$40.92	\$35.88	10	23
Epson Stylus Photo 1270	10.5%	\$166.00	\$384.38	\$333.99	35	45
Compaq iPaq H3650 Pocket PC	10.5%	\$169.71	\$565.68	\$490.28	29	1
Canon G1	9.9%	\$190.00	\$796.73	\$709.99	32	28
Palm Vx	9.2%	\$103.00	\$340.05	\$296.00	45	2
AMD ATHLON_900	9.1%	\$49.30	\$158.97	\$140.88	23	27
Plextor PlexWriter 12/10/32A CD_RW	9.1%	\$64.07	\$211.84	\$185.88	33	4
Kodak DC4800 Zoom	9.1%	\$135.01	\$528.66	\$464.98	48	5
HP DeskJet 970CXI	9.0%	\$115.00	\$288.32	\$234.00	38	32
Nikon Coolpix 950	8.7%	\$250.01	\$684.41	\$559.99	48	42
Nikon Coolpix 880	8.5%	\$211.00	\$597.12	\$498.00	56	30
Asus A7V (Socket A)	8.5%	\$24.12	\$144.18	\$133.88	5	9
Plextor PlexWriter 16X/10X40X	8.5%	\$71.07	\$244.05	\$208.88	30	20
Windows ME Upgrade	8.5%	\$26.98	\$90.88	\$75.00	27	35
Sony Cyber Shot DSC_S70	8.0%	\$221.02	\$714.68	\$609.99	46	12
Canon PowerShot S100	7.8%	\$111.55	\$445.44	\$392.00	53	8
Sony VAIO PCG_F650	7.6%	\$371.00	\$1,723.85	\$1,628.00	13	31
AMD ATHLON 1GHZ	7.4%	\$57.59	\$213.17	\$189.88	18	13
Sony VAIO PCG_F590	7.1%	\$424.00	\$2,219.56	\$2,075.00	9	16
Athlon Thunderbird 1200 MHz	7.0%	\$44.12	\$225.27	\$204.88	11	39
Sony VAIO PCG_F630	6.7%	\$241.00	\$1,258.86	\$1,158.00	14	38
Paint Shop Pro 7.0: Win9X/2K/NT4	6.7%	\$20.00	\$91.80	\$84.95	16	26
ATX ATHLON/DURON	6.6%	\$29.12	\$150.70	\$136.88	11	14
Palm IlIc	6.6%	\$68.25	\$311.50	\$271.00	44	29
ATi Radeon 64MB DDR	6.3%	\$51.07	\$197.42	\$183.88	21	17
Nomad Jukebox Audio Player 6GB Silver	6.2%	\$101.99	\$443.53	\$398.00	24	15
Nikon Coolpix 990	6.2%	\$180.00	\$795.25	\$719.99	48	3
ViewSonic PF790	6.0%	\$104.07	\$420.92	\$365.88	26	44
ATi All_in_Wonder Radeon 32MB	5.8%	\$52.06	\$256.92	\$237.89	29	25
Samsung SyncMaster 770 TFT	5.7%	\$210.99	\$1,046.13	\$929.99	27	36
Rio Volt Portable CD Player	5.4%	\$20.04	\$163.06	\$149.95	12	41
Palm M105	5.1%	\$29.11	\$192.53	\$173.95	27	47
Linksys EtherFast 4_port Cable/DSL Router	4.9%	\$22.95	\$135.60	\$127.00	29	18
Adobe Acrobat 4.0: Win9X/NT4 SP3	4.7%	\$35.11	\$226.68	\$206.89	22	49
Adobe Photoshop 6.0 UPG	4.1%	\$24.01	\$188.64	\$177.99	23	40
Wireless Access Point	3.8%	\$43.60	\$252.45	\$235.00	24	48
Sonicblue Multimedia Rio PMP300 MP3	3.7%	\$10.95	\$153.47	\$149.00	3	22
Compaq Deskpro EX P3/800 10GB	1.9%	\$49.85	\$1,029.36	\$999.99	5	50
Dell Dimension 4100	0.0%	\$0.00	\$1,588.00	\$1,588.00	1	37
Voodoo5 3dfx 5500 Graphics Card	0.0%	\$0.00	\$179.00	\$179.00	1	46

Table 2. Parameter Estimates and Implied Values of Epsilon

Experiment	Rounds	Parameter Estimates		Implied Values of Epsilon	
		λ	θ	US\$	As a Percentage of Monopoly Profits
Dutch Duopoly	All	.784	13.199	\$.026	.9%
US Duopoly	All	.761	15.643	\$.004	.8%
Dutch Duopoly	Last	.457	20.214	\$.062	2.0%
US Duopoly	Last	.907	10.297	\$.002	.4%
Dutch Triopoly	All	.468	1.484	\$.004	.1%
Dutch Triopoly	Last	.488	.870	\$.002	.1%
Dutch Quadopoly	All	.287	.678	\$.003	.1%
Dutch Quadopoly	Last	.325	.149	\$.000	.0%

Table 3. Tests of Equality of Means, Medians, and Distributions: Observed Prices versus those predicted under Epsilon, QRE, and Random Behavior

Experiment	Rounds	Observed	Means Predicted Under			Observed	Medians Predicted Under			Distributions Chi-square Statistics Under		
			Epsilon	QRE	Random		Epsilon	QRE	Random	Epsilon	QRE	Random
Dutch Duopoly	All	38.325	40.362 (.237)	35.651 (.121)	51.000 (.000)	29.500	26.000 (.090)	33.000 (.153)	50.000 (.000)	2.152	13.863	64.922
US Duopoly	All	48.513	47.947 (.628)	42.807 (.000)	60.000 (.000)	37.000	31.000 (.000)	40.000 (.001)	59.000 (.000)	47.602	53.586	340.241
Dutch Duopoly	Last	49.458	52.931 (.401)	46.952 (.634)	51.000 (.769)	41.000	40.000 (.832)	45.000 (.839)	50.000 (.286)	4.284	2.535	6.93
US Duopoly	Last	39.833	36.039 (.586)	29.275 (.134)	60.000 (.006)	25.000	20.000 (.233)	27.000 (.461)	59.000 (.000)	0.209	1.816	36.346
Dutch Triopoly	All	18.492	23.303 (.000)	14.833 (.007)	51.000 (.000)	11.000	5.000 (.000)	12.000 (.290)	50.000 (.000)	61.743	8.282	269.631
Dutch Triopoly	Last	15.125	18.315 (.480)	8.771 (.167)	51.000 (.000)	5.000	3.000 (.000)	7.000 (.093)	50.000 (.000)	11.106	7.099	25.783
Dutch Quadopoly	All	25.746	28.559 (.163)	19.507 (.002)	51.000 (.000)	10.500	5.000 (.000)	16.000 (.001)	50.000 (.000)	51.368	31.409	211.092
Dutch Quadopoly	Last	17.167	18.036 (.896)	6.932 (.134)	51.000 (.000)	5.000	2.000 (.000)	5.000 (1.000)	50.000 (.000)	9.414	2.848	30.664

Numbers in parentheses are P-values associated with the test of the relevant null hypothesis. Bold cells indicate a failure to reject the null hypothesis at the 5% significance level.

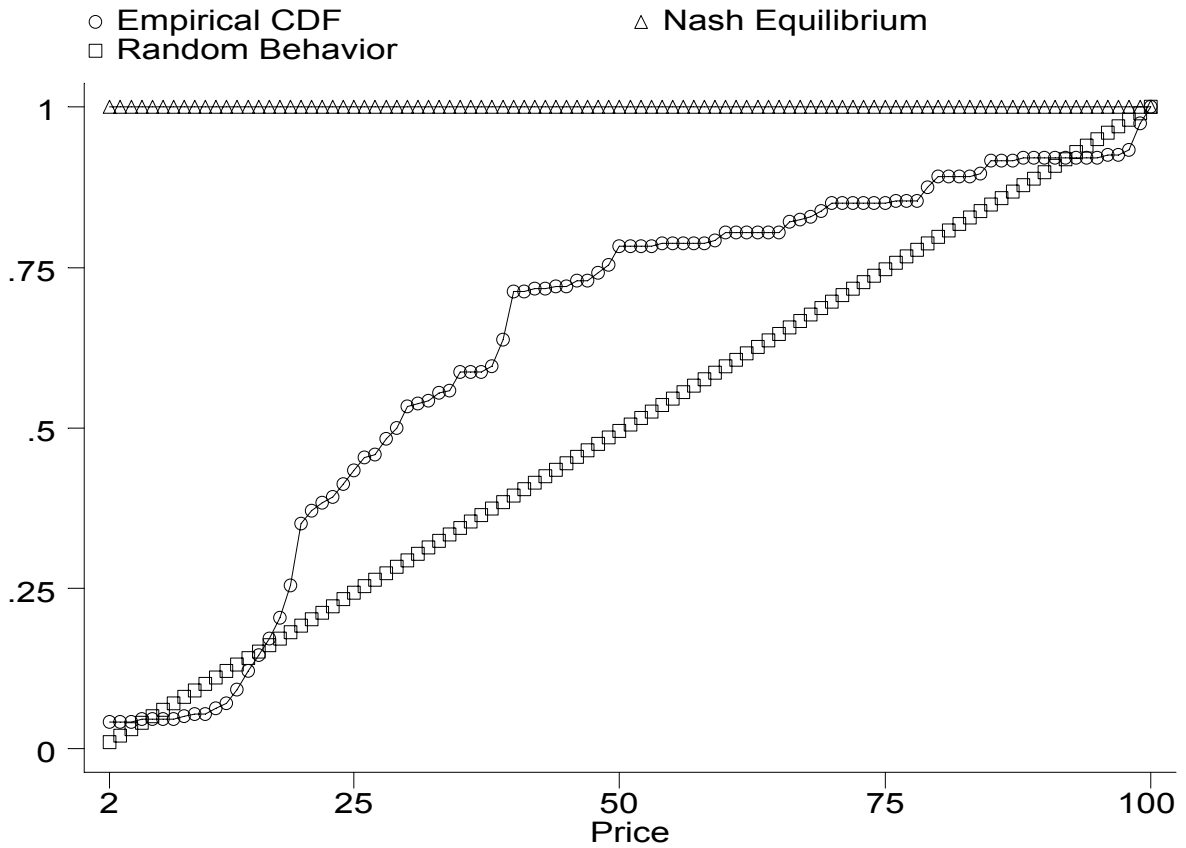


Figure 1. Cumulative Distributions of Prices under Nash and Random Behavior Compared to Actual Empirical Distribution of Prices: Duopoly Experiments with Dutch Subjects, All Rounds. *Data source: Dufwenberg and Gneezy (2000).*

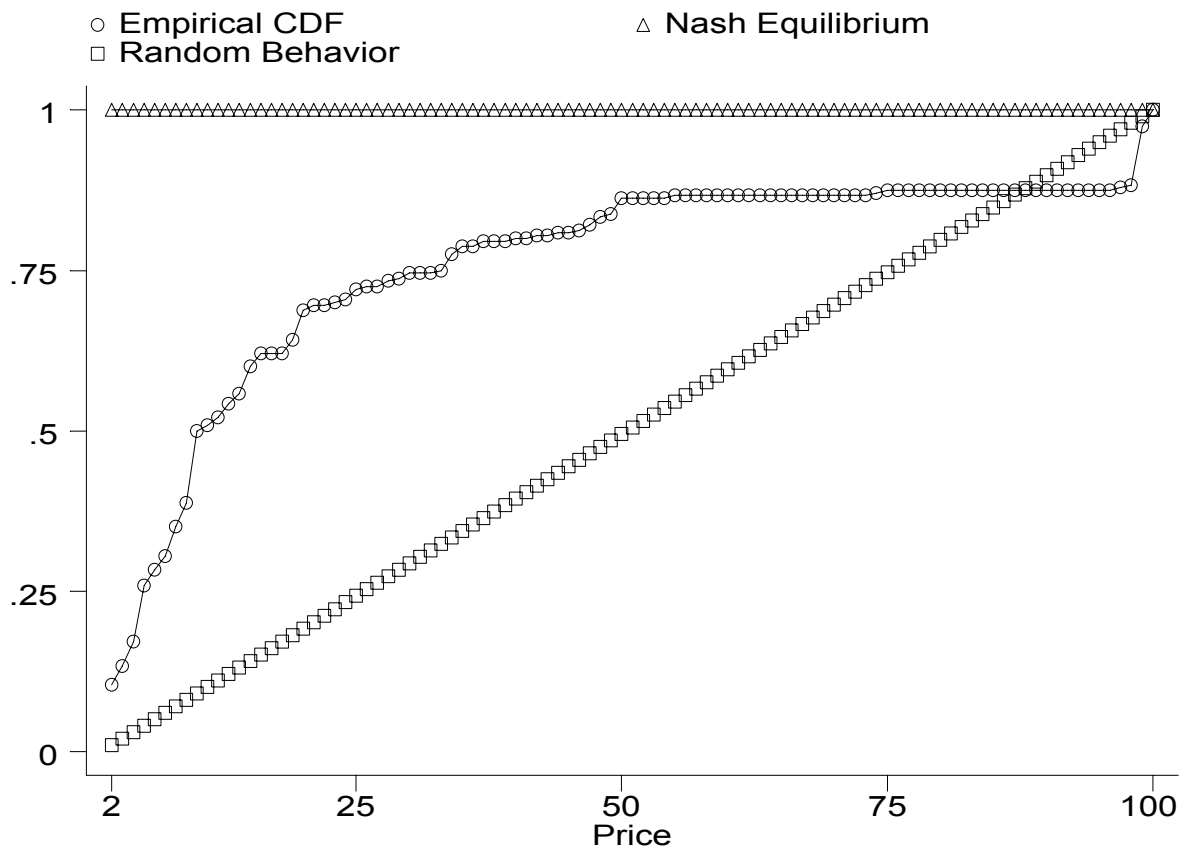


Figure 2. Cumulative Distributions of Prices under Nash and Random Behavior Compared to Actual Empirical Distribution of Prices: Quadopoly Experiments with Dutch Subjects, All Rounds. *Data source: Dufwenberg and Gneezy (2000).*

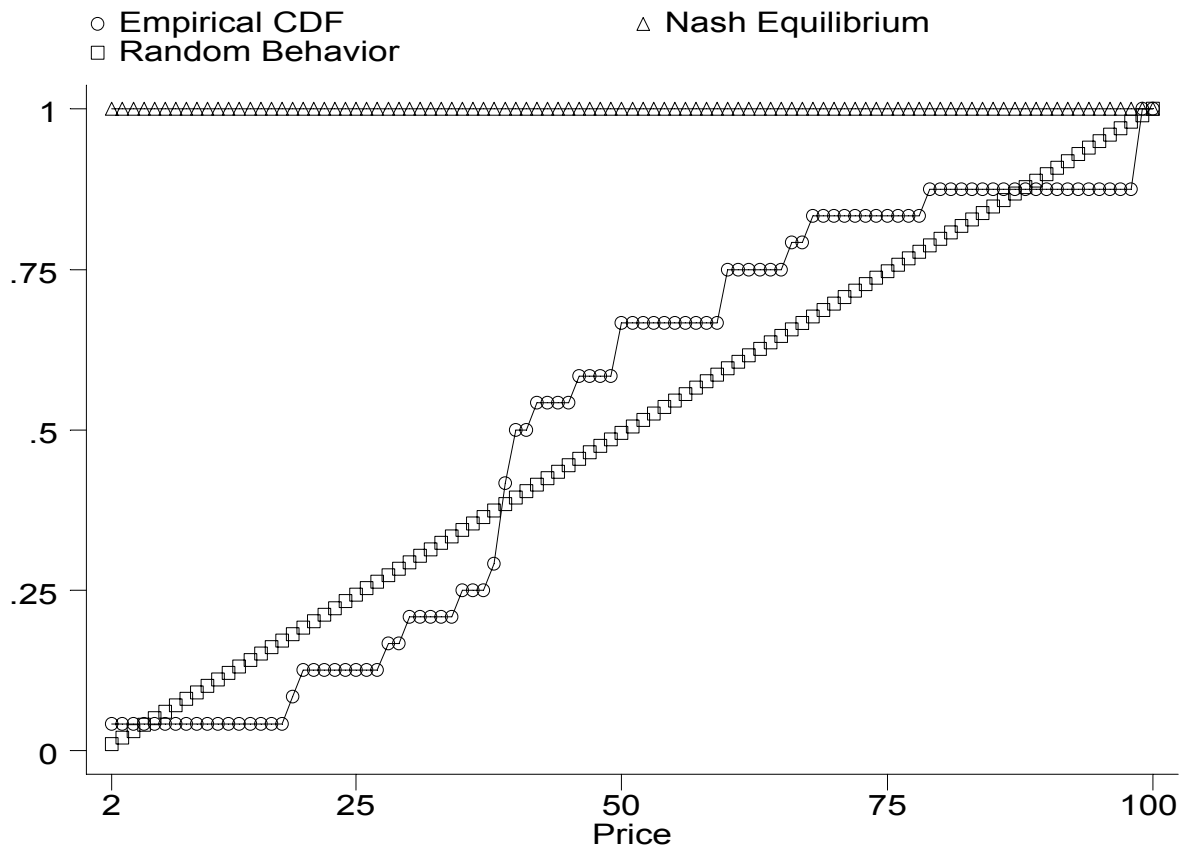


Figure 3. Cumulative Distributions of Prices under Nash and Random Behavior Compared to Actual Empirical Distribution of Prices: Duopoly Experiments with Dutch Subjects, Last Round. *Data Source: Dufwenberg and Gneezy (2000).*

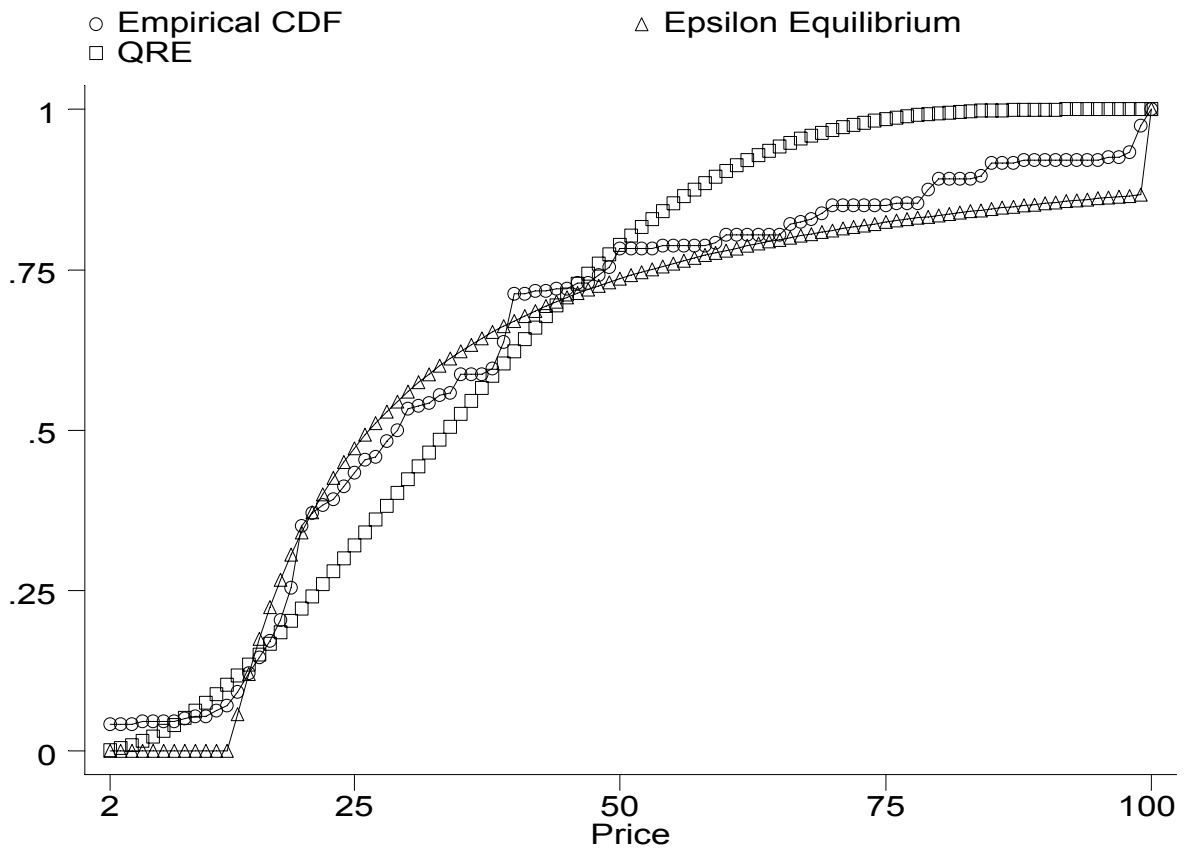


Figure 4. Cumulative Distributions of Prices under QRE and Epsilon Equilibrium Compared to Actual Empirical Distribution of Prices: Duopoly Experiments with Dutch Subjects, All Rounds. *Data source: Dufwenberg and Gneezy (2000).*

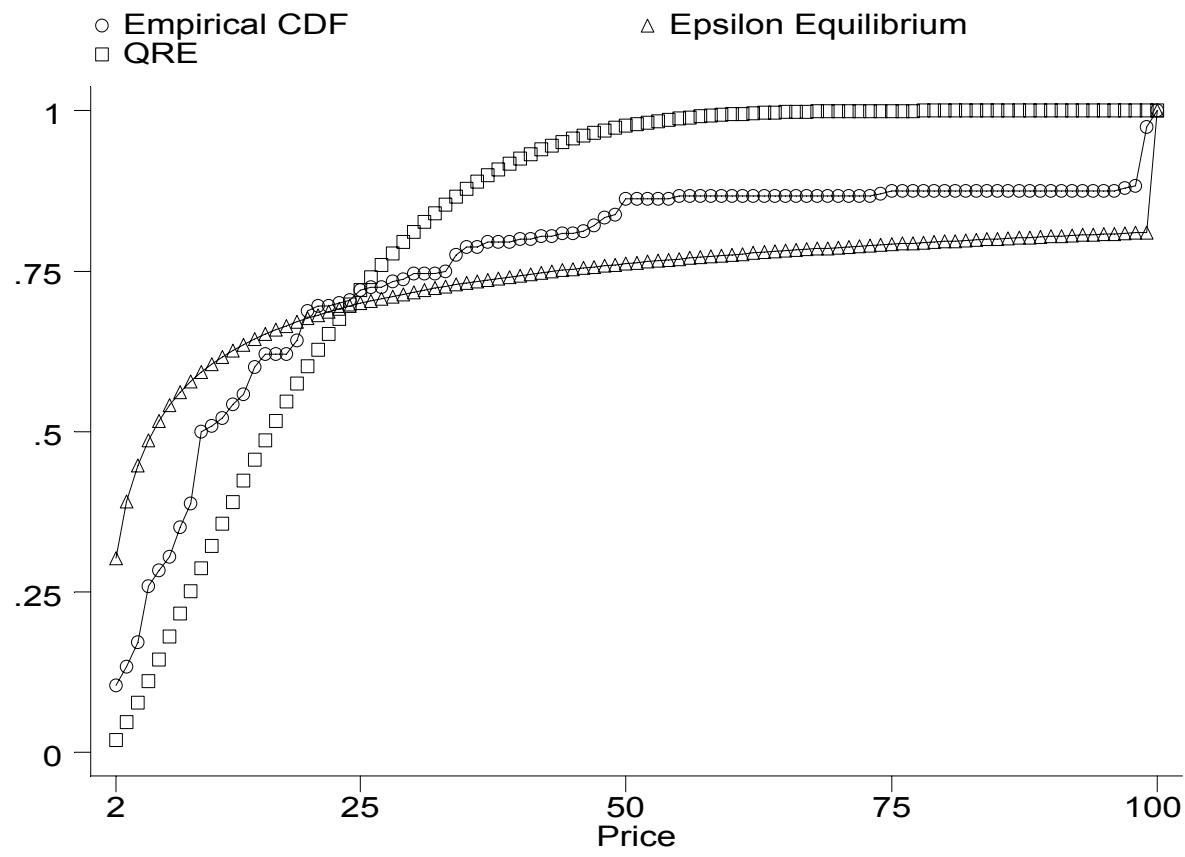


Figure 5. Cumulative Distributions of Prices under QRE and Epsilon Equilibrium Compared to Actual Empirical Distribution of Prices: Quadopoly Experiments with Dutch Subjects, All Rounds. *Data source: Dufwenberg and Gneezy (2000).*

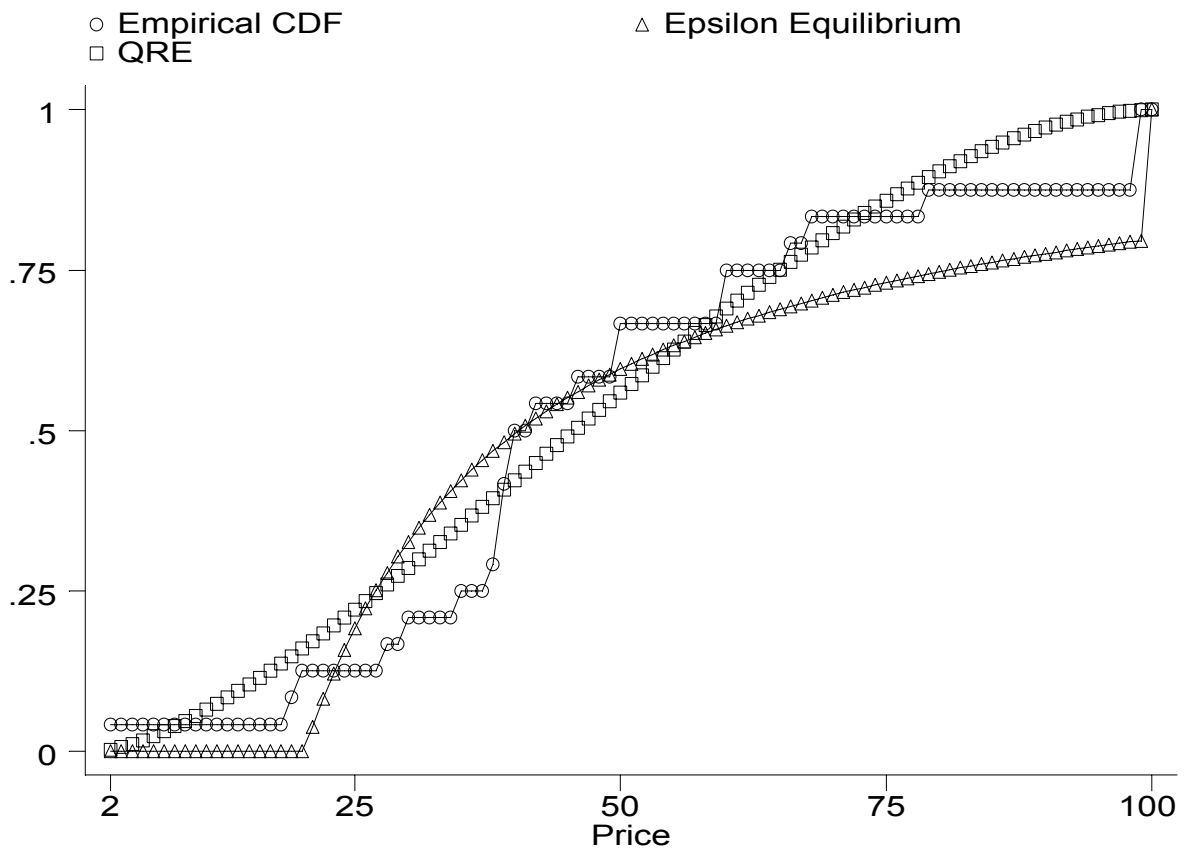


Figure 6. Cumulative Distributions of Prices under QRE and Epsilon Equilibrium Compared to Actual Empirical Distribution of Prices: Duopoly Experiments with Dutch Subjects, Last Round. *Data Source: Dufwenberg and Gneezy (2000).*