

Inappropriate Detrending and Spurious Cointegration

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Abstract

The empirical literature is abundant with detrended cointegration, where cointegration relationships are tested and estimated with deterministic trend terms. Cointegration is, however, critically dependent on whether time series is detrended or not. A series of Monte Carlo experiments show that inappropriately detrended time series tend to exhibit a spurious cointegration. Although true time series are known not to be cointegrated, inappropriately detrended series tend to be cointegrated. Foreign exchange rates are analyzed to demonstrate the relevance and importance of the inappropriate detrending in the cointegration analysis.

Key Words: Deterministic trend, Foreign exchange rates, Monte Carlo study, Stochastic trend

JEL Classification: C22, C15, E31

Inappropriate Detrending and Spurious Cointegration

1. Introduction

The empirical literature is abundant with cointegration studies, in which deterministic trend is commonly included in the test and in the estimation of cointegration relationships. In fact, in a popular software procedure in the test of cointegration relationship, a linear time trend is included as a default procedure. That is, the linear time trend is routinely removed when cointegration relationships are investigated. Two time series are cointegrated, according to Granger (1986, 1991) and Engle and Granger (1987), when a linear combination of the two $I(1)$ series becomes an $I(0)$ series. By definition, an $I(0)$ time series does not contain a unit root and an $I(1)$ time series does contain one.

This paper will investigate the consequence of an inappropriate detrending in the cointegration analysis. In particular, what happens to the test of cointegration and to the estimation of cointegration relationships if a deterministic time trend is inappropriately included? In Nelson and Kang (1981), an inappropriately detrended univariate time series is shown to introduce a certain spurious periodicity. When a time series is $I(1)$ without containing a linear time trend, but the series is nevertheless detrended, the resultant detrended time series exhibits some periodic behavior. More importantly, Nelson and Kang (1981, 1984) show that a pure $I(1)$ time series appears to contain a linear trend. When a random walk time series is regressed on a linear trend and when this “regression” equation is estimated by ordinary least squares (OLS), then the time trend term appears to be statistically significant. For instance, the average coefficient of determination is about 0.45 when 1,000 observations of a random walk time series is regressed on a linear time trend.

The importance of the role a deterministic trend plays in the study of regression or cointegration relationships has been extensively investigated. Nelson and Kang (1984) show that inappropriately detrended nonstationary time series will contain spurious regression relationships. Harvey and Jaeger (1988, p. 231) warn us that “the uncritical use of mechanical detrending can lead investigators to report spurious cyclical behaviour.” Nelson (1988) shows how a state space decomposition is influenced by detrending and Ohanian (1988) illustrates how vector autoregression analyses are affected by the treatment of detrending. Hamilton (1994, pp. 611-612) shows the importance the presence of a deterministic trend makes in the test of cointegration. Yet, Hamilton’s example of a cointegration between income and consumption indicates that whether the two time series are cointegrated or not does not really depend on the detrending. In this study, we show the impact of an inappropriate detrending on cointegration relationships and whether nonstationary time series are cointegrated or not often materially depends on the presence or the absence of the deterministic time trend.

Since most nonstationary $I(1)$ time series appear to contain a linear time trend, most analysts routinely and uncritically detrend nonstationary time series. If inappropriately detrended, however, the time series properties of the underlying series are then distorted. We thus expect that an inappropriate detrending may change the cointegration relationships between two $I(1)$ time series. Through a series of Monte Carlo sampling experiments, this paper will show that two $I(1)$ time series, that are not cointegrated, would appear to be cointegrated when both series are inappropriately detrended. In addition, inappropriately detrended $I(1)$ time series would create spurious structural changes. In particular, there would appear to be a structural break in the series when time series are inappropriately detrended.

Detrending and its consequence on cointegration are applied to three foreign exchange rates of the Japanese yen, British pound, and Italian lira, in terms of the United States dollar. Foreign exchange rates contain a unit root, but they in general appear to contain a linear time trend. Whether or not foreign exchange rates are detrended will have very different conclusions on their cointegration relationships. Three exchange rates from January 1986 to December 2001 are analyzed to show the impact of the detrending on their cointegration relationships. Whether exchange rates are detrended or not critically change the conclusion of the cointegration test.

The structure of the paper is as follows. The consequence of the inappropriate detrending will be investigated for univariate $I(1)$ time series in Section 2. The conventional t -statistics will show the presence of a significant linear trend in the pure $I(1)$ time series. Such significant trends are also well documented in the literature. Unlike earlier studies, the nature of the inappropriateness will be demonstrated by showing that there will appear to be a structural break in the series. Although a time series is generated to be a pure random walk in the entire time period without a trend, a spurious linear trend in the first half of the sample period will often be significantly different from another in the last half. Two $I(1)$ time series, not cointegrated by construction, will be shown to appear to be cointegrated when $I(1)$ time series are inappropriately detrended in Section 3. The role of the inappropriate detrending in the investigation of structural changes will be discussed in Section 4. Although two $I(1)$ time series are not cointegrated, inappropriately detrended series may appear to be cointegrated in one segment of time series but not in another thus incorrectly to suggest a structural change. Analysis of three foreign exchange rates is given in Section 5 and concluding remarks in Section 6.

2. Inappropriate Detrending of I(1) Time Series

Consider the following random walk series:

$$(1) \quad z_t = z_{t-1} + u_t,$$

where u_t is independently and identically distributed (iid) with mean zero and unit variance. By construction, z_t in (1) is I(1) to contain a stochastic trend, but not a (linear) deterministic time trend. Suppose such a time series is nevertheless detrended by estimating the following linear regression equation,

$$(2) \quad z_t = \alpha + \beta T_t + \varepsilon_{1t},$$

where T_t is a linear deterministic time trend. The OLS estimate of β will show a statistical significance.

To demonstrate the statistical significance of the linear trend, a series of Monte Carlo sampling experiments are conducted. First, 2,000 standard normal deviates of u_t are generated by Regression Analysis of Time Series (RATS), Version 5. By setting the initial value of z_0 to zero, a random walk series, z_t , of 2,000 observations is generated by using (1). A total of 100,000 replications are tried not only for this but also for all the other experiments. In order to reduce the influence of the zero initial value, the first 1,000 observations are discarded and only the last 1,000 observations are subsequently analyzed. The series is regressed on the linear deterministic time trend as in (2), and OLS is used for the estimation. Table 1 reports the conventional t-statistics and their absolute values. The figures are the mean values over the 100,000 replications. The standard deviations of the corresponding mean values are provided in parentheses.

[Insert Table 1 about here]

The mean value of t-statistics is close to zero with -0.36678 , because many trend coefficients are positive and many others are negative. The absolute values of t-statistics are relevant in testing a hypothesis for a linear deterministic time trend. The mean $|t(b)|$ is 35.57198 with the standard deviation of 28.3326 . Out of the $100,000$ replications, the linear time trend shows a statistical significance in 96.422% . The significance level of 5% is used throughout the paper unless otherwise specified. The average R^2 is 0.44749 which implies that the OLS estimation of (2) yields statistically significant results. It should be mentioned that a careful analyst would detect the inappropriateness of the OLS estimation of (2), because its Durbin-Watson statistics would indicate that the OLS residuals have strong autocorrelations. The exercise here, however, is to demonstrate how an inappropriate detrending would produce a significant linear trend.

In the next experiments, the number of observations is changed from $1,000$ to 500 and then to 200 . The average values of R^2 remain about the same regardless of the sample size, but a fewer number of $100,000$ replications show a significant linear trend when the sample size decreases. Nevertheless, even with $n = 200$, over 91% exhibit a significant linear trend.

A potential structural change is investigated next. The same random walk time series generated by (1) are now estimated by OLS for a linear regression equation:

$$(3) \quad z_t = \alpha_1 + \alpha_2 D_t + \beta_1 T_t + \beta_2 D_t T_t + \varepsilon_{2t}.$$

In (3), D_t is a dummy variable taking a value of zero in the first half of the sample and one in the last half. A hypothesis if $\beta_2 = 0$ is tested to investigate if the time trend significantly changes over the two sub-periods.

In the Monte Carlo experiments, the same data generated earlier are further estimated by

OLS. The results from (3) are provided at the bottom of Table 1. When $n = 1,000$, the average value of the absolute t-statistics for the estimate of β_2 is 23.58924 and the average R^2 is 0.69305. Moreover, 94.522% show a significant structural break. This average percentage drops as the number of observations gets smaller, but even when $n = 200$, still over 87% show a significant trend change.

The series generated thus far are pure random walk series. Such restrictive assumptions are relaxed to investigate more general I(1) series. In the next series of sampling experiments, either a drift term or an additional random component or both are added as in,

$$(4) \quad z_t = \delta + z_{t-1} + u_t \text{ and } y_t = z_t + v_t,$$

where u_t and v_t are mutually independent, iid normal deviates. The variance of u_t is set to one, but that of v_t , σ^2 , is allowed to vary. A time series y_t of 1,000 observations is generated by following the same procedure as in the earlier experiments. The time series is regressed on a linear trend as in (2) to test for a significant trend or on a linear trend with dummy variables as in (3) to test for a significant structure change.

Table 2 reports the OLS results. When there is a positive drift, linear time trends appear to be more significant, as expected. In Case 3, all 100,000 replications show a significant deterministic linear trend. In all three situations with different parameter values, over 94% exhibit a statistically significant structural change over the two sub-periods. Both experiments shown in Tables 1 and 2 indicate that when an I(1) time series is inappropriately detrended, the linear deterministic time trend would appear to be statistically significant and spurious structural changes are induced as statistical artifacts.

[Insert Table 2 about here]

3. Spurious Cointegration

When a linear combination of two or more I(1) series becomes I(0), then those series are said to be cointegrated. Since an I(1) time series appears to have a linear trend as shown above, the inappropriate detrending is expected to critically influence on whether or not I(1) series are cointegrated. Consider two time series that are generated by

$$(5) \quad z_{1t} = z_{1t-1} + u_{1t},$$

$$z_{2t} = z_{2t-1} + u_{2t},$$

$$y_{1t} = z_{1t},$$

$$y_{2t} = y_{1t} + z_{2t},$$

where z_{1t} and z_{2t} are two different random walk series. When u_{1t} and u_{2t} are uncorrelated, then z_{1t} and z_{2t} are not cointegrated. Likewise, y_{1t} and y_{2t} are not cointegrated. The reason why y_{2t} is generated by adding y_{1t} and z_{2t} as in (5) is to make it a cointegration regression so that OLS is an adequate method to apply.

In the literature, the maximum likelihood estimation method by Johansen (1988, 1991) and Johansen and Juselius (1990) is commonly used in the test and the estimation of cointegration relationships. Yet, Engle and Granger (1987) use so-called the cointegration regression as in

$$(6) \quad y_{2t} = \alpha + \beta y_{1t} + \varepsilon_t.$$

If y_{1t} and y_{2t} are cointegrated, then the OLS residual series from (6) will be an I(0) time series, whereas if they are not cointegrated, then it will be an I(1) time series. The test of a unit root in the OLS residuals, by using the Dickey-Fuller procedure, will show whether or not y_{1t} and y_{2t} are

cointegrated. Throughout the paper, unit root test is conducted by the Dickey-Fuller procedure, although there are numerous extensions and variations in the literature.

In order to investigate the role of the inappropriate detrending on the cointegration, two I(1) time series are first generated by using (5). In one experiment, 2,000 observations are first generated by using the initial values of zero for z_{1t} and z_{2t} and only the last half of 1,000 observations are used by regressing y_{2t} on y_{1t} . This cointegration test is conducted for the entire sample period and separately for the first and the last half of the sample period. The two time series are created such that they are not cointegrated in each and every one of those 100,000 replications in either two sub-samples as well as in the entire sample period. This condition imposed in order to investigate the impact of the inappropriate detrending on cointegration and structural change.

Since time series are generated with no cointegration relationships and without detrending, results from only detrended time series are reported in Table 3. Each of two time series is separately detrended and the detrended y_{2t} is regressed on the detrended y_{1t} as in

$$(7) \quad \det y_{2t} = \alpha + \beta \det y_{1t} + \varepsilon.$$

After (7) is estimated by OLS, the residuals are tested for a unit root. If it is I(0), then detrended y 's are cointegrated.

[Insert Table 3 about here]

In the entire period of 1,000 observations, 20.365% reveal that the two detrended series are spuriously cointegrated. Moreover, each of the two time series is again separately detrended in either half of the sub-samples. When sub-samples are analyzed for a possible cointegration by using (7), 23.476% of the first half and 23.338% of the last half show a significant spurious

cointegration.

A possible structural change is investigated next. Here a structural change is defined as a change in cointegration. A structural change exists if there is cointegration in the first half, but not in the last half, or vice versa. If there is a cointegration in both sub-sample periods or in neither sub-sample period, then there is no structural change. The results are provided at the bottom of Table 3. For Case 1, 64.104% show no structural change, because 5.0459% show cointegration in both sub-periods and 58.645% show no cointegration in either sub-period. Yet, a total of 35.896% of times, there is a structural change because 18.017% of cases change from cointegration to no cointegration and additional 17.879% of cases change from no cointegration to cointegration over the two sample periods. That is, over one-thirds of those 100,000 replications tried, the detrending has improperly introduced a spurious structural change.

In the next two columns in Table 3, two additional sampling experiments are conducted by using, respectively, 500 and 200 observations. In each case, twice as many observations are first generated to discard the first half of them in order to eliminate or reduce the influence of the zero initial starting values in the data generation. The results are very similar to those in the earlier experiment. The percentages of the spurious cointegration and the percentages of the spurious structural change are about the same whether the number of observations is 1,000 or 500 or 200. The inappropriate detrending does alter the properties of I(1) time series such a significant way as to create a spurious cointegration or a spurious structural change.

In the Monte Carlo experiments in Table 3, rather a simple data generation mechanism has been used. Both z_{1t} and z_{2t} are random walk series without any drift terms and y_{2t} is a simple addition of those two random walk series. In Table 4, more complex schemes are tried in the data

generation. Either one or both time series have non-zero drift terms and a third random term, which is normally distributed, is added to y_{2t} . All three cases in Table 4 have 1,000 observations. Otherwise, data generation and data analysis are exactly the same as in the earlier experiments. As can be seen in Table 4, the occurrences of a spurious cointegration change from one experiment to another. Yet, the percentages of the spurious cointegration are never below 20% in the entire samples or in either sub-sample periods. The percentage is about 30% in Case 2.

[Insert Table 4 about here]

Likewise, the detrending spuriously introduces apparent structural changes in over 36% for Case 1. Changes from cointegration to no cointegration or those from no cointegration to cointegration are about the same because of symmetric data generation. Those fractions far exceed the nominal 5% in the test. In the experiments conducted in Tables 3 and 4, each of 100,000 replications has been selected not to be cointegrated at the 5% significance level. If this pre-selection criterion had not been imposed, the cointegration cases would have been even greater than those reported in the tables. Such pre-selection would not, however, change the main conclusion of the impact of the inappropriate detrending on the spurious cointegration and spurious structural change.

4. Appropriate Detrending

Only inappropriate detrending has thus far been investigated. Needless to say, when the underlying data generating process truly contains a linear deterministic trend, then such a trend has to be removed or incorporated in the cointegration analysis. Consider a situation:

$$(8) \quad z_t = z_{t-1} + u_{1t}, \text{ where } u_{1t} \sim N(0, 1)$$

w_t is detrended z_t which is tested to be I(1)

$y_{1t} = \lambda T_t + w_t$ and

$y_{2t} = 0.1 T_t + y_{1t} + u_{2t}$, where $u_{2t} \sim N(0, 1)$.

By design, the I(1) series of w_t does not contain a linear deterministic trend because it is actually the outcome of detrended z_t . When λ in (8) is zero, then y_{1t} and y_{2t} will not be cointegrated because their linear combination contains a linear deterministic trend. Both y_{1t} and y_{2t} are detrended to obtain $\det y_{1t}$ and $\det y_{2t}$ and those detrended series are analyzed for cointegration. If λ is not zero, then depending on its numerical value, y_{1t} and y_{2t} without detrending will sometimes be cointegrated. No matter what the value of λ is, detrended y_{1t} and detrended y_{2t} will be cointegrated by design. That is, the detrending is necessary and appropriate in the cointegration investigation.

In order to understand the influence of the numerical values of λ in the cointegration investigation, a data generation process in (8) is used in the following Monte Carlo sampling experiments. In Table 5, three cases are studied with different values of λ . In each case, 2,000 observations are first generated with the initial value of zero in creating z_t and as before the first 1,000 observations are discarded in the analysis. A random walk series is detrended to obtain a trendless I(1) time series of w_t . A linear deterministic trend, λ times T_t , is then added to this I(1) series to generate y_{1t} . Finally, the linear time trend of 0.1 times T_t and the standard normal deviates of u_{2t} are added to generate y_{2t} as in (8)

[Insert Table 5 about here]

Cointegration is investigated with and without detrending those two time series. As before, cointegration regression is estimated by OLS and the OLS residuals are studied to check

if they are $I(1)$ or $I(0)$. Table 5 shows the results. In all cases, detrended time series are cointegrated in each and every replication, as expected from the data generation, and as shown at the bottom of the Table 5. When the time series are not detrended, however, they are sometimes cointegrated or sometimes not cointegrated. Only 0.868% in Case 1 when λ 0.01, 25.073% in Case 2 with λ 0.05, and substantial 77.270% in Case 3 with λ 0.1 become cointegrated. Since time series are known to contain linear time trends, such trends should be removed before time series are investigated for their cointegration. Failure to detrend the time series often erroneously indicates they are not cointegrated. In Case 1, over 99% show no cointegration, although the time series are generated to be cointegrated. The series of experiments show that whether time series should be detrended or not must critically be based on how the true underlying time series are assumed to be generated. Naturally, the incorrect and inappropriate treatment of a linear time trend will produce incorrect cointegration conclusions.

In the literature, the impact of the detrending on unit-roots has been discussed by Hansen (1992), Gonzalo and Lee (1998), Hassler (2000), Perron and Rodriguez (2003), and many others. General conclusion is that researchers have to pay a close attention to the potential presence of deterministic trends. In this paper, we have extended their work to illustrate how inappropriately detrended time series would introduce a spurious cointegration and a spurious structural change.

5. Foreign Exchange Rates

Monthly exchange rates of the Japanese yen, British pound, and Italian lira, all in terms of the U.S. dollar, are from St. Louis Federal Reserve Bank database, FRED, from April January 1986 to December 2001. The end period is dictated by the introduction of the euro in January

2002. Those three rates are shown in Figure 1.

Many studies (e.g., Cushman, Lee, and Thorgeirsson, 1996; Diebold, Gardeazabal, and Yilmaz, 1994; Lajaunie and Naka, 1997; and Sephton and Larsen, 1991) have investigated whether various foreign exchange rates have a unit root and still many others (e.g., Copeland, 1991; Hakkio and Rush, 1989; Karfakis and Parikh, 1994; and Rapp and Sharma, 1999) have studied if they are cointegrated.

Each series is investigated for its unit roots. Natural logarithmic transformation is taken for each exchange rate of 192 observations. All the analyses are performed by using RATS, Version 5, and CATS (Cointegration Analysis of Time Series), Version 1. All three exchange rates are tested to be I(1), having one unit root. In the following, those three logarithmically transformed exchange rates are denoted as, respectively, x_1 , x_2 , and x_3 . When x 's are regressed on a linear time trend and estimated by OLS, the trend is not significant for the pound with t-value of 1.38, but are very significant the lira and the yen with t-statistics of, respectively, 19.90 and -10.78 .

When the lira is regressed on the yen and the pound, the OLS result of this cointegration regression is $x_3 = 11.3012 (0.2062) + 1.2114 (0.0537) x_1 - 0.9645 (0.0452) x_2 + e_t$, $R^2 = 0.7809$, where standard errors are shown in parentheses. The OLS residual is tested to be I(1) so that the three foreign exchange rates are not cointegrated. The same three series are used in the Johansen and Juselius (1990) maximum likelihood test. With the lag length of four and by using three different methods to treat the deterministic parts – CIMEAN, DRIFT, and CIDRIFT -- the series is tested not to be cointegrated at the 10% significance level by using the trace statistics. It

should be noted that this popular software for cointegration analysis uses detrending as a built-in default feature. Users should instruct the program not to detrend the series automatically. From both OLS and MLE, those three exchange rates are thus tested not to be cointegrated.

Next, a linear time trend is added in the regression equation to yield, $x_3 = 8.8580 (0.2035) + 0.001733 (0.0001083) T_t + 0.8720 (0.0410) x_1 - 0.4522 (0.0435) x_2 + e'_t$, $R^2 = 0.9072$. The OLS residual is $I(0)$ indicating that that the three exchange rates are cointegrated when the linear trend term is added.

It should be mentioned that some trial-and-error experiments have been conducted to arrive at this contrasting cointegration result by trying different starting time periods. January 1986 is the outcome of this experiment to demonstrate that detrending can and does make a difference in the cointegration conclusion. When the three detrended exchange rates are used in the Johansen/Juselius maximum likelihood estimation, the trace statistic does show a significant cointegration at the 10% significance level when the lag length of four and the DETTREND = CIMEAN are used. That is, both OLS and MLE show that detrended exchange rates are cointegrated whereas they are not cointegrated without the detrending.

Since we do not know for sure if the actual exchange rates do indeed contain a linear deterministic trend, we cannot definitively conclude on detrending inappropriateness or cointegration spuriousness. What the application here demonstrates is that detrending does make a difference in the study of cointegration. Those series not cointegrated without detrending do often become cointegrated with detrending. Researchers should be careful about the presence of the linear deterministic trend in the cointegration analysis.

6. Conclusions

Cointegration is an important time series property among $I(1)$ time series. Cointegrated time series do not deviate from each other for a long time. It has long been established, and as shown in a series of Monte Carlo experiments here, the detrending changes the properties of the underlying nonstationary $I(1)$ time series. The detrending introduces a spurious periodicity. More importantly, an $I(1)$ time series, which is truly generated without containing a linear time trend would appear to have a statistically significant deterministic trend when an $I(1)$ series is regressed on time trend. The inappropriate detrending would also spuriously introduce a structural change. When the sample period is divided into two sub-periods, as shown in a series of sampling experiments, the behavior of time trend appears to statistically and materially change over the two time periods. This apparent structural change is a statistical artifact generated by the inappropriate detrending.

Cointegration properties also change with the inclusion of the time trend. Even if the underlying $I(1)$ time series are created not to be cointegrated, an inappropriate detrending would often make them to be cointegrated. In addition, the presence of the cointegration would appear to change from one sub-sample period to another. There would thus appear to be a structural change when time series are inappropriately detrended. The impact of the detrending in the study of cointegration is demonstrated with an application of three foreign exchange rates of the yen, pound, and lira, in terms of the U.S. dollar. The exchange rates are cointegrated with detrending, but not cointegrated without detrending. Since cointegration is important in understanding the long term behavior of nonstationary time series, researchers should pay a great attention to the

inclusion of the deterministic trend.

Our Monte Carlo experiments, like any such exercises, are limited in many dimensions. Only two time series are investigated in the cointegration and only limited variations are tried in the parameter values. Throughout, we have used rather the simple Dickey-Fuller procedure in the test of a unit root. There are many other test procedures. The use of the Dickey-Fuller procedure is justified, because the purpose of the sampling experiments is not to quantify the statistical properties of the inappropriate detrending, but to demonstrate the importance of the detrending procedure. It is well known that the inappropriate detrending changes the time series properties of nonstationary time series. In this study, cointegration and structural change are added into those statistical properties.

This paper has shown that whether a time series is trend-stationary or difference-stationary, as discussed in Nelson and Kang (1984) and others, should be resolved prior to the investigation to check if nonstationary time series are cointegrated.

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Table 1. Inappropriately Detrended Nonstationary Time Series.

Data generation: $z_t = z_{t-1} + u_t$, where $u_t \sim N(0, 1)$

Estimation by OLS:

$$z_t = \alpha + \beta T_t + \varepsilon_{1t}, \text{ where } T_t = 1, 2, \dots, n.$$

Estimation by OLS to test for a structural change:

$$z_t = \alpha_1 + \alpha_2 D_t + \beta_1 T_t + \beta_2 D_t T_t + \varepsilon_{2t},$$

where $D_t = 0$ if $T_t \leq n/2$ and $D_t = 1$ if $T_t > n/2$.

	Case 1	Case 2	Case 3
NOB (n)	1,000	500	200
OLS estimation			
t(b)	-0.36678 (45.4751)	0.04240 (32.2215)	-0.02052 (20.1908)
t(b)	35.57198 (28.3326)	25.19745 (20.0825)	15.78245 (12.5928)
Fraction of significant b	0.96442	0.94932	0.91893
R ²	0.44749 (0.2982)	0.44855 (0.2979)	0.44571 (0.2986)
OLS estimation for a potential structural change			
t(b ₂)	0.06488 (29.9283)	-0.02407 (21.0654)	-0.00501 (13.2601)
t(b ₂)	23.58924 (18.4187)	16.61684 (12.9472)	10.43918 (8.1764)
Fraction of significant b ₂	0.94522	0.92380	0.87688
R ²	0.69305 (0.2066)	0.69312 (0.2065)	0.69162 (0.2071)

Note: Means are followed by standard deviations in parentheses computed over 100,000 replications. Fractions are out of 100,000 replications.

Table 2. Inappropriately Detrended Nonstationary Time Series with Drifts.

Data generation: $z_t = \delta + z_{t-1} + u_t$, where $u_t \sim N(0, 1)$
 $y_t = z_t + v_t$, where $v_t \sim N(0, \sigma^2)$

Estimation by OLS:
 $y_t = \alpha + \beta T_t + \varepsilon_{1t}$, where $T_t = 1, 2, \dots, n$.

Estimation by OLS to test for a structural change:
 $y_t = \alpha_1 + \alpha_2 D_t + \beta_1 T_t + \beta_2 D_t T_t + \varepsilon_{2t}$,
 where $D_t = 0$ if $T_t \leq n/2$ and $D_t = 1$ if $T_t > n/2$.

	Case 1	Case 2	Case 3
NOB (n)	1,000	1,000	1,000
δ	0.0	0.1	0.5
σ	0.5	0.0	0.5
OLS estimation			
t(b)	-0.33629 (47.1445)	126.32508 (56.2245)	628.00449 (177.4439)
t(b)	37.50828 (28.5627)	126.37074 (56.1218)	628.00449 (177.4439)
Fraction of significant b	0.97017	0.99945	1.00000
R ²	0.47198 (0.2945)	0.89743 (0.1188)	0.99677 (0.0021)
OLS estimation for a potential structural change			
t(b ₂)	-0.12707 (30.6641)	0.38867 (29.1681)	0.23378 (29.1435)
t(b ₂)	24.29789 (18.7058)	22.95720 (17.9969)	22.94103 (17.9750)
Fraction of significant b ₂	0.94846	0.94379	0.94442
R ²	0.71805 (0.1874)	0.94599 (0.0663)	0.99837 (0.0008)

Note: Means are followed by standard deviations in parentheses computed over 100,000 replications. Fractions are out of 100,000 replications.

Table 3. Inappropriate Detrending and Spurious Cointegration.

Data generation: $z_{1t} = z_{1i-1} + u_{1t}$, where $u_{1t} \sim N(0, 1)$
 $z_{2t} = z_{2i-1} + u_{2t}$, where $u_{2t} \sim N(0, 1)$
 $y_{1t} = z_{1t}$ and $y_{2t} = y_{1t} + z_{2t}$
 y_{1t} and y_{2t} are not cointegrated.

Estimation of OLS:
 $\det y_{2t} = \alpha + \beta \det y_{1t} + \varepsilon_t$, where $\det y_{it}$ is detrended y_{it} .

Unit root test for the residual from the OLS estimation by Dickey-Fuller test

	Case 1 1,000	Case 2 500	Case 3 200
Cointegration between $\det y_{1t}$ and $\det y_{2t}$			
Entire period	0.20365	0.19901	0.19600
First half	0.23476	0.23233	0.23147
Second half	0.23338	0.23404	0.23288
Structural change from the first half to the second half Yes is for cointegration and No is for no cointegration			
No structural change	0.64104	0.64241	0.64407
Yes/Yes	0.05459	0.05439	0.05421
No/No	0.58645	0.58802	0.58986
Structural change	0.35896	0.35759	0.35593
Yes/No	0.18017	0.17794	0.17726
No/Yes	0.17879	0.17965	0.17867

Note: Figures for cointegration or structural change are the fractions out of 100,000 replications.

Table 4. Inappropriate Detrending and Spurious Cointegration with Drifts.

Data generation: $z_{1t} = \delta_1 + z_{1t-1} + u_{1t}$, where $u_{1t} \sim N(0, 1)$
 $z_{2t} = \delta_2 + z_{2t-1} + u_{2t}$, where $u_{2t} \sim N(0, 1)$
 $y_{1t} = z_{1t}$ and $y_{2t} = y_{1t} + z_{2t} + \gamma u_{3t}$, where $u_{3t} \sim N(0, 1)$

y_{1t} and y_{2t} are tested not to be cointegrated.

Estimation of OLS:

$\det y_{2t} = \alpha + \beta \det y_{1t} + \varepsilon_t$, where $\det y_{it}$ is detrended y_{it} .

Unit root test for the residual from the OLS estimation by Dickey-Fuller test

	Case 1	Case 2	Case 3
NOB (n)	1,000	1,000	1,000
δ_1	0.0	0.0	0.1
δ_2	0.0	0.1	0.1
γ	0.1	0.0	0.0

Cointegration between $\det y_{1t}$ and $\det y_{2t}$

Entire period	0.20732	0.30577	0.24633
First half	0.24300	0.29936	0.25134
Second half	0.24173	0.29971	0.25292

Structural change from the first half to the second half

Yes is for cointegration and No is for no cointegration

No structural change	0.63249	0.58101	0.62298
Yes/Yes	0.05861	0.09004	0.06362
No/No	0.57388	0.49097	0.55936
Structural change	0.36751	0.41899	0.37702
Yes/No	0.18439	0.20932	0.18772
No/Yes	0.18312	0.20967	0.18930

Note: Figures for cointegration or structural change are the fractions out of 100,000 replications.

Table 5. Appropriate Detrending and Cointegration.

Data generation: $z_t = z_{t-1} + u_{1t}$, where $u_{1t} \sim N(0, 1)$
 w_t is detrended z_t which is tested to be I(1)
 $y_{1t} = \lambda T_t + w_t$ and $y_{2t} = 0.1 T_t + y_{1t} + u_{2t}$, where $u_{2t} \sim N(0, 1)$

Estimation of OLS:

$$y_{2t} = \alpha + \beta y_{1t} + \varepsilon_{1t}.$$

$\det y_{2t} = \alpha' + \beta' \det y_{1t} + \varepsilon_{2t}$, where $\det y_{it}$ is detrended y_{it} .

Unit root test for the residual from the OLS estimation by Dickey-Fuller test

	Case 1	Case 2	Case 3
NOB (n)	1,000	1,000	1,000
λ	0.02	0.05	0.1

Cointegration between y_{1t} and y_{2t}

Entire period	0.00868	0.25073	0.77270
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Cointegration between $\det y_{1t}$ and $\det y_{2t}$

Entire period	1.00000	1.00000	1.00000
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Note: Figures for cointegration are the fractions out of 100,000 replications.

Figure 1. Exchange Rate Movements

