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Incentives, Solidarity, and the Division of Labor

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Abstract

In this paper, we consider a version of the Holmström-Milgrom linear model with two tasks, production and administration, where performance is harder to measure in the latter. Both the principal and agent can devote effort to these tasks. We assume there are gains from specialization and that players have a preference for solidarity in work. As the gains from specialization increase, the principal eventually prefers to hire the agent solely for production purposes over autarky. As these gains increase still further, the principal increasingly specializes in administration and in the limit there is a complete division of labor. At the same time, the nature of the employment contract is transformed from one based on solidarity to one based on incentives. We therefore formalize aspects of the thought of Smith and Marx, who held that a division of labor leads to exchange and a deterioration in social relations.

Keywords: alienation, cooperation, division of labor, incentives, Marxism, reciprocity, and solidarity.

JEL Classification Numbers: B12, B14, D86, L23, M52.

1. Introduction

In his wide-ranging survey, Bowles (1998) discusses a substantial body of experimental and field evidence from a variety of disciplines which indicates that markets and other economic institutions can affect people's preferences. E.g., Mallon (1983) documents the erosion of solidarity and community institutions for the provision of public goods when markets were extended to the central highlands of Peru during the early twentieth century. Likewise, Alesina and Fuchs-Schündeln (2006) report statistical evidence from panel data which suggests that former East Germans raised under communism have stronger preferences for redistribution and state intervention than West Germans, even after controlling for their economic interests.

In his survey and subsequent theoretical work, Sobel (2005, 2007) asks the question more directly: "do markets make people selfish?" In Sobel (2007) he develops a general class of interdependent preferences and shows that reciprocal agents are generally unable to affect the equilibrium in market-like settings and will therefore appear selfish. This is because agents have little influence over prices in such settings and it is difficult to punish someone on the other side of the market when there are other agents on one's own side who are willing to transact at the terms on offer. In essence, these results extend those by Fehr and Schmidt (1999), Bolton and Ockenfels (2000), and Falk and Fischbacher (2006). Sobel's point is that people's preferences can only be elicited to the extent that the specific environment allows.

In this paper, we consider a version of the Holmström and Milgrom (1991) linear agency model with two tasks, "administration" and "production," where performance is harder to measure in the former. In contrast with the existing literature, both the principal (she) and agent (he) can devote effort towards these tasks.[1] As in Sobel (2007), the player's preferences are interdependent and in our model express a preference for solidarity: the principal and agent prefer matching efforts in both tasks, *ceteris paribus*. Since the principal can commit to a contract that states her own efforts in each task, as

[1] An exception is the literature on *double moral hazard* [e.g., see Kim and Wang (1998) and their references], where the principal and agent work together on a single task and the principal's effort is also subject to moral hazard. In this paper, we assume the principal can commit to her own effort levels as part of the contract in order to study the effects of solidarity.

well as incentives for each, these preferences can be interpreted in terms of an endogenous *norm* established by the principal as part of the contract.

Like Holmström and Milgrom (1991, p. 36), we distinguish two extreme and polar institutions: the first involves zero incentives, so the players' efforts are motivated entirely by solidarity, and the second is supported by high-powered incentives alone. We call the former the *solidarity equilibrium* (similar to the "employment" contract in Holmström and Milgrom) and the latter the *market equilibrium* (similar to their "contracting" institution), although there is a continuum of possibilities in between. In this paper, we ask: under what conditions does the optimal contract reflect solidarity, and under what conditions does it reflect a market arrangement? This question is quite similar to Sobel's, except that now the *institution* is endogenous. In other words, Sobel's question can be rephrased as: how do reciprocal players affect the equilibrium in different institutions, whereas here we ask the complementary question: what institutions do reciprocal players choose?

In this paper, we explore an answer that dates back to Adam Smith and Karl Marx. In the *Wealth of Nations,* Smith explains how markets and the division of labor are mutually reinforcing: a division of labor leads to exchange because the producer can no longer subsist on her own specialized product, while the division of labor is limited by the extent of the market. Although Smith extolled the virtues of specialization, he also thought it deprived workers of their "intellectual, social, and martial virtues," an admission that paved the way for Marx's conception of *alienation* in capitalism [see Blaug (1997, p. 35)]. Although most economists are familiar with the main outlines of *Capital* (e.g., the labor theory of value, the falling rate of profit, etc.), the theory of alienation developed in his early writings seems less well-known. A major theme, described briefly in the next section, is that the division of labor leads to market exchange and a subsequent deterioration in social relations.

Marx's ideas on alienation have had a lasting influence on the broader social sciences outside economics, as well as popular discussions on globalization and capitalism.[2] E.g., Thomas L. Friedman expresses the main theme of his bestseller *The Lexus and the Olive*

[2] Although he does not blame capitalism, Putnam's *Bowling Alone* (2000) [also see the review by Sobel (2002)], attempts to document the depreciation in social capital in the US over the past 30 years or so, in the sense of a significant decline in group activities such as bowling leagues, voting, clubs, organized religion, etc. The theoretical literature on social capital in economics includes Mailath and Postlewaite (2003, 2006).

Tree (2000, p. 34) as follows:

> And what we are looking at and for is how the age-old quests for material betterment and for individual and community identity — which go all the way back to Genesis — play themselves out in today's dominant international system of globalization. This is the drama of the Lexus and the olive tree.

Likewise, Sylvia Ostroy (2001, p. 11-2) has remarked that

> One is struck in reading on the subject of anti-globalization how often two words appear — alienation and anomie. The first is from Marx and is essentially a moral critique of capitalism. He argued that because labor becomes a commodity, the worker loses all power to control the processes by which decisions are made that affect his life. Anomie, first used by the French sociologist Emile Durkheim writing at the beginning of the twentieth century, stressed the individual's sense of powerlessness and the loss of social cohesion... I think alienation and anomie are useful concepts to explain the rise of the anti-globalization movement.

To formalize these ideas, we assume that production exhibits gains from specialization, so output is greater when the players specialize in separate tasks. As these gains increase, the principal eventually prefers to hire the agent over autarky. Since the agent is risk averse and administration is harder to measure, she hires him solely for production work. As the gains from specialization increase still further, the principal re-allocates her effort out of production and into administration. At the same time, she increases the agent's incentives, whose motivation has been weakened by the loss of solidarity. *An increase in the gains from specialization leads to the division of labor, exchange (market relations), and a deterioration in social relations.* In the limit, we obtain the market equilibrium where both players completely specialize and the agent is motivated solely by incentives. A sufficient condition for these results is that the players' preferences for solidarity must be sufficiently weak: selfish players choose to transact in markets. We also sketch a mechanism that can achieve the solidarity equilibrium which formalizes certain utopian aspects of Marx's views on the transition from capitalism to communism.

Solidarity preferences are similar to standard reciprocal preferences in that an increase (decrease) in effort by the principal motivates the agent to increase (decrease) his own effort in the same task. The experimental literature on reciprocity in agency settings is surveyed in Fehr and Gächter (2000) and Fehr and Fischbacher (2002). This literature is important for our purposes because (i) it establishes that reciprocity is a major feature of agency relationships; (ii) Fehr, Klein, and Schmidt (2001, 2007) report that subjects acting as principals chose implicit contracts based on reciprocity (similar to our solidarity equilibrium) over incentive contracts 88% of the time; and (iii) Fehr and Gächter (2001) show that incentives can "crowd out" reciprocity, which is broadly consistent with our comparative statics results with respect to the gains from specialization.

The papers closest to ours in the literature on specialization are Lindbeck and Snower (1996, 2000), who consider two tasks, 1 and 2, that are complementary in production and two types of workers, where type 1 (2) workers have a comparative advantage in task 1 (2). After paying the workers' reservation wages, the firm allocates them between the two tasks. This allocation is determined by three main forces: (i) the gains from specialization, (ii) "informational task complementarities" (the more time a worker spends on one task, the better he does the other), and (iii) workers' preferences with respect to specialization. The authors show that a complete division of labor results when the gains from specialization increase sufficiently fast relative to the gains from informational task complementarities and workers have a sufficiently strong preference for specialized work.[3]

Rob and Zemsky (2002) consider a dynamic version of the linear agency model with a continuum of risk neutral agents. In each period, agents allocate effort between an individual and a cooperative task, where the latter is more profitable but the former is more easily observed by the principal. Their contributions to the cooperative task are driven by reciprocity, so an increase in incentives today leads to less cooperation today and therefore less cooperation tomorrow. The model can have two long-run steady-state equilibria: a "good" equilibrium with high cooperation, high profits, and low incentives, and a "bad" equilibrium. If the initial level of cooperation is low, the bad equilibrium results. As this initial condition increases, however, eventually the good equilibrium obtains. There is therefore a negative relationship between incentives and cooperation, whereas in our model this is driven by the gains from specialization.

The plan for the rest of the paper is as follows. In section 2, we briefly describe Marx's theory of alienation. In section 3, we present the model and results. Section 4 concludes.

[3] Holmström and Milgrom (1991, section 5.1) and Itoh (1991) also consider cooperation in agency relationships, without gains from specialization or social preferences. This "cooperation" is really joint production enforced by incentives, and not cooperation in a truly cooperative sense.

2. Alienation and the Division of Labor

In this section, we discuss those aspects of Marx's theory of alienation that are centered on the division of labor.[4] For more complete discussions, see Elster (1985, 1986), Kolakowski (2005), and Ollman (1976). For analytical treatments of other aspects of Marx's thought, see Roemer (1981, 1982). Before proceeding, we repeat Archibald's (1992, p. 61) remark that Marx's conception of alienation "in no way depends on the labor theory of value, which is simply irrelevant."

First of all, Marx takes the individual's objective to be *self-realization* rather than utility maximization [for a comparison, see Elster (1986, Chapter 3)]. In other words, the individual seeks to develop her potential and to manifest her abilities in society. In Marx's view, one can only achieve self-fulfillment in society.[5]

> The human being is in the most literal sense a *zoon politikon* [a political animal], not merely a gregarious animal, but an animal which can individuate himself only in the midst of society. Production by an isolated individual outside society – a rare exception which may well occur when a civilized person in whom the social forces are already dynamically present is cast by accident into the wilderness – is as much of an absurdity as is the development of language without individuals living *together* and talking to each other... The point could go entirely unmentioned if this twaddle, which had sense and reason for the eighteenth-century characters had not been earnestly pulled back into the center of the most modern economics by Bastiat, Carey, Proudhon, etc.
>
> Marx, the *Grundrisse*, p. 223.

For Marx, the most important arena for self-realization is joint production with others. Its implications extend beyond material sustenance, because it also defines a way of life.

> This mode of production must not be considered simply as being the reproduction of the physical existence of the individuals. Rather it is a definite form of activity of the individuals, a definite form of expressing their life, a definite *mode of life* on their part. As individuals express their lives, so they are. What they are, therefore, coincides with their production, both with *what* they produce and with *how* they produce.
>
> Marx and Engels, *The German Ideology*, p. 150.

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[4] The worker's alienation is traditionally divided into alienation from his product (issues stemming from the fact that the product belongs to the capitalist and not to him), his activity, man-from-man alienation, and species alienation. In this paper, we neglect alienation from the product because, as Elster (1986, p. 49) points out, modern production methods make it difficult for a worker to identify any product as exclusively his and mass market production breaks the personal bond between consumer and producer. As in this paper, Elster goes on to argue that joint production offers the most plausible vehicle for self-realization in modern economic life.

[5] All citations of Marx and Engels refer to page numbers in Tucker (1978). In *Capital* (p. 324), Marx writes: "Since Robinson Crusoe's experiences are a favorite theme with political economists, let us take a look at him on his island."

To complete his view of human nature, Marx had an exaggerated sense of the relationship between humanity and the external world, or nature. In an ideal state (i.e., communism), the human race will work together to transform the external world and will see it as the work of its own hands. At the same time, people are also a product of nature, which includes other people. According to Marx, idealized humanity can only be understood in terms of this mutual interaction, which he calls the "life of the species."

> It is an anthropocentric viewpoint, seeing in humanized nature a counterpart of practical human intentions; as human practice has a social character, its cognitive effect – the image of nature – is the work of social man. Human consciousness is merely the expression in thought of a social relationship to nature, and must be considered as a product of the collective effort of the species.
>
> Kolakowski (2005, p. 114).

We now come to the division of labor. Although Smith was emphatically positive overall, his negative remarks formed the starting-point for Marx's later views.

> The man whose life is spent in performing a few simple operations, of which the effects too are, perhaps, always the same, or very nearly the same, has no occasion to exert his understanding, or to exercise his invention in finding out expedients for removing difficulties which never occur. He naturally loses, therefore, the habit of such exertion, and generally becomes as stupid and ignorant as it is possible for a human creature to become. The torpor of his mind renders him, not only incapable of relishing or bearing a part in any rational conversation, but of conceiving any generous, noble, or tender sentiment, and consequently of forming any just judgment concerning many even of the ordinary duties of private life.
>
> Smith, *Wealth of Nations*, quoted in Archibald (1992, p. 64).

Marx agrees with Smith that a division of labor leads to exchange.[6]

> In *The German Ideology* the root of all evil is the division of labor, private property being once again a secondary phenomenon... Marx's view is that the division of labor... is the first source of the alienating process and, through it, of private property. This happens because the division of labor leads necessarily to commerce, i.e. the transformation of objects produced by man into vehicles of abstract exchange-value. When things become commodities, the basic premise of alienation already exists.
>
> Kolakowski (2005, p. 141).

[6] This is an instance of Marx's *historical materialism*, which states that the productive forces (e.g., technology, human capital, etc.) determine everything else, including property relations, the legal and political framework, and intellectual endeavors such as philosophy. In contrast, in Marx's historical analysis of the transition from feudalism to capitalism (as opposed to his theoretical statements), he also emphasized other factors such as the discovery of the new world [see Elster (1986, Chapter 6)].

The rise of markets results in a deterioration of social relations, where people perceive each other only as owners of their respective commodities.

> The bourgeoisie... has left no other nexus between man and man than naked self-interest, than callous "cash payment." It has drowned the most heavenly ecstasies of religious fervor, of chivalrous enthusiasm, of philistine sentimentalism, in the icy water of egotistical calculation. It has resolved personal worth into exchange value...
>
> Marx and Engels, *The Communist Manifesto*, p. 476.

As Elster (1986, p. 53) puts it, "markets operate by arm's-length transactions that subvert communitarian values."

In comparison with communism, the "life of the species" (labor) becomes only a way to make money. The social bond established by cooperative production no longer exists, and with the division of labor each individual has his or her own separate sphere. Since communal self-actualization is what separates human beings from the animals, work in capitalism becomes "individual animalized life."

> Labor, which is the life of the species, becomes only a means to individual animalized life, and the social essence of man becomes a mere instrument of individual existence. Alienated labor deprives man of his species-life; other human beings become alien to him, communal existence is impossible, and life is merely a system of conflicting egoisms.
>
> Kolakowski (2005, p. 115)

The capitalist is also alienated. In particular, the principal views the agent simply as a factor of production, while the agent perceives the principal merely as a source of income.

> The reification (as it would be called later) of the worker – the fact that his personal qualities of muscle and brain, his abilities and aspirations, are turned into a "thing," an object to be bought and sold on the market – does not mean that the possessor of that "thing" is himself able to enjoy a free and human existence. On the contrary, the process has its effect on the capitalist, too, depriving him of personality in a different way. As the worker is reduced to an animal condition, the capitalist is reduced to an abstract money-power: he becomes a personification of this power, and his human qualities are transformed into aspects of it.
>
> (*ibid.*, p. 115)

Marx was fond of pointing out aspects of capitalist society that conformed to the prisoner's dilemma (although obviously he did not use that terminology). Elster (1986) discusses several examples. E.g., although it is individually rational for each capitalist to effect a division of labor (indeed, competition compels it), the result is a society that no one wants.

> the division of labor offers us the first example of how... as long as a cleavage exists between the particular and the common interest, as long, therefore, as activity is not voluntarily but naturally, divided, man's own deed becomes an alien power opposed to him, which enslaves him instead of being controlled by him. For as soon as the distribution of labor comes into being, each man has a particular, exclusive sphere of activity, which is forced upon him and from which he cannot escape. He is a hunter, a fisherman, a shepherd, or a critical critic, and must remain so if he does not want to lose his means of livelihood... this consolidation of what we ourselves produce into an objective power above us, growing out of our control... is one of the chief factors of historical development up till now.
>
> Marx and Engels, *The German Ideology*, p. 160.

Elsewhere, he makes an analogy between capitalist society and the sorcerer's apprentice in Goethe's poem, who called up magical forces that he could not control [Kolakowski (2005, p. 131)]. True freedom is essentially a coordinated decision to cooperate in the prisoner's dilemma.

> The aggregate outcome of individual actions appears as an independent and even hostile power, not as freely and jointly willed... Only by coordinating their choices according to a common plan can people achieve freedom with respect not only to action but the consequences of action. Otherwise, they are condemned in perpetuity to playing the sorcerer's apprentice.
>
> Elster (1986, p. 49, 52-3)

The continuing influence of these ideas in the social sciences is poignantly illustrated by Jon Elster's (1986, p. 4) autobiographical statement that

> If, by a Marxist, you mean someone who holds all the beliefs that Marx himself thought were his most important ideas, including scientific socialism, the labor theory of value, the theory of the falling rate of profit, the unity of theory and practice in revolutionary struggle, and the utopian vision of a transparent communist society unconstrained by scarcity, then I am certainly not a Marxist. But if, by a Marxist, you mean someone who can trace the ancestry of his most important beliefs back to Marx, then I am indeed a Marxist. For me this includes, notably, the dialectical method and the theory of alienation, exploitation, and class struggle, in a suitably revised and generalized form.

In his list (*ibid.*, Chapter 10) of what is living and what is dead in Marx's thought, Elster lists the theory of alienation among the living.

3. Model and Results

The model is a version of the linear model in Holmström and Milgrom (1991). There are two tasks, "production" and "administration," although the only formal distinction between them is that performance is harder to measure in the latter. Let e^A and a^A denote the agent's production and administration efforts, respectively, and similarly for e^P and a^P corresponding to the principal. The degree of specialization of player $i = A, P$ is given by $|e^i - a^i|$. Let $G(e^A - a^A, e^P - a^P)$ represent the gains from specialization. Throughout the paper, subscripts refer to partial derivatives.

Assumptions 1. *(i) $G(0,0) = 0$, (ii) $G_1 < 0$ when $e^A < a^A$, $G_1 > 0$ when $e^A > a^A$, and $G_1 = 0$ when $e^A = a^A$. A similar statement applies to G_2. (iii) $G_{11}, G_{22} > 0$ and the players' degrees of specialization are complements for G. (iv) G is symmetric in each argument:*

$$G(x, y) = G(-x, y) = G(x, -y) \tag{1}$$

for all x and y.

There are no gains from specialization if neither player specializes. Otherwise, (ii) and (iii) state that G is increasing at an increasing rate, which encourages full specialization. Furthermore, an increase in specialization by one player increases the marginal benefit to specialization for the other. E.g., if $e^A > a^A$ and $a^P > e^P$ then $G_{12} < 0$. For simplicity, (iv) states that G is symmetric in the sense that what matters is a player's degree of specialization and not the specific task the player is specializing in. Note that we do not assume symmetry across players in the form of $G(x, y) = G(y, x)$ for all x and y.

We normalize the price of output to be one, so the principal's revenue and output are given by

$$P(e^A + e^P, a^A + a^P) + \gamma G(e^A - a^A, e^P - a^P), \tag{2}$$

where P is a standard production function that depends on total administration and production effort and γ parameterizes the magnitude of the gains from specialization. The following assumptions are standard.

Assumptions 2. *(i) $P(0,0) = 0$, (ii) $P_1, P_2, P_{12} > 0$, (iii) $P_2(\cdot, 0) = \infty$, and (iv) $P_{11}, P_{22} < 0$.*

Note that administration and production efforts are complementary since $P_{12} > 0$ and optimality requires non-zero administration.

The agent's efforts in the two tasks generate signals

$$x_e = e^A + \epsilon_e$$
$$x_a = a^A + \epsilon_a, \tag{3}$$

observed by the principal, where ϵ_e is normally distributed with mean zero and variance V_e and similarly for ϵ_a. We also assume ϵ_e and ϵ_a are independent. We take production to be a somewhat mechanical activity, whereas administration involves creativity and leadership and is therefore harder to measure in the sense that $V_a > V_e$.

For simplicity, we assume all efforts are uniformly bounded from above[7]

$$0 \le e^i, a^i \le E. \tag{4}$$

Let $C_i(e^i + a^i)$ be player i's disutility of effort, where $C_i(0) = 0$ and $C_i', C_i'' > 0$. As in Holmström and Milgrom (1991, p. 33) we assume $C_i'(0) > 0$, which will have important implications for the principal's choice between employment and autarky, as well as the division of labor.

Specialization entails two different kinds of costs. First, the players have a preference for cooperation or solidarity in the form of a benefit

$$M = m_e(e^A - e^P) + m_a(a^A - a^P) \tag{5}$$

that satisfies the following assumptions.

Assumptions 3. *(i)* $m_e' > 0$ *when* $e^A < e^P$, $m_e' = 0$ *when* $e^A = e^P$, *and* $m_e' < 0$ *when* $e^A > e^P$, *(ii)* $m_e'' < 0$, *and (iii)* m_a *satisfies similar assumptions.*

The components m_e and m_a in (5) are therefore strictly concave and attain their maxima when there is full solidarity in the sense that $e^A = e^P$ and $a^A = a^P$, respectively. Each player shares in half the total benefit as in (6) below.

[7] We think of production as an essentially physical activity and administration as mainly cognitive, so the two are not perfect substitutes as suggested by the disutility of effort. E.g., if $e^i = E$ then i's physical reserves are completely exhausted and she cannot increase her production effort, although she may still be able to increase administration effort. From a technical perspective, the constraint set defined by $e^i + a^i \le E$ is not a lattice, which we need for Proposition 5 below.

We can interpret this form of solidarity in terms of norms for hard work as in Rob and Zemsky (2002), where agents choose individual and cooperative efforts and suffer a cost in the form of guilt or shame if the latter falls short of an exogenous fixed target. In our model, however, the principal chooses her efforts first. The targets e^P and a^P can therefore be interpreted as endogenous norms chosen by the principal as part of the (implicit) contract. Alternatively, Marx believed that self-realization could only occur in society: "only in community with others has each individual the means of cultivating his gifts in all directions" (Marx and Engels, *The German Ideology*, p. 197). In that case, a loss of solidarity implies a lack of self-realization. Whatever the specific interpretation, (5) incorporates a basic form of reciprocity where an increase (decrease) in effort by the principal motivates the agent to increase (decrease) his own effort in the same task.

The other cost associated with specialization is that the players have a preference for variety in work. This takes the form $s_i(e^i - a^i)$ for i, which satisfies conditions similar to those in Assumptions 3. In particular, s_i is strictly concave and is maximized when $e^i = a^i$. We can interpret Smith's comments in the previous section in this sense, although they seem more relevant for factory work. In Marx's view, uniform work "disturbs the intensity and flow of man's animal spirits, which find recreation and delight in mere change of activity" (Marx, *Capital I*, p. 391). Note that Lindbeck and Snower (1996, 2000) allow a preference for either specialization or variety in work.

The agent's utility function is

$$-\exp\{-r[I - C_A + \sigma s_A + (1/2)\mu M]\}, \tag{6}$$

where $r > 0$ is the coefficient of absolute risk aversion, I is the agent's income, $\sigma \geq 0$ the weight on the preference for diversity in work, and $\mu \geq 0$ the weight on the preference for solidarity. The principal is risk neutral and her choice of contract is restricted to the set of linear compensation rules of the form

$$I = \alpha + \beta_e x_e + \beta_a x_a, \tag{7}$$

where α is the fixed component and β_e and β_a are the commission rates on the signals.[8] As is well-known, linear contracts are generally suboptimal but are assumed for tractability.

[8] The classical economists did not think of wage formation solely in terms of perfect competition. As

For more discussion on this point, see Holmström and Milgrom (1991) and Bolton and Dewatripont (2005). Under these assumptions, the agent's certainty equivalent is given by

$$U = \alpha + \beta_e e^A + \beta_a a^A - C_A + \sigma s_A + (1/2)\mu M - R, \tag{8}$$

where

$$R = (1/2)r(\beta_e^2 V_e + \beta_a^2 V_a) \tag{9}$$

is the agent's risk premium, which expresses the cost of risk. The only differences between (8) and the standard multi-tasking linear model are the two terms related to the agent's preferences for solidarity and diversity in work.

We assume the principal can commit to a contract $(\alpha, \beta_e, \beta_a, e^P, a^P)$ which stipulates her own production and administration efforts, as well as the lump-sum payment and piece rates. The timing of the game is as follows. First, the principal decides whether or not to offer the agent a contract. If not, the principal solves the autarky problem [see (16)] and the agent receives his outside option $\overline{U}$. In classical economics, as well as for Marx [Kolakowski (2005, p. 114)], $\overline{U}$ is the subsistence payoff: that which is just necessary for the worker to maintain himself and successfully reproduce. If the principal does make an offer, the agent must decide whether or not to accept it. If he accepts, the agent maximizes (8) subject to (4). Otherwise, he receives $\overline{U}$. As usual, the principal chooses α such that the participation constraint always binds, so the agent always accepts the contract if one is offered. After the agent solves his problem, the outcome is realized and both players receive their payoffs.

Proposition 1 below is a technical result for the agent's problem which will play an important role throughout the paper. All proofs are in the Appendix.

Proposition 1. *(i) The agent's maximization problem has a unique solution. (ii) If*

$$\underline{\beta_e} \equiv C_A'(0) - \sigma s_A'(-E) - (1/2)\mu m_e'(-E) > 0 \tag{10}$$

Archibald (1992) notes, Smith, J.S. Mill, and Marx all discussed the bargaining problem between employers and employees. In particular, Mill was well aware of the moral hazard problem and advocated piece rates whenever possible. Likewise, Marx believed that piece rates were cheaper than monitoring and therefore more suitable for capitalist production [Elster (1986, p. 87, 91)]. None of them, however, considered the risk-reward tradeoff.

then $e^A = 0$ at the optimum for all $\beta_e \leq \underline{\beta_e}$. A similar statement holds for β_a. (iii) There exists $\overline{\beta}_e > 0$ and $\overline{\beta}_a > 0$ such that the principal can achieve any effort allocation by the agent satisfying (4) by choosing $(\beta_e, \beta_a) \in [0, \overline{\beta}_e] \times [0, \overline{\beta}_a]$.

After stating that a unique solution exists, (10) provides a condition that rules out cases where the agent would be willing to work without incentives (or even negative ones) because of his preference for solidarity. Note that (10) requires $C'_A(0)$ to be positive, as assumed previously, and sufficiently large. Since the agent will not supply positive effort unless $\beta_e > \underline{\beta_e}$, $\beta_a > \underline{\beta_a}$, or both, the agent's risk premium is bounded from below by

$$\underline{R} = (1/2) r \min \left\{ (\underline{\beta_e})^2 V_e, (\underline{\beta_a})^2 V_a \right\} \tag{11}$$

in any nontrivial contract. As in Holmström and Milgrom (1991, p. 33), the existence of a positive quasi-fixed cost for nontrivial employment contracts will have important implications in what follows. In (iii), we show that the principal's constraint set can be restricted without loss of generality to be a compact lattice [see Vives (1999, p. 17)].

We now turn to the comparative statics of the agent's problem. In Proposition 2 below, we only report results for e^A since the results for a^A are similar. The proof is a standard comparative statics exercise and is therefore omitted.

Proposition 2. *At an interior solution,*

$$\begin{aligned}
\frac{\partial e^A}{\partial \beta_e} &= \frac{2(2C''_A - \mu m''_a - 2\sigma s''_A)}{D} > 0 \\
\frac{\partial e^A}{\partial \beta_a} &= -\frac{4(C''_A + \sigma s''_A)}{D} \\
\frac{\partial e^A}{\partial e^P} &= \frac{\mu m''_e(\mu m''_a + 2\sigma s''_A - 2C''_A)}{D} > 0 \\
\frac{\partial e^A}{\partial a^P} &= \frac{2\mu m''_a(C''_A + \sigma s''_A)}{D},
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
D = -2C''_A \Big[\mu(m''_a + m''_e) + 8\sigma s''_A \Big] + \\
\mu \Big[m''_a(\mu m''_e + 2\sigma s''_A) + 2\sigma m''_e s''_A \Big] > 0.
\end{aligned} \tag{13}$$

The intuition is straightforward. Given previous assumptions, the agent's production effort is increasing in production incentives, but the effect of administration incentives is ambiguous and depends on the weight σ on the preference for diversity in work. If the latter is positive and sufficiently large, then an increase in administration incentives will increase the agent's administration effort and therefore his production effort because of the preference for diversity. The agent's production effort is also increasing in the principal's production effort because of their preference for solidarity. It follows that the principal can motivate the agent's production effort through production incentives, the principal's own production effort, or some combination of the two. Their relationship can therefore take the form of a *market* (incentive-based) relation or a *social* relation based on solidarity (although not exclusively by Proposition 1) and the central question of the paper is how the gains from specialization affect this balance. The effects of the principal's administration effort are ambiguous and depend on σ in similar fashion.

If the principal offers the agent a contract, she sets α so the participation constraint $U \geq \overline{U}$ binds. In that case, the agent accepts the contract and the principal's payoff is

$$P + \gamma G - C_P + \sigma s_P + (1/2)\mu M - I. \tag{14}$$

Substituting the agent's participation constraint, the principal's expected payoff becomes

$$W = P + \gamma G - C_A - C_P + \sigma S + \mu M - R - \overline{U}, \tag{15}$$

where $S = s_A + s_P$. In the case of autarky, the principal's payoff is

$$\hat{W} = P(e^P, a^P) + \gamma G(0, e^P - a^P) - C_P + \sigma s_P + (1/2)\mu[m_e(-e^P) + m_a(-a^P)]. \tag{16}$$

Although the principal can still reap the gains from her own specialization, she also bears the cost of the lack of solidarity.

We now synthesize two classical ideas. The first is that a division of labor leads to exchange and markets, as expounded by Smith and Marx. The second involves the classical risk-reward tradeoff from agency theory. As Archibald (1992, p. 72) notes, the classical economists (including Smith, J.S. Mill, and Marx) were well aware of the moral hazard problem in agency, although they never considered risk-sharing in that context. In contrast, in proposition 2 of Holmström and Milgrom (1991) it is the risk associated with asset ownership that determines whether the agency relationship will be an employment relationship (zero incentives and the principal owns the asset) or a market one (positive incentives and the agent owns the asset). After stating in Proposition 3(i) below that a solution to the principal's problem exists, (ii) shows that the principal prefers autarky over employment when $\underline{R}$ and $\overline{U}$ are sufficiently large, which is clear from (15) and (16). Of course, $\overline{U}$ does not pose much of a hurdle if it equals the subsistence payoff.[9] In (iii), the principal prefers to hire the agent exclusively for production work when the gains from specialization are sufficiently large and administrative performance is sufficiently hard to measure.

Proposition 3. *(i) A solution to the principal's problem exists. (ii) The principal prefers autarky over employment when $\underline{R}$ and $\overline{U}$ are sufficiently large relative to other parameters. (iii) Given fixed values for all other parameters, and assuming (10), there exists $(\gamma_1, \overline{V}_a)$ such that for all $\gamma \geq \gamma_1$ and $V_a \geq \overline{V}_a$ the principal prefers to hire the agent with positive production effort and zero administration effort over autarky. In particular, $\beta_e > \underline{\beta_e}$ and $\beta_a = 0$ at the optimum.*

Note that (iii) is stronger than $a^A \to 0$ as $\gamma \to \infty$, which is a statement about what happens in the limit. Instead, (iii) states that the agent fully specializes in production for finite values of γ (the parameter emphasized in classical economics) and V_a (emphasized in modern agency theory). To prove this result, it seems we need *both* γ and V_a to be large (see the proof).

If we take the hypotheses in Proposition 3 as our starting-point, Proposition 4(i) shows that as $\gamma \to \infty$ each player fully specializes in their own separate task. In that case, there is a complete division of labor and the absence of any solidarity or diversity in work. The limiting case is therefore the market equilibrium, where there is 'no other nexus between man and man than naked self-interest, than callous "cash payment."' 4(ii) is an important

[9] From a historical perspective, Marx argued that the enclosure movement in England from the sixteenth through the eighteenth centuries was deliberately intended to force the small peasants from the land so that they would be available for capitalist production [Elster (1986, p. 82-3)]. In terms of the model, the principal could create the necessary conditions for profitable employment by reducing $\overline{U}$.

ingredient in the proof of Proposition 5.

Proposition 4. *Assume (10), $\gamma \geq \gamma_1$, and $V_a \geq \overline{V}_a$. (i) As $\gamma \to \infty$, $a^P_\gamma \to E$. (ii) In particular, there exists $\gamma_2 \geq \gamma_1$ such that $a^P_\gamma > e^P_\gamma$ for all $\gamma \geq \gamma_2$.*

We now turn to the comparative statics of the principal's problem. To derive these, we would like to use lattice programming methods [e.g., see Vives (1999, Chapter 2)] since the problem is too complicated for standard techniques. Unfortunately, the surface defined by the incentive compatibility constraint is not generally a compact lattice, so our strategy will be to assume the following functional forms:

$$\begin{aligned} C_i(e^i + a^i) &= (1/2)(e^i + a^i)^2 + c(e^i + a^i) \\ M &= -(e^A - e^P)^2 - (a^A - a^P)^2 + 1 \\ s_i(e^i - a^i) &= -(1/2)(e^i - a^i)^2 + 1, \end{aligned} \tag{17}$$

where $i = A, P$ and $c > 0$. It is easy to verify that $c - E(\mu + \sigma) > 0$ implies (10), so all our previous assumptions hold. Given these functional forms, we can explicitly solve the agent's maximization problem in (8) and then substitute into the principal's objective function (15). After eliminating the incentive compatibility constraint in this way, the principal's problem becomes a standard lattice program.

To state Proposition 5, we recall some terminology. A twice-differentiable function is *supermodular* if all of its cross-partial derivatives are nonnegative. This concept therefore captures the notion of complementarities between variables. Given two vectors x and y, we write $x \geq y$ if $x_j \geq y_j$ for each component. In general, the set of maximizers for a supermodular function has a largest and smallest element [see Vives (1999, p. 30)], which we refer to as the *extremal optima.* Given the usual ordering $\geq$ on the real line, we define the *reverse ordering* $\geq'$ as follows: $x \geq' y$ iff $y \geq x$. In what follows, we assume an upper bound $\overline{\gamma}$ on the magnitude of the gains from specialization.

Proposition 5. *Assume the hypotheses of Proposition 4 and the functional forms in (17). Furthermore, assume $\sigma < 1$ and*

$$P_{12} - \gamma G_{22} < 0 \tag{18}$$

on the interval $[\gamma_2, \overline{\gamma}]$. (i) If $\mu = 0$ (or is sufficiently close) then the principal's objective function (15) is supermodular in $(\beta_e, e^P, a^P, \gamma)$ with the reverse ordering for e^P. (ii) In that case, the extremal optima are monotonically increasing in γ given the reverse ordering for e^P. In particular, if the principal's problem has a unique solution then β_e and a^P are increasing in γ, while e^P is decreasing in γ.

The first result 5(i) reveals the basic structure of the paper. In Propositions 3 and 4, we showed that $e^A > 0$, $a^A = 0$, and $a^P > e^P$ when $\gamma \geq \gamma_2$ and $V_a \geq \overline{V}_a$ and in 5(i) we show that the principal's objective function is supermodular (with the reverse ordering for e^P) under those conditions, along with some others. In 5(ii), we show that as $\gamma \to \infty$ starting from the situation in Proposition 4, the extremal optima for incentives and the principal's administration effort increase. If the principal's problem has multiple solutions, then we have monotone comparative statics in the sense that the intervals containing the optimal values for those variables shift up. At the same time, the extremal optima for the principal's production effort decrease. In the case of uniqueness, we obtain monotone comparative statics in the usual sense.

The intuition is as follows. Since the principal is already doing more administration than production, she will only be willing to re-allocate her effort out of production and into administration if her preference for variety in work is sufficiently weak, which explains the condition $\sigma < 1$. Furthermore, (18) states that the corresponding loss in terms of production complementarity P_{12}[10] is more than offset by the corresponding gain G_{22} from increased specialization in administration. With respect to incentives, $\sigma < 1$ again implies the agent has little interest in diversity in work, so he is willing to increase his production effort. Furthermore, the degrees of specialization of the players are complements (see Assumptions 1), so the principal will want the agent to increase his production effort as she increasingly specializes in administration. Since she cannot motivate the agent by increasing her own production effort, she must increase incentives. As a result, the players increasingly specialize in their own separate tasks and market relations gradually replace

[10] If two variables are complements, then an increase in one raises the marginal benefit of increasing the other. As a result, optimality generally requires complementary variables to be set at comparable levels, but in this case the principal is creating an increasing imbalance between her administration and production efforts.

social ones. Of course, this could never be efficient if μ were sufficiently large. In the limit, we obtain the market equilibrium where, in effect, the principal regards the agent as merely a factor of production and the agent views the principal as an "abstract money-power."

From now on, we revert to the general expressions for C_i, M, and s_i, since we only need the explicit functional forms in (17) for Proposition 5. In the full information context, the principal can observe the agent's efforts and can use forcing contracts to achieve any effort allocation (e^A, a^A) she desires, subject only to the participation constraint and (4). A first best solution therefore solves the problem

$$\max_{e^A, a^A, e^P, a^P} P + \gamma G - C_A - C_P + \sigma S + \mu M - \overline{U} \tag{19}$$

subject to (4).

We now define the concepts of *solidarity allocation* and *solidarity equilibrium.*

Definitions 1. *(i) A solidarity allocation is a first best solution to the problem in (19) where all efforts are equal* $e^A = a^A = e^P = a^P$*. (ii) A solidarity equilibrium is an optimal linear contract such that* α *implies both players receive half the total surplus,* $\beta_e = \beta_a = 0$*, and the allocation of efforts is a solidarity allocation.*

In this definition, we follow a tradition extending back to Lange and Lerner in our insistence that the solidarity equilibrium entail a first best allocation of efforts. Since these efforts are all equal, it is characterized by full solidarity and full diversity in work (no division of labor). Furthermore, in keeping with its cooperative nature, it cannot be implemented using forcing contracts or incentives and can only be supported by reciprocity. Finally, it requires an equal division of the surplus, which is (19) plus $\overline{U}$. Up to this point, we have considered reciprocity solely in terms of work effort, but we now also insist on reciprocity with respect to payoffs. The solidarity equilibrium is therefore the complete opposite of the market equilibrium.

Although Marx depicted the division of labor as an "objective power above us, growing out of our control" (see the previous section), in our model we cannot think of it in terms of the prisoner's dilemma. Indeed, as $\gamma \to \infty$ in Proposition 5, the agent's payoff is fixed at $\overline{U}$ whereas it is clear that eventually the principal is increasingly better off. There is, however, another way in which capitalism can be thought of as a prisoner's dilemma which is consistent both with Marx's views and elementary contract theory. Suppose the principal were to offer a solidarity contract as in Definitions 1(ii). In principle, the agent could accept this contract and supply $e^A = a^A = e^P = a^P$. Although the outcome would be the solidarity equilibrium, it is not really an equilibrium because it is not incentive compatible: according to Proposition 1 the agent will shirk. Instead, both players "defect" and play their second best equilibrium strategies.

How, then, can the solidarity equilibrium be achieved? An important element of Marx's critique of capitalism is that man's social relations are determined by material forces – the production technology and the division of labor – and that the abolition of the latter will require (i) a development in the production technology that renders the division of labor obsolete and (ii) a fundamental change in human nature, such that the private and public interest coincide. In particular, we note the extreme contrast between labor in communism, which is the "life of the species," and its counterpart in capitalism, which provides no self-fulfillment and therefore "as soon as no physical or other compulsion exists, labor is shunned like the plague" (Marx, *The 1844 Manuscripts,* p. 74). In other words, the agent derives disutility from effort only in capitalism and the transition from capitalism to communism will necessarily involve a fundamental shift in the agent's attitude towards work. We now sketch a formal mechanism for achieving the solidarity equilibrium which is not only broadly consistent with Marx's views, but also with Elster's (1986, p. 159-66) characterization of them as "massively utopian."

The mechanism is as follows. First, the principal offers the solidarity contract in Definitions 1(ii). As things now stand, the agent will shirk. Assume, however, that the agent interprets the principal's offer as an act of solidarity and reciprocates by adopting the new disutility of effort $\hat{C}_A$ defined in Proposition 6 below. In essence, this idea is similar to that in Casadesus-Masanell (2004), who shows that it is in the agent's best interest to develop intrinsic motivation in the form of norms, ethical standards, or altruism. The specific form of $\hat{C}_A$ is taken from Holmström and Milgrom (1991, p. 34), in their analysis of low-powered incentives in firms. In our model, the agent's reciprocity is represented by $\hat{C}_A$, which implies that he derives positive utility from supplying his first best effort levels because he is participating in the cooperative establishment of the solidarity equilibrium and his efforts are therefore an act of self-realization. This is the first utopian element in our mechanism for the achievement of the solidarity equilibrium: it is assumed the prisoner's dilemma can be overcome through reciprocity. As discussed in the previous section, Marx's definition of freedom is essentially the coordinated decision to cooperate in prisoner's dilemmas, as opposed to the classical notion of freedom that leads to defection. It would be a mistake, however, to overemphasize the utopian nature of this mechanism since, as Fehr and Fischbacher (2002, p. C14) point out, cooperation rates are frequently between 40-60% in one-shot prisoner's dilemmas.

Proposition 6. *Let $t = e^A + a^A$ be the total effort required of the agent in some fixed solidarity allocation. If the agent adopts the new disutility of effort $\hat{C}_A$, where $\hat{C}'_A(x) < 0$ when $x < t$, $\hat{C}'_A(t) = 0$, and $\hat{C}'_A(x) > 0$ when $x > t$, then the agent will supply his first best effort levels $e^A = a^A = t/2$ if he accepts the solidarity contract.*

We motivate our next assumption based on experimental evidence from the *ultimatum game*, where one player (the proposer) offers another player (the responder) some amount p of a fixed surplus normalized to one. If the responder accepts, she receives a payoff of p and the proposer receives $1-p$. If the responder rejects, then both receive zero. A stylized fact from experiments with this game [see Sobel (2005, p. 397-8)] is that low proposals are rare and usually rejected, while equal or nearly equal splits occur more than 50% of the time. These findings are inconsistent with purely selfish preferences.

If we assume the agent rejects all offers where he receives less than half of the associated surplus, and make the reasonable assumption that half of the solidarity surplus exceeds the principal's autarky payoff, as well as $\overline{U}$, then the solidarity equilibrium is the unique subgame perfect equilibrium. If no offer is made, or if one is made where the agent receives less than half of the surplus, then the principal receives her autarky payoff and the agent receives $\overline{U}$. Of all those contracts the agent would accept, the principal clearly prefers the solidarity equilibrium because she receives half of the first best surplus computed using $\hat{C}_A$ (which is better than the first and second best with C_A). In all other acceptable contracts, the agent uses C_A to determine his second best efforts, but in the specific case of the solidarity contract he uses $\hat{C}_A$ because he regards such an offer as an act of solidarity.

In the previous argument, we implicitly assumed a unique solidarity equilibrium. We now show that this is indeed the case under reasonable assumptions, where the unique solidarity equilibrium involves all efforts at their maximum feasible level E.

Proposition 7. *Assume P is symmetric[11] and strictly concave, γ is sufficiently small that $P + \gamma G$ is strictly concave, the agent's disutility of effort is $\hat{C}_A$ as in Proposition 6, and $m_e \equiv m_a$. If*

$$P_1(2E, 2E) + P_2(2E, 2E) - 2C'_P(2E) \geq 0 \tag{20}$$

then the unique first best is the solidarity equilibrium where $e^A = a^A = e^P = a^P = E$.

The intuition is straightforward. Given that the agent is prepared to adopt any $\hat{C}_A$, it is optimal for him to adopt the one where he supplies maximum feasible effort in both tasks. In order for the principal to match these efforts, however, the production technology must entail sufficiently large marginal products P_1 and P_2 so as to dominate the gains from specialization. This is the second utopian element in Marx's thought – a sufficiently advanced production technology – which is reminiscent of Elster's characterization in terms of a "scarcity-free utopia."[12]

We cannot, however, completely dismiss these ideas. Indeed, a less extreme version of the solidarity equilibrium is a frequent outcome of experiments in agency settings. In particular, Fehr and Gächter (2001) found that experimental subjects in the role of agents supplied an average effort level of 0.37 on a scale of 0 to 1 in their "trust treatment" where principals could only offer fixed payments like α in our model. In contrast, in the "incentive treatment" the average effort level was 0.27 which is lower in a statistically significant sense. The authors explain this difference in terms of incentives "crowding-out" the agents' reciprocity. In similar experiments, Fehr, Klein, and Schmidt (2001, 2007) found that subjects acting as principals chose implicit contracts based on reciprocity over explicit incentive contracts 88% of the time, which suggests that solidarity may well be a

[11] I.e., $P(x, y) = P(y, x)$ for all x and y.

[12] An even more utopian mechanism would involve a similar $\hat{C}_P$ for the principal, or even eliminating C_A and C_P altogether. In this paper, however, we are attempting to identify the least utopian mechanism.

viable mechanism when implemented in small groups. Finally, Lindbeck and Snower (1996, 2000) document several fundamental changes that have occurred in production technologies over the last two decades that increasingly favor integrated tasks and a reduction in the degree of specialization.

4. Conclusion

In this paper, we developed a version of the linear agency model in Holmström and Milgrom (1991) with two tasks, production and administration, where performance is harder to measure in the latter. Both the principal and the agent can devote effort to these tasks. In addition, the model incorporates: (i) the gains from specialization – output is higher when the players specialize in separate tasks; (ii) social preferences – their choice of efforts is governed at least in part by norms, or the players have a preference for cooperation and solidarity; and (iii) a preference for diversity in work.

We showed that as the gains from specialization increase, eventually the principal prefers to hire the agent over autarky. In that case, the agent specializes in production because he is risk averse and performance in the production task is relatively easy to measure. Since the agent has a preference for solidarity, his production effort can be motivated either by the principal's production effort or incentives. As the gains from specialization increase further, the principal re-allocates her effort out of production and into administration. At the same time, she increases incentives to compensate for the agent's weakened motivation due to the loss of solidarity. In the limit, we obtain the market equilibrium where there is a complete division of labor, an absence of solidarity, and the agent is motivated solely by incentives. In a nutshell, a division of labor leads to exchange and a deterioration in social relations. These results hold when the players' preferences for solidarity and diversity in work are sufficiently weak and there are complementarities in production and specialization. In contrast with Sobel (2007), who shows that reciprocal types generally appear selfish in markets, we have shown that selfish types generally prefer to transact in markets. Finally, we showed that the solidarity equilibrium can be achieved when the agent derives positive utility from supplying his first best effort levels and the production technology exhibits high marginal products.

Appendix

Proof of Proposition 1

To prove (i), we note that a solution exists because all the functions in (8) are continuous on the agent's compact constraint set defined by (4). Since $D > 0$, U has a negative definite Hessian and is therefore strictly concave, so uniqueness follows. To prove (ii),

$$\begin{aligned} U_{e^A} &= \beta_e - C'_A + \sigma s'_A + (1/2)\mu m'_e \\ U_{a^A} &= \beta_a - C'_A - \sigma s'_A + (1/2)\mu m'_a. \end{aligned} \tag{A.1}$$

Since the agent's first-order conditions are necessary and sufficient, the corner solution $e^{A*} = 0$ obtains iff

$$U^*_{e^A} = \beta_e - C'_A(a^{A*}) + \sigma s'_A(-a^{A*}) + (1/2)\mu m'_e(-e^P) \leq 0. \tag{A.2}$$

A sufficient condition is therefore

$$\begin{aligned} \beta_e &\leq C'_A(0) - \sigma s'_A(-E) - (1/2)\mu m'_e(-E) \\ &\leq C'_A(a^{A*}) - \sigma s'_A(-a^{A*}) - (1/2)\mu m'_e(-e^P). \end{aligned} \tag{A.3}$$

We now prove (iii). Given uniqueness, the Maximum Theorem in Aliprantis and Border (1999, p. 539) implies e^{A*} and a^{A*} are continuous. If $\beta_e = \beta_a = 0$ then $e^{A*} = a^{A*} = 0$. It is clear from (A.1) that there exists $\overline{\beta}_e > 0$ such that $e^{A*} = E$ when $\beta_e = \overline{\beta}_e$ and likewise for $\overline{\beta}_a > 0$. Now, $[0, \overline{\beta}_e] \times [0, \overline{\beta}_a]$ is path-connected and the continuous image of a path-connected set is path-connected. Since the principal can achieve all 4 corners of the agent's square constraint set using $(0,0)$, $(\overline{\beta}_e, 0)$, $(0, \overline{\beta}_a)$, and $(\overline{\beta}_e, \overline{\beta}_a)$, she can achieve the entire set. ∎

Proof of Proposition 3

We first prove (i). If the principal offers a contract, her expected payoff is (15) after substituting e^{A*} and a^{A*}. Since the latter are continuous, (15) with substitutions and (16) are both continuous. The principal's efforts are constrained by the compact sets defined in (4) and incentives by $[0, \overline{\beta}_e] \times [0, \overline{\beta}_a]$, so a solution exists for both maximization problems. Since (ii) is obvious, we proceed to (iii). Suppose not. Then there exists a sequence $\{(\gamma_n, V_a^n)\}$ such that $\gamma_n \to \infty$, $V_a^n \to \infty$, and the corresponding optima $\{(\beta_e^n, \beta_a^n, e_n^P, a_n^P)\}$ involve either $\beta_a^n > \underline{\beta_a} > 0$ or autarky. Note that we omit the agent's efforts from the sequence of optima since they have already been substituted into (15). Let $(\hat{e}_n^P, \hat{a}_n^P)$ denote the principal's autarkic effort levels that maximize (16). We first focus on the subsequence corresponding to autarky and consider the following alternative: for all $n \geq 1$, the principal chooses her autarkic effort levels, $\beta_e = \overline{\beta}_e$, and $\beta_a = 0$, so $e^A = E$ and $a^A = 0$. The difference between the principal's payoff evaluated at the alternative and at the optimum is

$$\begin{aligned} P(E + \hat{e}_n^P, \hat{a}_n^P) - P(\hat{e}_n^P, \hat{a}_n^P) + \gamma_n \left[G(E, \hat{e}_n^P - \hat{a}_n^P) - G(0, \hat{e}_n^P - \hat{a}_n^P)\right] \\ - C_A(E) + \sigma s_A(E) \\ + \mu \left[m_e(E - \hat{e}_n^P) - m_e(-\hat{e}_n^P)\right] - (1/2) r \overline{\beta}_e^2 V_e - \overline{U}. \end{aligned} \tag{A.4}$$

Since all functions are continuous and all efforts are bounded, all the terms in (A.4) are either fixed or bounded except for

$$\gamma_n \left[G(E, \hat{e}_n^P - \hat{a}_n^P) - G(0, \hat{e}_n^P - \hat{a}_n^P)\right]. \tag{A.5}$$

Since the term in square brackets is strictly positive, the alternative is preferred for n large enough, which is a contradiction. We now turn to the subsequence $\{(\beta_e^n, \beta_a^n, e_n^P, a_n^P)\}$ of optima where $\beta_a^n > \underline{\beta_a} > 0$ and consider the alternative $\{(\overline{\beta}_e, 0, e_n^P, a_n^P)\}$. Let

$$\begin{aligned} e_n^A &= e^{A*}(\beta_e^n, \beta_a^n, e_n^P, a_n^P) \\ a_n^A &= a^{A*}(\beta_e^n, \beta_a^n, e_n^P, a_n^P), \end{aligned} \tag{A.6}$$

the agent's optimal efforts along the sequence of optima. Omitting all fixed and bounded terms, the difference in the principal's payoff is

$$\gamma_n[G(E, e_n^P - a_n^P) - G(e_n^A - a_n^A, e_n^P - a_n^P)] + (1/2) r (\beta_a^n)^2 V_a^n. \tag{A.7}$$

Since the term in square brackets is nonnegative and

$$(1/2)r(\beta_a^n)^2V_a^n \geq (1/2)r(\underline{\beta_a})^2V_a^n \to \infty, \qquad (A.8)$$

we have a contradiction for n sufficiently large. Note that we cannot claim the term in square brackets in (A.7) is strictly positive, so it appears we need both $\gamma_n \to \infty$ and $V_a^n \to \infty$ for the result. ∎

Proof of Proposition 4

Since $\gamma \geq \gamma_1$ and $V_a \geq \overline{V}_a$, the principal hires the agent exclusively for production purposes by Proposition 3, where $\beta_e > \underline{\beta_e}$ and $\beta_a = 0$. Let $\gamma \to \infty$ and $(\beta_e^\gamma, e_\gamma^P, a_\gamma^P)$ be the associated optima. Consider the alternative $\beta_e = \overline{\beta}_e$ and $|e^P - a^P| = E$, so the principal completely specializes in a single task to be determined. After omitting all fixed and bounded terms, the difference in the principal's payoff is

$$\gamma[G(E,E) - G(e_\gamma^A, e_\gamma^P - a_\gamma^P)], \qquad (A.9)$$

where e_γ^A is the agent's optimal effort. Note that (A.9) uses Assumption 1(iv). Since G is maximized at (E,E), we must have $|e_\gamma^P - a_\gamma^P| \to E$ and $e_\gamma^A \to E$. In the limit, the principal must completely specialize in a single task. Since $P_2(\,\cdot\,, 0) = \infty$, we must have $a_\gamma^P \to E$. ∎

Proof of Proposition 5

Since $\gamma \geq \gamma_2$ and $V_a \geq \overline{V}_a$, we have $\beta_a = 0$ and $a^A = 0$ from Proposition 3 and $e^{P*} < a^{P*}$ from Proposition 4. Given the functional forms in (17), we have

$$e^{A*} = \frac{\beta_e - c + \mu e^P}{1 + \mu + \sigma}. \qquad (A.10)$$

We then substitute (A.10) and (17) into (15) to obtain the principal's objective function. Note that the constraints for the principal's maximization problem $0 \leq \beta_e \leq \overline{\beta}_e$, $0 \leq e^P \leq E$, and $0 \leq a^P \leq E$ form a compact cube and hence a compact lattice. The cross-partial derivatives of the principal's objective function are

$$W_{e^P,a^P} = (\sigma - 1) + (P_{12} - \gamma G_{22}) + \frac{\mu}{1+\mu+\sigma}(P_{12} - \gamma G_{12}) < 0$$

$$W_{e^P,\beta_e} = \frac{\mu(1+\sigma)+(1+\mu+\sigma)(P_{11}+\gamma G_{12})+\mu(P_{11}+\gamma G_{11})}{(1+\mu+\sigma)^2} < 0$$

$$W_{a^P,\beta_e} = \frac{P_{12}-\gamma G_{12}}{1+\mu+\sigma} > 0$$

$$W_{e^P,\gamma} = G_2 + \frac{\mu G_1}{1+\mu+\sigma} < 0$$

$$W_{a^P,\gamma} = -G_2 > 0$$

$$W_{\beta_e,\gamma} = \frac{G_1}{1+\mu+\sigma} > 0. \tag{A.11}$$

Since $\sigma < 1$, (18) holds, $\mu = 0$ (or sufficiently close), $G_{12} < 0$, $G_2 < 0$, and $G_1 > 0$, we have the above inequalities. If we take the reverse ordering for e^P, all these inequalities are positive, and we can apply theorem 2.4 and its corollary in Vives (1999, p. 30) to complete the proof. ∎

Proof of Proposition 6

Assume the principal offers a contract where $\beta_e = \beta_a = 0$ and $e^P = a^P = t/2 > 0$ and the agent's disutility of effort is $\hat{C}_A$. A monotonic transformation of (6) gives

$$\alpha - \hat{C}_A + \sigma s_A + (1/2)\mu M. \tag{A.12}$$

Its partial derivatives with respect to e^A and a^A are, respectively,

$$\begin{aligned} &-\hat{C}'_A + \sigma s'_A + (1/2)\mu m'_e \\ &-\hat{C}'_A - \sigma s'_A + (1/2)\mu m'_a. \end{aligned} \tag{A.13}$$

These expressions vanish at $e^A = a^A = t/2$, so the agent's first-order conditions (interior or corner) are satisfied. Since the agent's objective function with $\hat{C}_A$ is strictly concave, we are done. ∎

Proof of Proposition 7

We first consider the problem

$$\max_{e^A, e^P, a^A, a^P} P + \gamma G + \sigma S + \mu M \tag{A.14}$$

subject to (4) and constraints $e^A + a^A = t^A$ and $e^P + a^P = t^P$ on the total effort of each player. Substituting $a^A = t^A - e^A$ and $a^P = t^P - e^P$ into (A.14), we obtain a problem involving only e^A and e^P. The partial derivative with respect to e^A is

$$P_1 - P_2 + 2\gamma G_1 + 2\sigma s'_A + \mu\left(m'_e - m'_a\right). \tag{A.15}$$

After substituting $e^A = a^A = t^A/2$ and $e^P = a^P = t^P/2$ into (A.15), we obtain

$$\begin{aligned} P_1(T/2, T/2) - P_2(T/2, T/2) + 2\gamma G_1(0,0) + 2\sigma s'_A(0) \\ + \mu\left\{m'_e\big[(t^A - t^P)/2\big] - m'_a\big[(t^A - t^P)/2\big]\right\} = 0, \end{aligned} \tag{A.16}$$

where $T = t^A + t^P$. This follows because $m_e \equiv m_a$ and P is symmetric, so $P_1 - P_2 = 0$ when all arguments are equal. Likewise, the proposed solution also satisfies the first-order condition for e^P. Since (A.14) is strictly concave, it is therefore the unique optimum: given fixed total time endowments, the players should split their time equally between the two activities. Now consider the problem

$$\begin{aligned} \max_{t^A, t^P} P(T/2, T/2) - \hat{C}_A(t^A) - C_P(t^P) \\ + \mu\left\{m_e\big[(t^A - t^P)/2\big] + m_a\big[(t^A - t^P)/2\big]\right\} \end{aligned} \tag{A.17}$$

subject to the constraints $0 \leq t^A/2 \leq E$ and $0 \leq t^P/2 \leq E$. The partial derivative of (A.17) with respect to t^A is

$$(1/2)P_1 + (1/2)P_2 - \hat{C}'_A + (1/2)\mu\left(m'_e + m'_a\right). \tag{A.18}$$

A solidarity allocation requires $t^A = t^P = t$. After substituting this into (A.18) and using $\hat{C}'_A(t) = 0$, we obtain $(1/2)(P_1 + P_2) > 0$. It follows that the only solidarity allocation that can possibly solve the problem is the corner solution $t = 2E$. The partial derivative of (A.17) with respect to t^P is

$$(1/2)P_1 + (1/2)P_2 - C'_P - (1/2)\mu\left(m'_e + m'_a\right). \qquad (A.19)$$

After substituting $t^A = t^P = 2E$, we find that this is nonnegative iff (20) holds. In that case, the unique first best solution is the corner solidarity allocation. ∎

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