A Fixed Effect Model of Endogenous Integration Decision and Its Competitive Effects

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Abstract

This paper studies endogenous integration decisions of firms and its competitive effects in a complementary market setting where downstream firms sell a product which must have a compatible variety of products that are supplied by upstream firms. I present the conditions under which a downstream firm will prefer integrating with an upstream firm, and conditions under a counter merger of firms occur. The analysis shows that a vertical merger is more likely to occur whenever one of the upstream firm is significantly productive than the other. Competitive effect of a integration of two firms can lead to a counter integration of rivals post integration. Counter integration is likely whenever both upstream firms are highly productive. In addition to a vertical merger and two vertical mergers, contracting under independent ownership can also be the method of procuring. As a result, no integration activity can be observed. The results are obtained in a general two downstream firms and two upstream firms market setting that allows efficient compatibility contracts between upstream and downstream producers.

Keywords:*Endogenous Vertical Integration, Positive Externality, Complementary Products, Product Variety* JEL Classifications: D21, L22, L4

1 Introduction

The mergers and acquisitions literature has examined how a price of a product or market concentration is affected post mergers, such as vertical integration or horizontal merger, to understand the competitive effects not only on the participant firms, but also on firms that are excluded from the merger. A typical assumption in most of the existing literature is that, the type of merger structured by participant firms is exogenous. However, a merger

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is a type of contract that must be agreed on by the both participant firms; thus mergers are strategic decisions. For example, when a firm decides to acquire a supplier to enhance its product quality, both the firm and the supplier assess the value of the merger before any executive decisions are made. The firms may or may not agree on the merger under the current conditions or future conditions which will arise post merger. Moreover, the merger proposed by a firm affects the excluded firms' ex post incentives to do business with the merged firm. The purpose of this paper is to study how different type of mergers or sequence of mergers can be explained in complementary markets when the merger decision is endogenous. In addition, the paper examines the competitive effect(s) of a merger on compatibility and supply decisions.

Particularly, I consider a model where downstream firms sell a product which must be supported with a compatible variety of products that are supplied by an upstream firm. In this setting, downstream firms product does not have any value unless it is supported by a set of compatible products. A downstream firm can integrate or maintain contractual relations with an upstream firm. The timing of the game is as follows: first, a downstream firm announces whether to integrate or not with an upstream firm; after observing the new market structure, an excluded downstream firm can counter integrate with the remaining upstream firm if a vertical integration occurred in the first stage. Then, each downstream firm that is not integrated with an upstream firm offers a compatibility contract to one of the upstream firms, while integrated firms will be supplied internally. Based on the contracts that are offered, upstream firms determine whether to produce compatible products. Then, each upstream firm, which agrees on a compatibility contract commits to a firm specific research and development investment that can enhances the upstream production. Finally firms compete and prices are determined.

If a downstream firm decides to integrate with an upstream supplier, remaining independent downstream firm may offer a contractual relation with the upstream division of the integrated firm even though two firms compete in the downstream market. On the other hand, an integration can also lead to a counter integration which will stiffen the downstream competition. A downstream firm's objective is to structure an organizational form that increases its expected profits. The optimal decision of a downstream firm in the complementary market setting is the focus of this paper.

In order to study this problem, I adopt Heavner's (2004) reduced form profit framework.[1] Heavner shows that, under some conditions, firms may remain independently owned because an integrated firm can not commit to supplying a better quality of a product for its downstream competitor when integration is the only alternative for the downstream firm and no counter integration is allowed. When counter integration is pos-

[1] In his reduced form setting, a firm's profit only depends on the quality its product and the quality of competitor's product.

sible, however, a downstream firm faces an additional consequence: an integration may also trigger a counter integration which makes the integration more costly ex ante.

An immediate finding of this paper is that a downstream firm's incentive to integrate with an upstream firm increases as productivity gap between the upstream firms increases. To see this, consider a setting with two downstream firms and a very high productivity upstream firm U_h, and very low productivity upstream firm U_l. First, the downstream firm D_1 decides to integrate with U_h, and then the other downstream firm D_2 considers whether to offer a compatibility contract to the upstream division of the integrated firm or integrate with U_l. If the productivity of U_h is very high, a counter integration decision of D_2 is unlikely to change the integration decision of D_1. That's why, D_2 can not be an ex ante threat to D_1 in the downstream market. When D_2, U_l counter integrate, D_2 will be supplied with less enhanced products by U_l than the case which D_2 maintains a contractual relation with U_h post D_1, U_h integration. In contrast, if D_2 extends a contract to the upstream division U_h of the integrated firm, D_2 will be supplied by more enhanced products even though the integrated firm has the incentive to invest less for its downstream competitor. On the other hand, it might be the case that D_2 will be a tougher competitor to the downstream division D_1 of the integrated firm if it signs a contract with the upstream division U_h. In this case, integrated firm either would not agree on a compatibility contract or would not invest for its downstream competitor. This is valid in a general setting as well.

Following this observation, an integration of a downstream and an upstream firm will be the equilibrium outcome if one of the upstream firms is highly productive than other. I present conditions under which only one of the downstream firms integrates with an upstream firm, conditions under which a counter merger will be observed, and conditions under which downstream and upstream firms are better off with only contractual relations.

This paper contributes to the mergers and acquisitions literature with endogenous merger decision and can be a theoretical basis to the existing literature on mark-up analysis after different types of mergers. Most of the existing literature assumes that an integration decision is exogenous and analyzed the equilibrium mark-up or welfare effects. In contrast, endogenous integration decision is analyzed in this paper: downstream firms can choose which firm to acquire prior to any business contracts or price competition. The market structure I study is quite general. In my model, I allow downstream firms to contract with or to acquire a complementary good producer.

To illustrate the model, I first study endogenous integration decision with a vertical merger option and no counter merger. A downstream firm either can integrate with an upstream firm or can contract with an upstream firm. An integrated downstream firm's profit is contingent upon the compatibility decisions. An integrated firm can acquire an efficient contract from an independent downstream firm. The downstream profits depend on the variety of its complementary product and variety of its downstream competitor's complementary product.

A downstream firm and an upstream firm may have conflicting interests in the merger because an upstream firm incurs the investment cost but may have to give up the business of the independent downstream firm. On the other hand, the downstream firm increases its variety of complementary products post integration. Moreover, an integrated downstream firm can solve that its competitor will be supplied by less variety of complementary products if downstream competitor extends a contract to the upstream division of the integrated firm ex post. Therefore, integration decision is a strategic decision for both the downstream firm and upstream firm.

Intuitively, it may seem that a more productive upstream firm always gives a downstream firm a higher incentive to integrate because the downstream firm always prefers being compatible with more efficient supplier. However, this kind of thinking is wrong because the downstream firm's incentive to acquire the upstream firm depends on his relative gain from integration rather than its complete gain. The marginal downstream profits post integration increases as the productivity asymmetry between upstream producers increases.

In this setting, the equilibrium outcome depends on not only the production asymmetries of upstream firms, but also independent downstream firms contractual relations. The following is the reason. If independent upstream firm can produce enough variety of complementary products to independent downstream firm after integration, the upstream division of the integrated firm can not acquire the business of the its downstream competitor. The independent downstream firm can extend a compatibility contract to the less productive upstream firm because upstream division of the integrated firm would invest less for its competitor in order to induce the complementary product variety of its downstream competitor. The upstream firm that is a candidate for an integration can loose the business of the independent downstream firm and loose profits. That's why participant firms may forego an integration. On the other hand, if independent upstream firm can not produce enough variety of complementary products to independent downstream firm after integration, vertical integration can not possibly hurt the participant firms' ex post profits. Again, the upstream division of the integrated firm will invest less for its downstream competitor, but this time the integrated firm still acquires the business of its downstream competitor.

To summarize, the basic model that allows for only a vertical merger has two main findings. First, the downstream firm's incentive to acquire the upstream firm is higher when the production asymmetry is higher, in other words, when the participant upstream firm becomes more and more productive than the excluded one. Second, with compatibility decision of the independent downstream firm, participant parties may forego an integration. An analysis of a more generalized model which includes upstream and downstream price competition is more subtle and complicated because each upstream firm competes and adjusts its downstream firm specific investment in order to attract downstream firms. With an outside option for an upstream firm, the optimal upstream price can be pointed out from the value of the outside option.

In order to analyze a more generalized model which includes a counter merger, I first provide sufficient conditions under which a downstream firm benefits from a vertical integration. Second, I analyzed that, under the sufficient conditions provided, whether an integration is still the equilibrium outcome whenever counter merge is also possible. I provided the sufficient conditions under which one integration or two integrations will be observed in the market. Finally, I map each of different complementary market setting, which are characterized by upstream productivity, to a market structure outcome.

1.1 A Complementary Market

One of the biggest and fast growing markets in which complementarity exists is the video game console market. In 1984, the home video game market crashed and thus, many hardware and software game manufacturers, such as Apollo, US Games, Spectravision, declared bankruptcy. However, Sega is acquired by investors and re-launched as a software producer in U.S. One year later, Nintendo released its first home video game system. In 1986, Sega decided to launch its Sega Master System and the system was supplied by Sega itself. On the other hand, Atari released its own video game system that features backward compatibility so that Atari could increase the number of titles available to its new video game system.

In the following years, Atari, Nintendo and Sega launched new versions of their home video game systems. In 1994, Sony launched its first home video game system, PlayStation. Sony's Playstation was going to be supported by independent software developers. The corporation agreed to pay a share of the game softwares revenues to independent software suppliers. Consequently, Playstation increased the number software titles which are exclusive to Sony's console Playstation in the console market. At the end of the third business year, Sony Corporation increased its market share to almost 50% in the home video game market. In 2000, Sony launched a backward compatible new home video game system, PS2. The same year video game industry grew 30%. At the end of 2000, Sony Corporation announced that its 50% of profits was generated by PlayStation although only 15% of Sony's total revenue is generated by PlayStation sales. Eventually, the console market consolidated and Atari was acquired by Sega systems.

In 2001, Microsoft decided to launch a new video game system, however, the corporation's main concern was the high variety of game softwares that are exclusive to Sony's

PS2. Microsoft was stressed because of the sustainable advantage in the console market Sony's PS2 had. As a market solution, Microsoft adopted a strategy of acquisition of some independently owned software firms. The corporation started to create a portfolio of its own game softwares that are developed under the name of the company.[2] On the other hand, Sega decided to exit the market, nevertheless, the firm resumed producing game softwares for the rival console producers.

Recently, Nintendo, Microsoft and Sony launched their next generation of video game consoles, Wii, XBOX 360 and PS3 respectively. DFC Intelligence's research on game industry reports that there has been a strong sales increase in the video game market over the past few years and there is still plenty of room for growth. The report also indicates that the generation who grew up with Atari and Nintendo is switching to Microsoft and Sony. The report suggests that an industry consolidation, which has not yet occurred as many as predicted, is on its way. A merger an acquisition wave is expected between the biggest game software producers and the biggest home video game console producers.[3]

2 Related Literature

This paper is related to the literature on mergers & acquisitions, competitive effects of mergers and complementarity & compatibility . First of all, I am presenting a more general framework than the existing literature has. Each of the papers from existing literature considers a special case. In my modeling framework, each of these special cases corresponds to a different set of underlying parameters in the paper. Second, my analysis also related with the studies on competitive effects in given merger types. Finally, this paper is close to the existing literature on competition among complementary products.

The first set of literature related to this work studies compatibility decisions in complementary markets. Matutes & Regibeau (1988) examines a two stage game in which two fully integrated firms make their compatibility decisions before competing in prices. They found that full compatibility is the symmetric perfect Nash equilibrium which leads to higher prices and also increases the variety of systems. This paper assumes the firms are fully integrated. Economides & Salop (1992) analyzes the competition and integration among complementary products that can be combined to create composite goods or systems. The model generalizes the Cournot duopoly complements model and analyzes the

[2]The recent Halo 2 is a tremendous hit and generated over $300,000,000 revenue. Microsoft announced that the latest Halo installment is available. GameSpot reported that 4.2 million units of Halo 3 were in retail outlets on September 24, 2007, a day before official release, a world record volume release. Halo 3 also holds the record for the highest grossing opening day in entertainment history, making US$170 million in its first 24 hours,[9] and US$300 million in its first week.

[3]Nintendo, Sega, EA, Acclaim and Capcom are five biggest software producers. Microsoft, Sony, Nintendo are three biggest home video game producers.

equilibrium prices for a variety of organizational and market structures. They solve the Nash equilibrium prices for different types of market structures such as vertical integration, horizontal merger, conglomerate merger. Gandal & Kende and Rob (2000) examines the influence of the different titles of CD on the CD industry. They estimated the elasticity of buying a hardware with respect to CD player prices and the cross elasticity with respect to the variety of CD titles. They showed that the influence of the variety is significant and there is a positive relationship between the profits of a firm and the total variety of the firm's complementary product. They assume that the software industry is competitive and there exists no vertical or horizontal integration in the market. These models examine similar market structures with the one this paper analyzes and focus on influence of complementarities on prices, but the market structures are taken as granted unlike my model which studies the endogenous merger decisions. In each of these papers, certain types of merger and contractual relationship are ruled out by assumption.

A second branch of the literature studies vertical mergers. Beggs(1994) examines competition between groups of firms selling products which are complementary within the group but substitutes across groups, such as components of different computer systems. The paper shows that firms within a group will often prefer to stay as separate companies rather than merge. McAfee(1999) analyzes the reaction of the other input suppliers to vertical integration. In his paper, he examines the price competition in input market post integration and the competitive effects of a vertical merger. The paper shows the input suppliers may reduce the its rival's cost instead of raising the input cost. Chen(2001) shows how the pricing incentive of a downstream producer and the incentive of a competitor in choosing input suppliers is effected by vertical integration. The paper develops an equilibrium theory of vertical merger which can provide a framework in which the competitive effects of vertical mergers are measured and compared. Heavner(2004) examines integrating firms' trading opportunities post integration. The model analyzes the integration decision of a downstream firm if downstream units must commit to suppliers before contracting on the final terms of trade. The paper shows integration can alter the supplier decisions of upstream units. As a result, firms may remain independently owned. In addition to Heavner (2004), the current paper studies the merger decision whenever counter merger and horizontal merger are also possible.

In the literature, the organizational forms are often taken for granted. The organizational forms that are included in the models can be summarized as

	Vertical Integration	Counter Merger	Contractual Relations
Matutes & Regibeau (1988)	Assumes	Assumes Away	Assumes
Economides & Salop (1992)	Assumes	Assumes	Assumes
Gandal & Rob & Kende (2000)	Assumes Away	Assumes Away	Assumes
Beggs (1994)	Assumes	Assumes Away	Assumes
McAfee(1999)	Assumes	Assumes Away	Assumes
Chen (2001)	Assumes	Assumes	Assumes
Heavner (2004)	Assumes	Assumes Away	Assumes

Unlike some of these papers, this paper identifies the conditions under which vertical merger, a counter merger or contracting is the only equilibrium organizational form. My paper falls into the same category as Chen(2001) & Heavner(2004). Different than those, the analysis considers a market setting in which downstream and upstream firms produce complementary goods. As RGK(2000) suggested, my paper takes the variety effect into account in merger decision analysis. This paper provides an equilibrium theory of organizational forms which can explain why different types of mergers can be observed. Vertical merger, a counter merger and contracting can all be organizational forms in equilibrium. This paper can provide the underlying framework for Economides & Salop paper and the merger literature which take the merger types for granted.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 is the analysis of the model with no counter integration. Section 4 contains the analysis of endogenous integration decision with counter merger. Section 5 discusses the welfare effects. Section 6 summarizes and concludes. All proofs are relegated to Appendix.

3 The Model

The model consists of two downstream firms (D_1&D_2) which produce a "Base Product" and two upstream firms (U_1&U_2) which produce "Side Products". Consumers need to buy a base product in order to utilize the side products, i.e. the products are perfect complements in the market. Thus, a downstream firm has to be supported by a variety of a side products which must be compatible with the base product. Each upstream firm develops a variety of a complementary good which will be compatible with a downstream firm's product. The variety depends on a firm specific R&D investment of an upstream firm for a downstream firm, and how efficient the upstream firm's production is.

An upstream firm, which signs a compatibility contract with a downstream firm, will choose a firm specific investment level to supply some variety of goods for its compatible base product. If U_i is compatible with D_j then the upstream firm supplies the variety v_j

to D_j where,

$$v_j = \varepsilon_i + \tau(r_{ij})$$

A downstream firm specific variety is determined by three factors: First, downstream firm D_j specific R&D investment r_{ij} of upstream firm U_i, second, upstream production technology $\tau(r)$, and third upstream firm U_i's production efficiency ε_i.[4] The amount of variety an upstream firm could supply when the upstream firm has no R&D investment defines the efficiency parameter ε of the upstream firm. In particular, $\{\varepsilon_i\}_{i=1,2}$ is the parameter space that characterizes different market schemes and generates the different results of the model. The cost of launching v_j upstream product is the level of investment (r_{ij}) that the upstream firm U_i invests for its compatible D_j.

The technology function $\tau(x)$ satisfies the conditions:

$$\tau(0) = 0, \;\; \frac{\partial \tau}{\partial r} > 0, \;\; \frac{\partial \tau^2}{\partial^2 r} < 0, \;\; \tau^{'}(0) = \infty, \;\; lim_{x \to \infty} \tau^{'}(x) = 0$$

Variety can be regarded as the number of different accessories that are available for a downstream product in the market.[5] *Variety* can also have different aspects, such as the quality among the various upstream products or how well the upstream firm's distribution in the market. Another point of view can be that *variety* may be considered as the quality of an upstream firm's product. The model analyzes *variety* as a scalar term, however it could be also represented as a vector.[6]

Both upstream firms and downstream firms are independently owned prior to any merger decision. Each independently owned firm maximizes its own profit. In case of a merger, the merged entity will have one central management which makes the production decision in order to maximize the merged entity's profit. I am interested in the endogenous merger decision of downstream firms.

One vertical integration, which is D_1, U_1 to merge and D_2, U_2 to remain independent, will be observed in the market if post integration profit of the integrated firm $U_1^v D_1$ is greater than the sum of independently owned D_1's and U_1's profits before the integration and profit of the integrated firm $U_2^v D_2$ would be less than sum of independently owned D_2's and U_2's profits post integration.

$$\Pi(U_1^v D_1) > \Pi(U_1) + \Pi(D_1) \quad \text{and} \quad \Pi(U_2) + \Pi(D_2) > \Pi(U_2^v D_2)$$

On the other hand, one vertical integration and a counter integration, which is D_1, U_1 to merge and D_2, U_2 to merge, will be observed in the market if post integration profit of

[4] $\tau(r)$ can be interpreted as the production function which satisfies diminishing marginal return.

[5] For instance, different game titles for Sony's Playstation or different accessories that are available to Apple's IPod.

[6] Variety also can have other attributes such as popularity in the market which has the impact on both the sale performance and the durability of a side product.

the integrated firm $U_1^v D_1$ is greater than the sum of independently owned D_1's and U_1's profits before the integration and profit of the integrated firm $U_2^v D_2$ would be greater than sum of independently owned D_2's and U_2's profits post integration.

$$\Pi(U_1^v D_1) > \Pi(U_1) + \Pi(D_1) \quad \text{and} \quad \Pi(U_2^v D_2) > \Pi(U_2) + \Pi(D_2) \quad \text{given}$$

Finally, there will be only contractual relations between upstream and downstream firms if a downstream firms would not benefit from a vertical integration. Independently downstream firms will contract with independently owned upstream firms.

I study the endogenous integration decision of a downstream firm by solving the subgame perfect Nash equilibrium in a setting in which the firm must decide to merge or not before any investment or compatibility decisions made in the market. At time zero, D_1 has to decide on a merger strategy.[7] The downstream firm can integrate with U_1. In case of vertical integration, the integrated firm will produce both a base product and side products that are compatible with its base product. On the other hand, if D_1 decides to merge with D_2, then the merged firm will be the only base product supplier in the market. In case of independent D_1, D_1 produces a base product and offers compatibility contract to either of the side product firms to be compatible with its base product.

At time one, D_2 may counter integrate with U_2 if D_1 integrated with U_1 at time zero. At time two, each independently owned downstream firm offers a compatibility contract to one supplier. A downstream unit of an integrated firm will be supplied by the upstream division of the integrated firm. Upstream firms (or divisions) observe the offers and decide whether to be compatible with the downstream firm or not. At time three, if an upstream firm decides to be compatible with a downstream firm, then the upstream firm will invest on firm specific R&D to establish a variety of the side products for its compatible downstream product. The firm specific R&D investment will determine the variety of the side products. At time four, upstream firms announce and launch the products they have developed. The variety of the side products is going to be observed and firms will compete in the market. The figure illustrates the timing of the model. The left section describes the model when there is a vertical merger. The middle section describes in case of a counter merger. The right section describes when there is only contractual relations.

[7] Aaker considered the market strategy of a firm in two parts. According to him, a firm can expand its product market by increasing his market share or a firm can vertically integrate by either forward integration or backward integration. Another common point of view is increasing the customer share.

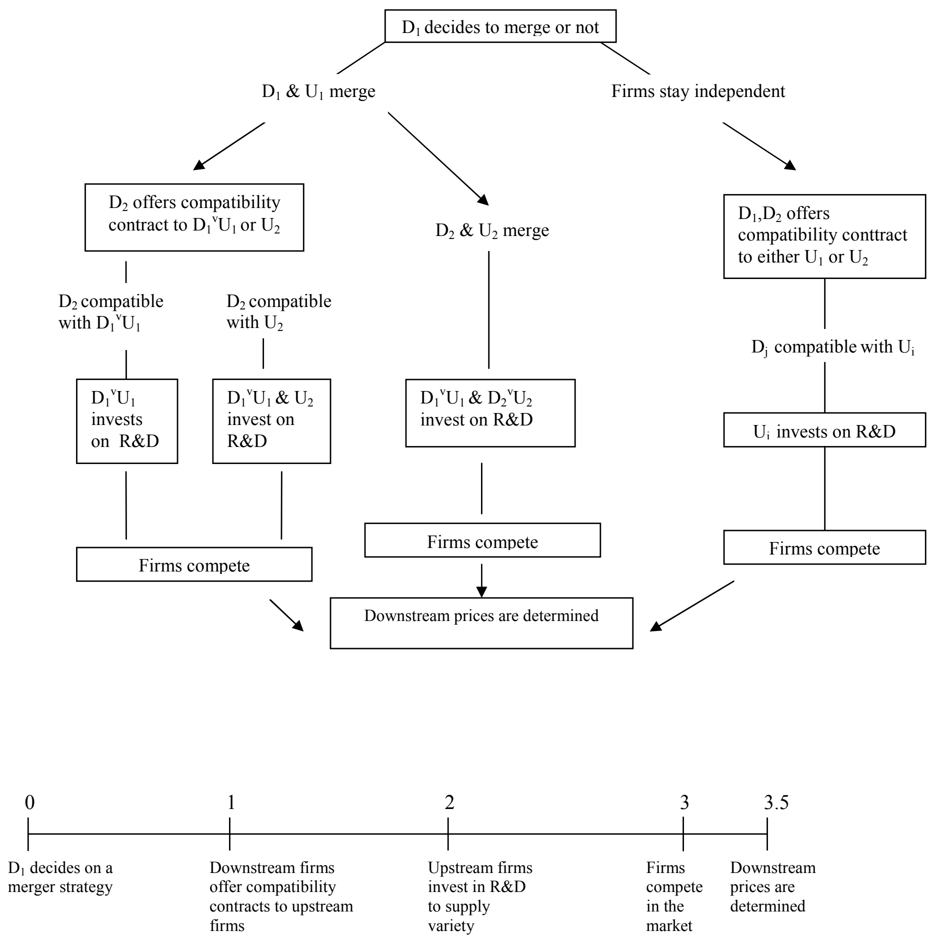

Figure 1: Timeline

This paper examines a setting in which the upstream firms can not alter their supply decision. That is, the investment decision of the upstream firms is a one time decision instead of series of decisions. However, sometimes, the producers increase the variety of a product by adding components which enhances the obsolete upstream products. On the other hand, It is unlikely for firms to forecast the enhancements in the future prior to any investment decision.[8]

[8]The Sony Corporation never expected that Grand Theft Auto, one of Sony's PlayStation title, will be a tremendous hit. Independently owned Electronic Arts, a game developer, launched various extensions of the title which increased the PS2 hit titles in the market

Downstream profits are assumed as reduced form functions such that the downstream firm D_j's profit $\Pi(D_j)$ is the equilibrium profit function for a subgame where firms compete on price

$$\Pi_{D_j} = \pi + \alpha v_j - \mu v_i \quad \text{where} \quad \mu \in [0,1] \quad \text{and} \quad \alpha = 1 - \beta$$

A compatibility decision between an upstream firm U_i and an downstream firm D_j increases the downstream profits by v_i. If U_i and D_j do not integrate, D_j pays a share of its gain, βv_j, to its compatible upstream firm as a contract fee where $\beta \in [0.1]$. [9] One way to think β as just a sharing parameter which is exogenous. The other way is that, without loss of generality, we can assume that the downstream firms have the bargaining power over the upstream firms. U_1 and U_2 are ex ante identical but ex post different. That's why, downstream producers can hold the upstream suppliers to their opportunity costs. Hence they can have the same opportunity cost to outside opportunities, but different productivity in this market.

The sensitivity U_i's and D_j's revenues to D_j' s variety whenever U_i supplies to D_j is measured by the parameter β. In equilibrium, β depends on the outside options of the upstream firms. We can think β as a market price per variety for given varieties to cover upstream firms' opportunity cost. In many markets, the expected value of an always available outside option for a firm can be determined. Each firm would have the same expected outside option, especially when they have significantly close production efficiencies. One can do a "local" analysis along the efficiency parameter with same expected outside options from different random draws for their market. In particular, this is true even if U_1 and U_2 draw different values of ε_1 and ε_2 in their particular market. This paper assumes that downstream firms have all the bargaining power that's why β can be determined by the outside option, θ, which is exogenous. [10]

For better exposition purposes, I will assume $\beta \in [\mu, 2\mu]$. However the assumption would not change many results. The assumption suggests that any integrated firm would supply to an independently owned downstream firm. If $\beta < \mu$, then the integrated firm would supply zero to an independently owned downstream firm. However, there may exist β and μ such that $\beta < \mu$. One must be aware that,there might be a region of $(\varepsilon_1, \varepsilon_2)$ in which no merger activity happens at all. Moreover, the assumption suggests that the price of an upstream firm is not too high that would harm a downstream firm, i.e. $\beta < 2\mu$.[11] That's why I am going to assume $\beta \in [\mu, 2\mu]$ for better exposition of the results.

Downstream profits depend on a fixed term π, variety of the firm v_i and variety of

[9] Heavner(2004) suggested that 50/50 bargaining rule implies that upstream and downstream firms share the profit v_j. i.e. $\alpha = 1 - \beta = \beta = 1/2$.

[10] In perfectly competitive markets,p=MC

[11] The assumption also suggests that outside option of an upstream firm is not high.

a downstream competitor v_j, and the sensitivity measures β and μ. [12] The exogenous parameter μ measures the sensitivity of D_i's revenues to D_j's variety.[13]

Reduced form of downstream profit functions come from a pricing game (given varieties) such that each downstream product's demand is

$$Q_i(p_i, p_j) = A + v_i - p_i + \delta(p_j - v_j) \quad i, j = 1, 2$$

where $v_i \;\; i = 1, 2$ is the variety of upstream products that are supplied by a contracted upstream firm. The profit of a downstream firm that confirms a compatibility contract with an upstream firm $U_j$is

$$\Pi(D_i; U_j) = (p_i - c_i)Q_i(p_i, p_j) - C(\beta, v_j) \quad i = 1, 2$$

where c_i is the marginal cost of production and $C(\beta, v_j)$ is the cost of acquiring side products from an upstream firm. $C(\beta, v_j) = cv_j$ is the payment that a downstream firm must make to sign a compatibility contract with an upstream firm.

The unique Nash equilibrium exists for the relevant $\delta \in (0, 1)$ and the equilibrium prices and profits are,

$$\begin{aligned} p_i(v_i; v_{-i}) &= \frac{A(2+\delta) + (2-\delta^2)v_i - \delta v_{-i}}{4-\delta^2} \\ \Pi(D_i; U_j) &= (f(v_i, v_{-i}))^2 - cv_i \end{aligned}$$

,where

$$f(v_1, v_2) = \frac{A}{2-\delta} + \frac{(2-\delta^2)v_1 - \delta v_2}{4-\delta^2}$$

The downstream profit function is increasing in its variety and decreasing in competitor's variety. As a result, downstream profit functions can be represented by the reduced form functions $\Pi(D_j) = \pi + \alpha v_j - \mu v_i$ that carries the same characteristics. The constant number π depends on the demand variables A and λ. The reduced form enables me to avoid any complex price analysis which has done in the literature quite often.[14]

The profit function of a upstream firm depends on the variety the firm supplies to the market and its R&D cost.

$$\begin{aligned} \Pi_{U_1} &= \Sigma_j \lambda_j (\beta v_j - r_{1j}) \\ \Pi_{U_2} &= \Sigma_j (1-\lambda_j)(\beta v_j - r_{2j}), \;\; j = 1, 2 \end{aligned}$$

[12] $\alpha = 1 - \beta$ represents the externality effect which is proven in Gandal & Kende and Rob (2000).

[13] One way that μ could measure the sensitivity of D_i's revenues to D_j's variety is if the elasticity of demand for D_i's product with respect to price per unit of variety of D_j's product is increasing in μ, i.e. $D_i = A - \frac{p_i}{v_i} + \mu \frac{p_j}{v_j}$

[14] Chen(2001),Ordover,Saloner& Salop(1990)

λ_j is an indicator function which is positive when U_1 is compatible with D_j and which is zero when U_2 is compatible with D_j.[15] β is a measure of the marginal upstream profit increase due to a variety effect in the market.

In case of a vertical integration, the integrated firm's profit is the combined profits of upstream and downstream divisions.

$$\Pi(D_i^v U_j) = \pi + v_i - \mu v_j - r^v + \lambda_j(\beta v_j - r_{ij})$$

r^v is the investment of the upstream division for the downstream division and λ_j is an indicator function which is positive when the independent downstream firm is compatible with $D_i^v U_j$ and which is zero when the independent downstream firm is compatible with the independent upstream firm.

3.1 Discussion

This paper analyzes a value added model of downstream and upstream firms. In the model, the added value is represented by the term variety, $v_j = \varepsilon_i + \tau(r_{ij})$, which is a characteristic of a downstream firm that is supplied by an upstream firm. The added value is modeled with a fixed effect ε, which is specific for each upstream firm, and technology function.[16]. My paper constructs a theory of mergers on Gandal & Rob and Kende (2000). GRK studied a model of fixed effects. However, they take the number of titles as an exogenous value from the data they used. They utilized the number of titles as a proxy to estimate the price and cross price elasticities of the profit function. [17] The fixed effect model controls for the differences between upstream suppliers. A fixed effect model is plausible in many industries in which the suppliers (upstream) has the same cost structure but different production capacities (Qualities). Telecommunication industry (AT&T, Lucent Technologies), video game console industry (EA, Konami, Sega) can be some examples. One can argue a downstream and upstream specific fixed effect. Modeling ε_{ij} instead of ε_i is not plausible because ε_1 and ε_2 is the production characteristics of upstream firms. The production efficiency of a producer can not depend on any efficient contract. In addition, the parameter space $(\varepsilon_1, \varepsilon_2)$ infers U_1's technological advantage or disadvantage relative to U_2. The model has an empirical implication. A structural model which estimates elasticity values of a log profit function by analyzing v_i as a proxy whenever the data contains the number of different titles associated with a downstream product and the supplier's cost in the upstream market. The empirical model can forecast a wave of mergers in a well structured complementary market. The next section starts analyzing the fixed effect model of endogenous integration decision when rival firms can not possibly counter integrate.

[15] D_j can be compatible with either U_1 or U_2

[16] Heavner (2004) also studies an added value model

[17] ($\Pi = \pi + \alpha_1 v_i + \alpha_{-1} Y$ where v_i is exogenous)

4 Endogenous Integration Decision with No Counter Integration of Rivals

The following sections analyze the determinants of integration and contractual decisions conditional on certain aspects of organizational choice being ascended away. The first part analyzes the integration decision under the assumptions U_2 and D_2 can not counter integrate. The following section analyzes the endogenous integration decision when the market rivals U_2 and D_2 can also integrate.

This section imposes two restrictions on the model. First, only one downstream firm may integrate with an upstream firm and two downstream firms can not merge. Second, remaining independent downstream and upstream firms can not counter merge. Thus, the section analyzes the integration decision of D_1 and U_1 in a setting in which D_2 and U_2 can not integrate as a market reaction. D_2 may preserve its contractual relations with either the new integrated firm or remaining upstream firm U_2 post integration.

Upstream firms U_1 and U_2 are not necessarily symmetric. A downstream firm would extend a compatibility contract to an upstream firm if the return of the option is the highest. If $\Pi(D_i; U_j)$ denotes the profit of downstream firm D_i which contracts with upstream firm U_j and $D_i \sim U_j$ denotes that D_i is compatible with U_j which suggests upstream firm U_j will supply a variety of side products to D_j's product, then

$$\begin{aligned} D_j \sim U_1 \quad & if \quad \Pi(D_j; U_1) > \Pi(D_j; U_2) \\ D_j \sim U_2 \quad & if \quad \Pi(D_j; U_2) > \Pi(D_j; U_1) \end{aligned}$$

Before starting the analysis, one must determine a tie breaking rule that should explain which of the independent upstream firms is going to be associated with an independent downstream firm in case the profits are equal. In this case, intuitively, a downstream firm would offer a compatibility contract to the upstream firm that is more efficient. That's why, tie breaking rule favors the more efficient upstream firm.

$$\begin{aligned} D_j \sim U_1 \quad & if \quad \varepsilon_1 \geq \varepsilon_2 \ \& \ \Pi(D_j; U_1) = \Pi(D_j; U_2) \\ D_j \sim U_2 \quad & if \quad \varepsilon_2 > \varepsilon_1 \ \& \ \Pi(D_j; U_1) = \Pi(D_j; U_2) \ \ , j = 1, 2 \end{aligned}$$

The integration decision of D_1 and U_1 depends on two aspects. First, how profitable the new integrated firm can be in both upstream and downstream markets. Second, how the downstream competitor will be supplied post integration. D_1 and U_1 integrate if the profit of the integrated firm is at least as high as the the sum of U_1's and D_1's profits in case of no integration. An integration can be foregone if the profit of integration is less than the individual profits.

I will follow rules of backward induction to analyze the endogenous integration decision under the condition that no counter merger is allowed. The analysis presents the conditions under which an integration is a sub-game perfect Nash equilibrium. This section also exposes the conditions under which firms can renounce vertical integration and maintain only contractual relations.

If each firm decides to have a contractual agreement, each firm is going to maximize its own profit. Under independent ownership, the profits are going to be,
[18]

$$\Pi(D_j;U) = \pi + \alpha v_j - \mu v_{-j} \quad , j = 1,2$$
$$\Pi(U_1) = \lambda_1(\beta v_1 - r_{11}) + \lambda_2(\beta v_2 - r_{12})$$
$$\Pi(U_2) = (1-\lambda_1)(\beta v_1 - r_{21}) + (1-\lambda_2)(\beta v_2 - r_{22}) \quad , (\alpha,\beta) \in \{(0,1)\text{x}(0,1) : \alpha+\beta = 1\}$$

λ_j is an indicator function which is determined by the compatibility agreements between the downstream and upstream firms. It is positive if the D_j is compatible with U_1 ($\lambda_j = 1$). The downstream firm can offer a contract to only one upstream firm, so the D_j would offer a contract to U_2 if the firm did not offer to $U_1(\lambda_j = 0)$. Thus, U_1's goal is to maximize profit. That is,

$$\max_{r_{11},r_{12}} \quad \lambda_1(\beta(\varepsilon_1 + \tau(r_{11})) - r_{11}) + \lambda_2(\beta(\varepsilon_1 + \tau(r_{12})) - r_{12}) \tag{1}$$

The first order conditions imply,

$$\frac{\partial \Pi_{U_1}}{\partial r_{1i}} = \beta\lambda_i\tau^{'}(r_{1i}) - 1 = 0 \tag{2}$$

U_1's optimal investment level which maximizes its profit is $r_{1j} = \gamma(\beta^{-1})$ if λ_j is positive, where $\gamma(x)$ is the inverse function of the derivative of the investment function, i.e. $\gamma(x) = \tau^{'-1}(x)$.

As a result, the optimal investment r_{ij} depends on two factors: Marginal product of investment and the sensitivity U_i's revenue to D_j's variety. Marginal product of investment, $\tau^{'}(x)$, is a decreasing function and so $\gamma(x)$ is. That's why, the D_j's variety increases as the value of upstream firms' outside option increases because revenue sensitivity parameter β increases as the outside option's value increases. If no merger occurs in the market, U_2 solves a similar profit maximization problem, and chooses a firm specific investment level $r^*_{2i} = \tau^{'-1}(\beta^{-1})$ whenever D_i is compatible with U_2.[19] Any upstream firms agrees on a compatibility contract if the net gain from the contract will be positive. [20]

[18] $\lambda_1 = 1 \; if \; D_1 \sim U_1, \quad \lambda_1 = 0 \; if \; D_1 \sim U_2, \quad \lambda_2 = 1 \; if \; D_2 \sim U_1, \quad \lambda_2 = 0 \; if \; D_2 \sim U_2$

[19] If we allow ε_i to be negative as well , then an independently owned upstream firm accepts a compatibility contract if $\varepsilon_i \geq \beta^{-1}r^*_{ij} - \tau(r^*_{ij})$

[20] $\Pi(U_i;D_j) > 0$

Moreover, downstream firms' contractual relations must maximize their profits. Thus, an independent downstream firm's main goal is to be supported by as various side products as possible, which necessarily maximizes its profit. In particular, a downstream firm would extend a compatibility contract to the upstream firm which can commit to supply the most variety.

At the time, each downstream firm prefers contracting with the upstream firm which has more efficient productivity. Thus, being ε_1 greater than ε_2 leads D_1 and D_2 to extend a contract to U_1; while, D_1 and D_2 extend a compatibility contract to U_2 if ε_2 is greater than ε_1. Moreover, comparing the upstream profits shows that both U_1 and U_2 prefer accepting as many compatibility contracts as possible.[21] Lemma 1 summarizes the equilibrium compatibility contracts when U_1 and D_1 are independent.

Lemma 1. *Assume that*
(A1) Upstream firms are independently owned
(A2) Downstream firms are independently owned
Both downstream firms extend a compatibility contract offer to the more efficient upstream firm. The more efficient upstream firm accepts the downstream firms' contract offers.

Should no upstream firm and downstream firm merge, downstream producers offer compatibility contract to the more efficient upstream firm. The more efficient upstream firm can supply more variety than the less efficient one which necessarily increases downstream profits. Hence, the only supplier in the market will be the more efficient upstream firm in any equilibrium in which no downstream firm integrate with an upstream firm.

Next, I will analyze the equilibrium compatibility contracts when U_1 and D_1 are integrated. The integration decision of two firms does not have any affect on the remaining independent upstream firm U_2's gain from any compatibility contracts. That's why, the integration decision of U_1, D_1 does not affect U_2's firm specific optimal R&D investment in case U_2 is compatible with the remaining independent downstream firm D_2.

Post integration, the upstream division U_1 will supply for the downstream division D_1 without signing a compatibility contract. Vertical integration of firms has two effects; one direct effect and one indirect effect on the upstream division U_1's incentives. The direct effect is increasing the optimal R&D investment which increases the downstream profit of the integrated firm. Thus, the total variety supplied to its downstream division will be higher post integration. The indirect effect is being reluctant to invest for its downstream competitor D_2 in case D_2 extends a compatibility contract to the upstream division of the integrated firm. The integrated firm may increase its upstream profits and earn $\beta(\varepsilon_1 + \tau(r_{12}) - r_{12})$. On the other hand, the integrated firm can loose some of

[21] If we let $\varepsilon_i < 0$ then U_i may not accept a contract because an independent U_i can not profitably produce.

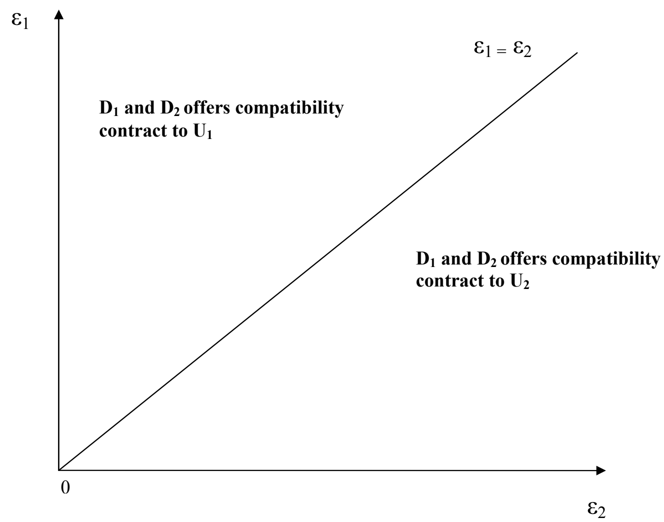

Figure 2: Compatibility Contracts under Independent Ownership

its downstream profit if the upstream division supplies a high variety for D_2 and loose $(\mu(\varepsilon_1 + \tau(r_{12})))$. As a result, U_1 will have less incentive to supply D_2. Consequently, this effect increases D_2's incentives to extend a compatibility contract to the independent upstream firm U_2, even if U_2 is less efficient than U_1

The profit of $U_1^v D_1$ will be

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(r_{11}) - \mu(\varepsilon_2 + \tau(r_{22}^*))) - r_{11} \quad if \quad D_2 \sim U_2$$
$$\Pi(U_1^v D_1; D_2) = \pi + \varepsilon_1 + \tau(r_{11}) + (\beta - \mu)(\varepsilon_1 + \tau(r_{12}^*))) - r_{11} - r_{12} \quad if \quad D_2 \sim U_1^v D_1$$

Since U_1, D_1 integration decision has no effect on U_2's gain, U_2 invests $r_{22}^* = \gamma(\beta^{-1})$ if D_2 extends a compatibility contract to U_2. Post integration, the integrated firm $U_1^v D_1$ invests to solve

$$\max_{r_{11}, r_{12}} \ \pi + \varepsilon_1 + \tau(r_{11}) - r_{11} - (1 - \lambda_2)\mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) + \lambda_2((\beta - \mu)(\varepsilon_1 + \tau(r_{12})) - r_{12})$$

U_1 invests for D_2 if the independent downstream firm extends a contract to the upstream division of the integrated firm and the integrated firm accepts the contract. U_1 invests more for D_1 post integration, whereas U_1 will be more reluctant to invest for D_2. Lemma 2 summarizes the equilibrium optimal investments when U_1, D_1 integrate.

Lemma 2. *Assume that*
(A1) U_1 *and* D_1 *merge*
(A2) No counter merger
(A3) $\beta > \mu$
If remaining independent downstream firm D_2 *signs a compatibility contract with the upstream division of the integrated firm, then the integrated firm invests* $r_{11} = \gamma(1)$ *and* $r_{12} = \gamma((\beta-\mu)^{-1})$

D_2 will be supplied by U_2 if the integrated firm does not accept the compatibility contract from D_2. In this case, D_2 will be supplied by $\varepsilon_2 + \tau(r_{22}^*)$. The integrated firm agrees to be the supplier to its downstream competitor if the potential value of the agreement is positive.

$$(\beta-\mu)(\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1}))) - \gamma((\beta-\mu)^{-1}) \geq -\mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) \quad \text{if} \quad \beta > \mu$$
$$(\beta-\mu)\varepsilon_1 \geq -\mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) \quad \text{if} \quad \beta < \mu$$

The integrated firm will supply to its downstream competitor if there is an economic gain. In other words, the integrated firm accepts to produce for D_2 if the gain from supplying, which are; a portion of D_2's sales, the competitive effect and the cost of R&D, is higher than the opportunity cost of not supplying, which is the competitive effect when U_2 supplies D_2. Lemma 3 summarizes the integrated firm's compatibility decision with D_2 when U_1, D_1 integrate.

Lemma 3. *Assume that*
(A1) U_1 *and* D_1 *merge*
(A2) No counter merger
The integrated firm's upstream division $U_1^v D_1$ *would supply for the remaining independent downstream firm* D_2 *if and only if*

$$\varepsilon_1 + \frac{\mu}{\beta-\mu}\varepsilon_2 \geq F_1(\mu,\beta) \qquad if\ \beta > \mu \tag{3}$$
$$\varepsilon_1 + \frac{\mu}{\beta-\mu}\varepsilon_2 \geq F_2(\mu,\beta) \qquad if\ \beta < \mu \tag{4}$$

,where

$$F_1(\mu,\beta) = \frac{\gamma((\beta-\mu)^{-1})}{\beta-\mu} - \frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu} - \tau(\gamma((\beta-\mu)^{-1}))$$
$$F_2(\mu,\beta) = -\frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu}$$
$$F_2(\mu,\beta) > 0 > F_1(\mu,\beta)$$

That's why, $U_1^v D_1$ always agrees on compatibility contract with D_1 whenever D_2 extended a contract if $\beta > \mu$. On the other hand, there may still be a positive economic gain, although $\beta < \mu$ in the downstream market so that the integrated firm can agree on a contract with its downstream competitor. However, the integrated firm would have no incentive to invest for D_2 since any positive investment would decrease the overall profit. Supplying D_2 won't have any affect on U_1's incentives to supply D_1. Post integration, upstream division U_1 invests more for downstream division D_1 even if $U_1^v D_1$ is compatible with D_2. In conclusion, the optimal level of U_1's investment for D_1 is higher than the optimal level of investment U_1 has for the downstream competitor. Consequently, the incentive of the independent downstream firm D_2 to be compatible with the integrated firm is reduced due to the integrated firm's unwillingness to invest for its competitor. Thus, $U_1 D_1$ integration increases the likelihood of D_2 to be compatible with U_2.

Post integration, the profit function of the independent downstream firm D_2 is,

If $\beta > \mu$

$\Pi(D_2) = \pi + \alpha(\lambda_2(\varepsilon_1 + \tau(\gamma((\beta - \mu)^{-1}))) + (1 - \lambda_2)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))) - \mu(\varepsilon_1 + \tau(\gamma(1)))$

If $\beta < \mu$

$\Pi(D_2) = \pi + \alpha(\lambda_2(\varepsilon_1) + (1 - \lambda_2)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))) - \mu(\varepsilon_1 + \tau(\gamma(1)))$

D_2 extends a compatibility contract to the integrated firm if

$$\Pi(D_2; U_1^v D_1) > \Pi(D_2; U_2)$$

Lemma 4 summarizes the D_2's equilibrium decision to be compatible with $U_1^v D_1$ when U_1, D_1 integrate.

Lemma 4. *Assume that*
(A1) U_1 and D_1 merge
(A2) No counter merger
The remaining independent downstream firm D_2 offers a compatibility contract to upstream division of the integrated firm if and only if

$$\varepsilon_1 - \varepsilon_2 \geq G_1(\beta, \mu) \qquad \text{if } \beta > \mu \tag{5}$$
$$\varepsilon_1 - \varepsilon_2 \geq G_2(\beta) \qquad \text{if } \beta < \mu \tag{6}$$

,where

$$G_1(\beta, \mu) = \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta - \mu)^{-1})) \tag{7}$$
$$G_2(\beta) = \tau(\gamma(\beta^{-1})) \tag{8}$$

Otherwise, the downstream firm will offer compatibility contract to the independent upstream firm U_2.

D_2's willingness to extend a compatibility contract to $U_1^v D_1$ increases as the upstream division U_1's relative efficiency to U_2's efficiency increases. D_2 extends a contract to $U_1^v D_1$ if the U_1's efficiency advantage is adequate. On the other hand, D_2 offers a compatibility contract to U_2 if U_2 is more efficient than U_1 or adequate efficient so that D_2 will be supplied with a higher variety post integration.

The analysis partitions the $(\varepsilon_1, \varepsilon_2)$ parameter space into three different strategic regions.[22]

$$R_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \frac{\mu}{\beta - \mu}\varepsilon_2 \geq F_1(\mu, \beta) = \frac{\gamma((\beta-\mu)^{-1})}{\beta-\mu} - \frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu} - \tau(\gamma((\beta-\mu)^{-1}))\}$$
$$R_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \varepsilon_2 \geq G_1(\beta, \mu) = \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta-\mu)^{-1}))\}$$
$$R_3 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \varepsilon_2 \geq 0\}$$

$$\text{Note that} \quad R_2 \subset R_3 \quad \text{and} \quad R_2 \subset R_1$$
$$\xi_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in R_2\}$$
$$\xi_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in R_3 \quad \text{and} \quad (\varepsilon_1, \varepsilon_2) \notin R_2\}$$
$$\xi_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \notin R_3\}$$

ξ_1 is the first strategic region. If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, then D_2 offers a compatibility contract to $U_1^v D_1$ because upstream division U_1 has the efficiency superiority. Second strategic region is ξ_2. If $(\varepsilon_1, \varepsilon_2) \in \xi_2$, then D_2 offers a compatibility contract to U_2 even though upstream division U_1 has the efficiency superiority because the integrated firm would invest less for its downstream competitor. The last strategic region is ξ_3 in which U_2's efficiency is more than U_1's. If $(\varepsilon_1, \varepsilon_2) \in \xi_3$, D_2 offers a compatibility contract to U_2. U_1, D_1's integration decision does not have any affect on D_2's incentive to be compatible with U_2 in ξ_3.

Until this part, I have examined the equilibrium compatibility contracts given U_1 and D_1's integration decision. The next part will examine the U_1's and D_1's integration decision. For better exposition purposes, the rest of the paper assumes that $\beta > \mu$.

If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, in which U_1 has an efficiency superiority, then U_1, D_1's integration decision has no affect on D_2's incentives. Both downstream firms will be compatible with U_1 when U_1, D_1 do not integrate. D_2 will be supplied by U_1 even when U_1, D_1 integrate.

Moreover, if $(\varepsilon_1, \varepsilon_2) \in \xi_2$, both downstream firms will be compatible with U_1 when U_1, D_1 do not integrate but D_2 will be supplied by U_2 when U_1, D_1 integrate. U_2 can attract D_2's business post integration that's why integration decision will change the D_2's incentive to offer a compatibility contract to U_1. Should D_2 compatible with the

[22] $R_2 \subset R_1$ since $G_1(\beta, \mu) > F_1(\beta, \mu)$. The slope of the line that defines R_1 is negative while the slope of the line that defines R_2 is one. One can proof that every $(\varepsilon_1, \varepsilon_2)$ which is in R_3 also is in R_2

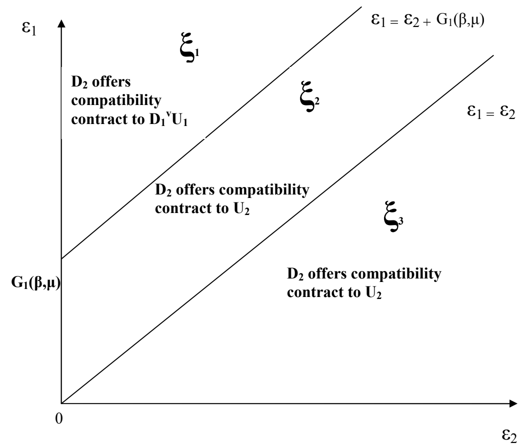

Figure 3: Compatibility Contracts if Vertical Integration Occurs with No Counter Merger

upstream division U_1, integration harms D_2's profit because $U_1^v D_1$ invests less for D_2 post integration. Hence, D_2 can switch its first choice supplier when U_1, D_1 integrate.

Furthermore, if $(\varepsilon_1, \varepsilon_2) \in \xi_3$, both downstream firms will be compatible with U_2 when U_1, D_1 do not integrate. As a matter of fact, the willingness of D_2's to extend a compatibility contract to U_2 also can not be altered when U_1, D_1 integrate. Nevertheless, $U_1^v D_1$ would accept an offer from D_2 if D_2 offered when U_1, D_1 integrate.

The analysis shows that for every $\varepsilon_1 < G_1(\beta, \mu)$ D_2 will be compatible with U_2 when U_1, D_1 integrate. The upstream division U_1's efficiency ε_1 must be at least $G_1(\beta, \mu)$ to attract D_2. Note that Lemma 4 suggests that $G_1(\beta, \mu)$ is increasing in μ. Hence, the more competitive downstream market is (higher μ), the more efficient the integrated firm should be in order to alter D_2's incentives. In addition, a higher μ causes U_2 to reduce optimal level of investment that U_1 has for D_2. That's why, a higher μ expands the region ξ_2 and shrinks the region ξ_1.

Theorem 1. *Assume that*
(A1) No counter merger
(A2) No horizontal merger of downstream firms
If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, *then* U_1 *and* D_1 *integrate.* D_2 *is supplied by the upstream division* U_1.

Lemma 1 states that downstream firms offer compatibility contract to U_1 if $(\varepsilon_1, \varepsilon_2) \in \xi_1$. Lemma 4 states that D_2 is supplied by $U_1^v D_1$ when U_1, D_1 integrate, if $(\varepsilon_1, \varepsilon_2) \in \xi_1$. Theorem 1 summarizes the firms' strategy when U_1 has an efficient superiority over U_2.

As a result, U_1, D_1 integrate under the conditions because of both gain in upstream and downstream markets. D_2 extends a contract offer to $U_1^v D_1$ since integration decision of U_1, D_1 does not affect D_2's incentives. To summarize, D_1 and U_1 integrate in ξ_1 if no counter merger and no horizontal merger of downstream firms are allowed.

Next, U_1, D_1's integration decision can alter the D_2's motives if $(\varepsilon_1, \varepsilon_2) \in \xi_2$. U_1 is more efficient but does not have the superior productivity to attract D_2 when U_1, D_1 integrate in ξ_2. Lemma 4 suggests D_2 extends a compatibility contract to U_2 if U_1, D_1 integrate in ξ_2. We define the strategic region Λ_1 as

$$
\begin{aligned}
&\Lambda_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \frac{\mu}{\beta - \mu}\varepsilon_2 \leq X_1(\beta, \mu)\} \quad \text{where} \\
&X_1(\beta, \mu) = \frac{\tau(\gamma(1)) - \gamma(1) - (1+\beta)\tau(\gamma(\beta^{-1})) + 2\gamma(\beta^{-1})}{\beta - \mu} \quad \text{and} \\
&\xi_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in R_3 \quad \& \quad (\varepsilon_1, \varepsilon_2) \notin R_2\}
\end{aligned}
$$

Theorem 2 summarizes the the firms' strategies employed if $(\varepsilon_1, \varepsilon_2) \in \xi_2$.

Theorem 2. *Assume that*
(A1) No counter merger
(A2) No horizontal merger of downstream firms
If $(\varepsilon_1, \varepsilon_2) \in \xi_2$,then U_1 and D_1 merge if and only if $(\varepsilon_1, \varepsilon_2) \in \Lambda_1$. D_2 will be supplied by U_2. Otherwise, U_1 and D_1 remains independently owned. D_1 and D_2 will be supplied by U_1.

The strategic region Λ_1 defines the region in which the total gain of both U_1, D_1 from integration is sufficiently large. Theorem 2 partitions the region ξ_2 into two strategic regions. In the first region , $\Lambda_1 \cap \xi_2$, U_1 and D_1 integrate even though D_2 chooses to be supplied by U_2 post integration. The reason is the gain due to U_1's higher investment for D_1 after integration being more than U_1's cost of loosing D_2's business. In second region, U_1 and D_1 forego integration, mostly because it is more costly for U_1 not to acquire a contractual relation with D_2.

As a result, U_1, D_1 integrate U_2's efficiency ε_2 is significantly low. On the other hand, U_1 is reluctant to integrate for high values of ε_2 because D_2 would switch its complementary good supplier in case of integration. Hence, U_1, D_1 integration has two opposite effects on upstream division U_2's profit. Positive effect is the bilateral gain due to higher variety. Negative effect is the loss of a potential business that would be acquired in case U_1 remains independent.

To sum up, U_1, D_1 always integrate whenever U_2 can not provide the required competition to lower the integrated firm's profit. Intuitively, U_1, D_1 always integrate if $U_1^v D_1$

has no incentive to accept a compatibility contract from D_2. Meanwhile, U_1 can forego integration even if $U_1^v D_1$ is willing to supply D_2. D_2's supplier decision heavily depends on the efficient asymmetry between upstream suppliers. $U_1^v D_1$ will not be offered a compatibility contract by D_2 in ξ_2 although U_1's best interest is to supply D_2. U_1, D_1 may forego integration because of the competitive effect post integration.

The threshold values $(\varepsilon_1, \varepsilon_2)$ which partitions the strategic regions heavily depends on the sensitivity of D_1's profits to D_2's variety (i.e. μ). The more sensitive the profits are, the more efficient U_1 should be in order to sustain a contractual relation equilibrium.[23]. Consequently, if μ is initially large, increasing μ makes U_1, D_1 integration more likely.

On the other hand, if μ is not high, then increasing μ makes $X_1(\mu, \beta)$ closer to zero, and the region $\Lambda_1 \cap \xi_2$ shrinks.[24] That's why, U_1, D_1 integration will be less likely to observe as an equilibrium outcome. In this case, D_1 is always eager to integrate, however U_1 would not integrate because the opportunity cost of loosing D_2's business is higher than the bilateral gain due to integration. For this reason, there is no general relationship between μ and integration decision if $(\varepsilon_1, \varepsilon_2) \in \xi_2$.

Moreover, U_1, D_1 integration decision also depends on how sensitive $U_1's$ profits to D_1's variety (i.e. β). If β is high. U_1's optimal investment and the variety of D_2 and increases, that's why U_1 will be less willing to integrate. In this case, U_1, D_1 integrate if U_1's efficiency is not high so that the loss of D_2's business does not harm U_1's profits significantly.

The last strategic region to analyze is the region in which U_2 has the efficiency advantage. U_1, D_1's integration decision do not alter D_2' incentives. Lemma 1 states, D_1 and D_2 extends contracts to U_2 when U_1, D_1 do not integrate and lemma 4 states that D_1 will be compatible with U_1 while D_2 extends a contract to U_2 when U_1, D_1 integrate, if $(\varepsilon_1, \varepsilon_2) \in \xi_3$. We define the strategic region Λ_2 as

$$\begin{aligned}
&\Lambda_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \alpha\varepsilon_2 \geq X_2(\alpha)\} \quad \text{where} \\
&X_2(\alpha) = (1-\beta)\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) + \gamma(1) \quad \text{and} \\
&\xi_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \notin R_3\}
\end{aligned}$$

Theorem 3 summarizes the firm's strategies employed if $(\varepsilon_1, \varepsilon_2) \in \xi_3$

Theorem 3. *Assume that*
(A1) No counter merger
(A2) No horizontal merger of downstream firms
If $(\varepsilon_1, \varepsilon_2) \in \xi_3$,*then* U_1 *and* D_1 *merge if and only if* $(\varepsilon_1, \varepsilon_2) \in \Lambda_2$. D_2 *will be supplied by*

[23] The slope of the inequality $\varepsilon_1 + \frac{\mu}{\beta - \mu}\varepsilon_2 \leq X_1(\beta, \mu)$ increases as the value $X_1(\beta, \mu)$ if we increase μ

[24] $X_1(\beta, \mu)$ is a decreasing function of μ.

U_2. Otherwise, U_1 and D_1 remains independently owned. D_1 and D_2 will be supplied by U_2.

D_1's best interest is always to integrate if U_1 has the efficiency advantage (i.e $(\varepsilon_1, \varepsilon_2) \in \xi_1 \cup \xi_2$). If U_1, D_1's decision is not to integrate in the equilibrium, the only reason is the U_1's cost of loosing D_2's business in $\xi_1 \cup \xi_2$. On the other hand, D_2's best interest is not always to integrate if U_2 has the efficiency advantage(i.e. $(\varepsilon_1, \varepsilon_2) \in \xi_3$). Not being supplied by the more efficient upstream firm U_2 may harm D_1's profits, especially when its downstream competitor D_2 will be supplied by U_2.

Unlike D_1, D_1's best interest may not be always to integrate if U_1 has the efficiency advantage (i.e $(\varepsilon_1, \varepsilon_2) \in \xi_1 \cup \xi_2$) because of the cost of integration to U_1. However, D_2's best interest is always to integrate if U_2 has the efficiency advantage(i.e. $(\varepsilon_1, \varepsilon_2) \in \xi_3$). U_1 can supply to any of the downstream firms unless U_1, D_1 integrate because both downstream firms extend a contract to U_2 under contractual relations.

Theorem 3 partitions ξ_3 into two strategic regions. Λ_2 defines the region in which both U_1's and D_1's best interest is to integrate in ξ_3. U_1, D_1's integration causes U_1 to invest more for D_1, that's why the downstream division D_1 may increase profit of $U_1^v D_1$.

Moreover, theorem 3 states that U_1, D_1's integration is more likely when $\beta$is small enough (or $\alpha = 1 - \beta$ is big enough). As βbecomes smaller, the strategic region Λ_2 shrinks.[25]. We can define the value of the sensitivity of U_1's profits to D_1's variety (β^*) so that for every βless tan β^*, U_1, D_1 integration is less likely if $(\varepsilon_1, \varepsilon_2) \in \xi_3$. β^* solves the equation $X_2(1 - \beta^*) = 0$. An immediate result is; U_1, D_1's integration is more likely if $\beta$is big enough. [26]. That's why there exists a critical efficiency parameter ε_2^* which necessarily implies U_1, D_1's integration decision.

Corollary 1. *Assume that*
(A1) No counter merger
(A2) No horizontal merger of downstream firms
If $\beta < \beta^*$ and $(\varepsilon_1, \varepsilon_2) \in \zeta_3$, U_1, D_1 integrate if $\varepsilon_2 < \varepsilon_2^*$, where $\varepsilon_2^* = \frac{X_2(1-\alpha)}{\beta - 1}$

In conclusion, this section summarizes the firms' strategies under the conditions that no counter merger and no horizontal merger of downstream firms is allowed. The $(\varepsilon_1, \varepsilon_2)$ parameter space is partitioned into five strategic regions in which U_1, D_1 integrate or do no integrate in equilibrium. U_1, D_1 integration becomes more likely as ε_1 increases whenever $\varepsilon_1 > \varepsilon_2$ and ε_1 is not close to ε_2. In addition, U_1, D_1 integration becomes less likely as ε_1 increases whenever $\varepsilon_2 > \varepsilon_1$ and ε_1 is significantly smaller than ε_2. Figure 4 illustrates the

[25] X_2 is a decreasing function of β

[26] If $\beta = 1$, then Λ_2 is characterized by $\varepsilon_1 - \geq X_2(0)$. Then, Λ_2 is the whole $(\varepsilon_1, \varepsilon_2)$ parameter space since $X_2(0) < 0$

predictions of Theorems 1,2 and 3. The next section will analyze D_1's and U_1's integration decision if a counter merger of U_2, D_2 is possible.

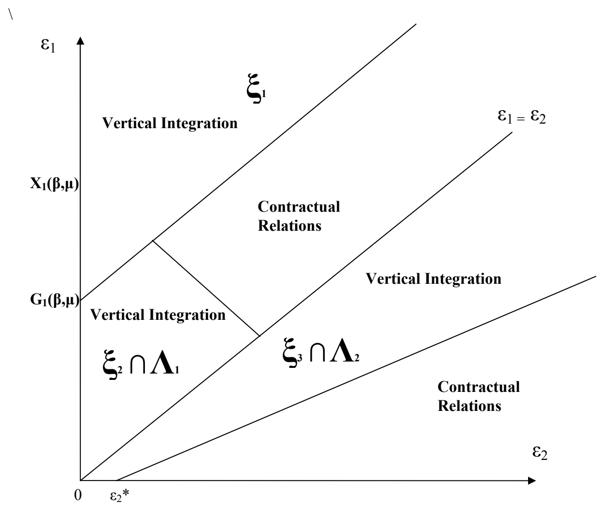

Figure 4: U_1, D_1 Integration Decision with No Counter Merger.

5 Endogenous Integration with Counter Integration of Rivals

In this section, I study U_1, D_1's integration decision if U_2, D_2 can counter integrate as a reaction when U_1, D_1 integrate. I am going to assume that,without loss of generality, D_1's first choice to integrate is always U_1 instead of U_2.

I use backward induction to analyze D_1's and D_2's endogenous integration decisions. At time three, each independent downstream firm shares its marginal profit with its compatible upstream firm. However, an integrated downstream division is supplied by the integrated upstream division and the joint profit is maximized by a central management.

If U_1 and D_1 integrate, then an integrated U_2 invests to solve

$$\max_{r_2} \quad \pi + \varepsilon_2 + \tau(r_2) - \mu(\varepsilon_1 + \tau(r_1^*)) - r_2$$

The first order conditions imply,

$$\frac{\partial \Pi_{U_2}}{\partial r_2^*} = \beta\tau^{'}(r_2) - 1 = 0$$

U_2, D_2 counter integration affects U_2's investment incentives. U_2 invests $r_2^* = \gamma(\beta^{-1})$. Lemma 2 states that U_2 also invests $r_1^* = \gamma(\beta^{-1})$.

Lemma 5. *Assume that*
(A1) U_1 and D_1 merge
(A2) U_2 and D_2 merge
Both upstream divisions invest $r^* = \gamma(\beta^{-1})$.

I now examine the equilibrium U_2 and D_2's counter integration decision when U_1 and D_1 are integrated. U_2, D_2 counter integrate if and only if the stand alone profits are less than an integrated firm's profits.

$\Pi U_2^v D_2 > \Pi(D_2) + \Pi(U_2)$

U_1, D_1 integration affects both D_2's and U_2's gain in the three strategic regions ξ_1, ξ_2 and ξ_3 differently.

First, I examine the equilibrium U_2 and D_2's counter integration decision if $(\varepsilon_1, \varepsilon_2) \in \xi_1$. Lemma 4 states that D_2 extends a contract to $U_1^v D_1$ and lemma 3 states that $U_1^v D_1$ accepts the contract if U_1, D_1 integrate and D_2 remains independent in ξ_1. U_2 supplies to the market if and only if D_2 and U_2 integrate in ξ_1. That's why U_2 always prefers a counter integration. On the other hand, D_2's ex post gain can be negative because it might be better for D_2 to be compatible with the more efficient upstream unit U_1 which might increase D_2's profits. That's why D_2 and U_2 might forego counter integration. If we define the strategic regions C_1 and CM_1 as

$$\begin{aligned}
C_1 &= \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{\varepsilon_2}{1-\beta} \leq S_1(\beta, \mu) \text{ where} \\
S_1(\beta, \mu) &= \frac{\tau(\gamma(1)) - \gamma(1)}{1-\beta} - \tau(\gamma((\beta-\mu)^{-1}))\} \\
CM_1 &= \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in C_1 \cap \xi_1\}
\end{aligned}$$

Following lemma summarizes the equilibrium U_2 and D_2's counter integration decision if $(\varepsilon_1, \varepsilon_2) \in \xi_1$ when U_1, D_1 integrate.

Lemma 6. *Assume that*
(A1) U_1 and D_1 merge
If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, U_2 and D_2 integrate if and only if $(\varepsilon_1, \varepsilon_2) \in CM_1$

Lemma 6 states that U_2's gain is not adequate for D_2 to integrate with U_2 if $(\varepsilon_1, \varepsilon_2)$ is not in the parameter space CM_1. CM_1 is the strategic regions in which U_2, D_2 counter integrate when U_1, D_1 integrate. A larger variety sensitivity β makes U_2, D_2 counter integration more likely. The main reason is a bugger U_2, D_2 bilateral gain when U_1, D_1

integrate because an independent D_2's contract fee βv_2 increases with a larger β. Lemma 6 also states that there exists always a strategic region CM_1 in ξ_1.

Second, I examine the equilibrium U_2 and D_2's counter integration decision if $(\varepsilon_1, \varepsilon_2) \in \xi_2$ and $(\varepsilon_1, \varepsilon_2) \in \xi_3$. In these two strategic regions, D_2 is going to be supplied by U_2 even if D_2 remains independent. Counter integration with U_2 is always a weakly dominant strategy for D_2 because investment decision made by a central management necessarily increases the joint profits when U_1, D_1 integrate.

$$\Pi(U_2^v D_2) \geq \Pi(D_2; U_2) + \Pi(U_2; D_2) \quad \text{if} \quad (\varepsilon_1, \varepsilon_2) \in (\xi_2 \cup \xi_3)$$

Next lemma summarizes the equilibrium U_2 and D_2's counter integration decision if $(\varepsilon_1, \varepsilon_2) \in \xi_2$ and $(\varepsilon_1, \varepsilon_2) \in \xi_3$

Lemma 7. *Assume that*
(A1) U_1 *and* D_1 *merge*
U_2 *and* D_2 *integrate if* $(\varepsilon_1, \varepsilon_2) \in (\xi_2 \cup \xi_3)$

Figure illustrates the equilibrium U_2 and D_2's counter integration decision when U_1, D_1 integrate.

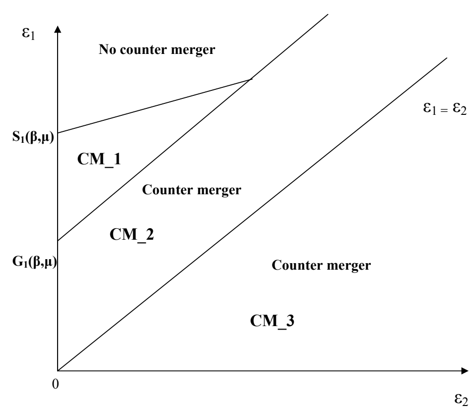

Figure 5: U_2, D_2 Counter Integration Decision when U_1, D_1 Integrate with No Horizontal Merger

Up to this point, I have not discussed the equilibrium integration decision of U_1, D_1. D_1's decision to integrate can be altered when U_2, D_2 counter merger is possible because downstream variety of D_2 increases when U_2, D_2 integrate.

First, I examine U_1, D_1 integration decision when D_2 would not integrate with U_2 when U_1, D_1 integrate. Intuitively, D_1's best interest is to integrate with U_1 because vertical integration would not lead to a counter integration. Theorem 1 claims U_1, D_1 integrate if $(\varepsilon_1, \varepsilon_2) \in \xi_1$. In other words, $U_1, D_1's$ bilateral gain is positive in ξ_1. In particular, D_1's best choice does not alter if D_2 would integrate with U_2. Another reason is that the efficiency difference between upstream producers is adequately high so that D_2 prefers being independent rather than counter integration. Theorem 4 summarizes the equilibrium integration decision of U_1, D_1 if U_2, D_2 would not integrate.

Theorem 4. *Assume that*
(A1) No horizontal merger of D_1, D_2 is allowed
If $(\varepsilon_1, \varepsilon_2) \in \xi_1$ and $(\varepsilon_1, \varepsilon_2) \notin CM_1$, U_1 and D_1 integrate and D_2 remain independently owned. D_2 is supplied by upstream division U_1.

Note that U_1, D_1's integration decision does not have any affect on D_2's incentives. The main reason is the upstream efficiency asymmetry. In this case, U_2 can not compete with U_1 even though U_1 would invest less for D_2. As a result, D_2 will be supplied by $U_1^v D_1$ in equilibrium.

Second, I examine U_1, D_1 integration decision when D_2 would not integrate with U_2 when U_1, D_1 integrate and U_1 is more efficient than U_2 (i.e., $(\varepsilon_1, \varepsilon_2) \in CM_1 \cup xi_2$). Although CM_1 and ξ_2 has different strategic implications when U_2, D_2 integration is not allowed, CM_1 and ξ_2 has the same strategic implications when when U_2, D_2 integration is allowed. U_2, D_2 would counter integrate when U_1, D_1 integrate if U_1 does not have adequate efficiency advantage even though U_1 is more efficient than U_2. U_2, D_2 counter integration decision can threat the profits of $U_1^v D_1$. Thus, D_1 and U_1 may forego vertical integration. If we define the strategic regions TM_1 and T_1 as

$$T_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \frac{\mu\varepsilon_2}{\beta - \mu} \leq S_2(\beta, \mu) \text{ where}$$
$$S_2(\beta, \mu) = \frac{(1-\mu)(\tau(\gamma(1)) - \tau(\gamma(\beta^{-1}))) - \gamma(1) - \beta\tau(\gamma((\beta-\mu)^{-1})) + 2\gamma((\beta-\mu)^{-1})}{\beta - \mu}\}$$
$$TM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in T_1 \cap (CM_1 \cup \xi_2)\}$$

Theorem 5 summarizes the equilibrium strategies employer by the firms.

Theorem 5. *Assume that*
(A1) No horizontal merger of D_1, D_2 is allowed
If $(\varepsilon_1, \varepsilon_2) \in CM_1 \cap \xi_2$, U_1 and D_1 integrate and U_2, D_2 integrate if and only if $(\varepsilon_1, \varepsilon_2) \in TM_1$.
Otherwise, firms remain contractual relations. D_1 and D_2 is supplied by U_1.

Theorem 5 partitions the strategic region $CM_1 \cap \xi_2$ into two. The first partition is TM_1. Even though U_2, D_2 counter integration is a threat to $U_1^v D_1$, U_1, D_1 integrates in TM_1.

As a result, two vertical integrations will be observed in the market. On the other hand, both D_1's and U_1 loss is higher than a possible bilateral gain from integration whenever $(\varepsilon_1, \varepsilon_2) \in CM_1 \cup \xi_2$ but $(\varepsilon_1, \varepsilon_2) \notin TM_1$. Consequently, neither of the downstream firms integrates with an upstream firm.

The likelihood of a independent ownership increases as U_2's efficiency increases in ξ_2. First reason is that D_1 would loose more downstream profits for higher efficiencies of U_2 when U_1, D_1 and U_2, D_2 integrate. Second reason is that an independent U_1 would acquire the business of an independent D_2. Post integrations, D_2 will be supplied by U_2. Theorem 5 and theorem 4 state that U_1, D_1 never integrate if U_1 is significantly efficient but not too efficient so that D_2 would not counter integrate. (i.e., $min(G_1(\beta, \mu, S_2(\beta, \mu))) < \varepsilon_1 < S_1(\beta, \mu)$)

Furthermore, U_1, D_1 integration decision under possibility of a counter integration depends on how sensitive the profits to variety and downstream competition. $U_1^v D_1$ would loose downstream profits when U_2, D_2 if the downstream market is highly competitive, while independent D_1 and U_1 would not be hurt as much by a high competitive downstream market since D_2 would not counter integrate and increase its variety. Thus, a large μ value makes independent ownership more likely and U_1, D_1 and U_2, D_2 integrations less likely. In addition, an independent U_1 would loose a significant amount of upstream profits if the upstream profits are significantly sensitive to downstream variety. The main reason is that U_1 would supply both downstream firms under contractual relations, while U_1 would supply only D_1 if both downstream firms integrate.[27] Consequently, a higher β value makes independent ownership more likely and U_1, D_1 and U_2, D_2 integrations less likely. In general, the findings support the immediate intuitive sense that it is less likely for an upstream firm to integrate if the price for the upstream service or product is high, upstream firm's market share is significantly high and upstream firm would loose some of its customers after integration.

Third, I examine U_1, D_1's integration decision when D_2 would not integrate with U_2 when U_1, D_1 integrate and U_1 is less efficient than U_2 (i.e., $(\varepsilon_1, \varepsilon_2) xi_3$). U_2, D_2 would counter integrate when U_1, D_1 integrate especially if U_1 U_1 is less efficient than U_2. U_2, D_2 counter integration decision also threats the profits of $U_1^v D_1$. Thus, D_1 and U_1 may forego vertical integration in ξ_3. If we define the strategic regions TM_2 and T_2 as

$$T_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - (1-\beta)\varepsilon_2 \leq S_3(\beta, \mu) \text{ where}$$
$$S_3(\beta, \mu) = (1-\beta)(\tau(\gamma((\beta-\mu)^{-1})) - \tau(\gamma(1)) + \gamma(1)) - \beta\tau(\gamma((\beta-\mu)^{-1}))\}$$
$$\xi_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \notin R_3\}$$
$$TM_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in T_2 \cap \xi_3)\}$$

[27] If $\beta = 1$, U_1 would never integrate with D_1 in ξ_2

Theorem 6 summarizes the equilibrium strategies employed by the firms in ξ_3.

Theorem 6. *Assume that*
(A1) No horizontal merger of D_1, D_2 is allowed
If $(\varepsilon_1, \varepsilon_2) \in \xi_3$, U_1 and D_1 integrate and U_2, D_2 integrate if and only if $(\varepsilon_1, \varepsilon_2) \in TM_2$. Otherwise, firms remain contractual relations. D_1 and D_2 is supplied by U_2.

Theorem 6 states that there exits a strategic region TM_2 in ξ_3 in which both downstream firms integrate with an upstream firm. Theorem 6 also states that there exists a strategic region in ξ_3 in which firms maintain their contractual relations and both downstream firms are supplied by the more efficient upstream firm U_2.

D_1 would be supplied by the more efficient upstream firm U_2, whereas U_1 would not supply to any of the downstream firms under independent ownership. That's why, U_1 is always willing to integrate unlike D_1 which may prefer being independent. D_1 has no incentive to integrate whenever U_2 has a significant efficiency advantage. Consequently, a higher efficiency value ε_2 makes independent ownership more likely and U_1, D_1 and U_2, D_2 integrations less likely. However, theorem 6 states that U_1, D_1 always integrates if U_2 is not efficient enough in ξ_3 (i.e. $\varepsilon_2 < \frac{S_3(\beta,\mu)}{1-\beta}$) and U_2, D_2 counter integrate.

U_1, D_1 integration decision under possibility of a counter integration depends on how sensitive the profits to variety and downstream competition in ξ_3 similar to the other strategic regions. As the sensitivity measure β becomes large, D_1 must incur a higher cost to be compatible with U_2.[28] Thus, a higher β value makes U_1, D_1 and U_2, D_2 integration more likely and independent ownership less likely. In addition, D_1 would know that D_2 would be also supplied by U_2 if U_1, D_1 integrates. A higher competition would hurt the downstream profits of $U_1^v D_1$ more because $U_2^v D_2$ will have a higher variety in ξ_3. Consequently, a higher μ value makes U_1, D_1 and U_2, D_2 integration less likely and independent ownership more likely.

In conclusion, theorems 4 through 6 summarize the endogenous integration decision of U_1, D_1 and U_2, D_2 yet the equilibrium decision is conditional on not allowing a horizontal merger of D_1, D_2. The figure illustrates the model's predictions. The next section summarizes and concludes.

6 Conclusion

The goal of this paper is to investigate the endogenous integration decision of firms in complementary market setting such that a downstream firm must commit to a compatibility contract with one of the complementary good producers. Post contractibg, the

[28] If $\beta = 1$, then D_1 does not gain from any variety in the downstream market.

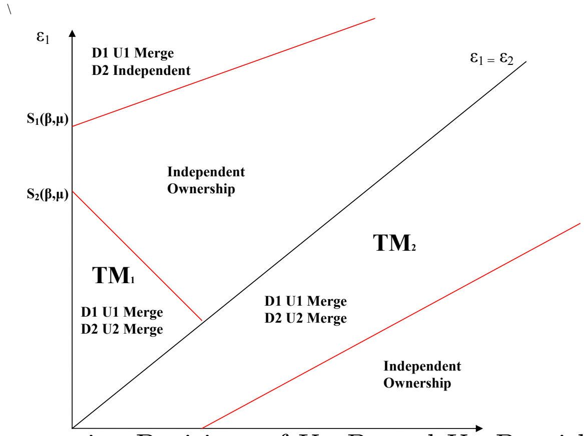

Figure 6: Integration Decisions of U_1, D_1 and U_2, D_2 with No Horizontal Merger

complementary good producers invest on firm specific R&D. I have identified the conditions under which integration with a complementary good producer and/or a counter merger of is a subgame perfect Nash equilibrium organizational form.

In following sections, I examine the conditional endogenous integration decision when a counter merger is not allowed. The first section analyzes the integration decision of firms when a horizontal merger is assumed away. Matutes&Regibeau(1998), Beggs(1994), Heavner(2004) are some examples of the literature which assumes the vertical integration option and assumes away a counter merger option.

The las section examines the unconditional endogenous integration decision when a counter merger is also allowed. Chen(2001), McAfee(1999)are some examples of the literature which assumes the vertical integration and counter integration and assumes away a horizontal merger option. An extension model can analyze the case when horizontal merger is also allowed.[29] The paper presents the model's predictions and claims that each of different organizational forms may be the equilibrium outcome. The paper provides a theory base for Economides & Salop(1992) which assumes different organizational forms.

To be specific, when U_1 has a high efficiency superiority($\varepsilon_1 >> \varepsilon_2$), vertical integration will always occur in absence of a horizontal merger option. Vertical integration changes the integrated firm's incentives to invest for its' downstream rival. The independent base product firm will be less apt to be compatible with the integrated firm post integration. Thus, the integration may not occur whenever the efficiency of the integrated firm is inadequate. Moreover, the base product firm may merge with its substitute good producer and forego vertical integration if we introduce horizontal merger as an alternative. Both

[29] Cakirer(2006)

of the upstream firms can supply variety to the market post merger, although the total variety will be less.

The model's extended predictions suggest that the counter merger may be more likely if variety competition is fierce when D_1 and D_2 can merge. Moreover, merger with substitute good producer is more likely whenever the rival downstream firm would not be compatible with the integrated firm in case of integration. On the other hand, the higher share the downstream gets from the profits generated by the variety effects, the more likely vertical integration to occur. If the variety competition is not fierce enough($\alpha > 2\mu$), independent ownership can not be sustained whenever U_1 has an efficiency advantage. In addition, independent ownership can not be a sustainable market structure whenever U_2 has the efficiency advantage and the competition is not fierce enough ($\alpha < 2\mu$). Nevertheless, the firms can stay independent for some efficiency levels even if both vertical integration and counter merger are available. In contrast to existing literature which takes the merger decision of the firm for granted, this paper suggests that some type of mergers are more likely than others in complementary markets under the factors such as degree of competition and distribution of the profit increment due to the externality effect.

The theory presented in this dissertation can provide a basis for a possible empirical work on mergers in complementary markets. One can use the fixed effect model of endogenous integration to estimate the efficiency values utilizing the available data on variety of a product. The profits also can be structured and estimated as a function of both the firm's variety and the rival's variety. One can observe the different kind of mergers and argue whether the predictions of this paper prevail or not. Further research is needed to test the predictions of the model. Moreover, one can investigate the effect of a pre-existing vertical merger on a horizontal merger and vice versa. Following the idea, one can investigate combination of mergers in complementary markets in a dynamic setting. The model also lacks a complete welfare analysis post merger. One can work work on a model which analyzes the welfare effects when the firm is supported with variety of complementary products using the paper as the model's basis.

Appendix

Proof of Lemma 1
If D_1, D_2, U_1 and U_2 are independent, then their profits are going to be

$$\begin{aligned}
\Pi(U_1) =&(\lambda_1 + \lambda_2)(\beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1})) \\
\Pi(U_2) =&(2 - \lambda_1 - \lambda_2)(\beta(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1})) \\
\Pi(D_i) =&\pi + \alpha(\lambda_i\varepsilon_1 + (1 - \lambda_i)\varepsilon_2 + \tau(\gamma(\beta^{-1}))) \\
&- \mu(\lambda_j\varepsilon_1 + (1 - \lambda_j)\varepsilon_2 + \tau(\gamma(\beta^{-1})))
\end{aligned}$$

If we compare the profits of a downstream firm when D_i is supplied by U_1 (i.e. $\lambda_1 = 1$) and D_i is supplied by U_2 (i.e. $\lambda_1 = 0$) then,

$$\Pi(D_i; U_i) > \Pi(D_i; U_j) \Leftrightarrow \varepsilon_i > \varepsilon_j$$

A downstream firm maximizes its profit by offering a contract to the upstream firm which is more efficient.

Proof of Lemma 2
When $\beta > \mu$,the first order conditions satisfy

$$\begin{aligned}
\frac{\partial \Pi}{\partial r_{11}} &= \tau'(r_{11}) - 1 = 0 \quad \Rightarrow r^*_{11} = \tau'^{-1}(1) \\
\frac{\partial \Pi}{\partial r_{12}} &= (\beta - \mu)\tau'(r_{12}) - 1 = 0 \Rightarrow r^*_{12} = \tau'^{-1}((\beta - \mu)^{-1})
\end{aligned}$$

If $\beta > \mu$, the upstream division of the integrated firm $U^v_1 D_1$ would invest $r^*_{11} = \gamma((1))$ and $r^*_{12} = \gamma((\beta - \mu)^{-1})$.

Proof of Lemma 3
If $\beta > \mu$, the integrated firm's upstream division $U^v_1 D_1$ would supply D_2 if and only if

$$\Pi(U^v_1 D_1; D_2) \geq \Pi(U^v_1 D_1)$$

where

$$\begin{aligned}
&\Pi(U^v_1 D_1; D_2) = \pi + (\varepsilon_1 + \tau(\gamma((1)))) - \gamma((1)) + (\beta - \mu)(\varepsilon_1 + \tau(\gamma((\beta - \mu)^{-1}))) - \gamma((\beta - \mu)^{-1}) \\
&\Pi(U^v_1 D_1) = \pi + (\varepsilon_1 + \tau(\gamma((1)))) - \gamma((1)) + -\mu(\varepsilon_2 + \tau(\gamma(\beta^{-1})))
\end{aligned}$$

Then, $U^v_1 D_1$ would supply D_2 if and only if

$$\varepsilon_1 + \frac{\mu}{\beta - \mu}\varepsilon_2 \geq F_1(\mu, \beta) = \frac{\gamma((\beta - \mu)^{-1})}{\beta - \mu} - \frac{\mu\tau(\gamma(\beta^{-1}))}{\beta - \mu} - \tau(\gamma((\beta - \mu)^{-1})) \; if \; \beta > \mu$$

$$F_1(\mu,\beta) = \frac{\gamma((\beta-\mu)^{-1})}{\beta-\mu} - \frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu} - \tau(\gamma((\beta-\mu)^{-1})) < 0 \Leftrightarrow$$
$$\gamma((\beta-\mu)^{-1}) - \mu\tau(\gamma(\beta^{-1})) - (\beta-\mu)(\tau(\gamma((\beta-\mu)^{-1}))) < 0$$

But we know that if $x > 0$ then $\beta\tau(x) > x$, otherwise an upstream firm would invest zero and maximize the variety.Hence,

$$\gamma((\beta-\mu)^{-1}) - \beta\tau(\gamma((\beta-\mu)^{-1})) < 0 \quad \text{and}$$
$$\mu[(\tau(\gamma((\beta-\mu)^{-1})) - \tau(\gamma(\beta^{-1}))] < \tau(\gamma((\beta-\mu)^{-1})) - \tau(\gamma(\beta^{-1})) < 0 \quad \text{then}$$

The two inequalities imply $F_1(\mu,\beta) = \frac{\gamma((\beta-\mu)^{-1})}{\beta-\mu} - \frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu} - \tau(\gamma((\beta-\mu)^{-1})) < 0$

If $\beta < \mu$, if the integrated firm decides to supply, the firm maximizes the total gain from accepting the compatibility contract. Thus, the integrated firm's problem is to maximize the gain if integrated firm is compatible with the independent base product firm.

$$\max_{r_{12}} \quad (\beta-\mu)(\varepsilon_1 + \tau(r_{12}^*)) - r_{12}^*$$

Since $\beta - \mu < 0$, the gain is maximized if and only if the firm produces the minimum amount of variety for its rival. Hence,the optimal level of investment for its rival base product firm will be

$$\tau(r_{12}^*) = 0 \Longleftrightarrow r_{12}^* = 0$$

If we plug the optimal level of investment, the integrated firm agrees on a compatibility contract if

$$(\beta-\mu)\varepsilon_1 > -\mu(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$
$$\varepsilon_1 + \frac{\mu}{\beta-\mu}\varepsilon_2 \leq F_2(\mu,\beta) = -\frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu}$$

such that $F_2(\mu,\beta) = -\frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu} > 0$ since $\beta - \mu < 0$.

Proof of Lemma 4

The independent downstream firm offers the compatibility contract to the upstream division of the integrated firm if the gain from the contract is higher than the downstream firm's gain from its outside option. D_2 offers a compatibility contract to $U_1^v D_1$ if and only if

$$\Pi(D_2; U_1^v D_1) > \Pi(D_2; U_2)$$

If $\beta > \mu$

$$\Pi(D_2; U_1^v D_1) = \pi + (1-\beta)(\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma((1))))$$
$$\Pi(D_2; U_2) = \pi + ((1-\beta)(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma((1)))))$$

If $\beta < \mu$

$$\Pi(D_2; U_1^v D_1) = \pi + (1-\beta)(\varepsilon_1) - \mu(\varepsilon_1 + \tau(\gamma((1))))$$
$$\Pi(D_2; U_2) = \pi + ((1-\beta)(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma((1)))))$$

Then,D_2 offers a compatibility contract to $U_1^v D_1$ if and only if

$$\begin{aligned} &\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1})) > \varepsilon_2 + \tau(\gamma(\beta^{-1})) && \text{if} \quad \beta > \mu \\ &\varepsilon_1 > \varepsilon_2 + \tau(\gamma(\beta^{-1})) && \text{if} \quad \beta < \mu \end{aligned}$$

or

$$\begin{aligned} &\varepsilon_1 - \varepsilon_2 \geq G_1(\beta, \mu) \quad \text{if} \quad \beta > \mu \\ &\varepsilon_1 - \varepsilon_2 \geq G_2(\beta) \quad \text{if} \quad \beta < \mu 7 \end{aligned}$$

,where

$$G_1(\beta, \mu) = \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta-\mu)^{-1})) \tag{9}$$
$$G_2(\beta) = \tau(\gamma(\beta^{-1})) \tag{10}$$

,and $\tau(\gamma(\beta^{-1})) > \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta-\mu)^{-1})) > 0$

Proof of Theorem 1
The strategic region ξ_1 is defined by $\xi_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \varepsilon_2 \geq G_1(\beta, \mu) = \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta-\mu)^{-1}))\}$. If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, lemma 4 states that both independent downstream firms would be supplied by U_1 when U_1, D_1 do not integrate and D_2 would be supplied by $U_1^v D_1$ when U_1, D_1 integrate. $U_1 D_1$ integrate if and only if

$$\Pi(U_1^v D_1; D_2) \geq \Pi(D_1; U_1) + \Pi(U_1; D_1, D_2)$$

If U_1 and D_1 remain independently owned then the profits of the firms will be,

$$\Pi(D_1; U_1) = \pi + (1-\beta)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma(\beta^{-1})))$$
$$\Pi(U_1; D_1, D_2) = 2(\beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1}))$$

When U_1, D_1 integrate, the profit function of the integrated firm will be,

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma(1)) + (\beta-\mu)(\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1}))) - \gamma(1) - \gamma((\beta-\mu)^{-1})$$

U_1, D_1 integrate if the total profit is higher than the sum of the two independent firms' profits.

$$\begin{aligned}
&\Pi(U_1^v D_1; D_2) \geq \Pi(D_1; U_1) + \Pi(U_1; D_1, D_2) \Leftrightarrow \\
&\pi + \varepsilon_1 + \tau(\gamma(1)) + (\beta - \mu)(\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1}))) - \gamma(1) - \gamma((\beta-\mu)^{-1}) \\
&\geq \pi + \varepsilon_1 + \tau(\gamma(\beta^{-1})) + (\beta-\mu)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - 2\gamma(\beta^{-1}) \qquad (11) \\
&\Leftrightarrow \tau(\gamma(1)) + (\beta-\mu)(\tau(\gamma((\beta-\mu)^{-1}))) - 2\tau(\gamma(\beta^{-1})) \geq \gamma((\beta-\mu)^{-1}) + \gamma(1) - 2\gamma(\beta^{-1})
\end{aligned}$$

Since $U_1^v D_1$ optimizes its profit, it must be true that

$$\begin{aligned}
&\tau(\gamma(1)) - \gamma(1) > \tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1}) \quad \text{and} \\
&(\beta-\mu)(\tau(\gamma((\beta-\mu)^{-1}))) - \gamma((\beta-\mu)^{-1}) > (\beta-\mu)\tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1})
\end{aligned}$$

The inequalities imply that inequality 11 holds. Thus, U_1, D_1 vertically integrate for $\forall(\beta, \mu)$ and $(\varepsilon_1, \varepsilon_2) \in \xi_1$.

Proof of Theorem 2

The strategic region ξ_2 is defined by $\xi_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in R_3 \ \text{ and } \ (\varepsilon_1, \varepsilon_2) \notin R_2\}$. If $(\varepsilon_1, \varepsilon_2) \in \xi_2$, lemma 4 states that both independent downstream firms would be supplied by U_1 when U_1, D_1 do not integrate and D_2 would be supplied by U_2 when U_1, D_1 integrate. $U_1 D_1$ integrate if and only if

$$\Pi(U_1^v D_1) \geq \Pi(D_1; U_1) + \Pi(U_1; D_1, D_2)$$

If the firms remain independently owned, the profit functions of the upstream firm and the downstream firm will be

$$\begin{aligned}
&\Pi(D_1; U_1) = \pi + (1-\beta)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) \\
&\Pi(U_1; D_1, D_2) = 2(\beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1}))
\end{aligned}$$

If U_1, D_1 integrate, the integrated firm will not earn profits surplus by supplying for its downstream rival. Post merger, the profit function of the integrated firm will be,

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma(1)) - \gamma(1) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$

U_1, D_1 integrate if and only if there is a positive gain,

$$\begin{aligned}
&\Pi(U_1^v D_1) \geq \Pi(D_1; U_1) + \Pi(U_1; D_1, D_2) \Leftrightarrow \\
&\pi + \varepsilon_1 + \tau(\gamma(1)) - \gamma(1) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) \geq \pi + (1+\beta-\mu)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - 2\gamma(\beta^{-1}) \\
&\Leftrightarrow (\beta-\mu)\varepsilon_1 + \mu\varepsilon_2 \leq \tau(\gamma(1))\ \gamma(1) - (1+\beta)\tau(\gamma(\beta^{-1})) + 2\gamma(\beta^{-1}) = X_1^m(\beta)
\end{aligned}$$

We can define $X_1(\beta,\mu) = \frac{X_1^m(\beta)}{\beta-\mu}$ and $F_1(\beta,\mu) = \frac{F_1^m(\beta,\mu)}{\beta-\mu}$

$$
\begin{aligned}
&X_1^m(\beta) \geq F^m(\beta,\mu) \Leftrightarrow \qquad (12)\\
&\tau(\gamma(1)) - \gamma(1) - (1+\beta)\tau(\gamma(\beta^{-1})) + 2\gamma(\beta^{-1}) \geq \gamma((\beta-\mu)^{-1}) - \mu\tau(\gamma(\beta^{-1})) - (\beta-\mu)\tau(\gamma((\beta-\mu)^{-1}))\\
&\Leftrightarrow \tau(\gamma(1)) - \gamma(1) - (1+\beta-\mu)\tau(\gamma(\beta^{-1})) + 2\beta\gamma(\beta^{-1}) + (\beta-\mu)\tau(\gamma((\beta-\mu)^{-1})) - \gamma((\beta-\mu)^{-1}) \geq 0
\end{aligned}
$$

We know that the integrated firm optimizes the investment levels for each downstream firm.Thus,

$$
\begin{aligned}
&\tau(\gamma(1)) - \gamma(1) \geq \tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1})\\
&(\beta-\mu)\tau(\gamma((\beta-\mu)^{-1})) - \gamma((\beta-\mu)^{-1}) \geq (\beta-\mu)\tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1})
\end{aligned}
$$

Rearranging the two inequalities,

$$
\begin{aligned}
&\tau(\gamma(1)) - \gamma(1) + (\beta-\mu)\tau(\gamma((\beta-\mu)^{-1})) - \gamma((\beta-\mu)^{-1}) \geq (1+\beta-\mu)\tau(\gamma(\beta^{-1})) - 2\beta\gamma(\beta^{-1})\\
&\Rightarrow X_1^m(\beta) \geq F^m(\beta,\mu)
\end{aligned}
$$

$$X_1^m(\beta) \geq F^m(\beta,\mu) \Leftrightarrow X_1(\beta,\mu) \geq F_1(\beta,\mu)$$

. As a result, the firms integrate if and only if

$$
\begin{aligned}
&\varepsilon_1 + \frac{\mu}{\beta-\mu}\varepsilon_2 \leq X_1(\beta,\mu)\\
&\text{where} \quad X_1(\beta,\mu) = \frac{\tau(\gamma(1)) - \gamma(1) - (1+\beta)\tau(\gamma(\beta^{-1})) + 2\gamma(\beta^{-1})}{\beta-\mu}
\end{aligned}
$$

,when $(\varepsilon_1,\varepsilon_2) \in \xi_2$.

Proof of Theorem 3
The strategic region ξ_3 is defined by $\xi_3 = \{(\varepsilon_1,\varepsilon_2) : (\varepsilon_1,\varepsilon_2) \in \varepsilon_1 - \varepsilon_2 < 0\}$. If $(\varepsilon_1,\varepsilon_2) \in \xi_3$, lemma 4 states that both independent downstream firms would be supplied by U_2 when U_1, D_1 do not integrate and D_2 would be supplied by U_2 when U_1, D_1 integrate. U_1D_1 integrate if and only if

$$\Pi(U_1^v D_1) \geq \Pi(D_1; U_2)$$

When the firms stay independent, the profit functions of the firms will be

$$
\begin{aligned}
&\Pi(D_1;U_2) = \pi + (1-\beta)(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1})))\\
&\Pi(U_1) = 0
\end{aligned}
$$

If the integration occurs, the profit function of the integrated firm will be,

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma(1)) - \gamma(1) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$

The integration takes place if and only if it is profitable for both parties.

That is, $\Pi(U_1^v D_1) \geq \Pi(D_1; U_2) \Leftrightarrow$

$$\pi + \varepsilon_1 + \tau(\gamma(1)) - \gamma(1) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) \geq \pi + (1 - \beta - \mu)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$
$$\Leftrightarrow \varepsilon_1 - (1-\beta)\varepsilon_2 \geq (1-\beta)\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) + \gamma(1)$$

If we define $X_2(\alpha) = (1-\beta)\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) + \gamma(1)$

Moreover $X_2(\alpha) < G_1(\beta, \mu)$.

$$X_2(\alpha) < G_1(\beta, \mu) \Leftrightarrow$$
$$(1-\beta)(\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) - \gamma(1) < \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta-\mu)^{-1})))$$

We know that

$$\tau(\gamma(1)) - \gamma(1) > \tau(\gamma((\beta-\mu)^{-1})) - \gamma((\beta-\mu)^{-1}) \quad \text{and}$$
$$\beta\tau(\gamma(\beta^{-1})) > \beta\tau(\gamma((\beta-\mu)^{-1})) > \gamma((\beta-\mu)^{-1}) \quad \text{since} \quad \gamma((\beta-\mu)^{-1}) > 0$$

Rearranging the inequalities, we get $G_1(\beta, \mu) > X_2(\beta)$.
U_1, D_1 vertically integrate if $(\varepsilon_1, \varepsilon_2) \in \Lambda_2 \cap \xi_3$ where,

$$\Lambda_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - (1-\beta)\varepsilon_2 \geq X_2(\beta)\}$$
$$X_2(\beta) = (1-\beta)\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) + \gamma(1)$$

Proof of Corollary 1
Let ε_2^* solves

$$\varepsilon_1 = 0$$
$$\varepsilon_1 - (1-\beta)\varepsilon_2^* = (1-\beta)\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) + \gamma(1) = X_2(\beta)$$

The $\varepsilon_2^* = \frac{X_2(\beta)}{\beta - 1}$ solves the equations. By theorem 3, any ε_2 which is less than ε_2^* lead to U_1, D_1 integration.

Proof of Lemma 5
Follows from lemma 2.

Proof of Lemma 6

If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, an independent D_2 would be supplied by $U_1^v D_1$ when U_2, D_2 do not integrate. $U_2 D_2$ integrate if and only if $\Pi(D_2^v U_2) \geq \Pi(D_2; U_1^v D_1) + \Pi(U_2)$.
The profits of $U_2^v D_2$ when U_2, D_2 integrate and U_2 and D_2 when U_2, D_2 do not integrate are

$$\Pi(D_2^v U_2) = \pi + \varepsilon_2 + \tau(\gamma((1))) - \mu(\varepsilon_1 + \tau(\gamma((1)))) - \gamma((1))$$
$$\Pi(D_2; U_1^v D_1) = \pi + (1-\beta)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma((1))))$$
$$\Pi(U_2) = 0$$

$\Pi(D_2^v U_2) \geq \Pi(D_2; U_1^v D_1) + \Pi(U_2)$ if and only if

$$(1-\beta)\varepsilon_1 - \varepsilon_2 \leq \tau(\gamma(1)) - \alpha\tau(\gamma((\beta-\mu)^{-1})) - \gamma(1)$$

Then,U_2, D_2 counter integrate when U_1, D_1 integrate in ξ_1 if $(\varepsilon_1, \varepsilon_2) \in CM_1$ where,

$$C_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{\varepsilon_2}{1-\beta} \leq S_1(\beta, \mu) \text{ where}$$
$$S_1(\beta, \mu) = \frac{\tau(\gamma(1)) - \gamma(1)}{1-\beta} - \tau(\gamma((\beta-\mu)^{-1}))\}$$
$$CM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in C_1 \cap \xi_1\}$$

Proof of Theorem 4
If $(\varepsilon_1, \varepsilon_2) \in \xi_1$ and $(\varepsilon_1, \varepsilon_2) \notin CM_1$, theorem 1 states U_1, D_1 integrate if there was no counter merger. In equilibrium, lemma 7 states U_2, D_2 would not integrate even if U_2, D_2 integration is possible. Hence, U_1, D_1 integrate and D_2 remain independently owned and is supplied by $U_1^v D_1$ in the equilibrium.

Proof of Theorem 5
If $(\varepsilon_1, \varepsilon_2) \in CM_1 \cup \xi_2$, U_2, D_2 would counter integrate when U_1, D_1 integrate. D_1 either integrates with U_1 or remains independently owned. D_1 remains independently owned if U_2, D_2 counter integration decreases U_1, D_1's profits. D_1 would be supplied by U_1 and U_1 would supply both D_1 and D_2 in $CM_1 \cup \xi_2$ in case of independent ownership. U_1, D_1 integrate if

$$\Pi(U_1^v D_1) \geq \Pi(U_1; D_1, D_2) + \Pi(D_1; U_1)$$

$U_1^v D_1$ profit when U_1, D_1 integrate and U_2, D_2 counter integrate is

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma((1))) - \gamma((1)) - \mu(\varepsilon_2 + \tau(\gamma((1))))$$

U_1 and D_1's stand alone profits are

$$\begin{aligned} &\Pi(D_1; U_1) = \pi + (1-\beta-\mu)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) \\ &\Pi(U_1; D_1, D_2) = 2(\beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1})) \end{aligned} \tag{13}$$

Then U_1, D_1 integrate if and only if

$$(\beta-\mu)\varepsilon_1 + \mu\varepsilon_2 \leq (1-\mu)(\tau(\gamma(1)) - \tau(\gamma(\beta^{-1}))) - \gamma(1) - \beta\tau(\gamma((\beta-\mu)^{-1})) + 2\gamma((\beta-\mu)^{-1})$$

Rearranging the inequality, U_1, D_1 integrate if and only if $(\varepsilon_1, \varepsilon_2) \in TM_1$ where,

$$\begin{aligned} &T_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \frac{\mu\varepsilon_2}{\beta-\mu} \leq S_2(\beta,\mu) \text{ where} \\ &S_2(\beta,\mu) = \frac{(1-\mu)(\tau(\gamma(1)) - \tau(\gamma(\beta^{-1}))) - \gamma(1) - \beta\tau(\gamma((\beta-\mu)^{-1})) + 2\gamma((\beta-\mu)^{-1})}{\beta-\mu}\} \\ &TM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in T_1 \cap (CM_1 \cup \xi_2)\} \end{aligned}$$

Proof of Theorem 6

If $(\varepsilon_1, \varepsilon_2) \in \xi_3$, U_2, D_2 would counter integrate when U_1, D_1 integrate. D_1 either integrates with U_1 or remains independently owned. D_1 remains independently owned if U_2, D_2 counter integration decreases U_1, D_1's profits. D_1 would be supplied by U_2 and U_1 would supply neither D_1 or D_2 in ξ_3 in case of independent ownership. U_1, D_1 integrate if

$$\Pi(U_1^v D_1) \geq \Pi(U_1) + \Pi(D_1; U_2)$$

$U_1^v D_1$ profit when U_1, D_1 integrate and U_2, D_2 counter integrate is

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma((1))) - \gamma((1)) - \mu(\varepsilon_2 + \tau(\gamma((1))))$$

U_1 and D_1's stand alone profits are

$$\begin{aligned} &\Pi(D_1; U_2) = \pi + (1-\beta-\mu)(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) \\ &\Pi(U_1;) = 0 \end{aligned} \tag{14}$$

Then U_1, D_1 integrate if and only if

$$\varepsilon_1 - (1-\beta)\varepsilon_2 \leq (1-\beta)(\tau(\gamma((\beta-\mu)^{-1})) - \tau(\gamma(1)) + \gamma(1)) - \beta\tau(\gamma((\beta-\mu)^{-1}))$$

Rearranging the inequality, U_1, D_1 integrate if and only if $(\varepsilon_1, \varepsilon_2) \in TM_2$ where,

$$\begin{aligned} &T_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - (1-\beta)\varepsilon_2 \leq S_3(\beta,\mu)\} \text{ where} \\ &S_3(\beta,\mu) = (1-\beta)(\tau(\gamma((\beta-\mu)^{-1})) - \tau(\gamma(1)) + \gamma(1)) - \beta\tau(\gamma((\beta-\mu)^{-1})) \\ &\xi_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \notin R_3\} \\ &TM_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in T_2 \cap \xi_3)\} \end{aligned}$$

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