Continuing Conflict

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Abstract

A relatively small but growing literature in economics examines conflictive activities where agents allocate their resource endowments between wealth production and appropriation. To date, their studies have employed a one period, static game theoretic framework. We propose a methodology to extend this literature to a dynamic setting, modeling continuous conflict over renewable natural resources between two rival groups. Investigating the system's steady states and dynamics, we find two results of general interest. First, Hirshleifer's "paradox of power" is self-correcting. Second, if productive activities cause damage to disputed resources, the introduction of a small amount of conflictive activity enhances social welfare.

JEL Numbers: D74, Q20

Keywords: Conflict, Dynamics, Paradox of Power, Renewable Resources

*We thank Michael Baye, Tom Lyon, Elinor Ostrom, Charles Anderton, two anonymous reviewers, associate editor Jack Hirshleifer and the editor of this journal for valuable suggestions and comments. We also thank seminar participants at 2nd World Congress of Environmental and Resource Economists 2002 (Monterey, CA), EAERE 2000 (Crete), the Eighth Junior Master Class for the Study of Formal Theory in Political Science at the University of Illinois, and the Indiana University workshop on Business Economics and Public Policy for useful discussions. We are responsible for any errors in the paper. Corresponding author J. W. Maxwell, Kelley School of Business, Indiana University, 1309 E. 10th St., Bloomington, IN 47405-1701. Tel.: +1 812 855 9219; fax: +1 812 855 3354. E-mail address: jwmax@indiana.edu.

1 Introduction

There is a relatively small but growing literature in economics, based on the seminal work of Hirshleifer (1988, 1989, 1991), that focuses on the allocation of endowed productive resources between wealth production and appropriation. Models in this literature, including ours, share three basic features. First, it is assumed that conflict is a rational activity. Second, a well-defined and enforced system of property rights over at least some resources does not exist. Third, the actors are assumed to be myopic, acting to maximize their current wealth.[1]

A fourth feature shared by the literature that follows Hirshleifer, but not by our model, is that these models are static. We offer a general method that takes an initial step towards extending Hirshleifer's framework to a dynamic setup by accounting for the full interplay across time over the disputed wealth and between conflict and productive activities.

Hirshleifer develops a one period game theoretic framework that augments the standard economic theory of production and exchange by treating wealth appropriation as a basic economic activity. Production is peaceful, whereas appropriation is conflictive. An underlying assumption of all the models that employ this framework is that at least the portion of the wealth of rival actors (typically two) that is open to appropriation lacks well-defined or enforceable property rights. Thus, a situation of anarchy prevails. Each actor's ultimate share of the contested wealth depends on its allocation of resources to appropriation. The contested wealth also depends on this allocation: the greater the amount of resources that is allocated to conflict, the smaller the amount of resources that is available for production of the contested wealth. Each group maximizes its current wealth by allocating its resources among production and appropriation with this basic tension in mind.

Hirshleifer's basic framework has been extended in various ways to include differentiation between defensive and offensive activities, trade, and the use of various functional forms. However, each of these extensions employs a similar one period game theoretic framework. Hirshleifer (1995) takes an initial step towards a dynamic approach through successive iterations of the one shot game, beginning with out-of-equilibrium resource allocations and examining conditions that ensure convergence to equilibrium allocations. However, this approach is not fully dynamic: it does not specify equations of motion for any variables, time is not a variable in the model, and the condition for dynamic stability is not derived based on standard dynamic analysis. The condition stated, instead, simply ensures the existence of a one-period-based internal solution. Several authors in this literature, including Hirshleifer,

[1] Works in this area include, among others, Hirshleifer (1995), Skaperdas (1992), Grossman and Kim (1995), Neary (1997), Anderton et al. (1999), and Reuveny and Maxwell (2001).

are aware that this is a limitation of the approach and have called for a dynamic extension of the basic framework (see e.g., Skaperdas, 1992; Hirshleifer, 1995; Grossman and Kim, 1995). The goal of this paper is to take an initial step in answering these calls.[2]

We develop a relatively simple method to extend Hirshleifer's static framework to a dynamic setting, acknowledging two important motivations for conflict. First, conflict spoils are used not only for instant gratification, but may also be invested to increase one's own pool of resources. This pool may then be used for future productive and conflictive activities. Second, parties find themselves in conflict because wealth is generated, at least partially, from disputed resources.

Our approach distinguishes between two types of resources. We label as "captive" those resources that cannot be appropriated by rival actors such as innate intellectual, physical, and human capital. We label as "disputed" those resources that may be appropriated by rivals, such as common pool natural resources or the human capital of employees (as opposed to that of firm owners). Generally, the usage rate of resources impacts their availability in future periods, which can generate tension when resources are disputed. Thus, we propose a model of continuing conflict that distinguishes between these two types of resources and tracks their interactions by modeling their growth and usage rates, both of which are impacted by conflict.

Our method assumes that the actors are myopic. A complete economic model of conflict in the presence of property rights might allow for optimal time-path decisions where the actors take into account the consequences of their future actions. This approach has not yet been taken in the Hirshleifer-based literature. We need to emphasize that our results only apply to myopic agents: the behavior of non-myopic agents may differ from the one presented here. We defer the development of a full dynamic model of conflict with non-myopic actors to future research. Such a model would likely be much more complex mathematically than our model.

[2] It is worth noting that the issue of conflict dynamics has been considered in prior literature, but not based on Hirshleifer's framework. Usher (1989) develops a model in which a society moves between anarchy and despotism. However, he provides no specific solution for the transition between these two states. Brito and Intriligator (1985) develop a two period game theoretic model that studies the circumstances under which conflict over the rights to a flow of a single good leads to the outbreak of war. This model is basically static, however, as the two period game is played only once. Powell (1993) models the guns-versus-butter problem for two states, using a repeated game. Alternating, each period, one state or the other decides on military spending, and in the next period on whether to attack the other state. Our model differs from his in that we distinguish between stock and flow variables, and our actors respond to each other's resource allocation decisions.

Our basic approach can be applied to study the determinants and implications of conflict in various political and/or economic settings. For example, consider two nations engaged in war. The ultimate goal of the conflict is to gain control over each other's productive infrastructure. Each nation will devote a portion of its resource endowment to the conflict. The remaining resources would be devoted to general wealth creation via production activities. These resources could be combined – for example, via trade – with the productive resources of third party nations to generate wealth. In the absence of total defeat, both the victor and the loser nations are likely to invest remaining wealth to develop their economies, which in turn generates resources available for future conflict. Since only portions of total resource endowments are devoted to conflict activities, all the resources in the model need to be tracked over time in order to determine the size of the wealth and the intensity of the conflict at stake at any point in time.

Similarly, consider two firms that compete over a pool of potential profits by investing in R&D and marketing. The victor firm in any period will be better positioned to capture potential profits in subsequent periods. It will have greater resources to devote to product development, and it may enjoy a greater level of customer loyalty. However, highly skilled managers and researchers can be thought of as a disputed resource because they could leave one firm to join another for the right price. Firms combine their wealth with managerial and research knowledge to generate future wealth.

Finally, in a domestic political setting consider two competing parties combining their political talents and campaign contributions to attract potential voters and, therefore, political power. While party funds and political talent may be thought of as captive resources (since the ideologies of individuals rarely change enough to cause them to switch parties), the common pool of potential voters can be thought of as a disputed resource. In each of the three examples, the property rights over the disputed resource are, respectively, disputed, weak, and nonexistent.[3]

Section 2 sets out our general modeling approach under conditions of conflict and cooperation (no conflict). The general set up allows us to illustrate the model's static and dynamic components and describe how they fit together. We shall see that the model generates complex dynamics that can only be analyzed via the use of specific functional forms.

[3] Our business example comes closest to a case where property rights over the disputed resource (managerial and research talent) exist. For example, firms might employ no-compete clauses in the employment contracts of their employees. However, these are of finite duration. Short of slavery, property rights over human resources cannot be complete.

Keeping in mind that our methodology is generalizable, we apply it to conflict and cooperation between two groups over renewable resources in Section 3.[4] Both the conflict and cooperation models employ a predator-prey framework in that the human populations prey on a natural resource stock that is essential for their procreation. In the conflict model, each period the groups divide their population endowments between harvest and conflict activities, ultimately resulting in a share of the combined harvest of the two groups. Periods are linked in two ways. First, the harvest of each group depends not only on its labor allocation but also on a common pool resource stock that changes over time depending on harvest activity and its own natural growth rate. Second, each group's population growth rate depends on its ultimate share of the total harvest. These links give rise to a complex dynamic interaction between conflict, harvest, population and natural resources. For comparative purposes, we also present a cooperative model in which the two groups act as one, devoting all their captive resources to harvesting. It is worth reiterating that although the two groups are cooperating, their basic relationship with nature is still one of predator and prey.

In Section 4, we study the statics and dynamics of the conflict and cooperation models. The conflict model has four corner steady states that exhibit either no population in one or both groups, or no resource stock. We focus on a fifth steady state of the model, in which both rival groups and the resource stock coexist. The cooperation model exhibits two corner steady states with no population, one with a stock of the common poll resource, and one without it. As with the conflict model, we focus on a third, internal steady state.

In Reuveny and Maxwell (2001), we develop a related application, using different specific functional forms in order to study conflict over resources in primitive historical societies, and apply this conflict to contemporary less developed societies. The current paper proposes a methodology to extend the general economic literature on conflict to a dynamic setting, and develops two propositions: one on the potentially positive welfare implications of conflict over renewable resources compared with cooperation, and the second on Hirshleifer's paradox of power.

The first proposition echoes Usher's (1992) general observation that a little violence is good in the long run. We find this both at the internal steady states *and* in the model's dynamics. Specifically, we find that both per capita income and the level of per capita resources are greater under conflict. We also find that the system's dynamics are less volatile under conflict. The second proposition deals with Hirshleifer's paradox of power. Similar to

[4] Several studies of conflict that employ Hirshleifer's framework use this case to motivate their analyses (e.g., Hirshleifer, 1995; Neary, 1997). For an extensive review of the literature, see Reuveny and Maxwell (2001).

Hirshleifer, we find a paradox of power in the sense that the group with less captive resources at the beginning of each period wins the same share of the disputed prize. However, our dynamic analysis shows that over time the paradox of power under the conditions identified by Hirshleifer is self-correcting and does not exist as a steady state phenomenon. In the steady state of our dynamic model, each group has the same strength and possesses the same level of captive resources. Conclusions and research extensions are discussed in Section 5.

2 The models

2.1 The conflict model

The model with conflict features two competing actors. Depending on the specific application actors could be individuals, groups, firms, political parties, or any two contenders. We begin our development of the model by examining its static aspects. Each period, each actor undertakes productive activities that generate potential wealth that is disputed via conflictive activities. To keep our focus on the methodology of linking the periods, we ignore the potentially destructive effects of conflict. This assumption also allows us to compare our results to the existing literature, which uniformly ignores these destructive effects.[5]

Each actor possesses a captive resource stock that embodies both human and physical capital. The captive stock possessed by each actor i at time t is denoted by R_{it}. Each period, each actor allocates its entire captive resource stock between productive and conflictive or appropriative activities.[6] We denote the portion of captive resources devoted to such activities each period as F_{it}.[7] We denote the remainder of the captive resource stock by E_{it}. Thus,

$$R_{it} = E_{it} + F_{it};\ i = \{1, 2\}. \tag{1}$$

The portion of the captive stock devoted to productive activity by each actor, E_{it}, is combined with a disputed pool of resources, S_t, in order to generate wealth for the actor, H_{it}. Thus,

$$H_{it} = f_i\left(S_t, E_{it}\right);\ i = \{1, 2\}, \tag{2}$$

[5] In many areas, such as business and politics, conflict need not be destructive at all.

[6] We shall use the term conflict and appropriation interchangeably throughout the paper.

[7] While appropriation encompasses activities aimed both at capturing the rival actor's wealth and at defending its own wealth, it is common in the conflict literature to refer to these activities as fighting.

where f_i denotes the wealth production function of actor i, with properties $f_S > 0, f_{SS} \leq 0, f_E > 0, f_{EE} \leq 0$ and $f_{SE} > 0$.[8] In Section 3, S_t represents a stock of renewable natural resources and H_{it} represent's actor $i's$ harvest. In the context of firm competition, S_t could represent a stock of managerial and research talent that firms draw on and combine with their captive resources (*e.g.,* buildings, machinery, production line workers) to produce profits. In a political context, S_t could represent the total number of potential voters who are transformed into voters for party i (H_{it}) after viewing the party's campaign commercials (E_{it}).

In the economic literature on conflict, it is common to assume, as we do, that the total wealth the actors produce in period t

$$H_t = H_{it} + H_{jt} \qquad i, j = \{1, 2\}, i \neq j \tag{3}$$

is at stake. That is, the actors derive income from the total yield of their productive activities (H_t) by fighting over it. Actor i's period t income, Y_{it}, is defined as the portion, P_{it}, of total wealth that actor i obtains from the conflict process:

$$Y_{it} = P_{it} H_t \qquad i = \{1, 2\}. \tag{4}$$

It is natural to assume that, *ceteris paribus*, the portion of wealth won by actor i is increasing in the level of captive resources it devotes towards conflict and is decreasing in the level of captive resources its rival devotes towards conflict. That is,

$$P_{it} = p_i(F_{it}, F_{jt}) \qquad i, j = \{1, 2\}, i \neq j \tag{5}$$

where $p_{_{F_{it}}} > 0$ and $p_{_{F_{jt}}} < 0$. In the conflict literature P_{it} is referred to as actor i's contest success function. Returning to our political example, one could think of Y_{it} as political power. Such power usually depends on the relative support of the party's policies as compared to the policies of rival parties. Thus, one can think of F_{it} and F_{jt} as party resources spent on developing robust policies. Developing such policies is costly in that it often involves detailed research.

In each period, each actor allocates its captive resources between productive and conflictive activities in order to maximize its period t income. Expression (6) states the two optimizations, for actors i and j, respectively:[9]

$$\max_{F_{it}} p_i(F_{it}, F_{jt})[f_i(S_t, R_{it} - F_{it}) + f_j(S_t, R_{jt} - F_{jt})] \qquad i, j = \{1, 2\}, i \neq j. \tag{6}$$

[8] In the general case examined here, each actor has access to a different production technology. For simplicity, we drop the subscript i when using derivative notation. Capital subscripts denote first and second order derivatives.

[9] In (6), it is possible to assume a single production function $F_{ij}(S_t, F_{it}, F_{jt})$ instead of adding f_i and f_j. This will not change the gist of our model. The important point is to allow for each actor to find (via optimization) its own optimal allocation of resource endowment between productive and conflictive efforts.

The reader will observe that we are implicitly examining conflict situations in which each actor fully discounts the future. That is, our players are assumed to be myopic. We examine this type of behavior for two reasons. First, we are interested in comparing our results to the existing static literature on conflict, which also assumes that the actors are myopic. In order to focus on the impact of the dynamic setting that recycles income into future wealth, we choose not to alter the standard within-period optimizing behavior. Second, to extend the model to a fully dynamic optimization requires more than simply having agents solve a differential game. It also may require the storage of at least some portion of income or the diminished use of the disputed resource, both of which assume enforceable property rights. While this may be possible in some settings, in many settings the enforcement of well-defined property rights is impossible. In fact, the lack of enforceable property rights over a common resource may be at the root of many disputes. Without such enforcement, dynamic optimization is not justified.

In solving (6) we assume that conflict resources are chosen simultaneously and that each actor holds Nash conjectures with regard to the level of captive resources its rival will devote to conflict activity.[10] We also assume that the levels of disputed resources and each actor's captive resources are common knowledge. Under these assumptions, optimization by each actor yields the following best response (BR) functions:

$$\tilde{F}_{it} = BR_i\left(S_t, R_{it}, R_{jt}, F_{jt}\right) \qquad i, j = \{1, 2\}, i \neq j. \tag{7}$$

Equating the two best response functions in (7) to solve for the Nash equilibrium levels of conflict, we see that

$$F_{it}^{*} = N_i\left(S_t, R_{it}, R_{jt}\right) \qquad i, j = \{1, 2\}, i \neq j. \tag{8}$$

Expression (8) illustrates that the level of conflict each period depends on the stocks of the disputed and captive resources (S_t and R_i and R_j , respectively). Combining (8) with (1), we see the Nash equilibrium levels of each actor's allocations of productive resources, E_{it}, are also functions of S_t, R_{it}, and R_{jt} only. As such, the evolutions of the allocations of effort to conflict and to productive activities depend on the underlying evolution of the disputed and captive resource stocks.

We turn now to the dynamic portion of the model. The growth rate of the disputed resource stock is assumed to be affected by its underlying exogenous growth rate and the productive activities of the two actors. Recalling that the period t Nash equilibrium levels

[10] Hirshleifer (1988, 1989, 1991, 1995) examines several types of conflict behavior including simultaneous and sequential Nash (or Stackelberg).

of captive resources devoted to productive activities are functions only of the disputed and captive resources, we write the growth rate of the disputed resource stock as

$$\frac{dS_t}{dt} = s(S_t) - H_t(S_t, R_{it}, R_{jt}) \qquad i, j = \{1, 2\}, i \neq j, \tag{9}$$

where the underlying growth rate $s(S_t)$ is assumed to be nonnegative. In the example studied in Section 3, S represents a stock of natural resources, and the harvesting activities of each actor is damaging to the resource stock. If we think of S as the stock of potential customers, H_t could represent the sum of each firm's sales of a *durable* good. Similarly, political campaigns aimed at attracting elder voters might alienate younger persons, negatively impacting the total number of persons who might consider voting in future elections.

We assume that the growth rates of captive resources are positive functions of each actor's current captive resources and income. That is,

$$\frac{dR_{it}}{dt} = r_i(R_{it}, Y_{it}). \qquad i = \{1, 2\} \tag{10}$$

where $r_{R_{it}} > 0$ and $r_{Y_{it}} > 0$. We also assume that $\frac{dR_{it}}{dt}|_{Y_{it}=0} < 0$, which implies that if actor i's income is zero each period, the actor's captive resources will eventually decline to zero. For example, successful firms can invest profits for future battles while successful political parties and candidates often find it easier to raise funds. However, unsuccessful firms and parties often fold. Using (4), (5) and (8) we may write each actor's current income:

$$Y_{it} = Y_i(S_t, R_{it}, R_{jt}) \qquad i, j = \{1, 2\}, i \neq j. \tag{11}$$

Thus the evolution of the system is described by (9), (10), and (11). The tensions in the model are now clear. From (10) future captive resources are rising in current income and resources, but from (9) large amounts of current resources may harm the amount of disputed resource available for wealth creation in future periods. These tensions will be investigated in Sections 3 and 4 for specific functional forms.

2.2 The cooperation model

The cooperation model assumes that the two actors work as one cohesive unit. For example, we can imagine that the rival actors have merged, come to a self-enforcing agreement regarding how to split total wealth or share power, one actor has left the area or market, or that tacit or overt collusion has been achieved. As in the conflict model, the basic setup is assumed to be such that the actors are myopic and there are no secure property rights.[11]

[11] All derivatives of the functions in the cooperative model carry the same signs as their counterparts in the conflict model.

We assume that each group's captive resource is fully employed. That is, it is not possible to leave these resources idle.[12] To maximize income, all captive resources are devoted solely to the productive activity. Total wealth is

$$H_t = f(S_t, R_t), \tag{12}$$

which also represents the unified actor's total income, *i.e.,*

$$Y_t = H_t(S_t, R_t). \tag{13}$$

The growth rate of the common resource is given by

$$\frac{dS_t}{dt} = s(S_t) - H_t(S_t, R_t), \tag{14}$$

and the captive resource growth rate is given by

$$\frac{dR_t}{dt} = r(R_t, Y_t). \tag{15}$$

The model's dynamics are governed by equations (13)-(15) and exhibit the same type of tension between the captive and (previously) disputed resource stocks as seen in the conflict case. Given that we have a single unified actor, we might imagine that the actor has established property rights institutions that might allow dynamic optimization. We do not consider this case because our focus is to examine the impact of conflict in the dynamic setting. Thus, we wish to compare the steady state and dynamics exhibited in (9)-(11) with those in (13)-(15).

3 Conflict and cooperation over renewable resources

3.1 The conflict model

This section applies our framework to the case of conflict over a common pool of natural renewable resources, which constitutes (using our terminology) the disputed resource stock in the model. The two rival actors are conceptualized as two groups of people. Each group's captive resources (again, using our terminology) are fully embodied in its human capital (population).

Each group combines its captive resource with the disputed natural resource by harvesting the disputed resource stock for food. The harvested food is subject to appropriation by the

[12] This assumption is important, as discussed in Section 4.2 below.

rival group.[13] Thus, a group may raise its income (food consumption) either by harvesting the disputed natural resource or by appropriating its rival's harvest. We assume that fertility is rising in food consumption, resulting in a linkage between current income and the future stock of captive resources (human capital embodied in population).

The structure of the application follows the framework laid out in Section 2. To apply our framework we need to specify functional forms for the contest success function ($p(F_{it,}F_{jt})$), the productive activity function ($f(S_t, E_{it})$), the disputed resource dynamics (dS_t/dt differential equation) and the captive resource dynamics (dR_{it}/dt differential equation). To simplify the notation, we now drop the time subscripts.

Beginning with the contest success functions, we follow Hirshleifer (1988, 1991) in defining P_1 and P_2 as

$$P_i = \frac{\alpha_i F_i}{\alpha_i F_i + \alpha_j F_j} \qquad i,j = \{1,2\} \quad i \neq j \tag{16}$$

where α_1 and α_2 denote the relative efficiency of conflict effort of the two groups respectively. A more general form of the contest success function is $P_i = (\alpha_i F_i^m) / \left(\alpha_i F_i^m + \alpha_j F_j^m\right)$, where m is a so-called decisiveness parameter that measures the effectiveness of fighting resources.[14]

Hirshleifer (1995) sets $\alpha_1 = \alpha_2 = 1$ and examines the impact of changes in m. In that special model, the internal (static) equilibrium solution exists only when $m < 1$. This result arises from the structure of that particular model (where all the resource endowments are open to appropriation). The condition $m < 1$ is not crucial for the existence of the internal solution when, as in our model, only a portion of resources is assumed to be open to appropriation.[15]

As indicated by expression (16), we set $m = 1$ and assume $\alpha_1 > 0$ and $\alpha_2 > 0$.[16] As discussed later, this assumption allows us to focus attention on the effect of the disputed and

[13] The assumption that the total harvest (H), and not the disputed resource stock (S), is open to appropriation is one of modelling convenience. One could alternatively imagine that each group allocates effort to capturing and defending a portion of the disputed resource stock and is able to consume all the harvest from that stock.

[14] As noted by Skaperdas (1996) and Garfinkel and Skaperdas (2000), many studies set $m = 1$ and $\alpha_1 = \alpha_2 = 1$.

[15] For a similar result, see Hirshleifer (1989, 1991).

[16] While many Hirshleifer-type models are based on (16), some studies specify these equations as general forms. For example, in Neary (1997) $P_1 = \frac{f(F_1)}{f(F_1)+f(F_2)}$, where f is twice continuously differentiable. Since we investigate the dynamics in numerical simulations, we employ a specific form to be able to compare to Hirshleifer's 1991 and 1989 papers. In Reuveny and Maxwell (2001), we employ the methodology of this paper in the investigation of the effect of m in a dynamic setting.

captive resource dynamics on one of Hirshleifer's (1989, 1991) central results: the paradox of power.[17]

The harvesting technology of each group is modeled as in Brander and Taylor (1998).

$$H_i = \beta S E_i \qquad i = \{1, 2\} \tag{17}$$

where β denotes harvesting efficiency.[18]

Substituting (16) into (5) and (17) into (2) and performing the optimization in (6) yields the following reaction functions for groups 1 and 2, respectively:

$$\frac{F_1}{F_2} = \frac{\alpha_2 (E_1 + E_2)}{\alpha_1 F_1 + \alpha_2 F_2} \tag{18}$$

and

$$\frac{F_2}{F_1} = \frac{\alpha_1 (E_1 + E_2)}{\alpha_1 F_1 + \alpha_2 F_2}. \tag{19}$$

Solving (18) and (19) for F_1 and F_2 and using (1), (3), (4) and (17) yields the Nash equilibrium income levels in each period:

$$Y_i = \frac{\sqrt{\alpha_i}}{\left(\sqrt{\alpha_i} + \sqrt{\alpha_j}\right)} S\beta \frac{(R_i + R_j)}{2} \qquad i, j = \{1, 2\} \quad i \neq j. \tag{20}$$

The dynamic paths of population (each group's captive resource) are assumed to evolve according to the following equation:

$$\frac{dR_i}{dt} = \delta_i R_i \quad i = \{1, 2\}, \tag{21}$$

where the growth rate of population (δ_i) rises with per capita income. Specifically, $\delta_i = \varepsilon + \varphi_i$, where ε denotes the natural net birth rate (i.e., the difference between natural birth and mortality rates), which is assume to be negative, and $\varphi_i = \phi \frac{Y_i}{R_i}$ captures the notion of positive dependence of fertility on the per capita income ($\phi > 0$).[19] An alternative interpretation

[17] In Hirshleifer's static setting, the paradox of power result holds for $m = 1$, but disappears when m gets larger. We will show that in our dynamic setting the paradox of power result does not hold when $m = 1$ (under similar conditions to Hirshleifer's setting).

[18] This technology was proposed by Schaefer (1957), and is popular in the resource literature (e.g., Clark, 1990: Chapter 1; Brander and Taylor, 1998). Expressions (17) assume that each group's harvest does not depend on the harvest of the rival group. While this assumption is likely to hold when the resource is in abundance, when the resource is scarce each group's harvest may impose a negative externality on its rival's harvest, thereby reducing each group's marginal return to harvesting. While our assumption has been made for analytical tractability, the reader may note that the marginal return to harvesting, βS, falls as the resource declines.

[19] Heerink (1994) finds support for this assumption in lesser developed countries. This assumption may not describe higher income countries where fertility seems to decline with consumption. However, this criticism applies less in our case. Clearly our application is amenable to rivalries in underdeveloped, primitive societies. Note also that one could assume that ε and ϕ differ across groups. As this would complicate the analysis without adding much insight, and there is no a priori reason to assume that the groups differ in these respects, we do not consider such differences.

of the dependency of fertility on resource consumption may be that natural resources are essential for procreation. For instance, when food or water decline, fertility will decline. Since we adopt the convention that ε is negative, population will decline to zero for sufficiently low rates of fertility. Incorporating the fertility assumption into (21) we obtain the following two population differential equations:

$$\frac{dR_i}{dt} = R_i(\varepsilon + \phi\frac{Y_i}{R_i}) \quad i = \{1, 2\}. \tag{22}$$

As is standard, we assume that the natural growth of the resource $s(S_t)$ is given by the logistic growth (the term inside the square brackets in (23) below). Combining the logistic resource growth assumption with the harvesting functions results in the following resource differential equation:

$$\frac{dS}{dt} = \left[rS\left(1 - \frac{S}{K}\right)\right] - \beta SE_1 - \beta SE_2 \tag{23}$$

where r is the intrinsic rate of growth of the resource and K is the resource carrying capacity.[20] Recalling that $E_1 = R_1 - F_1$ and $E_2 = R_2 - F_2$, equation (23) can be re-written as follows:

$$\frac{dS}{dt} = \left[rS\left(1 - \frac{S}{K}\right)\right] - \beta S\left(\frac{R_1 + R_2}{2}\right). \tag{24}$$

Substituting (20) into (22), the system of three differential equations in (22) and (24) describes the dynamic evolution of S, R_1, and R_2.

3.2 The cooperation model

Absent conflict, we assume that all captive resources are devoted towards production so as to maximize current period income (Subsection 2.2). Using the same functional forms as we used to derive the conflict scenario, straightforward calculations show that the cooperative system's dynamics are described by

$$\frac{dR}{dt} = R(\varepsilon + \phi S\beta) \tag{25}$$

and

$$\frac{dS}{dt} = \left[rS\left(1 - \frac{S}{K}\right)\right] - \beta SR, \tag{26}$$

where R denotes total population. In the following section we will compare and contrast both the statics and dynamics of the conflict and cooperative systems.

[20] For details on logistic growth see Clark (1990: 10). The logistic function implies that the natural growth rate of the resource stock will be greatest when the stock is low. As the stock rises to its carrying capacity, growth will slow, eventually to zero (at $S_t = K$).

4 The impact of continuing conflict

We first examine the system's steady states, and then investigate its dynamic behavior.

4.1 Steady states and comparative statics

The steady state solutions to the conflict system are found by setting the time derivatives of R_1, R_2 and S in (22), and (24) to zero.

$$R_1 \left(\varepsilon + \phi \frac{\sqrt{\alpha_1}}{\left(\sqrt{\alpha_1} + \sqrt{\alpha_2}\right)} S\beta \frac{(R_1 + R_2)}{2R_1} \right) = 0, \tag{27}$$

$$R_2 \left(\varepsilon + \phi \frac{\sqrt{\alpha_2}}{\left(\sqrt{\alpha_1} + \sqrt{\alpha_2}\right)} S\beta \frac{(R_1 + R_2)}{2R_2} \right) = 0, \tag{28}$$

$$rS \left(1 - \frac{S}{K} \right) - \beta S \left(\frac{R_1 + R_2}{2} \right) = 0. \tag{29}$$

This system of equations (27)–(29) has five solutions. The first steady state, in which $R_1 = 0, R_2 = 0$, and $S = 0$, depicts a situation in which both populations have declined to zero following exhaustion of the natural resource. In the second steady state, $R_1 = 0, R_2 = 0$, and $S = K$, which describes a situation in which both populations have declined to zero before the resource has been depleted, and the resource recovers to its carrying capacity. The next two steady states are "semi-corner" solutions. In one steady state $R_1 = 0, R_2 = R_2^*$, and $S = S^*$, and in the other $R_1 = R_1^*, R_2 = 0$, and $S = S^*$, where $*$ indicates some positive level. The fifth steady state is denoted as an "internal" solution since it depicts a situation in which $R_1 > 0, R_2 > 0$, and $S > 0$ may exist. In this case, the solutions are given by

$$R_1 = \frac{2r}{\beta}\left(\frac{2\varepsilon}{K\beta\phi} + 1\right)\frac{\sqrt{\alpha_1}}{\left(\sqrt{\alpha_1} + \sqrt{\alpha_2}\right)}, R_2 = \frac{2r}{\beta}\left(\frac{2\varepsilon}{K\beta\phi} + 1\right)\frac{\sqrt{\alpha_2}}{\left(\sqrt{\alpha_1} + \sqrt{\alpha_2}\right)}, S = \frac{-2\varepsilon}{\beta\phi}. \tag{30}$$

The effort allocations for appropriation and harvesting are then given by

$$F_1 = \frac{R_1}{2}\sqrt{\frac{\alpha_2}{\alpha_1}}, F_2 = \frac{R_2}{2}\sqrt{\frac{\alpha_1}{\alpha_2}}, E_1 = \frac{R_1}{2}\frac{\left(2\sqrt{\alpha_1} - \sqrt{\alpha_2}\right)}{\sqrt{\alpha_1}}, E_2 = \frac{R_2}{2}\frac{\left(2\sqrt{\alpha_2} - \sqrt{\alpha_1}\right)}{\sqrt{\alpha_2}}. \tag{31}$$

Using (30) and (31) one can discern the model's comparative statics through differentiation. Before turning to an examination of these comparative statics however, it is worth noting that for sufficiently small K, β, or ϕ, an internal solution will fail to exist and our corner solution with no population will prevail. This simulates the breakdown of anarchy in our model. Naturally, if we assumed differences in fertility and harvesting between the two groups, anarchy may breakdown as one of the two groups becomes extinct.[21]

Since our intention is to examine continuing conflict, we focus on the impacts of the model's parameters on the level of conflict resources. Differentiation reveals that the steady state levels of conflict resources are rising in r, K, and ϕ. The effect of each of these parameters on F_1 and F_2 works through its effect on each group's population (see (31)). Increases in r and K raise the growth rate of the natural resource. This in turn raises the steady state population. An increase in ϕ raises population growth and the rate of harvest. At the new steady state, population is greater while the natural resource is lower.[22]

The effect of harvesting efficiency, β, on conflict is given next:

$$\frac{\partial F_i}{\partial \beta} = \frac{\sqrt{\alpha_j}}{\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right) K\beta^3\phi}(-4\varepsilon - K\beta\phi) \quad i,j=\{1,2\}; i \neq j. \tag{32}$$

Recalling the final equation in (30), the sign of (32) is positive if in steady state $S > K/2$, and negative if in steady state $S < K/2$. That is, if the steady state S is relatively high (low), raising harvesting efficiency raises (lowers) conflict. The steady state S rises with net mortality rate and falls with harvesting efficiency and fertility. Hence, when the mortality rate is high and harvesting efficiency and fertility are low, a rise in harvesting efficiency is more likely to raise conflict in the steady state.

From (31) we see that when group 2 gets better at conflict (α_2 rises), group 1 allocates more resources to conflict:

$$\frac{\partial F_1}{\partial \alpha_2} = \frac{2r}{\beta}\left(\frac{2\varepsilon}{K\beta\phi}+1\right)\frac{0.5\sqrt{\frac{\alpha_1}{\alpha_2}}}{\left(\sqrt{\alpha_1}+\sqrt{\alpha_2}\right)^2} > 0. \tag{33}$$

This is so because as group 2 gets better at conflict (α_2 rises), its marginal return to harvesting has risen relative to that of conflict (as it retains more of its harvest). It then allocates more effort to harvesting. This results in $\frac{\partial E_2}{\partial \alpha_2} > 0$ and $\frac{\partial F_2}{\partial \alpha_2} < 0$. The improved conflict efficiency of group 2 lowers the marginal return to group 1 from harvesting (as it retains less of its harvest). This results in $\frac{\partial E_1}{\partial \alpha_2} < 0$ and $\frac{\partial F_1}{\partial \alpha_2} > 0$. The strength of these effects grows with K, r and ϕ, and falls with ε.

[21] Note that since fighting resources are smaller than total resources it is possible that fighting may stop (anarchy may breakdown) prior to the extinction of a group, if we imposed exogenous conditions that dictate the level of fighting resources must be an integer (i.e., at least one person must be fighting). For more on the breakdown of anarchy and its implications, see Hirshleifer (1995) and Reuveny and Maxwell (2001).

[22] The effect of an increase in the mortality rate (ε) is opposite that of ϕ.

Using (20) and the first equation in (30), we see that an increase in α_1 raises group 1's population and income, and an increase in α_2 reduces group 1's population and income. Hence, a group that becomes better at conflict is able to sustain a higher income and population. But when its rival gets better at conflict, the group's population and income decline.

4.2 Comparing conflict and cooperation

To this point, we have examined the impact of the model's parameters on the conflict decision, and the impact of that decision on the internal steady state. We now examine the impact of the introduction of conflict on the steady state. To do so we first examine the cooperative model's steady states. In this case the system exhibits two corner steady states and one interior steady state. The two corner solutions exhibit no population; one features the natural resource stock at its carrying capacity, while the other has a resource stock at zero.[23] We state the interior steady state since we wish to compare it to its conflict counterpart. The interior steady state of the conflict model is characterized by

$$R_{nc} = \frac{r}{\beta}\left(1 + \frac{\varepsilon}{\beta K \phi}\right), \; S_{nc} = -\frac{\varepsilon}{\beta \phi}, \tag{34}$$

where the subscript nc, denotes no-conflict.

Comparing the final equations in (30) and (34), we see that conflict raises the steady state stock of resources, *ceteris paribus*. This result arises from the fact that conflict diverts resources away from resource-damaging harvesting activities. The reader is cautioned that we are able to obtain this result due to our assumption that conflict activity does not directly damage the resource. The types of conflict activities we are considering, therefore, include stealing and deterrence activities such as patrolling or building fortifications. However, it is interesting to note that even in cases where conflict directly damages the stock of natural resources, it might be easy to overestimate the improvement in the level of resources arising from a suspension of conflict activities since such a suspension would likely cause an increase in productive activities that could damage the natural resource stock.

Turning to population, we let $R_{1nc} = R_{nc}/2$ and assume that in the conflict scenario both groups are equally efficient at fighting, *i.e.*, $\alpha_1 = \alpha_2 = \alpha$, which we normalize to one. In this case we can rewrite R_1 (with conflict) as

$$R_1 = \frac{r}{\beta}\left(\frac{2\varepsilon}{K\beta\phi} + 1\right), \tag{35}$$

[23] Each steady state solution is derived using the same methodology as that used in the conflict model. For the sake of brevity, we do not include the derivations here.

while

$$R_{1nc} = \frac{r}{\beta}\left(\frac{\varepsilon}{2K\beta\phi} + \frac{1}{2}\right). \tag{36}$$

Subtracting (36) from (35) we obtain

$$R_1 - R_{1nc} = \frac{r}{2\beta}\left(1 + \frac{3\varepsilon}{K\beta\phi}\right). \tag{37}$$

Equation (37) implies that conflict may raise or lower the steady state population.[24]

Turning to the impact of conflict on per capita income, under our assumptions regarding equal conflict efficiency and symmetric group size, straightforward calculation reveals

$$Y_1^{pc} = \beta S \text{ and } Y_{1nc}^{pc} = \beta S_{nc}, \tag{38}$$

where the superscript *pc* denotes a per capita value. Comparing the final equations in (30) and (34), we see from (38) that conflict *raises* the level of income per capita.

Summarizing, conflict activity that does not directly damage the underlying natural resource or human resources works to raise the steady state levels of the natural resource and per capita income. Further, from (37) we see that for ε sufficiently close to zero, or for sufficiently large K, β, or ϕ, population may rise with conflict. Thus, it is possible for conflict to create a Pareto improvement.[25] We record these observations in the following proposition.

Proposition 1 *A little conflict is good. Conflict that does not directly damage the natural resource stock nor directly harm the population stock raises the steady state resource stock and per capita income and may also raise the steady state population stock.*

The result contained in proposition 1 depends on three important assumptions. First, in the cooperative model, all of the population is employed in harvesting; there is no free disposal condition in the model (i.e., it is impossible to leave proportion of the population idle). Second, the two groups behave myopically in both the conflict and cooperative models. Third, as noted, conflict essentially has no destructive effects on the natural resource and human population stocks.

[24] The expression $1 + 3\varepsilon/K\beta\phi$ can be written as $1 + 2\varepsilon/K\beta\phi + \varepsilon/K\beta\phi$. Since we are comparing internal steady states, we know from our solution to the conflict steady state that $1 + 2\varepsilon/K\beta\phi > 0$. However, $\varepsilon/K\beta\phi < 0$.

[25] For a similar result in a different context, see Usher (1992).

If in the cooperative model, the two groups can restrain their productive activities by leaving some part of the population idle (and without engaging in conflict), it may be possible to obtain higher per-capita incomes in the steady state. However, we do not allow for the free disposal of population, which may in fact be desired so as to limit the detrimental effects of harvesting. This assumption is reasonable in the current application. The captive resource of each group is labor. If one assumes that each unit of labor is embodied in a different individual, there will likely be great incentive to defect from any plan under which some labor resources remain idle, as long as agreements cannot be enforced in the absence of a well-developed system of property rights.

The desire to control one's harvesting activities also requires foresightedness, since some of the benefits from not harvesting today arrive in the future. Proposition 1 is derived under the assumption that the two groups are myopic, both under conflict and under cooperation. If the two groups were non-myopic, it is more likely that a setting of cooperation, where harvesting is chosen in order to maximize the sum of discounted future utilities, would lead to higher welfare than conflict. In the cooperative model, the two groups are likely to be more successful in constraining their harvesting activities (although this is not guaranteed, as stated previously); in the conflict model achieving such constraint is less likely. The benefit from constrained harvesting is the protection of the common pool resource for future use. In the conflict model, each group is not likely to realize the full benefits of constraining harvesting because the benefits must be shared with the competing group. Fully non-myopic behavior will also likely involve consumption smoothing, which may require harvest storage (if physically possible). This is more likely in a cooperative setting than in a conflict setting, where any stored goods could be open to appropriation.

In the current application the assumption of myopic behavior is reasonable. Recall that investment in the current example takes place via human fertility. Consequently, each time period can be viewed as a generation, or at least a large number of years. As the simulations below illustrate (under reasonable parameterization), convergence to the steady state is not rapid. It is reasonable to assume that it is difficult for any current generation to see the full impact of its activities on the steady state outcome.

Finally, our welfare result is dependent on the assumption that conflict does not damage the disputed or captive resources. It is not uncommon to have seen such conflict in primitive societies. In some cases competing tribes settled disputes through ritualistic contests which involved little bloodshed.[26] It is also worth noting that the proposition would hold if conflict

[26] For an elaborated discussion of conflict in primitive societies, see Reuveny and Maxwell (2001). Maxwell

caused a small level of damage to both types of resources. This seems likely in primitive societies where weapons were much less destructive than they are today, although exceptions certainly exist. Clearly, if conflict caused a high level of damage to either the disputed or the captive resources, the cooperative setting would be more attractive. If conflict would have damaged the disputed resource, the steady state harvest and per capita income would likely be lower. If conflict would have damaged the captive resources, measured steady state income might rise, but losses to captive resources should also be included in the welfare measure, making conflict less desirable.

4.3 System dynamics

To the best of our knowledge, the system of non-linear differential equations (22) and (24) does not have an analytical solution. Two methods may be used in such cases to learn about the dynamics: local stability analysis and numerical simulations. A local stability analysis involves linearizing the system around each steady state and finding its eigenvalues. This method is not tractable analytically in our conflict case since the system's characteristic equation is cubic.[27] Consequently, we investigate the dynamics via numerical simulations.

In order to simulate the system, we must settle on a particular parameterization. There are, of course, many sets of parameters from which one could choose. It is also clear that any such set is arbitrary to some extent, and the simulated trajectory may only apply to that set. We base our choice of parameters on the studies of Hirshleifer (1989) and Brander and Taylor (1998). We refer the reader to these two studies for a fuller discussion. Here we briefly describe the parameters.

The conflict efficiency parameters are taken from Hirshleifer (1989). In the base case α_1 and α_2 are set to 1. In a second case, $\alpha_1 = 1.25$ and $\alpha_2 = 0.75$. Brander and Taylor choose parameters that roughly mimic historical estimated information about Easter Island. The carrying capacity, K, is set to 12,000. The resource growth rate, r, is set to 0.04; the natural mortality rate of the population, ε, is set to -0.1. The fertility parameter, ϕ, is set to 4, and the harvesting efficiency parameter, β, is set to 0.00001. The initial populations are set to 40 each, and the initial resource stock is set at 12,000.

Figure 1 presents the base case for group 1's population (R_1), people allocated to conflict

and Reuveny (2000) develop a model in which conflict damages the disputed and captive resources. Their model, however, is not based on Hirshleifer's approach and does not employ game theory.

[27] Note that since the system is of an order higher than two, the phase diagram approach is not appropriate here.

(F_1), the resource stock (S), and income (Y_1).[28] As shown, the system cycles dampen over time.[29] Income is a leading indicator of population and conflict, while changes in the resource stock lead income changes. This is so because income affects fertility, and a rise in S raises harvest, income and fertility. We also find that conflict resources ($F_1$and F_2) are high when S is low. This observation deserves further comment since some studies link resource scarcities to conflict.[30] While our model generates outcomes that accord with the notion that resource scarcity induces conflict (*i.e.,* measured conflict is relatively high when per capita resources are relatively low), we *do not* assume that conflict is driven by resource scarcity. Thus, one must be cautious in drawing causal links between resource scarcity and conflict solely from the observation that conflict is high when the natural resources stock is low.

[Insert Figure 1 here: Base Case]

Figure 2 presents a simulation of the system without conflict. In this case, we divided the population level by 2 in order to make the figure comparable to Figure 1. We see that the cooperative system exhibits more volatility, conflict having the effect of reducing the volatility of both population and the resource stock. This occurs because at its core our application can be characterized as a predator-prey model in which humans are the predator, and the natural resource stock is the prey. As noted, conflict activity diverts resources from productive (predatory) activities. As a result, the impact of population growth on the resource stock is diminished, and the system is less volatile. Figure 2 also demonstrates that the cooperative model exhibits higher peaks of per capita resource scarcity, and therefore poverty, over time. Thus, a little conflict is also good in dynamic sense.

[Insert Figure 2 here: No conflict]

4.4 The paradox of power in a dynamic setting

One of the central results derived by Hirshleifer's static framework focuses on the effect of disparity in the actors' initial effort endowments on the actors' wealth. Hirshleifer distinguishes between two forms of what he calls the paradox of power. In the *strong form*, a disparity in initial effort endowments, or (using Hirshleifer's terminology) power endowments, between

[28] In this case the two groups are similar in every respect. The values for group 2 are therefore identical. In addition, we plot income at ten times its actual level to be able to view it on the same graph with the other variables.

[29] Figure 1 presents only the first few cycles of the system's evolution over time in order to illustrate clearly the interplay among the system's variables. Simulating the system beyond the 280 periods presented, we find that the amplitude of the cycles continues to decline over time. The system converges to a steady state after approximately 500 periods.

[30] See, *e.g.,* Homer-Dixon (1999).

two groups does not imply a difference in their income in the model's solution.[31] As he notes, "the contending parties will end up with exactly identical incomes ($I_1/I_2 = 1$) [I denotes income] regardless of the initial resource allocation" (Hirshleifer, 1991: 186). In the *weak form* of the paradox, "the final distribution of income will have less dispersion than the initial distribution of resources" (Hirshleifer, 1991: 187), but the incomes are not equal.

Hirshleifer (1989, 1991) shows that for the case with $m = 1$ in the contest success function, the *strong form* of the paradox holds as long as the initial power endowment ratio is not too small, so that the solution of the static game is interior. When the initial power endowment asymmetry is too large (i.e., one actor is too weak relative to the other), the model has no internal solution, and then the *weak form* of the paradox holds.

Hirshleifer's paradox of power result arises because the party with a lower power endowment devotes relatively more of it to conflict, while the party with a higher power endowment devotes relatively less of it to conflict. In the case where the power endowment are interpreted as population, the smaller group realizes the same income as the larger group (in the strong form of the paradox), resulting in a larger income per capita for the small group than for the larger group.

Hirshleifer also shows that as m grows from unity, the *strong form* of the paradox fails, and as m grows further (in Hirshleifer's numerical simulations this happens for $m > 4$), both the *strong* and the *weak forms* of the paradox of power fail. In this case, the model does not have an internal solution when the initial resource endowments are not equal.

We focus on the case of $m = 1$ in order to investigate whether the strong version of the paradox of power (which holds in Hirshleifer's static framework) holds in our dynamic version of Hirshleifer's framework. In our model, the initial effort endowments are equivalent to human resource endowments. The gains from conflict are "invested" (via fertility) to raise the effort stock (or stock of power). The ultimate impact of this investment is seen by examining our steady state levels of human resources and income. For the case with $m = \alpha_1 = \alpha_2 = 1$, we see from (30) that

$$R_1 = R_2 = \frac{2r}{\beta}\left(\frac{2\varepsilon}{K\beta\phi} + 1\right), \tag{39}$$

and from (20) that

$$Y_1 = Y_2 = S\beta\left(\frac{R_1 + R_2}{2}\right). \tag{40}$$

[31] In the conflict literature it is common to equate the power of each group with its level of resources.

Thus, in contrast to Hirshleifer's one period (static) result, where $R_1 > R_2$ while $Y_1 = Y_2$, we see that in our dynamic setup the prerequisite for the strong version of the paradox of power ($R_1 > R_2$) is *endogenously* eliminated. This steady state result holds for *all* initial conditions, as long as the internal solution of the model exists, which requires that the condition $\frac{2\varepsilon}{K\beta\phi} < 1$ holds.[32]

To illustrate this point numerically, we plot the relative allocations of effort to conflict ($\frac{F_1}{R_1}$ and $\frac{F_2}{R_2}$) and the per capita incomes ($\frac{Y_1}{R_1}$ and $\frac{Y_2}{R_2}$) in Figure 3. The initial population of group 1 is set to 100 and the initial population of group 2 is set to 40 (as in the base case).

[Insert Figure 3 here: paradox of power with equal conflict efficiencies]

Figure 3 illustrates that initially group 2 (the group which has less captive resources in period 0), as dictated by the paradox of power, devotes relatively more of its captive resources to conflict than does group 1. As a result the level of disputed resources won by group 2 are greater relative to its captive resources than the relative level of disputed resources won by group 1. Since in this application the level of captive resources of each group is identical to the group's population, we can say that the per capita income of group 2 is greater than the per capita income of group 1 ($Y_2/R_2 > Y_1/R_1$) This same phenomenon also happens in Hirshleifer's (1989, 1991) static game. The group with fewer people ends up with the same income as the group with the larger number of people, which is the essence of the strong version of the paradox of power. It follows that the income per capita of the smaller group is larger than the income per capita of the larger group.

In our model the interaction does not stop after one cycle. Moreover, there is feedback from income to the groups' captive resources and, therefore, the disputed resources (via the fighting and the productive allocation channels). As a result of group 2 having a larger income per capita than group 1, the population of group 2 grows faster than that of group 1, via fertility (see equation (22)). As time passes, as shown in Figure 3, the extent to which group 2 allocates relatively more effort (F_2/R_2) to conflict diminishes, while that of group 1 (F_1/R_1) rises. Eventually, the relative effort allocations equalize across groups. At this point, the populations of the two groups are also equal, determined by the resource and the population parameters of the model. In this example, the initial population level of each

[32] The reader may note that the disappearance of the paradox of power over time does not happen in Hirshleifer's (1988, 1989, 1991) static framework if one reiterates the game because the effort endowment of each group (its power, e.g., its population) is constant. As noted, time is not a variable in Hirshleifer's work. In our model, population (the group's power) is changing over time according to its law of motion (its differential equation). The endogeneity of the captive resource (population) drives the disappearance of the paradox of power, an effect that is not present in Hirshleifer's work.

group is the initial power of each group. Hence, we see that the group with the lower power endowment (or population) has caught up with the group with the higher power endowment (or population).[33] This analysis yields the following result.

Proposition 2 *When contested resources are invested for future conflict and productive activities, Hirshleifer's strong version of paradox of power is self-correcting, and consequently the preconditions for the paradox of power do not exist in the steady state, ceteris paribus.*

The disappearance of the paradox of power preconditions in steady state for the case $m = 1$ is driven by two factors. The first is the paradox itself. Namely, the resource-poor group generates a greater return on its conflict allocation (i.e., a greater per capita income). The second factor is that the returns to investment in effort (i.e., population growth) are rising in terms of per capita income (i.e., fertility rises with per capita income). The latter factor is plausible for our setting of conflict between relatively underdeveloped groups over natural resources.[34]

The endogenous disappearance of the paradox of power preconditions is significant because it suggests that persistent disparities of resource endowments among groups in conflict must be driven by exogenous factors. These factors could include, for example, laws written by the dominant group that restrict access to investment opportunities (such as education opportunities) for resource poor groups.[35]

Finally, we investigate the impact of differential conflict efficiencies through the simulation illustrated in Figure 4. In this simulation, group 1 is more efficient at conflict than group 2 ($\alpha_1 = 1.25$ and $\alpha_2 = 0.75$) and holds an initial advantage in terms of greater resources. Interestingly Figure 4 illustrates that per capita incomes are equalized while we know from

[33] It is worth noting that this result is dependent on the assumption that fertility rises with income per capita. Other "investment" functions may not generate this result (e.g., when investment returns rise with total income).

[34] If the growth rate of the captive resource does not rise with the level of investment per unit of captive resource, which may be the case with physical capital, the paradox of power may not disapear. However, we can think about non population-related cases for which this assumption may be reasonable. For example, *once proper economic and political institutions are in place*, some lesser developed countries (LDCs) tend to generate greater returns on investments than developed countries (DCs). Their economies subsequently grow faster, so that the per capita capital disparity between DCs and LDCs declines.

[35] The reader may note that the disappearance of the paradox of power is not an artifact due to equation (22). This equation implies that in steady state ($\frac{dR_i}{dt} = 0$) income per capita of both players ($\frac{Y_i}{R_i}$) is given by $-\frac{\varepsilon}{\phi}$. In our model, however, two groups that begin with different populations ($R_1 \neq R_2$), end up in steady state with the same income ($Y_1 = Y_2$) and the same population ($R_1 = R_2$). The result that $Y_1 = Y_2$ and $R_1 = R_2$ implies that Hirshleifer's (1989, 1991) paradox of power can not exist in the steady state of our model. Our result is driven by the populations changing endogenously (i.e., investment), whereas in Hirshleifer's model they are constant.

(30) that $R_1 > R_2$ in the steady state. Have we recovered a steady state paradox of power? No. Observe from (20) that the conflict efficient group (group 1) always earns *more* income than its less efficient rival. As a result it is able to sustain a higher group population in the steady state, leading to an equalization of per capita incomes in the steady state, but not of total incomes.

[Insert Figure 4 here: Paradox of Power with Differential Conflict Efficiencies]

5 Conclusions

This paper extends the literature in economics that examines agent decisions to allocate resources between productive and appropriative (or conflictive) activities in a static setting. We suggest a method that allows for the study of such conflict in a dynamic setting. Rather than simply repeating the same static game, we link periods of conflict by allowing the competing groups to invest the spoils for future conflictive and productive activities, and we model the dynamic paths of both captive and disputed resources.

Our principal goal in writing this paper was to illustrate a methodology for modeling continuing conflict in a fully dynamic setting. Illustrating our method in a specific case, we obtain two interesting insights. First, we find that if productive activity harms a disputed common stock (a common occurrence), then moderate conflict may work to preserve the resources and raise social welfare. This positive result holds not only in the steady state, but also along the system's dynamic path. Second, our methodology confirms the intuition that absent exogenous intervention, when Hirshleifer's paradox of power holds, it is necessarily a short run phenomenon. Once investment of conflict spoils is taken into account, we see that an initially resource-poor group will eventually obtain an equal level of resources as the initially resource-rich rival. Thus, our model suggests that long-term consistent resource disparities may be driven by exogenous factors such as laws established by dominant groups that deny equal access to investment opportunities or common resources.

As our work represents an initial step in the study of conflict dynamics, worthwhile research extensions are numerous. We mention three important areas here. First, while our assumption of myopic agents is appropriate where property rights regimes are weak or where rights over the common resource cannot be assigned (e.g., where the resource is potential voters), conflict situations can also arise in setting where property rights can be assigned. In these situations, agents may face a dynamic optimization problem, and their behavior may differ from that studied here.[36] Second, many forms of conflict harm both disputed and captive resources. In these settings our finding that conflict is beneficial may fail to hold. Finally, future work could explore the sensitivity of our results to modifications considered by Hirshleifer and others including different conflict protocols (such as Stackelberg), distinguishing between defensive and offensive activities and using different conflict success functions.

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Figure 1: Base Case

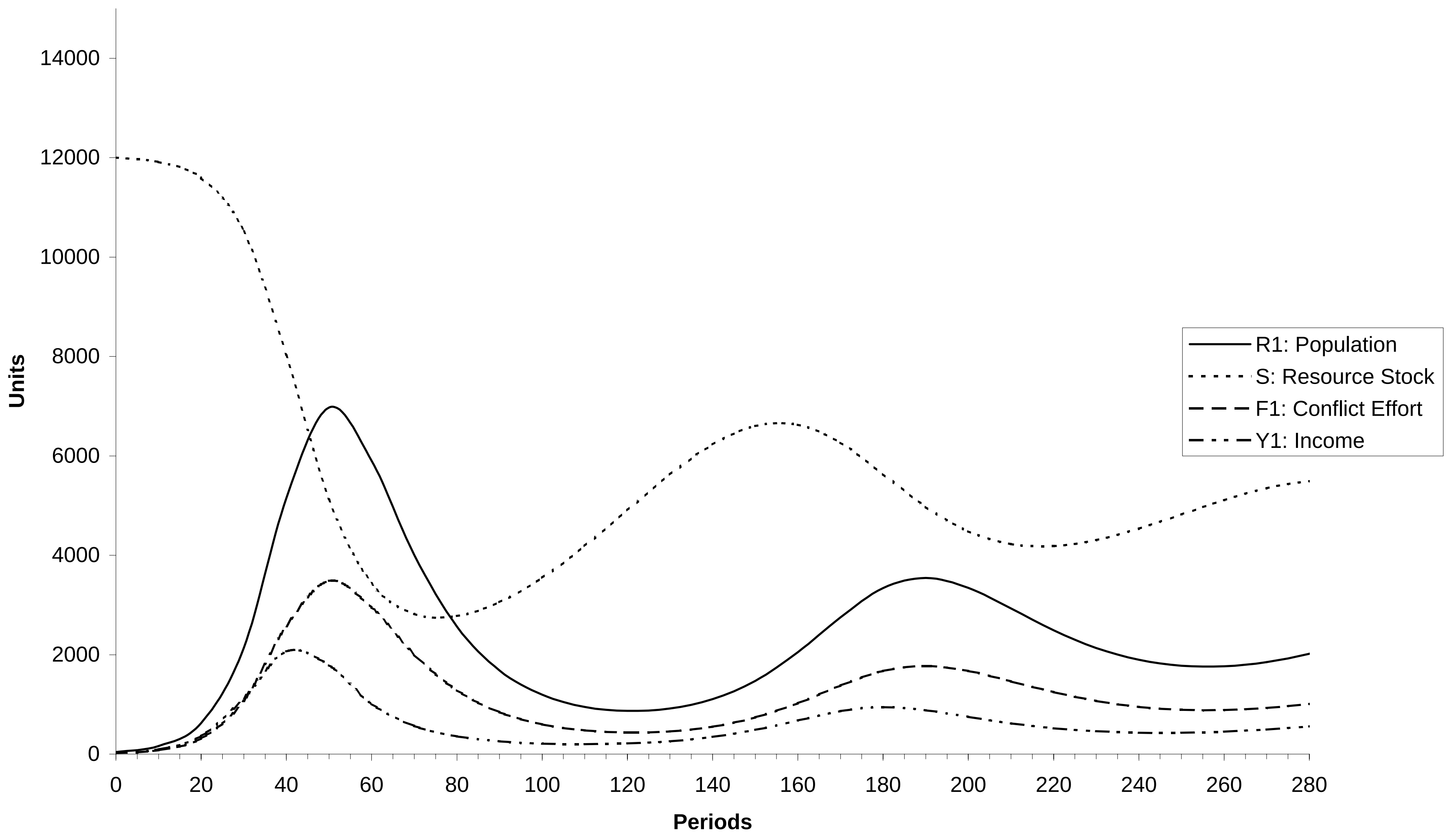

Figure 2: No Conflict

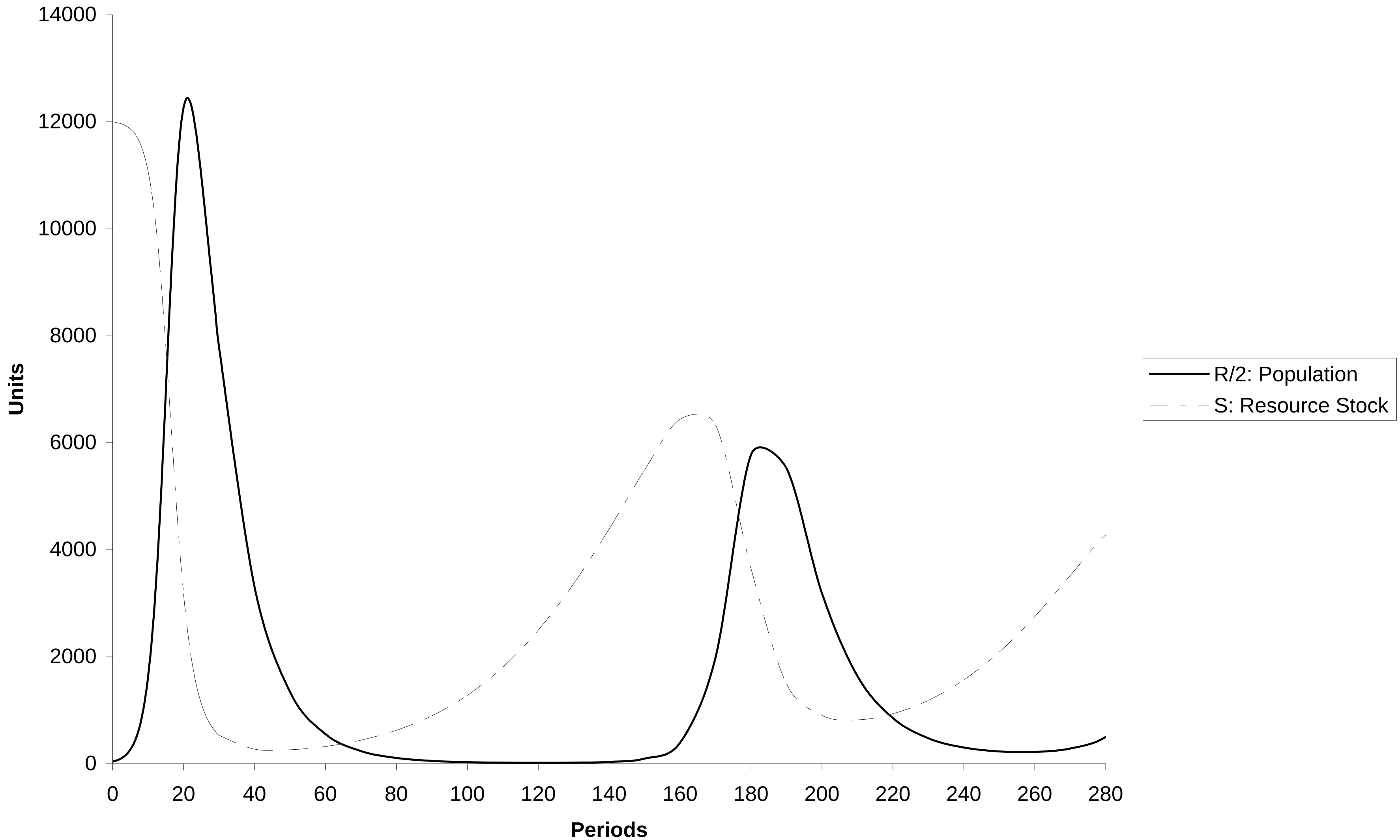

Figure 3: Paradox of Power with Equal Conflict Efficiencies

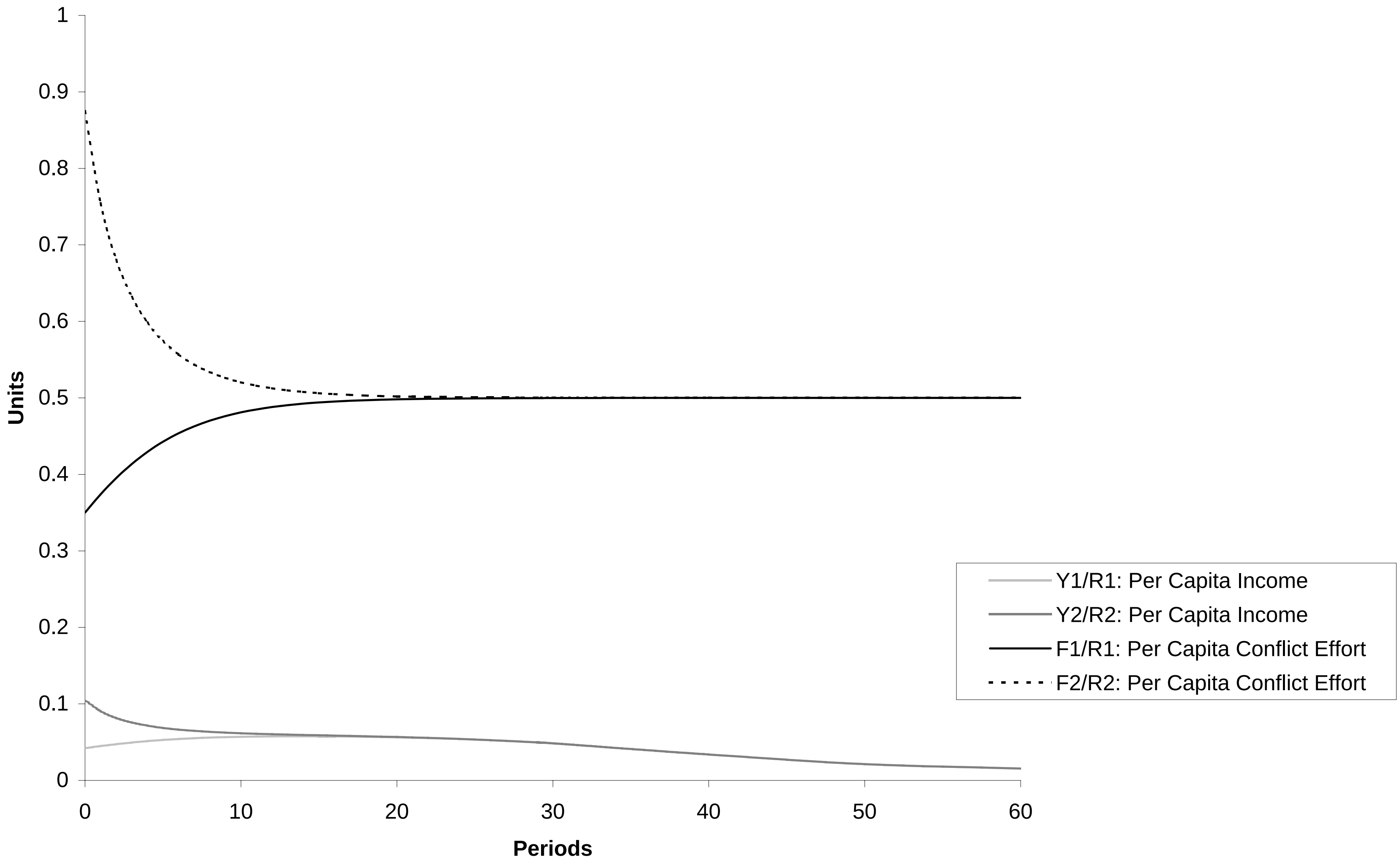

Figure 4: Paradox of Power with Differential Conflict Effenciencies