Multiple Equilibrium Problem and Non-Canonical Correlation Devices

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Abstract: A correlated equilibrium for a normal form game is a mediator whose recommendations the players find optimal to follow obediently. A (direct) correlation device or a mediator however may face the *multiple equilibrium problem* as the obedient strategy profile may be dominated by some other Nash equilibrium in the *extended game*. This paper addresses the question whether it is possible to avoid this problem by considering non-canonical correlation devices. The paper presents three different non-canonical correlation devices (one with and two without a *sunspot*) to understand this problem for a parametric version of the *Chicken* game.

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1. INTRODUCTION

A normal form game can be played using a correlation device. The correlation device first sends private messages to each player according to a probability distribution and then the players play the original normal form game. The original game is therefore extended. In this extended game, a pure strategy for any player is a map from the set of messages to the set of pure strategies of the original game. A correlation device is called *direct* or *canonical* if the set of messages is identical to the set of pure strategies of the original game, for each player. A (direct) correlated equilibrium (Aumann, 1974, 1987) can best be described as a mediator whose recommendations the players find optimal to follow obediently. In other words, for a correlated equilibrium, the strategy of following the mediator's recommendations constitutes a Nash equilibrium in the extended game.

Consider for example, the two-player game (*Chicken*) in Figure 1a. Each of the two players has two strategies, namely, *A* and *P*.

	A	*P*
A	0, 0	7, 2
P	2, 7	6, 6

Figure 1a

The direct correlation device in Figure 1b is indeed a correlated equilibrium for this game, i.e., the *obedient* strategy profile[1] (*AP*, *AP*) is a Nash equilibrium in the canonical extended game.

	A	*P*
A	1/5	2/5
P	2/5	0

Figure 1b

It is well known that an extended game, extended by a (direct) correlation device, may have

[1] *AP* represents the strategy of playing *A* when the recommendation is *A* and playing *P* when the recommendation is *P*.

equilibrium other than the obedient one. First of all, *babbling* equilibrium always exists; i.e., ignoring the messages from the device altogether and playing a Nash equilibrium of the original game constitutes trivially a Nash equilibrium in any such extended game. For example, (AA, PP) is a Nash equilibrium in the extended game in the above example. There may also be non-babbling Nash equilibrium in which players do not follow the mediator's suggestions. In the above example, the profile (PA, PA) in which the players play exactly the opposite of the recommended strategies is also a Nash equilibrium in the extended game. The other equilibrium may even dominate the obedient one, i.e., by playing the other Nash equilibrium, all players can obtain a higher payoff. Indeed, in the above example, the equilibrium (PA, PA) dominates the obedient equilibrium. A direct correlation device or a mediator therefore may face this *multiple equilibrium problem*.[2] The question thus arises whether it is possible to find a device that can implement the given outcome, however, does not suffer from this multiple equilibrium problem.

One should certainly mention here the recent advancement in the literature on mediated and unmediated (cheap) talk that can generate any correlated equilibrium of a given game (Aumann and Hart, 2002; Barany, 1992; Ben-Porath, 1998, 2002; Forges, 1990; Gerardi, 2000, 2001; Gossner, 1998; Gossner and Vieille, 2001; Lehrer, 1996; Lehrer and Sorin, 1997; Urbano and Vila 2002a, 2002b, 2002c). This literature uses sophisticated communication protocols or unmediated cheap talk that can take the place of a mediator. The central theme of this literature is that, depending on the specific conditions any correlated equilibrium can be obtained by a Nash equilibrium of a communication scheme. None of these papers however addresses the multiple equilibrium problem.

This paper simply takes the first step towards understanding the multiple equilibrium problem by restricting the attention to a particular type of multiple equilibrium problem and a

[2] The multiple equilibrium problem is well understood in other contexts, such as, implementation theory (Palfrey, 1992), principal-agent theory (Mookherjee, 1984), differential-information economies (Postlewaite and Schmeidler, 1986), mechanism design (Demski and Sappington, 1984). There also exists an extensive literature on mechanism design exploring mechanisms that can uniquely implement an outcome (Ma, 1988; Ma, Moore and Turnbull, 1988).

particular communication scheme. In a recent work Ray, Serrano and Vohra[3] formally address and study the issue of (full and virtual) implementation of correlated equilibrium distributions. This paper studies a specific two-person game (*Chicken*) and looks at a specific form of multiple equilibrium problem, involving *disobedience.* A player adopts a *disobedient* strategy if he always chooses the action that is not recommended by the mediator. A correlated equilibrium is said to suffer from the multiple equilibrium problem if the disobedient strategy profile is also a Nash equilibrium of the extended game and it generates (weakly) higher (ex-ante, expected) payoffs for both players. This paper asks the question whether there exists a communication scheme, more specifically, a *non-canonical* correlation device that can implement a correlated equilibrium and does not suffer from this multiple equilibrium problem.

This exercise is clearly different from identifying an efficient correlated equilibrium (Ray, 1996a) or characterising efficiency (Myerson, 2002). This paper tries to implement an equilibrium that is clearly not optimal in the first place. One might criticise the basic motivation of this research by asking: why should the players be interested in implementing such a sub-optimal outcome? The players should always select a correlation device that does not suffer from the multiple equilibrium problem (possibly using the concepts in Ray, 1996a; Myerson, 2002). One response to this fair criticism is that the correlated equilibrium that needs to be implemented could be the desire of a third party, or even the mediator.

Although any communication scheme can be thought of to avoid the multiple equilibrium problem in this context, this paper considers only *non-direct* mediators or *non-canonical* devices. A non-direct mediator or a non-canonical device is a device in which the messages are not the strategies of the original game. The paper offers three different non-canonical structures (one with and two without a *public message* or a *sunspot*) each of which, together with a particular strategy profile of the non-canonical extended game, induces the direct correlated equilibrium in

[3] Private communication; work in progress.

consideration. The non-canonical devices are characterised by certain parameters. For each of the non-canonical devices, one can precisely find the range(s) of the characterising parameter(s) such that the non-canonical extended game has the equilibrium that induces the correlated equilibrium, but not the other that corresponds to the disobedient equilibrium.

The paper is organised as follows. The next section presents a couple of examples of non-canonical devices to motivate this study. Section 3 collects all the relevant definitions. Section 4 presents the basic game, the correlation devices, and all the findings. Section 5 concludes.

2. EXAMPLES OF NON-CANONICAL DEVICES

Consider the non-canonical device[4] in Figure 2a, in which the message sets for two players are, respectively, $\{a, b, c\}$ and $\{d, e, f\}$.

	d	e	f
a	0	0	1/7
b	1/7	3/7	0
c	2/7	0	0

Figure 2a

Suppose this device is used to play a version of *Chicken*, as in Figure 2b.

	A	P
A	4/3, 4/3	7, 2
P	2, 7	5, 5

Figure 2b

Each player first gets a private message from the non-canonical device, and then plays the game. A pure strategy of a player in the extended game is a map from the set of messages to the set

[4] First appeared in Ray (1996b).

of pure strategies of the original game.

It is easy to check that the strategy profile (*PAP*, *APA*) is a Nash equilibrium in this non-canonical extended game. This equilibrium strategy profile induces a probability distribution over the outcomes of the original game, as illustrated in Figure 2c.

	A	*P*
A	1/7	3/7
P	3/7	0

Figure 2c

The distribution in Figure 2c can now be identified as a direct correlation device. It is well known as the *revelation principle* (Myerson, 1982, 1985) that this induced (direct) device is also a (direct) correlated equilibrium for the game in Figure 2b. However, note that, this correlated equilibrium suffers from the multiple equilibrium problem, as indeed, the disobedient strategy profile (*PA*, *PA*) is also a Nash equilibrium of the (canonical) extended game and it generates higher payoffs than the obedient equilibrium. It is more important to note that the non-canonical device in Figure 2a (that induces the canonical distribution in Figure 2c), does not suffer from this problem, as the strategy profile (*APA*, *PAP*), that corresponds to the disobedient profile in the canonical game is not an equilibrium in the non-canonical extended game.

The above example shows that it is possible to generate the obedient equilibrium and at the same time avoid the multiple equilibrium problem if one considers a non-canonical device such as the one in Figure 2a. One now might be interested in the structure of such non-canonical devices that would get rid of the multiple equilibrium problem. The device in Figure 2a does have a special structure. It is evident that the message profile (*a*, *f*) in this device is a public message or a sunspot. Introducing a public message or a sunspot in the non-canonical device is however not the only way to do this job, as the following example shows.

Consider for example, the game in Figure 2b and the correlated equilibrium in Figure 2c.

One can induce the correlated equilibrium using the non-canonical device in Figure 2d that involves no sunspot. It is easy to verify that in this non-canonical extended game, the strategy profile (*AAP*, *AAP*) is an equilibrium and it induces the direct correlated equilibrium in Figure 2c. However, the strategy profile (*PPA*, *PPA*) that corresponds to the disobedient equilibrium in the canonical game is not an equilibrium of this non-canonical extended game.

	d	*e*	*F*
a	0	1/14	3/14
b	*1/14*	0	3/14
c	2/7	1/7	0

Figure 2d

It is important to realise that the above non-canonical structures (as in Figures 2a and 2d) may not work to get rid of the multiple equilibrium problem for all such games. For example, for the game in Figure 1a and the correlated equilibrium in Figure 1b, one cannot find any non-canonical device, structurally similar to that in Figure 2a or Figure 2d, that does not suffer from the multiple equilibrium problem, as shown later in this paper.

3. FORMALITIES

Fix any normal form game, $G = [N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N}]$ with set of players: $N = \{1,...., n\}$, finite pure strategy sets: $S_1,...., S_n$; $S = \prod_{i\in N} S_i$, and payoff functions: $u_1,...., u_n$; $u_i: S \to \Re$, for all i.

Definition 1. (i) A *correlation device* is an $(n+1)$-tuple, $d = (M_1,...., M_n, \mu)$ where, M_i is a finite set of messages for player i and μ is a probability distribution over M $(= \prod_{i\in N} M_i)$. The device selects a message profile m $(= (m_1,...., m_n))$ according to μ, and send the private message m_i to each player i.

(ii) The extended game G_d is the game where the correlation device d selects and sends messages to the players, and then the players play the original game G. A pure[5] strategy for player i in the game G_d is a map s_i: $M_i \rightarrow S_i$ and the corresponding (*ex-ante*, expected) payoff is given by, $u_i^*(s_1,...., s_n) = \Sigma_{m\in M}\, \mu(m)\, u_i(s_1(m_1),...., s_n(m_n))$.

(iii) A *direct correlation device*, d, is a correlation device where $M_i = S_i$. For such a device, d would also denote the probability distribution over $S = \prod_{i\in N} S_i$.

Definition 2. In a given correlation device, a message profile m (= $(m_1,...., m_n)$), is called a *public message* or a *sunspot* if $\mu(m) > 0$, and the *conditional probability*[6] of $((m_{-i}))$ given m_i is 1 for all i.

Definition 3. (i) A *correlated equilibrium* of the game G is a pair $(d, (s_i)_{i\in N})$, where the (pure or behavioral) strategy profile $(s_1,...., s_n)$ is a Nash equilibrium of the extended game G_d.

(ii) A *direct correlated equilibrium* d of the game G is a correlated equilibrium where $M_i = S_i$, and s_i is the identity map, for all i. The corresponding payoff to player i is given by $\Sigma_{s\in S}\, d(s)\, u_i(s)$.

(iii) A *correlated equilibrium distribution* of the game G is a probability distribution on S which is induced by a correlated equilibrium $(d, (s_i)_{i\in N})$.

Remark 1. A direct correlated equilibrium can be identified with an element of ?(S). Clearly, it is also a correlated equilibrium distribution, induced by itself. Let $C(G)$ denote the set of all correlated equilibrium distributions for the game G.

Remark 2. Any Nash equilibrium and any convex combination of several Nash equilibrium of a

[5] One can also think of behavioral strategies in the game G_d. A behavioral strategy for player i is a map from M_i to ?(S_i) and the (*ex-ante*, expected) payoff for player i corresponding to a behavioral strategy vector is given by, $u_i^*(s_1,...., s_n) = \Sigma_{m\in M} P(m)[\Sigma_{s\in S}\{(\Pi_{j\in N} s_j(s_j \mid m_j))u_i(s_1,..., s_n)\}]$. Let us restrict ourselves to pure strategies only.

[6] Using the standard notion of conditional probability.

given game G, *corresponds* to a direct correlated equilibrium. Let $N(G)$ denote the set of all distributions that correspond to any Nash equilibrium and let $CONV(G)$ denote any convex combination of several Nash equilibrium. Clearly, $N(G) \subseteq CONV(G) \subseteq C(G)$. Let Γ be the set of all games for which $CONV(G) \subset C(G)$. Let us consider games only in Γ, i.e., games for which the set of correlated equilibrium is strictly bigger than the convex hull of Nash equilibrium outcomes.[7] Also, let us consider correlated equilibrium distributions that are outside the convex hull of Nash equilibrium outcomes, i.e., $d \in C(G) \backslash CONV(G)$.

Definition 4. Given any game $G \in \Gamma$, and a direct correlated equilibrium[8] $d \in C(G) \backslash CONV(G)$ for G, the set of *inducible distributions*, $I(d)$, is the set of all distributions over S that are induced by some (pure or behavioral) Nash equilibrium strategy profile (s_1,...., s_n) of the extended game G_d.

Remark 3. Clearly, any distribution in $I(d)$ is a correlated equilibrium for the game G. Thus, $I(d) \subset C(G)$, for any game[9] $G \in \Gamma$, and a direct correlated equilibrium $d \in C(G) \backslash CONV(G)$.

Remark 4. As $d \in C(G) \backslash CONV(G)$ is a direct correlated equilibrium for the game G, the obedient strategy profile forms a Nash equilibrium extended game G_d. Hence, $d \in I(d)$.

Remark 5. For any given game $G \in \Gamma$, and for any given direct device[10] $d \in C(G) \backslash CONV(G)$, the strategy profile (of the extended game) that *induces* a (pure or mixed) Nash equilibrium of the original game, is also a Nash equilibrium of the extended game. Particularly, for any game $G \in \Gamma$, and a direct correlated equilibrium $d \in C(G) \backslash CONV(G)$ for G, $N(G) \subset I(d)$. Note however that

[7] Certain games are hereby excluded, such as, Prisoners' Dilemma, Cournot Duopoly (see Yi, 1997). Also see Moulin and Vial (1978) in this context.

[8] Definition 4 and Remarks 3-5 are valid for any correlated equilibrium distribution $d \in C(G)$.

[9] Remark 3 is valid for any game G.

[10] Remark 5 is valid for any canonical or non-canonical device d.

$CONV(G) \not\subset I(d)$, as not all convex combination of Nash equilibrium points can be induced from a given distribution d. Clearly, $N(G) \subset I(d) \cap CONV(G)$.

Remark 6. Following Remark 5, for any given game $G \in \Gamma$ and for any given device $d \in C(G)\backslash CONV(G)$, let us concentrate only on $I(d)\backslash\{I(d) \cap CONV(G)\}$, the set of induced correlated equilibrium other than the inducible convex combinations of Nash equilibrium of the original game.

Definition 5. A direct correlated equilibrium $d \in C(G)\backslash CONV(G)$ of a game $G \in \Gamma$ is said to suffer from the *multiple-equilibrium problem* if there exists an induced correlated equilibrium distribution $d' \in I(d)\backslash\{I(d) \cap CONV(G)\}$, such that $\Sigma_{s\in S}\, d'(s)\, u_i(s) \geq \Sigma_{s\in S}\, d(s)\, u_i(s)$ for all i, with at least one strict inequality.

This paper analyses 2x2 games only and a particular form of multiple equilibrium problem. Therefore the following two definitions are the only two required notions for the rest of the paper.

Definition 6. (i) For a 2x2 game $G \in \Gamma$, and for a direct correlated equilibrium $d \in C(G)\backslash CONV(G)$, the *disobedient* strategy profile in the extended game G_d is the profile in which each player always chooses the action that is not recommended by the mediator.

(ii) For a 2x2 game $G \in \Gamma$, a direct correlated equilibrium $d \in C(G)\backslash CONV(G)$, is said to suffer from the *multiple equilibrium problem* if the correlated distribution, induced by the *disobedient* strategy profile, $d' \in I(d)\backslash\{I(d) \cap CONV(G)\}$, and $\Sigma_{s\in S}\, d'(s)\, u_i(s) \geq \Sigma_{s\in S}\, d(s)\, u_i(s)$ for all i, with at least one strict inequality.

Definition 7. Suppose for a given 2x2 game $G \in \Gamma$, a non-canonical correlation device *induces* a direct correlated equilibrium that suffers from the multiple equilibrium problem. We would say that the non-canonical device itself does (does not) suffer from the multiple equilibrium problem as well, if the strategy profile that *corresponds*[11] to the disobedient equilibrium of the direct extended game is (is not) an equilibrium in the non-canonical extended game.

4. ANALYSIS

This section studies a general version of the two-person game of *Chicken* as the basic game. Motivated by the example in the Introduction, it considers a particular direct correlation device which is characterised by a single parameter, p.

The Game: Chicken

Consider the two-player non-cooperative game of *Chicken* as in Figure 3a, where[12], $0 < a < b < c < d$. Each of the two players has two strategies, namely, A and P. This game has two pure Nash equilibrium, namely, (A, P) and (P, A) and a mixed equilibrium in which each player plays A with probability $(d\text{-}c)/\{(d\text{-}c) + (b\text{-}a)\}$.

	A	P
A	a, a	d, b
P	b, d	c, c

Figure 3a

The Canonical Correlation Device

Motivated by the examples in the Introduction, we here present a particular form of a direct

[11] There always exists such a strategy profile and can be easily identified by combining two maps.
[12] We have chosen strictly positive payoffs just for the sake of simplicity in calculations.

correlation device as in Figure 3b. The device is characterised by one parameter, p; $0 < p < 1$.

	A	P
A	p	$(1-p)/2$
P	$(1-p)/2$	0

Figure 3b

Fix a game of Chicken (i.e., fix values of a, b, c and d in the game in Figure 3a). Suppose now this game is played using a correlation device as in Figure 3b. Clearly for certain values of the parameter p, both the obedient and the disobedient strategy profile will be equilibrium of the canonical extended game. Note that, by the structure of the device and the payoffs in the original game (*Chicken*), the disobedient strategy generates higher payoffs than the obedient one, for both players. Thus, for a fixed game of *Chicken*, one can find a precise range of the parameter p, for which this correlated equilibrium suffers from the multiple equilibrium problem.

Proposition 1. The (direct) correlation device in Figure 3b is a correlated equilibrium and also suffers from the multiple equilibrium problem if $0 < p \leq \text{Min}\ [x, y]$, where, $x = (d-c)/\{(d-c) + 2(b-a)\}$ and $y = (b-a)/\{(b-a) + 2(d-c)\}$.

Proof. Assuming that the other player is following the strategy *AP*, it is obvious (by the symmetric structure of the device and the payoffs of the original game) that a player will play P when the recommendation is P and when the recommendation is A, will play A if $ap + d(1-p)/2 \geq bp + c(1-p)/2$, i.e., if $p \leq x$. Thus, the obedient profile (*AP*, *AP*) is a Nash equilibrium of the canonical extended game if $p \leq x$. Similar argument shows that the disobedient profile (*PA*, *PA*) is a Nash equilibrium if $p \leq y$. QED

To understand the above Proposition, let us consider the examples already discussed earlier.

Example 1.1. Consider the game in Figure 1a. Here, $a = 0$, $b = 2$, $c = 6$ and $d = 7$. For these parameter values, $x = 1/5$ and $y = ½$. Therefore, any direct device as in Figure 3b will suffer from the multiple equilibrium problem if $0 < p \leq 1/5$. As noticed earlier, for this game, the device in Figure 1b, which is characterised by $p = 1/5$, does suffer from the multiple equilibrium problem.

Example 2.1. Consider the game in Figure 2b. Here, $a = 4/3$, $b = 2$, $c = 5$, $d = 7$, $x = 3/5$ and $y = 1/7$. Any direct device as in Figure 3b will suffer from the multiple equilibrium problem if $0 < p \leq 1/7$. As noticed earlier, for this game, the device in Figure 2c, which is characterised by $p = 1/7$, does suffer from the multiple equilibrium problem.

For the rest of the paper, let us hereby fix a game of *Chicken* (i.e., fix values of a, b, c and d). Also, let us restrict ourselves to direct devices as in Figure 3b, with $p \leq$ Min $[x, y]$ only, so that the correlated equilibrium in question does suffer from the multiple equilibrium problem.

Let us now consider three different non-canonical correlation devices. In each of these devices, the message sets are respectively, $\{a, b, c\}$ and $\{d, e, f\}$. The devices are characterised by certain parameters. In each of these non-canonical extended games, there is a strategy profile that induces the direct correlated equilibrium in consideration and there is another strategy profile that *corresponds to* the disobedient equilibrium, (PA, PA) in the canonical extended game. Let us precisely find the range(s) of the characterising parameter(s) of the non-canonical devices for which the first strategy profile is an equilibrium of the non-canonical extended game but the second profile is not (i.e., the non-canonical device does not suffer from the multiple equilibrium problem).

The Non-Canonical Device #1 (With a sunspot)

Consider the non-canonical device in Figure 4. This device is motivated by the one in the Introduction (Figure 2a) and is characterised by the parameter $\varepsilon > 0$. The device is a combination of private messages and a sunspot as (a, f) is a *public message* or a *sunspot*.

	d	e	f
a	0	0	ε
b	p	(1-p)/2	0
c	(1-p)/2 - ε	0	0

Figure 4

Note that in the non-canonical extended game, the strategy profile (*PAP*, *APA*) induces the direct correlation device in Figure 3b and the strategy profile (*APA*, *PAP*), *corresponds to* the disobedient equilibrium in the canonical extended game.

Proposition 2. (i) Suppose $p \leq x < y$. There does not exist any non-canonical device as in Figure 4 for which the strategy profile (*PAP*, *APA*) is an equilibrium but the profile (*APA*, *PAP*) is not (i.e., the device does not suffer from the multiple equilibrium problem).

(ii) Suppose $p \leq y < x$. The non-canonical device as in Figure 4 does not suffer from the multiple equilibrium problem if $0.5(1 - p/y) \leq \varepsilon \leq 0.5(1 - p/x)$.

Proof. Let us first check if and when the strategy profile (*PAP*, *APA*) is a Nash equilibrium of the non-canonical extended game. Fix the strategy *APA* of player 2. Now, for player 1, playing *P* is optimal when the message is a. When the message is b, playing *A* is optimal if $ap + d(1\text{-}p)/2 \geq bp + c(1\text{-}p)/2$, i.e., if $p \leq x$, which is indeed the case (recall that $p \leq$ Min $[x, y]$). Finally, when the message is c, playing *P* is optimal. Now fix the strategy *PAP* of player 1. For player 2, playing *A* is optimal when the message is f. When the message is e, playing *P* is optimal. Finally, when the message is d, playing *A* is optimal if $ap + d((1\text{-}p)/2 - \varepsilon) \geq bp + c((1\text{-}p)/2 - \varepsilon)$, i.e., if $\varepsilon \leq 0.5\{1 - p/x\}$. Therefore, (*PAP*, *APA*) is an equilibrium if $\varepsilon \leq 0.5\{1 - p/x\}$. Similarly, the strategy profile (*APA*, *PAP*) is not an equilibrium if $0.5\{1 - p/y\} \leq \varepsilon$. If $p \leq x < y$, then it is easy to check that both conditions can not be met simultaneously. For $p \leq y < x$, one has the desired result. QED

Let us illustrate the above Proposition using the earlier examples.

Example 1.2. Consider the game in Figure 1a and the correlated equilibrium in Figure 1b. Here $p = x = 1/5 < y = ½$. Therefore, one cannot find any non-canonical device as in Figure 4 that does not suffer from the multiple equilibrium problem.

Example 2.2. Consider the game in Figure 2b. Here, $x = 3/5$ and $y = 1/7$. As noticed earlier, any correlated equilibrium as in Figure 3b with $0 < p \leq 1/7$, would suffer from the multiple equilibrium problem. One can find a non-canonical device as in Figure 4, with $\varepsilon \in [(1–7p)/2, (3–5p)/6]$, that would get rid of this problem. Note that the direct device in Figure 2c has $p = 1/7$. For such a device, the desired range of ε in the non-canonical device is (0, 8/21]. The device in Figure 2a has $\varepsilon = 1/3$ and therefore does not suffer from the multiple equilibrium problem, as noticed earlier.

The Non-Canonical Device #2 (Symmetric, Without any Sunspots)

Consider the non-canonical device in Figure 5, which is characterised by the single parameter $\alpha \in (0, 1)$. Clearly, this device does not have any sunspots; also, there is an amount of structural symmetry in it.

	d	*e*	*f*
a	0	αp	$\alpha(1-p)/2$
b	$(1-a)$p	0	$(1-\alpha)(1-p)/2$
c	$(1-\alpha)(1-p)/2$	$\alpha(1-p)/2$	0

Figure 5

Note that in this non-canonical extended game, the strategy profile (*AAP*, *AAP*) induces the direct correlation device in Figure 3b and the strategy profile (*PPA*, *PPA*), *corresponds to* the disobedient equilibrium in the canonical game.

Proposition 3. For any $p \leq$ Min [x, y], a non-canonical device as in Figure 5 always suffers from the multiple equilibrium problem.

Proof. Let us first check whether the profile (*AAP*, *AAP*) is a Nash equilibrium of the non-canonical extended game or not. Fix the strategy *AAP* of player 2. Now, for player 1, when the message is a, playing A is optimal if $ap + d(1\text{-}p)/2 \geq bp + c(1\text{-}p)/2$, i.e., if $p \leq x$. Also, when the message is b, playing A is optimal if $p \leq x$. Finally, when the message is c, playing P is optimal. By the symmetry of the game and the device, the above argument holds for player 2 and hence, the profile (*AAP*, *AAP*) is an equilibrium if $p \leq x$. Similar argument shows that the profile (*PPA*, *PPA*) is also an equilibrium if $p \leq y$. Hence the result holds. QED

The Non-Canonical Device #3 (Asymmetric, Without any Sunspots)

Consider the non-canonical device in Figure 6, which is characterised by two parameters, α and β, $\alpha \neq \beta$ and both α and $\beta \in (0, 1)$. The structure of this device is motivated by the one in Figure 2d. Clearly, this device, like the device #2, does not have any sunspots; however, unlike the device #2, it is not symmetric.

	d	*e*	*f*
a	0	αp	$\alpha(1\text{-}p)/2$
b	*(1-a)*p	0	$(1\text{-}\alpha)(1\text{-}p)/2$
c	$(1\text{-}\beta)(1\text{-}p)/2$	$\beta(1\text{-}p)/2$	0

Figure 6

Note that, here as well, the strategy profile (AAP, AAP) induces the direct correlation device in Figure 3b and the strategy profile (PPA, PPA), *corresponds to* the disobedient equilibrium.

Proposition 4. For any $p \leq$ Min [x, y], the non-canonical device as in Figure 6, does not suffer from the multiple equilibrium problem if $\alpha k_1 \leq \beta \leq \alpha k_1 + (1 - k_1)$ but not $\alpha k_2 \leq \beta \leq \alpha k_2 + (1 - k_2)$, where, $k_1 = \{p(b\text{-}a)\} / \{(1\text{-}p)(d\text{-}c)/2\}$ and $k_2 = \{p(d\text{-}c)\} / \{(1\text{-}p)(b\text{-}a)/2\}$.

Proof. To check whether the profile (AAP, AAP) is a Nash equilibrium, fix the strategy AAP of player 2. It is easy to verify that for player 1, playing AAP is optimal if $p \leq x$, which is indeed the case. Now fix the strategy AAP of player 1. For player 2, playing P is optimal when the message is f. When the message is d, playing A is optimal if $a(1\text{-}\alpha)p + d(1\text{-}\beta)(1\text{-}p)/2 \geq b(1\text{-}\alpha)p + c(1\text{-}\beta)(1\text{-}p)/2$, i.e., if $\beta \leq \alpha k_1 + (1 - k_1)$. Finally, when the message is e, playing A is optimal if $a\alpha p + d\beta(1\text{-}p)/2 \geq b\alpha p + c\beta(1\text{-}p)/2$, i.e., **f** $\alpha k_1 \leq \beta$. Therefore, the profile (AAP, AAP) is an equilibrium if $\alpha k_1 \leq \beta \leq \alpha k_1 + (1 - k_1)$. Similar argument shows that the profile (PPA, PPA) is also an equilibrium if $\alpha k_2 \leq \beta \leq \alpha k_2 + (1 - k_2)$. Thus, the result holds. QED

Let us revisit the earlier examples to illustrate the above Proposition.

Example 1.3. Consider the game in Figure 1a and the correlated equilibrium in Figure 1b. Here p = 1/5, $k_1 = 1$ and $k_2 = 1/4$. Therefore, one cannot find any α and β which would satisfy the conditions in Proposition 4 and hence, for this example, it is not possible to construct a non-canonical device as in Figure 6 that would not suffer from the multiple equilibrium problem.

Example 2.3. Consider the game in Figure 2b and the correlated equilibrium in Figure 2c. Here p = 1/7, $k_1 = 1/9$ and $k_2 = 1$. It is therefore easy to construct a non-canonical device as in Figure 6, (following the condition in Proposition 4) that would get rid of the multiple equilibrium problem. Any α and β such that $\beta \neq \alpha$ and $\beta \in [\alpha/9, \alpha/9 + 8/9]$ would generate the desired result. For example, one can consider $\alpha = ½$ and any $\beta \in [1/18, 17/18]$, other than ½. Note that, the device in Figure 2d is characterised by $\alpha = ½$ and $\beta = 1/3$ and as noticed earlier, it indeed gets rid of the multiple equilibrium problem.

5. REMARKS

This paper studies the multiple equilibrium problem in normal form games played using correlation devices and asks the question whether there exists a communication scheme, more specifically, a non-canonical correlation device that can implement a correlated equilibrium and does not suffer from the multiple equilibrium problem. A couple of early examples do suggest that it is indeed the case. The question that then arises is what kind of non-canonical devices one needs to achieve this. One possible way is to include a sunspot in the non-canonical device, as one of the examples indicates. However, one perhaps can do the same job without a sunspot, as another example confirms. This paper analyses three different non-canonical structures, one with and two without a sunspot, to understand this problem.

The paper does not provide any general result. It only considers a 2x2 game (*Chicken*) and a specific form of multiple equilibrium problem using a particular disobedient strategy. In this framework, sunspots in the non-canonical device turn out to be neither sufficient nor necessary to solve the problem. For some parameter values, device #1 (with a sunspot) does help, however, not for all games. Device #2 (without any sunspots) on the other hand, never gets rid of the problem, while device #3, which is an asymmetric version of the device #2, manages to do the job for some parameter values. There exist however, games and correlated equilibrium (Example 1) for which none of the non-canonical structures discussed in this paper would be enough to get rid of the problem. One definitely needs a different structure, possibly with larger message sets for each player, for these games.

It is worthwhile to mention that Ray (2001) has also discussed the multiple equilibrium problem in correlation devices to explain the failure of the revelation principle for coalition-proof correlated equilibrium. Using some conditions presented in that paper, Ray explained exactly how and where the multiple equilibrium problem becomes relevant.

For future research, one might consider a couple of different directions. First, as mentioned earlier, it is now well known that mediated and unmediated (cheap) talk can generate any correlated equilibrium of a given game. Particularly, any correlated equilibrium can be generated by a (one-shot public) mediated talk (Lehrer 1996; Lehrer and Sorin 1997) or pre-play unmediated communication (Ben-Porath 1998). Both these schemes are valid for 2x2 games. It would therefore be interesting to analyse the games and the correlated equilibrium distributions discussed in this paper in their framework. Second, one reckons that the non-canonical structures (particularly, the device #1) discussed here might be useful to model and analyse communication between (two) agents in any group decision problem in the bounded rationality literature (Chapter 6, Rubinstein 1998).

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