Growth and Welfare Effects of Business Cycles In Economies with Idiosyncratic Human Capital Risk

First Version: September, 2001
This Draft: October, 2002

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Abstract

This paper uses a tractable macroeconomic model with idiosyncratic human capital risk and incomplete markets to analyze the growth and welfare effects of business cycles. The analysis is based on the assumption that the elimination of business cycles eliminates the variation in idiosyncratic risk. The paper shows that a reduction in the variation in idiosyncratic risk decreases the ratio of physical to human capital and increases the total investment return and welfare. If the degree of risk aversion is less than or equal to one, then economic growth is enhanced. This paper also provides a quantitative assessment of the macroeconomic effects of business cycles based on a calibrated version of the model. Even for relatively small degrees of risk aversion (around one) the model implies that the elimination of business cycles has substantial effects on investment in physical and human capital, economic growth, and welfare.

JEL Classification: D52, E32, O40
Keywords: Cost of Business Cycles, Growth, Idiosyncratic Risk

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*I would like to thank for helpful comments Andy Atkeson, Oded Galor, Peter Howitt, Bob Lucas, Tomo Nakajima, Herakles Polemarchakis, Per Krusell, Tony Smith, David Weil and seminar participants at Arizona State University, Brown University, Carnegie-Mellon University, Columbia University, the Economic Theory Conference, Ischia 2001, the Stoney-Brook Incomplete Markets Workshop, 2001, the Minnesota Workshop in Macroeconomic Theory, 2002, and the SITE Workshop on Liquidity and Distribution in Macroeconomics, 2002. Financial assistance from the Solomon Research Grant, Brown University, is gratefully acknowledged. All errors are mine.

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I. Introduction

In a highly influential contribution, Lucas (1987) argues that the welfare costs of business cycles are likely to be very small, and that therefore the potential benefits from countercyclical stabilization policy are negligible. His argument is based on a calibrated representative-agent model with no production, that is, Lucas (1987) assumes that there is no uninsurable idiosyncratic risk and that economic growth and business cycles are unrelated. This paper asks to what extend the introduction of market incompleteness and production changes Lucas' conclusions regarding the welfare cost of business cycles.

The current analysis is based on an incomplete-markets version of the class of endogenous growth models analyzed by, among others, Jones and Manuelli (1990) and Rebelo (1991).[1] More specifically, households are ex ante identical and have CRRA-preferences, production displays constant returns to scale with respect to physical and human capital, and all markets are competitive. There are aggregate productivity shocks that affect the return to physical and human capital investment (stock returns and wages), and there are idiosyncratic human capital shocks that only affect human capital returns.[2] Conditional on the history of aggregate states, these idiosyncratic shocks are independently distributed over time and identically distributed across households. Finally, the financial market structure is incomplete in the sense that there are no assets with payoffs that depend on idiosyncratic shocks. However, households have the opportunity to trade stocks (accumulate physical

[1]See also Alvarez and Stokey (1998) and Caballe and Santos (1993). Lucas (1988) analyzes a human capital model with externalities. Krebs (2002) shows that the current incomplete-markets model admits for simple recursive equilibria and provides a useful characterization of equilibrium, and Krebs (2001) studies a version of the model with log-utility and no aggregate risk.

[2]A negative human capital shock (depreciation shocks) might occur when firm- or sector-specific skills are either destroyed or made obsolete in the event of a job loss. Jovanovic (1979) and Ljungqvist and Sargent (1998) analyze search models with specific human capital, but they assume risk-neutral workers and do not model the accumulation of physical capital. Empirically, the permanent wage loss of displaced workers is substantial (Jacobson, LaLonde, and Sullivan 1993, Neal 1995, and Topel 1991) suggesting that job displacement is associated with large losses of firm or sector-specific human capital.

capital) and any asset (bond) in zero net supply with payoffs that depend on the aggregate shock variable.

The incomplete-markets model analyzed in this paper is highly tractable in the sense that there is always one equilibrium allocation that is identical to the equilibrium allocation of an economy in which households consume and produce in autarky facing both aggregate and idiosyncratic risk. In other words, a recursive equilibrium of the incomplete-markets economy can be found by solving a one-agent decision problem, but this one-agent decision problem is not the one-agent decision problem associated with the complete-markets economy (idiosyncratic risk matters). Exploiting the tractability of the model, this paper first conducts a qualitative analysis of the growth and welfare effects of business cycles. The analysis assumes that the amount of idiosyncratic risk varies over the business cycle, and that the elimination of business cycles removes these variations in idiosyncratic risk leaving the average amount of idiosyncratic risk unchanged. The economic motivation for this assumption derives from recent empirical work (Storesletten, Telmer, and Yaron, 2001a, and Meghir and Pistaferri, 2001) that has documented strong cyclical variations in uninsured idiosyncratic labor income risk, and the notion that counter-cyclical stabilization policy might eliminate these variations in idiosyncratic risk without changing the average amount of idiosyncratic risk (Atkeson and Phelan, 1994, Imrohoroglu, 1989, Krusell and Smith, 1999, and Storesletten, Telmer, and Yaron, 2001b).

The qualitative analysis shows that business cycles have the following general effects on growth and welfare. First, a reduction in the variation in idiosyncratic human capital risk makes human capital investment less risky, and therefore induces households to invest more in high-return human capital. Thus, economic growth always increases if total investment in physical and human capital does not decrease, which is the case if the degree of relative risk aversion is less than or equal to one. However, if the degree of risk aversion is larger than one, then economic growth may decrease due to the reduction in total investment. Second, although the growth effect of eliminating business cycles might be negative for high degrees of risk aversion, the welfare effect is always positive since the equilibrium allocation is the solution to a one-agent decision problem.

In addition to the qualitative analysis, this paper also provides a quantitative assessment of the growth and welfare consequences of business cycle fluctuations. The quantitative analysis assumes that the elimination of business cycles eliminates both the fluctuations in aggregate productivity and the variation in idiosyncratic human capital risk, and the model economy is calibrated so that the implied variation in idiosyncratic labor income risk in the economy with business cycles matches the estimates obtained by recent empirical studies (Storesletten et al., 2001a, and Meghir and Pistaferri, 2001). The general finding is that even for moderate degrees of risk aversion (around one), business cycle fluctuations have a substantial impact on investment, growth, and welfare. In particular, the welfare gain from eliminating business cycles is almost two orders of magnitude larger than what Lucas (1987) has found for the complete-markets exchange economy. Most of the welfare gain is due to the elimination of the variation in idiosyncratic human capital risk, which improves welfare for two reasons: it eliminates the variation in the volatility of individual consumption growth and, for moderate degrees of risk aversion, it increases average consumption growth. Both of these welfare effects are substantial, but the volatility effect is somewhat stronger. In short, even without an endogenous output response (exchange economy) the welfare costs of business cycles are quite large once market incompleteness is introduced, but taking into account the endogeneity of output (production economy) increases the cost even further.

The welfare costs of business cycles found in this paper are much larger than the welfare costs found by Imrohoroglu (1989) and Krusell and Smith (1999), and also larger than the costs found by Storesletten et al.(2001b). In accordance with the current analysis, all these papers assume that the elimination of business cycles leads to an elimination of the variations in idiosyncratic risk without affecting the average level of idiosyncratic risk.[3] However, there are differences in modeling the interaction between business cycles and idiosyncratic risk that account for a substantial part of the diversion of results.

First, this paper follows Storesletten et al. (2001b) and assumes that there are permanent idiosyncratic income shocks. In contrast, Imrohoroglu (1989) and Krusell and Smith (1999) only consider income shocks with moderate degree of persistence, which implies that individual consumption fluctuations are relatively small and temporary. Second, in this paper, and to a certain extent also in Storesletten et al.(2001b), the level of aggregate economic activity affects the magnitude of idiosyncratic income losses. Put differently, this paper assumes that recessions are not only times in which the probability of job displacement goes up, but also times in which the cost of job displacement increases. In contrast, Imrohoroglu (1989) and Krusell and Smith (1999) assume that the level of aggregate economic activity only affects the probability of job displacement, but not the magnitude of the income loss experienced by displaced workers.[4] As has been pointed out by Atkeson and Phelan (1994) and Krusell and Smith (1999), in this case the elimination of business cycles only affects the equilibrium outcome if there are indirect price effects, which are absent in the current model and relatively small in Krusell and Smith (1999).

There is one feature of the present model that distinguishes it from all the previous work on the welfare cost of business cycles in incomplete-market economies, namely that

[3]The notion that counter-cyclical stabilization policy reduces the variations in idiosyncratic risk without affecting the average amount of idiosyncratic risk can be made more precise using the integration principle (Krusell and Smith, 1999) discussed in Section III. Beaudry and Page (2001) assume that the elimination of business cycles removes all uninsurable idiosyncratic risk.

[4]Clearly, the displacement probability increases during an economic downturn. However, there are also good reasons to expect the size of the income loss to be affected: the distribution of employment opportunities worsens inducing the unemployed worker to accept lower wage offers and to increase the average time of search.

business cycles are detrimental to economic growth (at least for moderate degrees of risk aversion). In contrast, the previous incomplete-markets literature has relied on a framework that either disregards the output response to changes in business cycle activity (Imrohoroglu, 1989) or implies that the elimination of business cycles decreases output due to a reduction in precautionary saving (Krusell and Smith, 1999, and Storesletten et al., 2001b). This growth effect is yet another reason why the welfare cost of business cycles reported here are substantially larger than the cost found by the previous incomplete-markets literature (including Storesletten et al., 2001b).

This paper emphasizes the normative analysis of business cycles. However, the human capital model with incomplete markets has also several interesting positive implications. In particular, it provides one of the few attempts in the literature to explain formally the observed negative relationship between macroeconomic volatility and economic growth (Ramey and Ramey, 1995).[5] Jones, Manuelli, and Stacchetti (1999) study this relationship within the context of the complete-markets version of the human capital model, in which case business cycles may affect total investment, but not the ratio of physical to human capital. Barlevy (2000) shows that a negative relationship between aggregate volatility and economic growth may arise in a representative-agent model with endogenous technological innovation (Aghion and Howitt, 1992), and Acemoglu and Zilibotti (1997) discuss a model in which non-diversifiable entrepreneurial risk affects aggregate volatility and economic growth. Finally, Boldrin and Rustichini (1994) discuss the relationship between growth and fluctuations using a model with deterministic fundamentals and indeterminate equilibria.

[5]But see also Kormendi and Meguire (1985) for the opposite finding. Acemoglu and Zilibotti (1997) and Quah (1993) find a strong negative link between per capita income and volatility of growth.

II. Model

II.A. Economy

Consider a discrete-time infinite-horizon economy with $i = 1, \ldots, I$ households. A complete description of the exogenous state of the economy in period t is a vector $(s_{1t}, \ldots, s_{It}, S_t)$, where we interpret s_{it} as a household-specific (idiosyncratic) shock and S_t as an economy-wide (aggregate) shock. The vector of idiosyncratic shocks is denoted by $s_t = (s_{1t}, \ldots, s_{It})$, and a (partial) history of idiosyncratic, respectively aggregate, shocks by $s^t = (s_0, \ldots, s_t)$, respectively $S^t = (S_0, \ldots, S_t)$. We assume that idiosyncratic and aggregate shocks are identically and independently distributed over time (unpredictable) and that idiosyncratic shocks are identically distributed across households (households are ex-ante identical). That is, we assume $\pi(s_{t+1}, S_{t+1}|s^t, S^t) = \pi(s_{t+1}, S_{t+1})$ and $\pi(\ldots, s_{i1}, \ldots, s_{jt}, \ldots) = \pi(\ldots, s_{jt}, \ldots, s_{it}, \ldots)$.

There is one non-perishable good that can be either consumed or invested, and there is one firm that produces this "all-purpose" good. If the firm employs K_t units of physical capital and H_t units of human capital in period t, then it produces $Y_t = A_t F(K_t, H_t)$ units of the good in period t. Here F is a standard neoclassical production function. More specifically, we assume that F displays constant-returns-to-scale, is twice continuously differentiable, strictly increasing, strictly concave, and satisfies $F(0, H) = F(K, 0) = 0$ as well as $\lim_{K\to 0} F_k(K, H) = \lim_{H\to 0} F_h(K, H) = +\infty$ and $\lim_{K\to\infty} F_k(K, H) = \lim_{H\to\infty} F_h(K, H) = 0$. Total factor productivity is a function $A : \mathbf{S} \to \mathbb{R}_{++}$ that assigns to each aggregate state S_t a (strictly positive) productivity level $A_t = A(S_t)$. The firm rents input factors (physical and human capital) in competitive markets. We denote the rental rate of physical capital by $\tilde{r}_{kt}$ and the rental rate of human capital (the wage rate per efficiency unit of labor) by $\tilde{r}_{ht}$. In each period, the firm hires capital and labor up to the point where current profit is maximized. Thus, the firm solves the following static maximization problem:

$$max_{K_t,H_t} \{ A_t F(K_t, H_t) - \tilde{r}_{kt} K_t - \tilde{r}_{ht} H_t \} \tag{1}$$

Let k_{it} and h_{it} stand for the stock of physical and human capital owned by household i at the beginning of period t, and denote the corresponding investment levels by x_{kit} and x_{hit}. If we denote household $i's$ consumption by c_{it}, then the sequential budget constraint reads:

$$\begin{aligned} c_{it} + x_{kit} + x_{hit} &= \tilde{r}_{kt} k_{it} + \tilde{r}_{ht} h_{it} \\ k_{i,t+1} &= (1-\delta_{kt}) k_{it} + x_{kit} \quad , \quad k_{it} \geq 0 \\ h_{i,t+1} &= (1-\delta_{ht} + \eta_{it}) h_{it} + x_{hit} \quad , \quad h_{it} \geq 0 \\ & \quad (k_{i0}, h_{i0}) \; given \; . \end{aligned} \tag{2}$$

In (2) δ_{kt} and δ_{ht} denote the average depreciation rate of human and physical capital, respectively. These average depreciation rates are defined by functions $\delta_k : \mathbf{S} \rightarrow \mathbb{R}_+$ and $\delta_h : \mathbf{S} \rightarrow \mathbb{R}_+$ assigning to each aggregate shock $S_t \, \epsilon \, \mathbf{S}$ a deprecation rate $\delta_{kt} = \delta_k(S_t)$, respectively $\delta_{ht} = \delta_h(S_t)$. The term η_{it} denotes a household-specific shock to the stock of human capital and is defined by a function $\eta : \mathbf{s} \times \mathbf{S} \rightarrow \mathbb{R}$ assigning to each $(s, S) \, \epsilon \, \mathbf{s} \times \mathbf{S}$ a realization $\eta_{it} = \eta(s_{it}, S_t)$. We assume $E[\eta_{it}|S_t] = 0$.[6] Since $\tilde{r}_{ht}\eta_{it}$ is labor income of household i, the random variable η_{it} determines the nature of idiosyncratic labor income risk.

The budget constraint (2) makes four implicit assumptions. First, it does not impose a non-negativity constraint on human capital investment. Second, it does not distinguish between general and specific human capital. In other words, η_{it} can be either a shock to general human capital or a shock to specific human capital. Third, it neglects the labor-leisure choice of workers. Finally, it models human capital investment as direct expenditures. See Krebs (2002) for a detailed discussion of these assumptions.

The current paper emphasizes variations of the distribution of η_{it} over the business cycle.

[6] Of course, we restrict the depreciation functions so that the depreciation rate of physical and human capital never exceeds 100 percent.

In particular, the empirical evidence suggests that the dispersion of η_{it} increases during an economic downturn. One interpretation of this increase in dispersion is that realizations $\eta_{it} < 0$ correspond to the loss of firm- or sector-specific human capital experienced by displaced workers, and that these losses become more likely and/or more severe during economic downturns. Notice that the budget constraint (2) assumes that the wage is paid in each period. Thus, the current version of the model disregards the forgone wage during the period of unemployment and focuses instead on the (permanent) wage loss due to the difference between the wage before job displacement and the starting wage after a new job has been found. Empirically, the permanent component of this wage differential is quite large (Jacobson, LaLonde, and Sullivan 1993, Neal 1995, and Topel 1991).

To simplify the analysis, we do not explicitly mention financial markets. However, the equilibrium allocation of the above economy in which households accumulate physical capital is also the equilibrium allocation of a stock market economy in which the firm is a stock company that makes the intertemporal investment decision.[7] If we normalize the number of outstanding shares to one, the stock price is $Q_t = K_{t+1}$, household $i's$ equity share is $\theta_{i,t+1}Q_t = k_{i,t+1}$, and the return to equity investment is $\tilde{r}_{kt} - \delta_{kt}$. Moreover, the equilibrium allocation is unchanged if households are given the opportunity to trade $j = 1, \ldots, J$ securities in zero net supply with payoffs $D_{jt} = D_j(S_t)$. In particular, the introduction of a risk-free asset in zero net supply (borrowing and lending at the risk-free rate) will not change the equilibrium allocation (Krebs 2002).

The budget constraint can be rewritten in a way that shows how the households's op-

[7] In general, this type of market arrangement might lead to conceptual problems when markets are incomplete because shareholders (households) do not agree on the optimal investment policy (Magill and Quinzii, 1996). This, however, is not the case for the economy analyzed in this paper since here we have agreement among shareholders in the sense that the equilibrium investment policy maximizes the expected present discounted value of one-period profits using any household's intertemporal marginal rate of substitution to discount future profits.

timization problem is basically a standard intertemporal portfolio choice problem. To this end, define the following variables: $w_{it} \doteq k_{it} + h_{it}$ (total wealth) and $\tilde{k}_{it} \doteq k_{it}/h_{it}$ (the capital-to-labor ratio). With this new notation, the fraction of total wealth invested in physical capital is $\tilde{k}_{it}/(1+\tilde{k}_{it})$ and the fraction of total wealth invested in human capital is $1/(1+\tilde{k}_{it})$. Introduce further the following (average) rate of returns on the two investment opportunities: $r_{kt} \doteq \tilde{r}_{kt} - \delta_{kt}$ and $r_{ht} \doteq \tilde{r}_{ht} - \delta_{ht}$. Using this notation, the budget constraint reads:

$$\begin{aligned} w_{i,t+1} &= \left[1 + \frac{\tilde{k}_{it}}{1+\tilde{k}_{it}} r_{kt} + \frac{1}{1+\tilde{k}_{it}} (r_{ht} + \eta_{it})\right] w_{it} \,-\, c_{it} \\ & w_{it} \geq 0 \;,\; \tilde{k}_{it} \geq 0 \;, \\ & (w_{i0}, \tilde{k}_{i0}) \; given \;. \end{aligned} \tag{3}$$

Households have identical preferences over consumption plans $\{c_{it}\}$. These preferences allow for a time-additive expected utility representation:

$$U(\{c_{it}\}) = E\left[\sum_{t=0}^{\infty} \beta^t u(c_{it})\right] \;. \tag{4}$$

Moreover, we assume that the one-period utility function, u, is given by $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, $\gamma \neq 1$, or $u(c) = log\, c$, that is, preferences exhibit constant degree of relative risk aversion γ.

II.B. Equilibrium

In general, a sequential equilibrium is a process of prices (returns) and actions defined by a sequence of functions mapping histories,(s^t, S^t), into current prices and actions so that i) the firm maximizes profit, ii) households maximize expected lifetime utility, and iii) markets clear. In this paper, however, we are only interested in sequential equilibria with a recursive (Markov) structure. Indeed, in this paper we focus attention on recursive equilibria that are stationary in the sense that the ratio of physical to human capital, $\tilde{K}_t \doteq K_t/H_t$, is constant. We now outline how to construct such an equilibrium. Details and proofs can be found in Krebs (2002).

In equilibrium, aggregate returns on physical and human capital investment are given by the first-order conditions associated with the static profit maximization problem (1)

$$
\begin{aligned}
r_k(\tilde{K}, S_t) &= A(S_t) f'(\tilde{K}) - \delta_k(S_t) \\
r_h(\tilde{K}, S_t) &= A(S_t) f(\tilde{K}) - \tilde{K} f'(\tilde{K}) - \delta_h(S_t) \ ,
\end{aligned}
\tag{5}
$$

where we introduced the intensive-form production function $f = f(\tilde{K})$ with $f(\tilde{K}) \doteq F(\tilde{K}, 1)$. Given the return functions defined in (5), from the budget constraint (3) it immediately follows that in a stationary recursive equilibrium any individually optimal plan is generated by a policy function $g : \mathbb{R}_+ \times \mathbf{s} \times \mathbf{S} \to \mathbb{R}^3_+$ that assigns to each state (w_{it}, s_{it}, S_t) an action $(c_{it}, \tilde{k}_{i,t}, w_{i,t+1})$. Finally, market clearing reads

$$
\frac{\sum_i \frac{\tilde{k}_{it}}{1+\tilde{k}_{it}} w_{it}}{\sum_i \frac{1}{1+\tilde{k}_{it}} w_{it}} = \tilde{K} \ . \tag{6}
$$

Notice that (6) simply says that the aggregate capital-to-labor ratio chosen by households must be equal to the capital-to-labor ratio chosen by the firm.[8]

A stationary recursive equilibrium can be found by considering an economy with no financial markets and "home-production" by each household (farmer) using the production function F and facing both aggregate and idiosyncratic risk. That is, equilibrium investment and consumption of each household i is the solution to

$$
\begin{aligned}
& max \sum_{t=0}^{\infty} E \left[\beta^t u(c_{it}) \right] \\
s.t. \ : \ c_{it} + x_{kit} + x_{hit} &= A_t F(k_{it}, h_{it}) \\
k_{i,t+1} &= (1 - \delta_{kt}) k_{it} + x_{kit} \ \ , \ \ k_{it} \geq 0 \\
h_{i,t+1} &= (1 - \delta_{ht} + \eta_{it}) h_{it} + x_{iht} \ \ , \ \ h_{it} \geq 0 \\
& (k_{i0}, h_{i0}) \ given \ ,
\end{aligned}
\tag{7}
$$

[8]Notice that this condition in conjunction with the individual budget constraint (2) implies goods market clearing: $Y_t = C_t + X_{kt} + X_{ht}$.

with $u(c_{it}) = \frac{c_{it}^{1-\gamma}}{1-\gamma}$, $\gamma \neq 1$, or $u(c_{it}) = log\, c_{it}$. The stochastic productivity and depreciation parameters in (7) are again defined by functions $A_t = A(S_t)$, $\delta_{kt} = \delta_k(S_t)$, $\delta_{ht} = \delta_h(S_t)$, and $\eta_{it} = \eta(s_{it}, S_t)$. The transition probabilities of the Markov process $\{s_{it}, S_t\}$ satisfy $\pi(s_{i,t+1}, S_{t+1}|s_{it}, S_t) = \pi(s_{i,t+1}, S_{t+1}|S_t)$ and are derived from the transition probabilities of the joint Markov process $\{s_t, S_t\}$ using the formula $\pi(s_{i,t+1}, S_{t+1}|S_t) = \sum_{-i} \pi(s_{1,t+1}, \ldots, s_{I,t+1}, S_{t+1}|S_t)$. Because of our previous assumption that the transition probabilities are symmetric with respect to households, these marginal transition probabilities are the same for all households.

Krebs (2002) shows that the solution to the autarky problem (7) exists if the following condition

$$sup_{\tilde{k}}\, \beta E\left[\left(1 + r(\tilde{k}, s_i, S)\right)^{1-\gamma}\right] < 1 \tag{8}$$

is satisfied[9], and that this solution is also the equilibrium allocation of the market economy described above. In this equilibrium, all households choose the same capital-to-labor ratio, $\tilde{k}$, and consumption-to-wealth ratio, $\tilde{c} = c_{it}/(1 + r_{it})w_{it}$, and the equilibrium values of these ratio variables are given by

$$\begin{aligned} E\left[\frac{r_h(\tilde{k}, S) + \eta(s_i, S) - r_k(\tilde{k}, S)}{\left(1 + r(\tilde{k}, s_i, S)\right)^{\gamma}}\right] &= 0 \\ \tilde{c} = 1 - \left(\beta E\left[\left(1 + r(\tilde{k}, s_i, S)\right)^{1-\gamma}\right]\right)^{1/\gamma} &\quad , \end{aligned} \tag{9}$$

where the functions r_h and r_k are defined as in (5) and $r(\tilde{k}, s_i, S) = \left(\tilde{k}/(1 + \tilde{k})\right) r_k(\tilde{k}, S) + \left(1/(1 + \tilde{k})\right)\left(r_h(\tilde{k}, S) + \eta(s_i, S)\right)$. The evolution of individual consumption, capital (investment), and wealth is given by

$$\begin{aligned} c_{it} &= \tilde{c}\left[1 + r(\tilde{k}, s_{it}, S_t)\right] w_{it} \\ w_{i,t+1} &= (1 - \tilde{c})\left[1 + r(\tilde{k}, s_{it}, S_t)\right] w_{it} \end{aligned} \tag{10}$$

[9]Condition (8) ensure the existence of a solution, but does not ensure that the solution exhibits positive growth. A condition for positive growth can easily be inferred from (13).

$$k_{it} = \frac{\tilde{k}}{1+\tilde{k}} w_{it} \quad , \quad h_{it} = \frac{1}{1+\tilde{k}} w_{it} \quad .$$

Finally, expected lifetime utility can be calculated by substituting the solution (10) into the expression (4). This yields

$$\begin{aligned} E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\gamma}}{1-\gamma} \right] &= \frac{c_{i0}^{1-\gamma}}{(1-\gamma)\left(1 - \beta E\left[\left(1+g(\tilde{k}, s_i, S)\right)^{1-\gamma}\right]\right)} \quad \gamma \neq 1 \qquad (11) \\ E_0 \left[\sum_{t=0}^{\infty} \beta^t log c_{it} \right] &= \frac{1}{1-\beta} log c_{i0} + \frac{\beta}{(1-\beta)^2} E\left[log\left(1+g(\tilde{k}, s_i, S)\right)\right] \ , \end{aligned}$$

where $c_{i0} = \tilde{c}[1 + r(\tilde{k}, s_{i0}, S_0)]w_{i0}$ is initial consumption and g the individual consumption growth rate $c_{i,t+1}/c_{it} = \beta[1 + r(\tilde{k}, s_{i,t+1}, S_{t+1})]$. Notice that in (8), (9), and (11) the expectation is taken over aggregate and idiosyncratic shocks. Notice also that for the case of log-utility preferences, (8) reads $\beta < 1$ and (9) implies $\tilde{c} = 1 - \beta$.

As mentioned before, the introduction of financial assets $j = 1, \ldots, J$ that are in zero net supply and have payoffs $D_{jt} = D_j(S_t)$ will not change the equilibrium allocation. More precisely, if we introduce these assets and modify the budget constraint (2) accordingly, then at equilibrium prices

$$\begin{aligned} Q_j &= E\left[M(S_{t+1})D_j(S_{t+1})\right] \ , \qquad (12) \\ M(S_{t+1}) &= \beta E\left[\left((1 + r(\tilde{k}, s_{i,t+1}, S_{t+1}))(1-\tilde{c})\right)^{-\gamma}\right] \ . \end{aligned}$$

households choose not to trade these assets (see Krebs 2002 for details). This no-trade result is the extension of Constantinides and Duffie (1996) to the case of a production economy. As in Constantinides and Duffie (1996), individual income growth rates a unpredictable, that is, individual income follows a logarithmic random walk (see equation 15).

The expression (10) for individual equilibrium consumption implies that aggregate consumption growth is given by (invoking the law of large numbers)

$$\begin{aligned} \frac{C_{t+1}}{C_t} &= E\left[\frac{c_{i,t+1}}{c_{it}} | S^{t+1}\right] \qquad (13) \\ &= \beta\left(1 + E[r(\tilde{k}, s_{i,t+1}, S_{t+1}) | S_{t+1}]\right) \ . \end{aligned}$$

Thus, aggregate consumption growth rates are i.i.d., that is, aggregate consumption follows (approximately) a logarithmic random walk and the risk-free rate is constant. Annual data on real short-term interest rates and consumption show only small deviations from these two properties (Campbell and Cochrane, 1999). Given that this paper uses annual data to calibrate the model economy (see below), the assumption of an i.i.d. aggregate process seems therefore a reasonable first approximation.

III. Qualitative Analysis

We are interested in the change in $\tilde{k}$ and $\tilde{c}$, and the corresponding changes in growth and welfare, when we move from an economy with business cycles to an economy without business cycles. This amounts to "removing" the aggregate shocks S, that is, replacing the joint probabilities $\pi(s_i, S)$ with probabilities $\pi'(s_i)$ and any function with values $f(s_i, S)$ (describing an exogenous economic variable in the economy with business cycles) by a function with values $f'(s_i)$ (describing the same exogenous economic variable in the economy without business cycles). The question that arises is what relationship holds between π and π' as well as f and f'. It seems natural to compute the probabilities in the economy without business cycles as the simple marginals of the probabilities in the economy with business cycles. Using an analogous principle for the function f' we have:[10]

$$\begin{aligned} \pi'(s_i) &= \sum_S \pi(s_i, S) \\ f'(s_i) &= \sum_S f(s_i, S)\pi(S|s_i) \end{aligned} \tag{14}$$

In words: the value of each exogenous parameter in the economy without business cycles is equal to the average value of the same parameter in the economy with business cycles,

[10]For random variables with uncountable support that have a probability density function, these two conditions become $\pi'(s_i) = \int \pi(s_i, S)dS$ and $\eta'(s_i) = \int \eta(s_i, S)\pi(S|s_i)dS$. More generally, the function f' is defined as $f'(s_i) = E[f(s_i, S)|s_i]$.

where the average (expectation) is taken over the aggregate shocks. This procedure was first suggested by Krusell and Smith (1999) as a general principle guiding the analysis of business cycle effects in incomplete-markets economies, and is a generalization of the principle adopted by Atkeson and Phelan (1994). Krusell and Smith (1999) call it the integration principle, a terminology we will follow in this paper. Notice that this procedure ensures that the random variable f is a mean-preserving spread of the random variable f', and in this sense the economy with business cycle fluctuations exhibits more risk with no change in the mean of exogenous economic variables.

Applied to economic variables that have no idiosyncratic shock component, the integration principle is a natural extension of the original analysis conducted by Lucas (1987). For example, if $f = A$, then the integration principle implies: $A' = \sum_S A(S)\pi(S)$. However, when it comes to economic variables that directly depend on idiosyncratic shocks (in our case η), then Lucas (1987) provides no clear guideline. To understand the economic meaning behind the integration principle for such variables, we now turn to a simple example.

Suppose there are two aggregate states, $S = L$ (low level of aggregate economic activity) and $S = H$ (high level of aggregate economic activity), that occur with probabilities $\pi(L)$ and $\pi(H)$, respectively. Suppose further that there are two idiosyncratic states, $s_i = b$ and $s_i = g$, corresponding to a bad, respectively good, idiosyncratic shock. Finally, assume that idiosyncratic shocks are independently distributed across households: $\pi(s_1, \ldots, s_I, S) = \Pi_i \pi(s_i, S)$. This is the structure used in Atkeson and Phelan (1994) and Krusell and Smith (1999), where $s_i = b$ is interpreted as unemployment and $s_i = g$ as employment. Using this interpretation, the conditional probability $\pi(b|L)$ is the probability of being unemployed when the economy is contracting and $\pi(b|H)$ is the probability of being unemployed when the economy is expanding. The integration principle implies $\pi'(b) = \pi(b|L)\pi(L) + \pi(b|H) + \pi(H)$. In words: the probability of being unemployed in the economy without business cycles is equal to the average probability of being unemployed in the economy with business cycles.

So far, we have only discussed probabilities π. We now turn to the outcome function η. The integration principle requires $\eta'(b) = \eta(b, L)\pi(L) + \eta(b, H)\pi(H)$. In words: the loss of human capital (income) when unemployed in the economy without business cycle is equal to the average loss of human capital (income) when unemployed in the economy with business cycles. Both Atkeson and Phelan (1994) and Krusell and Smith (1999) use the integration principle, but consider the special case $\eta(b, L) = \eta(b, H)$, that is, they assume that the human capital (income) loss is the same for all aggregate states. In this case, the elimination of variations in idiosyncratic risk has no effect on the equilibrium allocation and welfare (proposition below). More generally, the integration principle in conjunction with the assumption that aggregate shocks do not affect the size of idiosyncratic shocks implies neutrality. However, if $\eta(b, L) \neq \eta(b, H)$, then the elimination of variation in idiosyncratic risk has an effect on growth and welfare (proposition below). Notice also that in this example the elimination of business cycles amounts to the elimination of the correlation of shocks across individuals. Thus, the claim made by Atkeson and Phelan (1994) that eliminating the correlation of shocks across individuals has no effect is only correct if the outcome function does not depend on aggregate shocks.

In summary, the integration principle defines in a unique way how to move from an economy with business cycles to an economy without business cycles. However, the cost of business cycles depends in a subtle way on the interaction between business cycles and idiosyncratic risk, and different modeling approaches (S affects π vs S affects η) can lead to radically different conclusions regarding the cost of business cycles. Moreover, in Appendix 2 we show that these different approaches are observationally equivalent in the important case of normally distributed idiosyncratic shocks.[11]

[11] Notice that even though the two approaches lead to very different cost of business cycles, they have the same implications for the equity premium (if the observational equivalence holds). Thus, there is no simple

Proposition *Assume that* $A(S_t) = A$, $\delta_k(S_t) = \delta_k$, *and* $\delta_h(S_t) = \delta_h$. *Consider two economies, one economy with joint probabilities* $\pi(s_i, S)$ *and idiosyncratic human capital shocks* $\eta = \eta(s_i, S)$ *(the economy with business cycles) and another economy with probabilities* $\pi'(s_i) = \sum_S \pi(s_i, S)$ *and idiosyncratic human capital shocks* $\eta'(s_i) = \sum_S \eta(s_i, S)\pi(S|s_i)$ *(the economy without business cycles). Then the following comparative dynamics results hold, where all inequalities are strict if, and only if, the function* $\eta = \eta(s_i, S)$ *depends in a non-trivial way on* S.

- Capital-to-labor ratio: $\tilde{k} \geq \tilde{k}'$
- Average investment returns:

$$\begin{aligned} r_k(\tilde{k}) &\leq r_k(\tilde{k}') \\ r_h(\tilde{k}) &\geq r_h(\tilde{k}') \\ E\left[r(\tilde{k}, s_i, S)\right] &\leq E\left[r(\tilde{k}', s_i, S)\right] \end{aligned}$$

- Consumption-to-wealth ratio:

$$\begin{aligned} \tilde{c} \geq \tilde{c}' \quad &if \quad \gamma < 1 \\ \tilde{c} = \tilde{c}' = 1 - \beta \quad &if \quad \gamma = 1 \\ \tilde{c} \leq \tilde{c}' \quad &if \quad \gamma > 1 \end{aligned}$$

- Growth rates:

$$E\left[\frac{c_{i,t+1}}{c_{it}}\right] \leq E\left[\frac{c'_{i,t+1}}{c'_{it}}\right] \quad if \quad \gamma \leq 1 \ ,$$

- Welfare:

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_{it})\right] \leq E\left[\sum_{t=0}^{\infty} \beta^t u(c'_{it})\right]$$

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link between the cost of business cycles and the equity premium. Alvarez and Jermann (2000) provide a general discussion of the relationship between the equity premium and the welfare cost of business cycles in representative-agent economies.

Proof: See appendix.

IV. Quantitative Analysis

IV.A. Model Specification

We consider a two-state aggregate shock process, $S \in \{L, H\}$, with $\pi(L) = \pi(H) = 1/2$, where L stands for low level of aggregate economic activity and H stands for high level of aggregate economic activity. We assume that $s_{it} \sim N(0, 1)$ regardless of the aggregate shock, S. That is, we assume $\pi(s_i, S) = \pi(s_i)\pi(S)$, where $\pi(s_i)$ is the density function of a normal distribution with zero mean and unit variance. We further specify that $\eta(s_i, L) = \sigma_\eta(L)s_i$ and $\eta(s_i, H) = \sigma_\eta(H)s_i$. Thus, we have $\eta_{it} \sim N(0, \sigma^2_\eta(L))$ if $S_t = L$ and $\eta_{it} \sim N(0, \sigma^2_\eta(H))$ if $S_t = H$. The assumption of normally distributed human capital shocks with $S-$dependent variance allows us to relate the current model to recent empirical studies of idiosyncratic labor income risk (see below). Moreover, the assumption that the aggregate state S enters in a non-trivial manner into the function $\eta = \eta(s_i, S)$ ensures that the elimination of variation in idiosyncratic risk has an impact on the equilibrium allocation and welfare (see the proposition in Section III). Indeed, the assumption that all variations in idiosyncratic risk are due to the influence of S on the η-function tends to maximize the growth and welfare effect of variations in idiosyncratic risk. In this sense, the growth and welfare effects of business cycles reported in this section provide an upper bound. See the Appendix 2 for more details.

For the baseline economy, we assume log-utility preferences: $\gamma = 1$. In this case, $\tilde{c} = 1-\beta$ always solves the intertemporal Euler equation. We further assume a Cobb-Douglas production function:$f(\tilde{K}) = A(S)\tilde{K}^\alpha$. Aggregate depreciation rate of physical and human capital are equal and may depend on the aggregate state: $\delta_{kt} = \delta_{ht} = \delta(S_t)$. The introduction of aggregate depreciation shocks allows us to match the volatility of both aggregate output and aggregate consumption. Without these aggregate depreciation shocks, the simple human capital model analyzed here generates an excessively smooth aggregate consumption series (Jones et al. 1999). An increase in the aggregate depreciation rate might be the result of an increase in the rate of business failure if plant and business closure destroys specific physical and human capital.

IV.B. Calibration

We calibrate the model economy as follows. We assume that the period length is one year (annual data). This choice is made to ensure consistency with a number of empirical studies that estimate individual labor income risk using annual PSID data (see below). We choose $\alpha = .36$ to match capital's share in income. The remaining parameters $A(L), A(H), \delta(L), \delta(H), \sigma_\eta(L), \sigma_\eta(H)$, and β are determined in conjunction with the equilibrium value for $\tilde{k}$ by the following restrictions:

- The remaining Euler equation holds
- $E[\delta_t] = .06$
- $E\left[Y_{t+1}/Y_t - 1\right] = .02$
- $E\left[X_{kt}/Y_t\right] = .25$
- $\sigma\left[Y_{t+1}/Y_t\right] = .0237$
- $\sigma\left[C_{t+1}/C_t\right] = .0168$
- $\sigma\left[y_{hi,t+1}/y_{hit}|S_t = L\right] = .24 \quad , \quad \sigma\left[y_{hi,t+1}/y_{hit}|S_t = H\right] = .12$

The first restriction ensures that the portfolio choice $\tilde{k}$ is an equilibrium outcome. The next restriction pins down the average depreciation rate. The value .06 is a compromise between the probably higher depreciation rate of physical capital[12] and the probably lower depreciation rate of human capital. This value is also assumed in Jones et al. (1999). The next two restrictions ensure that the model generates a realistic average growth rate and average saving rate, and the following two restrictions imply that the model matches the observed volatility of aggregate consumption and output growth for the period 1949-1999. The last restriction requires the model's process for idiosyncratic labor income risk to be consistent with the evidence form microeconomic data, to which we now turn.

Individual labor income is $y_{hit} = \tilde{r}_{ht} h_{it}$. Using $\tilde{r}_{ht} = \tilde{r}_h(\tilde{k}, S_t)$, the equilibrium law of motion (10) for h_{it}, and the approximation $log(1+r) \approx r$, we find

$$log y_{hi,t+1} = \phi(\tilde{k}, S_t, S_{t+1}) + log y_{hit} + \tilde{\eta}_{it} \ , \tag{15}$$

where $\tilde{\eta}_{it} = \frac{1}{1+\tilde{k}} \eta_{it}$. In other words, the idiosyncratic component of individual labor income approximately follows a logarithmic random walk with error term $\tilde{\eta}_{it} \sim N(0, \sigma_y^2(S_t))$, where $\sigma_y(S_t) = \sigma_\eta(S_t)/(1+\tilde{k})$.[13] The random walk specification is often used by the empirical literature to model the permanent component of idiosyncratic labor income risk (Carroll and Samwick, 1997, Hubbard, Skinner, and Zeldes, 1995, Meghir and Pistaferri, 2001, and Storesletten et.al., 2001a). Thus, their estimate of the standard deviation of the error term for the random walk component of annual labor income corresponds to the value of $\sigma_y(S)$. For the average standard deviation, $E[\sigma_y(S)]$, Carroll and Samwick (1997) and Hubbard et al.(1995) estimate a value of .15, Meghir and Pistaferri (2001) find an estimate of .19, and Storesletten et al.(2001) have .25. For the baseline economy, we choose $E[\sigma_y(S)] = .18$. Meghir and Pistaferri (2001) and Storesletten et al.(2001a) are the only studies so far that

[12] A common choice is $\delta_k = .10$. However, Cooley and Prescott (1995) argue that $\delta_k = .05$ is more realistic.

[13] We have $\tilde{\eta}_{it}$ instead of $\tilde{\eta}_{i,t+1}$ in equation (15), and the latter is the common specification for a random walk. However, this is not a problem if the econometrician observes the idiosyncratic depreciation shocks with a one-period lag. In this case, (15) is the correct equation from the household's point of view, but a modified version of (15) with $\tilde{\eta}_{i,t+1}$ replacing $\tilde{\eta}_{it}$ is the specification estimated by the econometrician.

allow σ_y to vary with the aggregate state S. Meghir and Pistaferri (2001) estimate that the variation in σ_y, measured by $\sigma[\sigma_y(S)]$, is equal to .05. Storesletten et al.(2001b) find $\sigma[\sigma_y(S)] = .10$. For our baseline economy, we choose $\sigma[\sigma_y(S)] = .06$, but we also consider an economy with $\sigma[\sigma_y(S)] = .10$. For the two-state aggregate process used in this section, the two restrictions $E[\sigma_y(S)] = .18$ and $\sigma[\sigma_y(S)] = .06$ translate into $\sigma_y(L) = .24$ and $\sigma_y(H) = .12$.

The above approach might underestimate human capital risk if the lower tail of the actual distribution of idiosyncratic income shocks is fatter than suggested by the normal distribution framework. For strong evidence for such a deviation from the normal-distribution framework, see Brav, Constantinides, and Greczy (2002) and Geweke and Keane (2000). There is, however, also an argument that the current approach might overestimate human capital risk because it assumes that all of labor income is return to human capital investment. If some component of labor income is independent of human capital investment, and if this component is random (random endowment of genetic skills), then some part of the variance of labor income is not human capital risk.

Equilibria are computed using a simple two-state approximation of the normally distributed random variable η. More specifically, we replace the random variable $\eta \sim N(0, \sigma_\eta(S))$ by a discrete random variable with two possible realizations, $-\sigma_\eta(S)$ and $+\sigma_\eta(S)$, which occur with equal probability. This approximation procedure will be used throughout this section to compute equilibria. The implied parameter values for the baseline economy are $A(L) = .2869$, $A(H) = .3001$, $\delta(L) = .0756$, $\delta(H) = .0444$, $\beta = .9364$.

The above calibration procedure ensures that the model economy matches as many features of the U.S. economy as there are free parameters. It is also interesting to investigate how the calibrated model performs in matching additional features of the U.S. economy. For example, the implied value for the average return on physical capital is $E[r_k(S)] = 5.55\%$.

This is higher than the observed real interest rate on short-term US government bonds, but lower than the observed real return on U.S. equity.[14] The model's equity premium is .06 percent, which is a substantial improvement over the value found in Mehra and Prescott (1985) for logarithmic utility (.003 percent), but still very far away from the observed value for the U.S. stock market (7 percent). The model's Sharpe ratio, however, comes much closer to its observed counterpart (3.5 versus 35). Put differently, equity returns are excessively smooth in the model economy, and the implied equity premium is therefore quite small. The implied average return on investment in human capital, $E[r_h(S)] = 11.92\%$, is in line with the estimates of rate of returns to schooling.[15] Notice that the implied excess return on human capital investment is $E[r_h(S)] - E[r_k(S)] = 6.37\%$. Thus, the model generates a substantial "human capital premium".

Finally, notice that conditional on the aggregate shock, individual consumption growth is normally distributed, $g_{i,t+1} = c_{i,t+1}/c_{it} - 1 \sim N(\mu_g(S_{t+1}), \sigma_g^2(S_{t+1}))$. The calibrated model yields an average standard deviation of consumption growth of $\sigma_g = .5\,\sigma_g(L) + .5\,\sigma_g(H) = 16.83\%$. This amount of consumption volatility is somewhat lower than what is found in the data. For example, using CEX data on consumption of non-durables and services Brav et al.(2002) find that the standard deviation of quarterly consumption growth ranges from 6 percent to 12 percent for different household groups with an average of about 9 percent. If quarterly consumption growth is i.i.d., then this corresponds to a standard deviation of annual consumption growth of 18 percent.

IV.C. Results

Applying the integration principle discussed in Section III to the case analyzed in this section,

[14]The RBC literature usually strikes a compromise and chooses the parameter values so that the implied return on capital is 4%, which is somewhat lower than the value used here.

[15]The estimates vary considerably across households and studies, with an average of about 10% (Krueger and Lindhal, 2001).

we find that the elimination of business cycles amounts to replacing the functions $A = A(S)$ and $\delta = \delta(S)$ by the constants $A' = .5\,A(L) + .5\,A(H)$ and $\delta' = .5\,\delta(L) + .5\,\delta(H)$. Furthermore, the function $\eta = \eta(s_i, S)$ is replaced by the function $\eta'(s_i) = .5\,\eta(s_i, L) + .5\,\eta(s_i, H) = .5\,(\sigma_\eta(L) + \sigma_\eta(H))\ s_i$. In particular, this means that the elimination of business cycles changes the process of human capital risk from the heteroscedastic process $\{\eta_{it}\}$, where $\eta_{it} \sim N(0, \sigma_\eta^2(S_t))$, to the homoscedastic process $\{\eta'_{it}\}$, where $\eta'_{it} \sim N(0, \sigma_\eta^{2\,\prime})$.[16]

The first three rows in table 1 report the growth and welfare gains from eliminating business cycles, which are calculated as follows. Let $\{c_{it}\}$ stand for the individual consumption process in the equilibrium with business cycles. This process is defined by an initial consumption level, c_{i0}, and a sequence of individual consumption growth rates, $\{g_{it}\}$, that are distributed according to $g_{it} \sim N(\mu_g(S_t), \sigma_g^2(S_t))$ with $S_t = L, H$. When business cycles are removed, a new equilibrium is established with individual consumption process $\{c'_{it}\}$ defined by an initial consumption level c'_{i0} and a sequence of growth rates, $\{g'_{it}\}$, that are distributed according to $g'_{it} \sim N(\mu'_g, \sigma_g^{2\,\prime})$. The first row in table 1 shows the effect of eliminating business cycles on average per capita consumption growth, that is, it reports the difference $\Delta\mu_g = \mu'_g - \mu_g$, where $\mu_g = .5\,\mu_g(L) + .5\,\mu_g(H)$. The second row in table 1 reports the welfare gains from eliminating business cycles expressed in equivalent consumption-level terms with (production economy) and without (exchange economy) the change in the average growth rate. These welfare gains are calculated as follows.

Expected lifetime utility associated with the consumption plan $\{c_{it}\}$ or $\{c'_{it}\}$ is given by (11). The welfare cost of business cycles expressed in compensating consumption differential is the fraction of initial consumption, Δ_c, an agent is willing to give up in return for the

[16] Storesletten et al.(2001b) also assume that idiosyncratic shocks are normally distributed in both economies (with and without business cycles), and that the removal of business cycles changes the heteroscedastic distribution to a homoscedastic distributions. They, however, assume that $\sigma_\eta^{2\,\prime} = .5\,\sigma_\eta^2(L) + .5\,\sigma_\eta^2(H)$.

elimination of business cycle. More formally, it is the number solving

$$\begin{aligned} \frac{(c'_{i0}(1-\Delta_c))^{1-\gamma}}{1-\beta E\left[(1+g')^{1-\gamma}\right]} &= \frac{(c_{i0})^{1-\gamma}}{1-\beta E\left[(1+g)^{1-\gamma}\right]} \quad \gamma \neq 1 \qquad (16) \\ log\left(c'_{i0}(1-\Delta_c)\right) + \frac{\beta}{1-\beta} E\left[log(1+g')\right] &= log c_{i0} + \frac{\beta}{1-\beta} E\left[log(1+g)\right] \ , \end{aligned}$$

where the random variables g and g' are defined in the preceding paragraph.[17] Notice that the expectations in (16) extends over idiosyncratic risk and, in the case of business cycles, aggregate risk. Assuming $c_{i0} = c'_{i0}$,[18] equation (15) yields

$$\begin{aligned} \Delta_c &= 1 - \left(\frac{1-\beta E\left[(1+g')^{1-\gamma}\right]}{1-\beta E\left[(1+g)^{1-\gamma}\right]}\right)^{\frac{1}{1-\gamma}} \quad \gamma \neq 1 \qquad (17) \\ \Delta_c &= 1 - \frac{\beta}{1-\beta}\, exp\left(E[log(1+g')] - E[log(1+g)]]\right) \end{aligned}$$

Notice that the expression (17) is independent of c_{i0}. Thus, the welfare gain expressed in equivalent consumption terms is the same for all households. The second row in table 1 records the total welfare gain of eliminating business cycles using (17) and a growth rate $g' \sim N(\mu'_g, \sigma_g^{2\,\prime})$, where μ'_g and $\sigma_g^{2\,\prime}$ are the mean and variance of individual consumption growth after business cycles are eliminated. The third row reports the welfare gain for fixed average consumption growth, that is, we assume $\mu'_g = \mu_g$.

Table 1 shows that the growth effect of eliminating business cycles is positive and non-negligible: .07 percent for the baseline economy and .21 percent for the economy with large variations in idiosyncratic risk. Most of the increase in growth is due to the elimination of variation in idiosyncratic risk. However, in contrast to the complete-markets model, in the model with uninsurable idiosyncratic risk the elimination of aggregate productivity and depreciation shocks has an impact on growth.

[17]Equation (16) expresses the welfare change in terms of compensating differential (variation). In the current model, this is the same as expressing the welfare change in terms of equivalent variation.

[18]This amounts to assuming that households cannot readjust their choices in the same period in which business cycles are eliminated. Note, however, that the quantitative results are barely changed if one assumes that c_{i0} is affected since $c_{i0} = (1-\beta)(1+r_{i0})w_{i0}$ and $1-\beta$ is small.

The second row in table 1 shows that the welfare costs of business cycles are quite large: 3.21 percent of initial consumption for the baseline economy and 7.48 percent for the economy with large variations in idiosyncratic risk. This is an order of magnitude large than the welfare cost in the corresponding complete-markets economy, and almost two orders of magnitude larger than the .1 percent originally found by Lucas (1987).[19] A comparison of cases a and b reveals that fluctuations in aggregate productivity and depreciation only have a moderate effect on welfare, and that most of the welfare cost of business cycles is due to the variation in idiosyncratic risk. Thus, even if aggregate consumption growth were constant (no aggregate productivity and depreciation shocks), the model still generates welfare costs of business cycles that are almost two orders of magnitude larger than the ones found by Lucas (1987).

As mentioned in the Introduction, the elimination of variation in idiosyncratic risk affects welfare in two ways: it eliminates the variation in the volatility of individual consumption growth and it increases average consumption growth (if γ is not too large). To compare the magnitude of these two effects, the third row of table 1 reports the welfare gain of eliminating business cycles for fixed average consumption growth (fixed μ_g). A comparison of the second and third row in table 1 shows that both channels are important, although the elimination in the variation of individual consumption volatility has a somewhat stronger effect on welfare.

Table 1 also shows the effect of business cycles on asset returns and investment. Notice that these effects are quite substantial. The strong asset return effect indicates that general equilibrium effects are very important. Indeed, the growth effect of business cycles is much larger if we consider a partial equilibrium model with fixed asset returns. For example, for

[19] Notice that Lucas (1987) finds a welfare cost of only .1 percent for a complete-markets economy with log-utility preferences, whereas this paper finds .43 percent. This difference is due to the fact that in this paper aggregate consumption follows a logarithmic random walk, whereas Lucas (1987) assumes that aggregate consumption is trend-stationary.

the baseline economy the growth effect increases to .49 percent.

Finally, we investigate to what extent our results depend on the degree of relative risk aversion. Tables 2 and 3 show the results for $\gamma = .5$ and $\gamma = 1.5$, respectively. Not surprisingly, for $\gamma = .5$ the growth effect is larger than for $\gamma = 1$ since in the former case the elimination of business cycles has a positive effect on total investment. Correspondingly, for $\gamma = 1.5$ the growth effect is diminished since total investment is reduced. Notice also that even though an increase in γ reduces the growth enhancing effect of eliminating business cycles, the total welfare gain is roughly constant since an increase in γ increases the welfare benefit from reducing variations in individual consumption volatility. In this sense, the welfare cost of business cycles reported in this paper are relatively robust to moderate changes in the degree of relative risk aversion.

V. Conclusion

This paper has used a tractable macroeconomic model with idiosyncratic human capital risk and incomplete markets to analyze the qualitative and quantitative effects of business cycles on growth and welfare. The qualitative analysis has shown that the elimination of variations in idiosyncratic risk decreases the ratio of physical to human capital and increases the total investment return and welfare. Moreover, it was shown that growth is always enhanced if the degree of relative risk aversion is less than or equal to one. The quantitative analysis revealed that even for relatively small degrees of risk aversion, the elimination of business cycles has substantial effects on investment in physical and human capital, economic growth, and welfare.

The current paper does not address the issue of why certain insurance markets for idiosyncratic human capital risk are missing. One possible explanation for this lack of insurance might be the asymmetry of information with respect to idiosyncratic human capital shocks.

An interesting question for future research is to investigate under what conditions the equilibrium allocation of the incomplete-markets economy is also the constrained efficient allocation of an economy with asymmetric/private information. The results by Atkeson and Lucas (1992) and Cole and Kocherlakota (2001) taken together show that the relationship between competitive allocations and constrained-efficient allocations crucially depends on the nature of private information.

This paper has followed the previous literature by assuming that a reduction in business cycle activity will lead to a decrease in the variation in idiosyncratic risk, but has not attempted to go beyond the existing literature by providing an explicit model of the interaction between business cycles and idiosyncratic risk. Developing such a model is likely to lead to additional insights into the welfare costs of business cycles, and is an important topic for future research.

Appendix 1: Proof of Proposition

We begin with the case in which η is constant across different S-realizations. For the economy analyzed in this paper, the equilibrium allocation is the solution to the one-agent decision problem with values of $\tilde{k}$ and $\tilde{c}$ determined by (9). The solution to the equation system (9), however, is the same for all joint probability distributions $\pi(s_i, S)$ satisfying $\sum_S \pi(s_i, S) = \pi'(s_i)$ for fixed marginal probabilities $\pi'(s_i)$ since none of the functions r_k, r_h, η, and r that enter into (9) depend on S. Hence, the elimination of business cycles has no effect on the allocation since it simply amounts to replacing the joint probabilities $\pi(s_i, S)$ by the marginal probabilities $\pi'(s_i) = \sum_S \pi(s_i, S)$. It also has no effect on welfare because expected lifetime utility (11) is unchanged.[20]

Suppose now $\eta(s_i, S) \neq \eta(s_i, S')$ for some s_i, S, S'. Define

$$\varphi(\tilde{k}, \eta) \doteq \frac{r_h(\tilde{k}) + \eta - r_k(\tilde{k})}{\left(1 + r(\tilde{k}, \eta)\right)^{\gamma}} \ .$$

The Euler equation (9) determining $\tilde{k}$ then reads $E[\varphi(\tilde{k}, \eta(s_i, S))] = 0$. Let $\tilde{k}$ be the solution of this equation for η and $\tilde{k}'$ for η', that is, $\tilde{k}$ and $\tilde{k}'$ solve

$$\begin{aligned} E\left[\varphi(\tilde{k}, \eta(s_i, S))\right] &= 0 \qquad\qquad \text{(A1)} \\ E\left[\varphi(\tilde{k}', \eta'(s_i))\right] &= 0 \ . \end{aligned}$$

It is straightforward to show that $r_h(\tilde{k}) > r_k(\tilde{k})$ (agents are risk averse and human capital investment is riskier than physical capital investment). Using this and that r_h is increasing

[20] The proof still goes through if the state process follows a general Markov process as long as S_t is not useful in predicting $s_{i,t+1}$ (an assumption that is implicit in the set-up of the current model because we assume that idiosyncratic shocks are unpredictable). However, if S_t contains information about $s_{i,t+1}$, then the optimal investment decision in general depends on S_t, and changes in the joint probabilities will affect the equilibrium outcome even for fixed marginal probabilities. Notice also that the proof of the neutrality result hinges on the assumption that preferences allow for an expected utility representation (independence axiom) and that S does not directly enter into the utility function (no taste shocks). Finally, we note that the neutrality result does not hold if the equilibrium allocation is not the solution to a one-agent decision problem (indirect price effects).

and r_k is decreasing in $\tilde{k}$, we find that the function φ is increasing in $\tilde{k}$ and convex in η. Recalling that $\eta'(s_i) = E[\eta(s_i, S)|s_i]$, we derive

$$\begin{aligned} E\left[\varphi\left(\tilde{k}, \eta(s_i, S)\right)\right] &= E\left[E\left[\varphi\left(\tilde{k}, \eta(s_i, S)\right)|s_i\right]\right] \qquad \text{(A2)} \\ &> E\left[\varphi\left(\tilde{k}, E\left[\eta(s_i, S)|s_i\right]\right)\right] \\ &= E\left[\varphi\left(\tilde{k}, \eta'(s_i)\right)\right] \ . \end{aligned}$$

Combining (A1) and (A2) we therefore conclude that $E[\varphi(\tilde{k}, \eta'(s_i))] < E[\varphi(\tilde{k}', \eta'(s_i))]$, which implies that $\tilde{k} > \tilde{k}'$ because φ is decreasing in $\tilde{k}$. This proves the first part of the proposition. The statements about returns and investment then follow immediately.

It is left to show that the statement about $\tilde{c}$ is true (the growth-rate effect then immediately follows). From the formula for $\tilde{c}$ we infer that we need to discuss the effect on $E\left[(1 + r(\tilde{k}, \eta(s_i, S)))^{1-\gamma}\right]$. There are two effects: a direct effect (change in η) and an indirect effect due to the change in $\tilde{k}$. Recall that the movement from η to η' decreases $\tilde{k}$ and that r is decreasing in $\tilde{k}$. For $\gamma < 1$, both effects decrease the expression because in this case $(1 + r)^{1-\gamma}$ is a concave and increasing function of r. For $\gamma > 1$, both effects increase the expression because in this case $(1+r)^{1-\gamma}$ is a convex and decreasing function of r. From this the statement about $\tilde{c}$ follows. This completes the proof of proposition 3.

Appendix 2: Normally Distributed Idiosyncratic Shocks

This appendix discusses the important case of normally distributed idiosyncratic shocks. As in section IV, we assume that there are two aggregate states, $S \in \{L, H\}$, that occur with probability $\pi(L)$ and $1 - \pi(H)$, respectively. We also assume that in the economy with business cycles idiosyncratic human capital shocks are conditionally normally distributed: $\eta_i \sim N(0, \sigma_\eta^2(L))$ if $S = L$ and $\eta_i \sim N(0, \sigma_\eta^2(H))$ if $S = H$. We show next how this "observed" family of distributions can be generated in two different ways, and how these different approaches give rise to different answers regarding the welfare cost of business cycles even if we apply the same integration principle.

Approach 1.

In contrast to the assumption made in Section IV, assume that $\eta(s_i, S) = \eta'(s_i) = s_i$. Thus, aggregate shocks do not affect the η-function, and the proposition therefore implies that the elimination of business cycles has no effect on the equilibrium allocation and welfare. If we assume that $\eta_i = s_i \sim N(0, \sigma_\eta^2(L))$ if $S = L$ and $\eta_i = s_i \sim N(0, \sigma_\eta^2(H))$ if $S = H$, then we clearly match the "observed" distribution of idiosyncratic human capital shocks. Notice that the application of the integration principle leads to $\pi'(s_i) = \pi(s_i|L)\pi(L) + \pi(s_i|H)\pi(H)$, where $\pi(s_i|L)$, respectively $\pi(s_i|H)$, is the density function of a normal distribution with zero mean and standard deviation $\sigma_\eta(L)$, respectively $\sigma_\eta(H)$. Thus, when we eliminate business cycles, we move from an economy with heteroscedastic and normally distributed idiosyncratic shocks to an economy with non-normally distributed idiosyncratic shocks (the mixture of two normal distributions).

Approach 2.

In accordance with the assumption made in Section IV, assume that probabilities are independent of business cycle activity, $\pi(s_i, S) = \pi_1(s_i)\pi_2(S)$, choose $s_i \sim N(0, 1)$, and match the "observed" distribution of idiosyncratic human capital shocks by letting $\eta(s_i, L) = \sigma_\eta(L)\, s_i$ and $\eta(s_i, H) = \sigma_\eta(H)\, s_i$. This case is discussed in Section IV. Clearly, it does not lead to neutrality of business cycles.

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TABLE 1. Business cycle effects for $\gamma = 1$ (log-utility)

$Case$	$1a$	$1b$	$2a$	$2b$	3
$\Delta\mu_g$	.012 %	.071 %	.024 %	.210 %	0 %
Δ_{c1}	.693 %	3.21 %	.923 %	7.48 %	.435 %
Δ_{c2}	.513 %	2.16 %	.604 %	4.65 %	.435 %
$\Delta X_k/Y$	-.090 %	-.58 %	-.14 %	-2.02 %	0 %
$\Delta X_h/Y$	.13 %	.79 %	.20 %	1.97 %	0 %
Δr_k	.06 %	.38 %	.10 %	.99 %	0 %
Δr_h	-.05 %	-.32 %	-.09 %	-.88 %	0 %

TABLE 2. Business cycle effects for $\gamma = .5$

$Case$	$1a$	$1b$	$2a$	$2b$	3
$\Delta\mu_g$	.022 %	.123 %	.040 %	.379 %	.007 %
Δ_{c1}	.756 %	3.99 %	1.01 %	9.45 %	.392 %
Δ_{c2}	.327 %	1.65 %	.413 %	3.82 %	.197 %
$\Delta X_k/Y$	-.015 %	-.048 %	.013 %	-.141 %	.023 %
$\Delta X_h/Y$	.168 %	1.05 %	.261 %	2.66 %	.041 %
Δr_k	.031 %	.231 %	.053 %	.615 %	0 %
Δr_h	-.022 %	-.161 %	-.039 %	-.436 %	0 %

TABLE 3. Business cycle effects for $\gamma = 1.5$

$Case$	$1a$	$1b$	$2a$	$2b$	3
$\Delta\mu_g$	.005 %	.049 %	.089 %	.192 %	-.005 %
Δ_{c1}	.648 %	2.65 %	.858 %	5.93 %	.392 %
Δ_{c2}	.592 %	2.07 %	.627 %	3.98 %	.589 %
$\Delta X_k/Y$	-.199 %	-.980 %	-.653 %	-4.03 %	-.020 %
$\Delta X_h/Y$	.131 %	%	.765 %	3.23 %	-.037 %
Δr_k	.100 %	.545 %	.401 %	2.63 %	0 %
Δr_h	-.107 %	-.564 %	-.476 %	-2.72 %	0 %

1: starting from incomplete-markets economy with $\sigma_y(L) = .24$ and $\sigma_y(H) = .12$.
2: starting from incomplete-markets economy with $\sigma_y(L) = .28$ and $\sigma_y(H) = .08$.
3: starting from complete-markets economy
a: elimination of variation in aggregate productivity and depreciation
b: elimination of variation in idiosyncratic risk and variation in aggregate productivity and depreciation

$\Delta\mu_g$: change in aggregate consumption growth
Δ_{c1}: change in welfare (in terms of equivalent change in initial consumption)
Δ_{c2}: change in welfare for constant $\Delta\mu_g$ (in terms of equivalent change in initial consumption)
$\Delta X_k/Y$: change in the rate of saving in physical capital
$\Delta X_h/Y$: change in the rate of saving in human capital
Δr_k: change in interest rate
Δr_h: change in wage rate