Victim or Injurer: Negligence-Based Liability Rules Under Role-Type Uncertainty, With An Extension to Collisions Of Different-Sized Vehicles

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Working Paper No. 2003-17

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August 29, 2003

Abstract

This paper modifies the standard tort model by introducing role-type uncertainty, that is, it is assumed that neither party knows in advance whether she will become the victim or the injurer when an accident occurs. When the standards of care of the two parties are assumed to be set at the socially optimal levels, only pure comparative negligence and the equal division rule guarantee efficiency, while the rules of simple negligence, contributory negligence, and comparative negligence with fixed division (other than a 50:50 split) produce the possibility of inefficient equilibria. Since the pure comparative negligence rule splits liability between negligent parties according to each party's degree of fault, it makes the accident loss division independent of one's role-type. This produces a social efficiency advantage. We also extend the model to the choice of vehicle size, as a factor determining who will be the injurer and who the victim in motor vehicle collisions. In the extension we analyze various negligence-based liability rules, and tax rules, as instruments for mitigating the vehicle size "arms race."

Keywords: role-type uncertainty, negligence, comparative negligence, contributory negligence, equal division rule, motor vehicle collisions, vehicle size.

JEL classification: K13, D61.

1 INTRODUCTION

One of the stylized features of the standard economic model of tort law is that each party's potential role as a victim or an injurer in a possible accident is pre-determined. That is, each knows in advance the position - victim or injurer - she will be in if an accident occurs. This setup is consistent with some types of torts, such as product liability or medical malpractice, but it doesn't fit some other types of torts, like automobile accidents.[1]

Arlen (1990) extends the standard analysis to the "bilateral risk" situation in which injurers as well as victims suffer accident damages.[2] She shows that if each party is allowed to sue her counterpart for her own damages, the main implications of the basic model remain intact. That is, various forms of negligence-based rules[3] induce both parties to take efficient levels of care if the standard of care for each party is set equal to the socially efficient level.

In her model, however, the proportion of total damages each party would suffer from an accident is known in advance. Landes and Posner (1987, p.77) and Wittman et al. (1997) make the same assumption in dealing with the theory of bilateral risk accidents.[4] However, this may not be a satisfactory way to capture the essence of role-type uncertainty. In some cases, uncertainty about whether one will be the victim or the injurer is central.

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[1] According to the study of the U.S. Department of Justice, Bureau of Justice Statistics, automobile accidents or property liability claims accounted for more than 75% of all tort filings in state courts between July 1, 1991 and June 30, 1992. Medical malpractice, product liability, and toxic substances together comprised about 10% of all the cases filed. See Cooter and Ulen (2000, pp.356-357). Huber (1988) estimated that traffic accident claims account for around 40% of all tort cases.

[2] Note that a unilateral risk model with role-type uncertainty is in essence similar to a bilateral risk model, if we assume that everyone is entitled to sue the other party for his own damages from an accident. In other words, the situation where we know that the victim suffers the whole damages but do not know who will be the victim, is analytically similar to the situation where we know both can suffer damages but do not know what the proportions will be.

[3] Arlen (1990) does not include comparative negligence in her analysis, but the application of the model to comparative negligence is straightforward.

[4] White (1989) has a different approach for modeling automobile accidents cases, along the lines of Diamond (1974). Rather than considering gaming behavior between the two parties involved, she focuses on the representative driver's behavior, assuming that the other driver's care level is randomly chosen from a population. Her analysis also includes uncertainty in the court's decision-making, as well as role-type uncertainty.

This paper modifies the standard tort model by introducing role-type uncertainty. That is, neither party knows in advance whether she will be the victim or the injurer when an accident occurs. With imperfect information, each party makes choices based on a subjective belief about the probability that she will be the victim. The sum of the two parties' subjective victimization probabilities is not necessarily equal to 1.

In section 2 of this paper we show that, if the two victimization probabilities do not sum to 1, pure comparative negligence has better efficiency properties than other negligence-based rules. We show that role-type uncertainty enhances the efficiency claim of pure comparative negligence because it makes the liability assignment between two negligent parties independent of their role-types. In particular, we show that when standards of care are set at socially optimal levels, only the pure comparative negligence rule and the equal division rule guarantee efficiency, given role-type uncertainty. In contrast, the rules of simple negligence, negligence with contributory negligence as a defense (henceforth "contributory negligence"), and comparative negligence with a fixed division other than 50:50, all allow the possibility of inefficient equilibria in which both parties take insufficient care.

In section 3 of the paper we extend the model to a theory of negligence and choice of vehicle size. In this section there is again role-type uncertainty, but this is a theory of motor vehicle collisions, where one's probability of being the injurer depends on the size of one's vehicle. Drivers of big cars are more likely to be injurers, drivers of small cars, victims. The general conclusion of the section 3 model is that all the standard negligence-based liability rules we examine produce efficient equilibria in terms of the parties' *care levels*, but all produce an inefficient "arms race" (in White's (2003) phrase) in terms of *vehicle size*. We indicate how creating a size-based standard of care could ameliorate the inefficiency, and we compare that policy to the imposition of a vehicle size tax.

2 EFFICIENT LIABILITY RULES UNDER ROLE-TYPE UNCERTAINTY

2.1 The Model

Suppose that there are two risk-neutral people, X and Y, who engage in some activity that creates a risk of accidents. If an accident occurs, there is one victim, who suffers a monetary loss $L > 0$. The loss L is assumed to be constant. But there is uncertainty regarding the roles of X and Y in the possible accident: neither knows in advance whether she will be the *victim* or the *injurer*. We assume that each party has a subjective belief about the likelihood that she will be the victim or the injurer. We define:

$\alpha \equiv$ Person X's subjective probability that she will be the victim $(0 \le \alpha \le 1)$.
$\beta \equiv$ Person Y's subjective probability that she will be the victim $(0 \le \beta \le 1)$.

Note that since these are subjective probabilities, formed *ex ante*, the sum of α and β need not be equal to 1.[5] It will turn out that the value of $\alpha + \beta$ has a crucial role in determining game equilibria, and the efficiency, or lack thereof, of some liability rules.

Let x and y denote person *X'*s and person *Y'*s care levels, respectively, measured by their care expenditures. Following the standard modeling in the tort liability literature since Brown (1973), we assume that each person can choose any level of care between 0 and ∞, and that the probability of an accident $(\equiv p(x, y))$ is defined continuously for every possible combination of care levels chosen by each person. Increasing care levels reduce the probability of an accident, and thus the expected accident cost ($p_x < 0$, $p_y < 0$). We also assume that, for all x and y, $p_{xx} > 0$, $p_{yy} > 0$, $p_{xy} > 0$, and $p_{xx}p_{yy} - p_{xy}^2 > 0$. To insure an interior solution, we assume that, for all x and y, $p_x(0, y) < -1$ and $p_y(x,0) < -1$. For simplicity, we assume further that the care level a party takes does not affect her subjective probability of being the victim (or the injurer).[6]

Total social cost (TSC) is defined as the sum of care-taking costs of both parties and expected accident costs. That is, TSC = $x + y + p(x, y)L$. The social goal is, as usual, to minimize total social cost. Let (x^*, y^*) denote the solution to this TSC minimization

[5] For example, in the context of automobile collision cases, each driver's car is different in terms of its weight, bumper height, etc. Each driver may not know in advance with whom she will collide. A Hummer driver may expect that her probability of being the victim in a collision with another car will be small. If two Hummer drivers are involved in a collision, $\alpha + \beta$ may be much less than one. However, if the drivers do know with whom they will collide, then it is plausible that $\alpha + \beta$ will equal 1; see section 3 below.

[6] This implies, for example, as a driver reduces her speed, both the probability of hitting other cars and the probability of being hit by other cars are expected to decline by the same degree. This assumption can be relaxed without changing the main implication of this section of this section.

problem. By the assumptions on the $p(x, y)$ function, the efficient (x^*, y^*) is unique and strictly positive.

When an accident occurs, the entire loss L is initially born by the victim. The court then enforces a *liability rule*, which determines where L ultimately falls: on the victim, on the injurer, or on both. A negligence-based liability rule is defined in terms of which parties are negligent, and if both are negligent, possibly their degrees of negligence. A party is *negligent* if her care expenditure falls short of the court-enforced *standard of care*. We assume that everyone, including the court, knows the expected loss function and the governing liability rule, that the court can solve the TSC minimization problem, and that everyone, including the court, can observe each party's care level accurately. We assume that the standard of care for each party is set at the socially optimal level. That is, for instance, party X is found *negligent* by the court if and only if she spends $x < x^*$.[7]

We will analyze and compare the rules of simple negligence (section 2.2 below), contributory negligence (section 2.3), comparative negligence with fixed division (section 2.4), and pure comparative negligence (section 2.5). These rules are distinguished by their ways of assigning liability when (and only when) both the victim and the injurer are negligent. In other cases, all the liability rules listed above share the following characteristics:

(i) If neither the victim nor the injurer is negligent, all accident costs fall on the victim.
(ii) If the victim is non-negligent and the injurer is negligent, all accident costs fall on the injurer.
(iii) If the victim is negligent and the injurer is non-negligent, all accident costs fall on the victim.

[7] Note that the assumptions that the court knows the expected loss function, and sets the standard of care at the efficient levels (x^*, y^*), are exceedingly strong; we argue elsewhere that these assumptions, which are standard in the literature, are actually unrealistic. See Kim (2003) and Feldman and Kim (2003).

From the above three properties, person *X's* expected personal cost ($\equiv C_X$) under role-type uncertainty can be calculated as a function of both parties' care levels as follows:

If $x \geq x^*$ and $y \geq y^*$, then $C_X = \alpha(x + p(x,y)L) + (1-\alpha)x = x + \alpha\, p(x,y)L$ (1)

If $x < x^*$ and $y \geq y^*$, then $C_X = \alpha(x + p(x,y)L) + (1-\alpha)(x + p(x,y)L) = x + p(x,y)L$ (2)

If $x \geq x^*$ and $y < y^*$, then $C_X = \alpha x + (1-\alpha)x = x$ (3)

If $x < x^*$ and $y < y^*$, then C_X varies from rule to rule.

Person *Y's* expected personal cost function ($\equiv C_Y$) is derived in a similar way. Equations (1) through (3) enable us to establish that, as in the standard model with pre-determined role-type, the social optimum (x^*, y^*) is a Nash equilibrium even with role-type uncertainty, under the four liability rules. The result is Lemma 1 below. Closely related Lemma 2 says that under the four liability rules, there is no Nash equilibrium that involves either party taking *more* than due care, that is, spending more than her efficient x^* or y^*. Proofs of both lemmas are in the Appendix.

Lemma 1. *With role-type uncertainty, the social optimum* (x^*, y^*) *is a Nash equilibrium under the rules of simple negligence, contributory negligence, comparative negligence with fixed division, and pure comparative negligence.*

Lemma 2. *With role-type uncertainty, nobody takes more than due care, at any possible Nash equilibrium, under the rules of simple negligence, contributory negligence, comparative negligence with fixed division, and pure comparative negligence.*

Lemmas 1 and 2 do not rule out the possibility of a Nash equilibrium in which *both* parties take *less* than due care. Since each negligence-based rule specifies a different liability assignment in this case, the parties' expected cost functions vary from rule to rule. If one of the four liability rules could successfully rule out the possibility of an insufficient-care-equilibrium, then (x^*, y^*) would turn out to be the *unique* equilibrium of the game, and we could conclude that the rule would be efficient in the presence of role-type uncertainty. With this in mind, we examine each of the four negligence-based rules in turn.

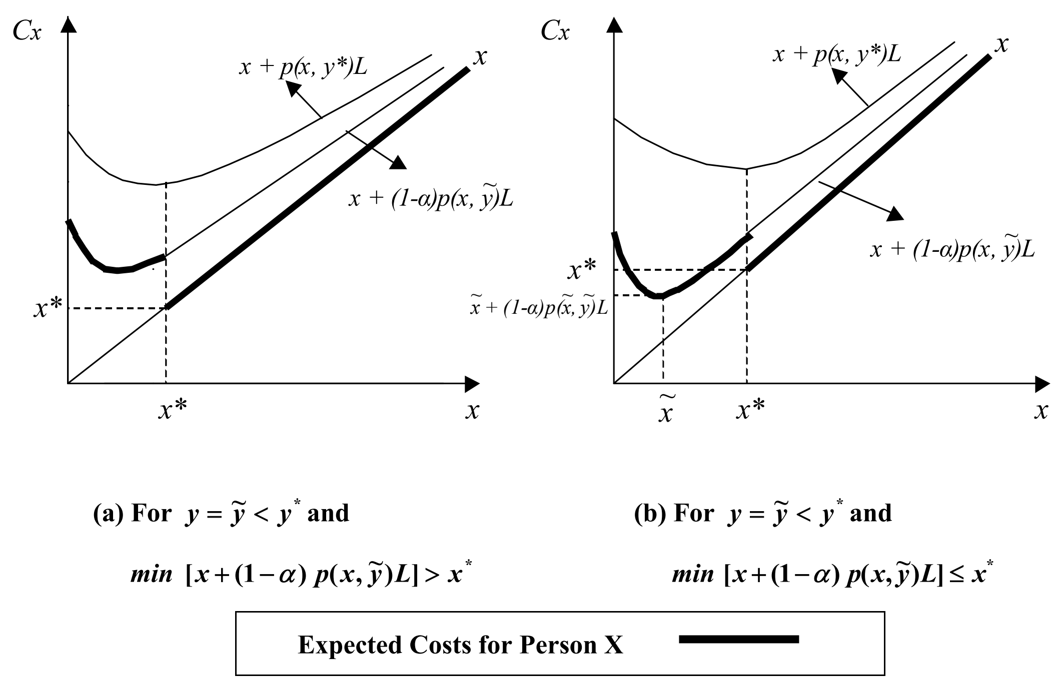
2.2 Simple Negligence

The simple negligence rule allows the victim to recover damages from a negligent injurer, even if the victim is negligent herself. So in addition to (1) through (3), person *X*'s expected cost function (when both parties are negligent) is given by:

If $x < x^*$ and $y < y^*$, then $C_X = \alpha x + (1-\alpha)(x + p(x,y)L) = x + (1-\alpha)\, p(x,y)L$ (4)

Note that when facing a negligent counterpart, person *X*'s best response for minimizing her own expected cost may be either to take due care (Figure 1 (a) below), or to choose negligence (Figure 1 (b)).

As shown in Figure 1 (b), if α is sufficiently high, there may exist care levels for *X* such that the condition $x + (1-\alpha)p(x,\tilde{y})L < x^*$ holds. In this case, person *X* would take insufficient care (at $\tilde{x}$), given person *Y*'s choice of insufficient care (at $\tilde{y}$).



(a) For $y = \tilde{y} < y^*$ and

min $[x + (1-\alpha)\, p(x,\tilde{y})L] > x^*$

(b) For $y = \tilde{y} < y^*$ and

min $[x + (1-\alpha)\, p(x,\tilde{y})L] \le x^*$

Figure 1: Expected Costs for Person X under Contributory Negligence When $y < y^*$

In short, the simple negligence rule under role-type uncertainty allows the possibility of inefficient equilibria, in which both parties are choosing negligent behavior. In addition to the efficient equilibrium at (x^*, y^*), the game has an inefficient equilibrium at ($\tilde{x}, \tilde{y}$), where $\tilde{x} < x^*$ and $\tilde{y} < y^*$, if the following conditions are satisfied simultaneously:

$$\tilde{x} = arg\ min[x + (1-\alpha\ )p(x, \tilde{y})L] \text{ and } \tilde{y} = arg\ min[y + (1-\beta)\ p(\tilde{x}, y)L] \tag{5}$$

$$\tilde{x} + (1-\alpha)\ p(\tilde{x}, \tilde{y})L \leq x^* \tag{6}$$

$$\tilde{y} + (1-\beta)\ p(\tilde{x}, \tilde{y})L \leq y^* \tag{7}$$

The higher is α [β], the higher is the likelihood that condition (6) [(7)] is met. The intuition behind conditions (6) and (7) is straightforward. Under the simple liability rule, if both parties are failing the standard of care, each thinks that she will bear no accident costs if she is the victim, but will bear all accident costs if she is the injurer. Therefore, if a party believes that the possibility of her being a victim is quite high, she heavily discounts the accident losses that are placed on her when she is the injurer. As a result, she may choose negligently low care expenditures.

For instance, suppose that party X is sure that she will be the victim, and that Y is also (i.e., $\alpha = \beta = 1$). In this case, the simple negligence game has two Nash equilibria; (0,0) and (x^*, y^*). One possible outcome here is an inefficient equilibrium, where neither party takes *any* precaution.

Lemma 3 below gives a necessary condition for the existence of an inefficient equilibrium, in terms of the sum of two subjective victimization probabilities.

Lemma 3. $\alpha + \beta > 1$ *is a necessary condition for the simple negligence rule to produce an inefficient equilibrium under role-type uncertainty.*

Proof. Assume to the contrary. By adding together inequalities (6) and (7), we have $\tilde{x} + \tilde{y} + (2-(\alpha+\beta))p(\tilde{x}, \tilde{y})L \leq x^* + y^*$. But since x^* and y^* solve the total social cost minimization problem, $x^* + y^* < x^* + y^* + p(x^*, y^*)L \leq \tilde{x} + \tilde{y} + p(\tilde{x}, \tilde{y})L$. If $\alpha + \beta \leq 1$, $\tilde{x} + \tilde{y} + p(\tilde{x}, \tilde{y})L \leq \tilde{x} + \tilde{y} + (2-(\alpha+\beta))p(\tilde{x}, \tilde{y})L$, a contradiction. Q.E.D.

2.3 Contributory Negligence

Under the rule of negligence with contributory negligence as a defense, the negligent injurer is liable for accident losses unless the victim is also negligent, in which case the burden falls on the victim. When both are negligent, each bears her own losses, i.e., her losses when she is the victim. So in addition to equations (1) through (3), person *X'*s expected cost function (when both parties are negligent) is now given by:

If $x < x^*$ and $y < y^*$, then $C_X = \alpha(x + p(x,y)L) + (1-\alpha)x = x + \alpha\ p(x,y)L$ (8)

By the same logic that was used in the preceding section, we can derive conditions for the existence of an inefficient equilibrium at ($\tilde{x}, \tilde{y}$), where $\tilde{x} < x^*$ and $\tilde{y} < y^*$:

$$\tilde{x} = arg\ min[x + \alpha p(x,\tilde{y})L] \text{ and } \tilde{y} = arg\ min[y + \beta p(\tilde{x}, y)L] \quad (9)$$

$$\tilde{x} + \alpha\ p(\tilde{x},\tilde{y})L \le x^* \quad (10)$$

$$\tilde{y} + \beta\ p(\tilde{x},\tilde{y})L \le y^* \quad (11)$$

Like the simple negligence rule, the contributory negligence rule does not necessarily lead to an efficient outcome under role-type uncertainty. Since the accident costs fall on the victim when both are negligent, the contributory negligence rule is likely to produce an insufficient-care-equilibrium when both parties believe they have high probabilities of becoming an injurer, that is, low probabilities of becoming a victim. Based on the same logic used in the proof of Lemma 3, we have the following:

> **Lemma 4.** $\alpha + \beta < 1$ *is a necessary condition for the contributory negligence rule to produce an inefficient equilibrium under role-type uncertainty.*

Lemma 4 makes it clear why Arlen (1990) and Wittman et al. (1997) conclude that efficiency is guaranteed under the contributory negligence rule. In their bilateral risk models, both parties suffer losses and each party's expected loss function is separable and known to everyone. Accordingly, the condition $\alpha + \beta = 1$ (in our context) is always satisfied, and the possibility of an insufficient-care-equilibrium is eliminated.

2.4 Comparative Negligence with Fixed Division

In contrast with "all-or-nothing" rules like simple negligence or contributory negligence, the comparative negligence rule with fixed division splits accident damages between two negligent parties, in some fixed proportion.[8] Let γ denote the fraction of accident damages borne by the victim when both parties are negligent ($0<\gamma<1$). In particular, if $\gamma=\frac{1}{2}$, this is "the equal division rule". The equal division rule was once the dominant doctrine in admiralty law, until it was replaced by the pure comparative negligence system.[9]

Under the comparative negligence rule with fixed division, person X's expected cost function, when both parties are negligent, is given by:

If $x<x^*$ and $y<y^*$, then
$$\begin{aligned} C_X &= \alpha(x+\gamma p(x,y)L)+(1-\alpha)(x+(1-\gamma)p(x,y)L) \\ &= x+[(2\alpha-1)\gamma+(1-\alpha)]p(x,y)L \end{aligned} \tag{12}$$

Now equation (12), combined with equations (1) through (3), fully describe person X's expected cost function under the fixed division rule.

It is easy to see that this rule may not exclude the possibility of an insufficient-care-equilibrium, for certain values of α, β, and γ. The conditions for the existence of an inefficient equilibrium at ($\tilde{x},\tilde{y}$), where $\tilde{x}<x^*$ and $\tilde{y}<y^*$, are given by:

$$\tilde{x}=arg\,min\,[x+((2\alpha-1)\gamma+(1-\alpha))p(x,y)L] \text{ and}$$

[8] It needs to be emphasized that the loss-sharing property of the comparative negligence with fixed division rule works only as a "defense". So this rule is different from what Shavell (1987) defines as the rule of "strict division of accident losses". Strict division of accident losses implies that the fraction of losses borne by the injurer and the victim is assumed to be independent of their levels of care and, in particular, independent of whether someone was negligent. It is straightforward that the strict division of accident losses does not provide each party with correct incentives of efficient care-taking in the standard model without role-type uncertainty.

[9] By about 1700 English courts were consistently applying the equal division rule in admiralty collision cases. This doctrine was replaced in 1911 in Great Britain by a statute providing for division of damages in proportion to the degree of fault of each vessel. The U.S. Supreme Court adopted the equal division rule in 1854, in a ship collision case, *The Schooner Catharine v. Dickinson* (58 U.S. (17 how.) 170, 15L. Ed. 233). In turn, the equal division rule was replaced by the pure comparative negligence rule in U.S. admiralty

$$\widetilde{y} = arg\,min[y + ((2\beta - 1)\gamma + (1-\beta))p(x,y)L] \quad (13)$$

$$\widetilde{x} + [(2\alpha - 1)\gamma + (1-\alpha)]p(\widetilde{x},\widetilde{y})L \leq x^* \quad (14)$$

$$\widetilde{y} + [(2\beta - 1)\gamma + (1-\beta)]p(\widetilde{x},\widetilde{y})L \leq y^* \quad (15)$$

However, note that if $\gamma = \frac{1}{2}$, the role-type uncertainty parameters α and β disappear in conditions (14) and (15). Under the equal division rule, (14) and (15) turn into $\widetilde{x} + \frac{1}{2}p(\widetilde{x},\widetilde{y})L \leq x^*$ and $\widetilde{y} + \frac{1}{2}p(\widetilde{x},\widetilde{y})L \leq y^*$, respectively. But these two conditions cannot be satisfied simultaneously, because adding the two inequalities together yields $\widetilde{x} + \widetilde{y} + p(\widetilde{x},\widetilde{y})L \leq x^* + y^*$, which is not possible since (x^*, y^*) is a unique TSC minimizing optimum. Therefore, the equal division rule guarantees efficiency. This is an intuitive result, since among the set of all fixed division rules, the equal division rule is the only one that makes one party's expected cost, assuming both parties are negligent, independent of her role-type.

With the exception of the equal division rule, all fixed division rules allow the possibility of insufficient-care-equilibria. By logic similar to that used in the proof of Lemmas 3, we can derivr necessary conditions for such equilibria:

> **Lemma 5.** *With role-type uncertainty, if there exists an inefficient equilibrium under the comparative negligence rule with fixed division, one of the following two conditions is* satisfied*: (i)* $\alpha + \beta > 1$ *and* $\gamma < \frac{1}{2}$*, or (ii)* $\alpha + \beta < 1$ *and* $\gamma > \frac{1}{2}$.

Lemma 5 makes it clear that, as with simple negligence and contributory negligence, inefficient outcomes may occur under the fixed division rule, but do not occur if $\alpha + \beta = 1$. By comparing the conditions for inefficient outcomes derived so far, we can compare simple negligence, contributory negligence, and comparative negligence with fixed division (except 50:50 split) in terms of social efficiency. We find that (i) if $\alpha + \beta = 1$, then all three rules are efficient, and (ii) if $\alpha + \beta \neq 1$, then none of these three rules guarantees efficiency, and, furthermore, none dominates the others on efficiency grounds.

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cases as a result of a 1975 U.S. Supreme Court case, *United States v. Reliable Transfer Co.* (421 U.S. 397, 44 L. Ed. 2d 251, 95 S. Ct. 1708). See Keeton et al. (1984) and Schwartz (1994).

2.5 Pure Comparative Negligence

The pure comparative negligence rule splits accident damages between two negligent parties according to their degrees of fault. In the context of continuous care models, the proportions of accident damages persons *X* and *Y* should bear are generally defined using the ratio of each party's deviation from her due care level. That is, *X'*s degree of fault is most naturally measured by $\frac{x^* - x}{x^* - x + y^* - y}$, and similarly for *Y'*s.

Now, in addition to (1) through (3), person X's expected cost function when both parties are found negligent is given by:

If $x < x^*$ and $y < y^*$,

then $C_X = \alpha(x + \frac{x^* - x}{x^* - x + y^* - y} p(x,y)L) + (1-\alpha)(x + \frac{x^* - x}{x^* - x + y^* - y} p(x,y)L)$

$$= x + \frac{x^* - x}{x^* - x + y^* - y} p(x,y)L \tag{16}$$

Note that when both take less than due care, each party's expected cost is completely independent of the role-type uncertainty parameters α and β. When both are negligent, one party's damage share depends only on her own care level, the given x^*, y^*, and the other party's care level. As before, the social optimum (x^*, y^*) is a well-defined Nash equilibrium under the pure comparative negligence rule. Furthermore, we can now show that it is a unique equilibrium of the game: Suppose to the contrary that there exists another equilibrium ($\tilde{x}, \tilde{y}$), where $\tilde{x} < x^*$ and $\tilde{y} < y^*$. Then, at ($\tilde{x}, \tilde{y}$), the following two conditions must be met simultaneously:

$$\tilde{x} + \frac{x^* - \tilde{x}}{x^* - \tilde{x} + y^* - \tilde{y}} p(\tilde{x}, \tilde{y})L \le x^* \tag{17}$$

$$\tilde{y} + \frac{y^* - \tilde{y}}{x^* - \tilde{x} + y^* - \tilde{y}} p(\tilde{x}, \tilde{y})L \le y^* \tag{18}$$

Adding the above two inequalities together yields

$$\tilde{x} + \tilde{y} + p(\tilde{x}, \tilde{y})L \leq x^* + y^* \tag{19}$$

But condition (19) contradicts the fact that (x^*, y^*) is the TSC minimizing optimum, since $x^* + y^* < x^* + y^* + p(x^*, y^*)L \leq \tilde{x} + \tilde{y} + p(\tilde{x}, \tilde{y})L$.

In conclusion, the pure comparative negligence rule guarantees efficiency even with role-type uncertainty, as long as the standards of care are set equal to the socially efficient levels. Since the pure comparative negligence rule splits liability between negligent parties according to each party's degree of fault, it makes the accident loss division independent of one's role-type. Whatever beliefs each party has regarding the probability of being the victim or the injurer are completely beside the point. No matter what α and β are, the pure comparative negligence rule always produces the socially optimal outcome as the unique equilibrium of the game. Our main findings so far are summarized in:

> **Proposition 1.** *Suppose that the standard of care for each party is set at the socially optimal level. In the standard tort model, with role-type uncertainty, under which each party forms a subjective belief regarding her probability of becoming an injurer or a victim:*
>
> *(i) If $\alpha + \beta = 1$, all the negligence rules we have examined guarantee efficiency.*
>
> *(ii) In general, if $\alpha + \beta$ is not constrained at 1, then the pure comparative negligence rule and the equal division rule both guarantee efficiency;*
>
> *(iii) However, the rules of simple negligence, contributory negligence, and comparative negligence with fixed division (other than the 50:50 split) all produce the possibility of an inefficient equilibrium in which both parties take less than due care. Among these three rules, no rule dominates others in terms of efficiency.*

3 AN APPLICATION OF THE MODEL TO VEHICLE SIZE

At this stage we add another dimension to the analysis by relating α and β to vehicle size. With the burgeoning use of SUV's and pickup trucks as passenger vehicles in recent years, it has become apparent that many people may be choosing large vehicles to reduce the likelihood that, in the event of a collision, they will be victims. If your GM Hummer or Lincoln Navigator collides with a Honda Civic, whose vehicle is totaled? More important, whose children go to the hospital? The auto companies don't advertise it explicitly, but it's implicit: Drive our big SUV, and *you'll* walk away from the crash; the *guy in the other car* will be carried to the hospital. White (2003) provides data about the external effects created by vehicle size and the failure of negligence-based liability rules to solve this externality problem; the modeling below is in the same spirit.

We now assume that there is a one-dimensional property that is chosen by each of our two parties, which we call "size." (In reality, motor vehicles have a number of characteristics that impact on the probability that their occupants will be victims or injurers, including weight, height of center of gravity, bumper height, horsepower, body reinforcements, airbags, and so on.)

We assume that size is measured from 0 (smallest) to 1 (largest). We assume that person *X* chooses size s, and that person *Y* chooses size t. We assume that the following victimization probability rule governs, and that all parties are aware of it:

If *X* and *Y* are involved in an accident, and $(s,t) \neq (0,0)$ the probabilities of each being the victim are given by:

$$\alpha = \frac{t}{s+t} \text{ and } \beta = \frac{s}{s+t} \tag{20}$$

To put it another way, the probability that person *X* is the *injurer* is given by $\frac{s}{s+t}$, the fraction of total vehicle weight comprised by *X*'s vehicle. The big cars rule! To complete the specification of the victimization probabilities, we assume that if $(s,t) = (0,0)$, the probabilities are both $\frac{1}{2}$. (The model is easy to extend to the case where, if $(s,t) = (0,0)$, the victimization probabilities are two non-zero constants summing to 1.)

Case 1. As a first simple case, assume that *X* and *Y* have fixed, pre-determined vehicle sizes, known to both, and that they are choosing care levels x and y, subject to one of the negligence-based rules analyzed above. Given the victimization probability rule of equation (20), it must be the case that $\alpha + \beta = 1$, and under our current assumptions, both parties know it. Therefore, under Lemmas 3, 4, and 5 above, no matter which negligence-based rule is in effect, the unique Nash equilibrium care levels are at (x^*, y^*).

When (x^*, y^*) is the Nash equilibrium, both parties are taking adequate care, and therefore under all the negligence-based rules examined, all of the accident losses fall on the accident victim. We conclude, therefore, that the bottom line for all costs for party *X* is now:

$$C_X = x^* + \frac{t}{s+t} p(x^*, y^*)L \tag{21}$$

Case 2. Next we assume that party *X* takes party *Y*'s vehicle size t as given, and is able to choose her own vehicle size s. We let r represent the cost per unit of vehicle size, so that a vehicle of size s costs rs. (Realistically, of course, size is one vehicle characteristic whose cost is added to the costs of other vehicle characteristics. We are assuming that the choice of a minimally-sized vehicle adds $r0$ to the vehicle cost, whereas choosing the maximally-sized vehicle adds $r1$ to the cost.)

By the arguments above, any Nash equilibrium requires that the care levels be at the efficient (x^*, y^*).

Anticipating a Nash equilibrium in the care levels at (x^*, y^*), and deciding what sized vehicle to choose, party *X* wants to choose s to minimize

$$C_X + rs = x^* + \frac{t}{s+t} p(x^*, y^*)L + rs$$

The first order condition for a minimum leads easily to

$$s = \left(\frac{t}{r} p(x^*, y^*)L \right)^{1/2} - t \tag{22}$$

Case 3. Now assume that both parties, *X* and *Y*, are minimizing expected accident costs plus vehicle size costs, each taking the other's (optimal) care as a given, and each taking the other's vehicle size as a given. Party *X* is solving equation (22), and party *Y* is solving an analogous equation. It is then easy to show that the resulting vehicle size choices, s^*, t^*, are given by

$$s^* = t^* = \frac{1}{4r} p(x^*, y^*)L \tag{23}$$

Total expenditure on vehicle size is of course a pure social waste in this model, since it has no function other than to throw fixed damages on the *other* party in the event of an accident. We will call this social cost *vehicle size social cost,* and abbreviate it VSSC; it is of course an addition to the previously defined TSC, which only comprised care costs plus expected accident costs. The vehicle size social cost is trivially derived from equation (23) above, as follows:

$$\text{VSSC} = rs^* + rt^* = \frac{1}{2} p(x^*, y^*)L \tag{24}$$

Equation (24) suggests that the social waste produced by White's (2003) "arms race" is quite substantial; in our model it amounts to half of the expected accident losses.

What policies might be adopted to reduce this loss? As White (2003) indicates, vehicle size *per se* is not a negligence issue under the legal rules with which we are familiar. However, in theory, it could be. If the designer of a judicial system were to establish standards of care for both care levels x and y, *and* vehicle sizes s and t, our model would proceed as follows: Vehicle size costs would be added to the previously defined TSC, producing a redefined total social cost of:

$$\text{TSC} = x + y + p(x, y)L + rs + rt\,.$$

This is minimized at $(x,s) = (x^*,0)$ and $(y,t) = (y^*,0)$. The court would set party *X*'s standard of care at $(x^*,0)$ and party *Y*'s standard of care at $(y^*,0)$. Failure to meet *either* the care expenditure standard *or* the size standard would result in a party's being found negligent.

Now consider whether or not party *X* would want to deviate from $(x^*,0)$, if party *Y* were at $(y^*,0)$. If X maintains s at 0, by the assumptions of this section $\alpha + \beta = \frac{1}{2} + \frac{1}{2} = 1$, and the analysis of the previous sections goes through, with $(x^*,0)$ and $(y^*,0)$ comprising a Nash equilibrium under any of the negligence-based liability rules. However, X might be tempted to choose $x = x^*$ and $s = \varepsilon > 0$, in order to throw the accident damages on the other party. But if she does so she becomes negligent, and under any of the negligence-based rules, all the damages get thrown back on her.

Therefore, she resists the temptation. It is also clear that no Nash equilibrium would be possible if party *Y* were at (y^*, t), with $t > 0$, because *X* would then choose $(x^*, 0)$; this would result in *X*, in the small vehicle, becoming the victim with probability 1, but the damages would always be thrown back on *Y*, whose too-large vehicle makes her negligent. The results of this section to this point are summarized in:

Proposition 2. *Suppose that parties X and Y choose both care-taking expenditures x and y, and vehicle sizes s and t. Assume that their probabilities of being victims in accidents are given by equation (20) above, and are known to both parties. Assume that standards of care for x and y are set at the socially optimal levels. Under the rules of simple negligence, contributory negligence, comparative negligence with fixed division (other than the 50:50 split), equal division, and pure comparative negligence:*

(i) If the court imposes no size standard, all the rules produce efficient Nash equilibria in terms of care-taking expenditures, but social waste from a vehicle size "arms race."

(ii) If the court also imposes a standard of care for size, at the optimal level of zero, all the rules produce efficient Nash equilibria in terms of care-taking expenditures and vehicle size.

The principal drawback of proposition 2's positive second part is that it is only a faint theoretical possibility. It's not likely that driving a large vehicle will ever be *per se* evidence of negligence, at least not where we live. A more plausible possibility is a tax on vehicle size, which could be easy to implement (state motor vehicle departments already know our vehicle weights, and some states impose higher registration fees on heavier vehicles), and for which there are precedents. In the rest of this section we model such a tax.

Case 4. We continue to assume that both parties, *X* and *Y*, are minimizing expected accident costs plus vehicle size costs, each taking the other's (optimal) care as a given, and each taking the other's vehicle size as a given. We assume that negligence is defined only in terms of x and y. We assume that there is an *ad valorem* tax τ on vehicle size, and therefore the cost of a unit of vehicle size, inclusive of the tax, is $(1+\tau)r$. We assume that each dollar of tax revenue produces a dollar's worth of social value, so the tax itself does not create its own social loss. The result is that each party replaces r with

$(1+\tau)r$ in her calculations regarding vehicle size to choose. Equations (23) and (24) are replaced with the following:

$$s^* = t^* = \frac{1}{4r(1+\tau)} p(x^*, y^*)L \,. \tag{25}$$

$$\text{VSSC} = rs^* + rt^* = \frac{1}{2(1+\tau)} p(x^*, y^*)L \,. \tag{26}$$

The case 4 conclusion is this: All the negligence-based liability rules produce efficient Nash equilibria in terms of care costs x and y. There remains a vehicle size "arms race" loss VSSC as shown in equation (26), but society can reduce this below any (positive) threshold by choosing the *ad valorem* size tax τ large enough.

4 CONCLUSION

The recently published *Restatement of the Law (Third), Torts* focuses on the comparative responsibility system, reflecting the fact that all but four states (Alabama, Maryland, North Carolina, and Virginia) plus the District of Columbia, have adopted comparative negligence in place of the older contributory negligence rule. Spurred by this major doctrinal switch in tort law, scholars of law and economics have actively looked for an efficiency gain from the switch. The existing literature favoring the comparative negligence rule on efficiency grounds generally focuses on the characteristic of comparative negligence as a "sharing rule," and contrasts it with the "all-or-nothing" feature of contributory negligence.[10]

In section 2 of this paper, we have focused on another aspect of comparative negligence that makes it superior to simple negligence, contributory negligence, or fixed-division negligence (other than equal division), namely the behavior of the rule under

[10] Cooter and Ulen (1986) and Haddock and Curran (1985) argue that comparative negligence is superior to contributory negligence under "evidentiary uncertainty". They contend that when courts make errors in evaluating care levels, over-precaution caused by evidentiary uncertainty is less under comparative negligence than it is under other negligence-based rules. Rubinfeld (1987) and Emons (1990) introduce heterogeneity in individual's care-taking costs. They show that a properly designed sharing rule can improve on all-or-nothing type negligence rules. But recently, Bar-Gill and Ben-Shahar (2003) critique these approaches, and question the validity of the efficiency justification for comparative negligence.

role-type uncertainty.[11] We have shown that when each party enters the game with a subjective belief regarding the probability that she will be the victim (or the injurer), and the victimization probabilities do not sum to 1, only pure comparative negligence and the equal division rule guarantee social efficiency, while the rules of simple negligence, negligence with contributory negligence, and comparative negligence with fixed division other than 50:50, all allow the possibility of inefficient equilibria. Since the pure comparative negligence rule splits liability between negligent parties according to each party's degree of fault, it makes the accident loss division independent of one's role-type – victim or injurer. We have shown that this often-overlooked characteristic implies that comparative negligence has a significant advantage over other rules in terms of social efficiency.

In section 3 of this paper we have extended our model to reflect the "arms race" in vehicle size, since vehicle size is an important factor in the determination of victimization probabilities. We find that in this extension of the model, where the assumptions imply that $\alpha + \beta = 1$ must hold, all of the standard negligence-based liability rules we have analyzed produce efficient care levels, but all result in another kind of inefficiency, because vehicles are too large. We find that if the notion of negligence could be broadened to encompass vehicle size, the vehicle size "arms race" inefficiency could be erased, and we also find that, without broadening the notion of negligence, a vehicle size tax could be used to mitigate the inefficiency.

[11] Wittman et al. (1997) claim that the prevalence of comparative negligence is directly associated with the fact that automobile accidents have come to dominate the tort system. The superiority of comparative negligence under role-type uncertainty may be interpreted as a supporting evidence for this claim.

APPENDIX

Proof of Lemma 1. For the purposes of the proofs of lemmas 1 and 2 we will assume that $0 < \alpha < 1$ and $0 < \beta < 1$; the arguments when α or β equal 0 or 1 are obvious.

Given that person *Y* satisfies her due care exactly (i.e., $y = y^*$), person *X'*s expected cost function is given by (1) and (2):

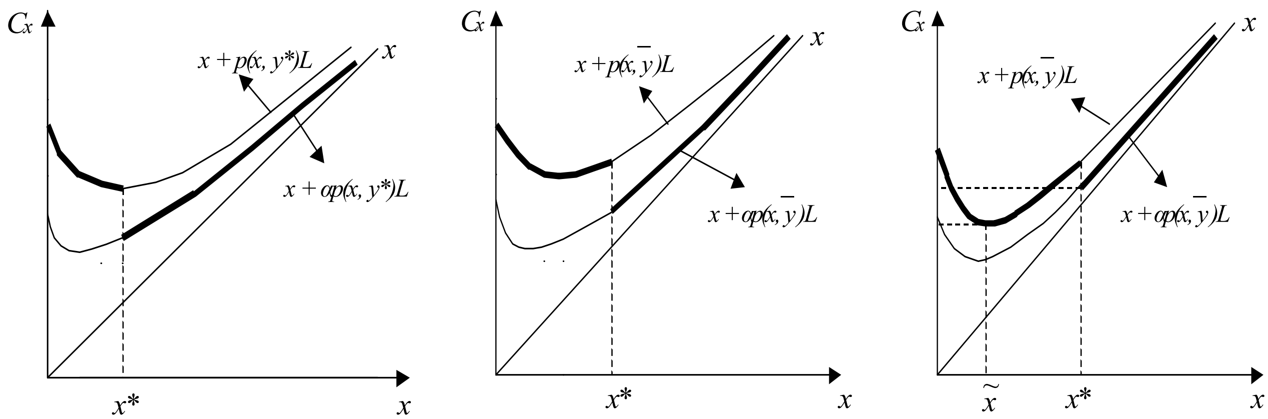
$$C_X = x + \alpha\ p(x, y^*)L\ , \text{ if } x \geq x^*;$$
$$= x + p(x, y^*)L\ , \quad \text{if } x < x^*.$$

By definition, $x^* = arg\ min[x + p(x, y^*)L]$. Also, $x^* > arg\ min[x + \alpha\ p(x, y^*)L]$ holds, since $p_{xx} > 0$. Hence, as Figure 2 (a) below shows, *X'*s expected cost function for $y = y^*$ jumps down at the due care level x^*, and then increases following the line $C_X = x + \alpha\ p(x, y^*)L$. As a result, given that *Y* satisfies her due care exactly (i.e., $y = y^*$), person *X'*s best response is to choose x^*. By the same logic, person *Y'*s best response to $x = x^*$ is to choose y^*. Q.E.D.

Proof of Lemma 2. Suppose to the contrary that *Y* chooses $\bar{y} > y^*$. Since $p_{xy} > 0$, $x^* > arg\ min[x + p(x, \bar{y})L]$. Figures 2 (b) and (c) below show the two possible graphs of person X's expected cost function when *Y* has chosen $\bar{y} > y^*$. If $min[x + p(x, \bar{y})L] > x^* + \alpha\ p(x^*, \bar{y})L$ holds (Figure 2 (b)), person *X* will choose x^*, in response to the given $\bar{y}$. However, $(x^*, \bar{y})$ is not an equilibrium, since by Lemma 1 person *Y'*s best response to x^* is to choose y^*.

Figure 2 (c) shows the case where $\min[x + p(x, \bar{y})L] \leq x^* + \alpha\ p(x^*, \bar{y})L$. In this case *X* may want to take less care than due care in response to *Y'*s choice of $\bar{y}$. To show that this is also not an equilibrium, it suffices to observe that *Y* would always prefer y^* to $\bar{y}$ if person *X* chooses $\tilde{x} < x^*$. By reducing her care level from $\bar{y}$ to y^*, person *Y* still escapes liability, while saving the care cost difference.

We conclude that for any Nash equilibrium choice of *y*, $y \leq y^*$ must hold. By the same logic, $x \leq x^*$ must hold at any Nash equilibrium choice of *x*. Q.E.D.



(a) For $y = y^*$

(b) For $y = \bar{y} > y^*$ and

$\boldsymbol{min\,[x + p(x, \bar{y})L] > x^* + \alpha\ p(x^*, \bar{y})L}$

(c) For $y = \bar{y} > y^*$ and

$\boldsymbol{min\,[x + p(x, \bar{y})L] \le x^* + \alpha\ p(x^*, \bar{y})L}$

Figure 2: Expected Costs for Person *X* (Shown in Bold), When $y \ge y^*$

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