

# **The Hand Rule And *United States v. Carroll Towing Co.* Reconsidered**

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**Allan M. Feldman  
Jeonghyun Kim**

Department of Economics  
Brown University  
Providence, RI 02912 USA  
Allan\_Feldman@brown.edu  
Jeonghyun\_Kim@brown.edu

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## **Abstract**

Judge Learned Hand's opinion in *United States v. Carroll Towing Co.* (1947) is canonized in the law and economics literature as the first use of cost-benefit analysis for determining negligence and assigning liability. This paper revisits the original case in which the famous Hand formula was born, and examines whether Judge Hand's ruling in that case would truly provide correct incentives for efficient precaution. We show that the original rule specified by Judge Hand is different from the usual application of the Hand formula by modern law-and-economics theorists in the standard continuous care model. Through a game theoretic analysis of the case, we show that Judge Hand's negligence rule from *United States v. Carroll Towing Co.* may in fact produce games with inefficient equilibria. Such a possibility of inefficiency does not depend on the specific liability rule that governed the original case. It is even more ironic that there exist cases where the equilibrium is efficient, but the equilibrium requires that the victim not have a "bargee" on board, which flies in the face of Judge Hand's opinion.

*Keywords:* Learned Hand rule, negligence, liability rules, negligence-based rules.

*JEL classification:* K13.

## I. Introduction

Judge Learned Hand's opinion in *United States v. Carroll Towing Co.*<sup>1</sup> is canonized in the law and economics literature. It is like the opening notes in Beethoven's 5<sup>th</sup> symphony or the third line of the Bible: *And God said, Let there be light: and there was light.* What is remembered, of course, is the judge's formula:

Since there are occasions when every vessel will break from her moorings, and since, if she does, she becomes a menace to those about her; the owner's duty, as in other similar situations, to provide against resulting injuries is a function of three variables: (1) The probability that she will break away; (2) the gravity of the resulting injury, if she does; (3) the burden of adequate precautions. Possibly it serves to bring this notion into relief to state it in algebraic terms: if the probability be called P; the injury, L; and the burden, B; liability depends upon whether B is less than L multiplied by P: i.e., whether  $B < PL$ .

Thus the judge throws mathematics, and rational cost-benefit analysis, into the law, and the theme has continued to play for 55 years in the literature of economic analysis of liability rules.

As the economic analysis now stands, it is generally believed that a determination of each party's negligence should be based on some cost-benefit test like Judge Hand's: i.e., if a party's cost of preventing accidents (what the judge calls the burden) is less than the expected losses from the accidents, that party should be deemed negligent; and that under certain general conditions, a negligence-based liability rule (like simple negligence, negligence with a defense of contributory negligence, or comparative negligence) will lead rational victims and injurers to an efficient outcome.<sup>2</sup>

The purpose of this paper is to revisit the specific case *United States v. Carroll Towing Co.*, and Judge Hand's ruling. We will examine the Hand test in that particular case, and we will show that liability according to the Hand rule in *United States v. Carroll Towing Co.* may in fact produce games with inefficient equilibria.

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<sup>1</sup> 159 F.2d 169 (2d Cir. 1947)

<sup>2</sup> See Brown (1973), Cooter and Ulen (2000), Landes and Posner (1987), Rea (1987), and Shavell (1987).

## **II. Structure of the case**

*United States v. Carroll Towing Co.* grew out of an accident that took place in New York Harbor on January 4, 1944. The parties involved were as follows:

Connors Marine Co., Inc., owner of the barge “Anna C”

Pennsylvania Railroad Co., charterer of the barge “Anna C”

The United States, owner of a cargo of flour aboard the “Anna C”

Carroll Towing Co., Inc., owner of the tug “Carroll”

Grace Line, Inc., charterer of the tug “Carroll”

Some relevant facts of the case, recounted in Judge Hand’s opinion, were as follows: The Pennsylvania Railroad Co. had chartered the barge Anna C from Connors Marine Co., and the charter arrangement included the services of a “bargee” attendant, provided by Connors Marine Co., between the hours of 8 a.m. and 4 p.m. On or before January 2, 1944, the barge was loaded with a cargo of flour owned by the United States. Sometime on or before January 4, 1944, the barge was moored to Pier 52 on the North River. At some time after Anna C was moored, 5 other barges were moored outside her. The next pier north of Pier 52 was the Public Pier. On January 4, 1944, the tug Carroll was sent down to the Public Pier to “drill” out one of the barges at that pier. On board the Carroll were her captain, employed by Connors Marine Co., and also a “harbormaster,” employed by the chartering company, Grace Line, Inc. There were lines between the tier of barges on the 2 piers, and the Carroll had to shift some of those lines in order to drill out the barge it was after on the Public Pier. The captain of the Carroll put a deckhand and the harbormaster on the Pier 52 tier of barges (including the Anna C). The deckhand and the harbormaster went aboard those barges, readjusted their fasts, and then reboarded the tug Carroll. Shortly thereafter, the tier of barges on Pier 52 broke adrift. Soon the Anna C struck a tanker, sprung a leak, careened, dumped her cargo, and sank. At the time this happened the bargee who was supposed to be on board the Anna C was absent. Had he been on board, he would have been able to see that she was leaking, he would have been able to get help from the tugs in the vicinity of the accident, and he would have been able to prevent the loss of the cargo and the sinking of the Anna C.

This complex accident gave rise to several issues of negligence and liability. Was Carroll Towing Co., owner of the tug and employer of her captain, negligent because of

inadequate readjustment of the fasts? Was Grace Line, charterer of the Carroll and employer of the harbormaster, negligent because of inadequate readjustment of the fasts? And was Connors Marine Co., owner of the Anna C, negligent because its employee the bargee was absent from his post? There was no claim that Pennsylvania Railroad Co., charterer of the Anna C, or that the United States, owner of the cargo, was negligent. There was also a complication in that the damages were two-tiered: After the Anna C broke her fasts, drifted, and stuck another vessel; she sustained what Judge Hand describes as “collision damages.” She then took on water, dumped her cargo and sank. But had a bargee been on board at the time of the collision, Judge Hand found that she wouldn’t have dumped the cargo and sank. The latter damages Judge Hand called “sinking damages.” So the questions are then: who was negligent in the creation of what damages? And how should the damages be parceled out among the various parties, under the governing Admiralty Law, which, in a straightforward single-defendant and single-plaintiff case where both are negligent, would split damages 50/50?

Ultimately Judge Hand apportioned the damages as follows: Of the “collision damages,” one half was put on Grace Line, the charterer of the tug, and one half was put on Carroll Towing Co., the owner of the tug. Of the “sinking damages,” one third was put on Grace Line, one third on Carroll Towing Co., and one third on Connors Marine Co., the owner of the barge Anna C. But why?

### **III. Three ways to apply Judge Hand’s test**

Judge Hand’s test is  $B < PL$ : the burden of adequate precaution is less than the product of the accident probability and the loss in case of an accident. If the inequality holds, the party to whom the test is being applied is negligent. But this is really a much more complicated issue than this simple inequality suggests. To explain the complications, let’s start by changing Judge Hand’s notation in a minor way. Instead of using B for “burden” we use  $c$  for “cost”. And we note immediately that most litigated accidents involve at least 2 parties, an injurer (defendant) and a victim (plaintiff). (Note that *United States v. Carroll Towing Co.* involves 5 parties.) In many accidents (including this one), preventive actions might have been taken by the defendant or by the plaintiff. We use  $c_I$  to represent the cost to the injurer of taking a preventive action, and  $c_V$  to

represent the cost to the victim of taking a preventive action. Now the Hand test might be applied to the injurer:  $c_I < PL$ , or to the victim:  $c_V < PL$ .

Several issues immediately come up, largely related to the connections among  $c_I$ ,  $c_V$ , and  $P$ . The first is whether the preventive actions, and the costs  $c_I$  and  $c_V$  of taking such actions, are continuously variable choices, or discrete choices. That is, is there some range of preventive actions, or is there only one, or perhaps a few, relevant actions? To put it another way, can the party at hand take any degree of care (or spend any amount of money) from \$0 to \$100,000, say, or is it a matter of taking one particular action or not? In the case of *United States v. Carroll Towing Co.*, Judge Hand applied his famous formula to the presence or absence of the barge on the Anna C. This appears to be a discrete dichotomous choice. The second issue is whether or not there is some meaningful interaction between the actions that are taken, or the amounts  $c_I$  and  $c_V$  that are expended. In the standard modern liability rule model, the three variables are explicitly interconnected, with  $P$  a function of  $c_I$  and  $c_V$ , often a continuous and differentiable function. The third issue is related to the first two, and has to do with how the Hand formula (for the injurer or for the victim) is applied. As we see it, there are 3 possible ways to apply the formula.

### **1. Independent application**

In some circumstances it is possible to apply the two tests  $c_I < PL$  and  $c_V < PL$  independently. In particular, suppose  $c_I$  and  $c_V$  are both discrete and dichotomous. That is, the injurer can either take some specific care (and spend  $c_I$ ), or not (and spend 0); the victim can either take some specific care (and spend  $c_V$ ), or not (and spend 0). Suppose that if neither takes care, the accident probability is  $P$ . Suppose that if either the injurer takes care (and spends  $c_I$ ), or the victim takes care (and spends  $c_V$ ), the accident probability is reduced to zero. (The accident is prevented.) We can call this the perfect prevention, alternative (redundant) care case.

In this case, the test can be applied to the injurer -- i.e.,  $c_I < PL$  -- independently of the behavior of the victim, or to the victim, independently of the behavior of the

injurer. That is, the question of party A's negligence or non-negligence can be determined *with no reference to party B's behavior*<sup>3</sup>.

## **2. Conditional application, contingent on the other party's *actual* behavior**

In some circumstances the injurer's test  $c_I < PL$  must be made contingent on the victim's action or inaction, and the victim's test  $c_V < PL$  must be made contingent on the injurer's action or inaction. Suppose for instance that if neither party acts,  $P = 0.1$  and  $L = \$1000$ , so that  $PL = \$100$ . Suppose that if the injurer acts to prevent, it costs \$60, and whenever the injurer spends  $c_I = \$60$ , the accident probability drops to 0. Suppose that if the victim acts, at a cost of  $c_V = \$40$ , the effect is to reduce the accident probability by half.

The proper logic of the Hand test is now as follows. If  $c_I = c_V = \$0$ , the test is applied to the injurer:  $\$60 < PL = \$100$ , and the injurer is negligent. The test is applied to the victim:  $\$40 < \Delta PL = \$50$ , and the victim is also negligent. (Note that in this example, the victim's action does not reduce the accident probability to zero. Therefore the right hand side of the inequality has the *change* in the accident probability  $\Delta P$ , times the loss  $L$ , rather than the accident probability  $P$ , times the loss  $L$ .) If  $c_I = \$60$ , the accident probability has already been reduced to zero, there will be no accidents and no lawsuits, and further action by the victim is redundant, as in the previous subsection. If  $c_V = \$40$ , the accident probability has been reduced to 0.05. The test for the injurer is now  $\$60$  vs.  $\$50$ , and the injurer is now *non-negligent* if he spends 0. In this example, therefore, the Hand test applied to the injurer is naturally *contingent on the actual behavior* of the victim.

In a general model with  $c_I$  and  $c_V$  possibly multi-valued, and possibly interactive in their effect on  $P$ , the *Learned Hand test contingent on actual behavior* would operate as follows: The injurer is negligent if, given the victim's choice of  $c_V$ , an incremental

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<sup>3</sup> See the formal analysis of Feldman and Frost (1998). Orr (1991) uses the independent application of the Hand formula in an "imperfect" prevention case. Early Posner and Landes use the independent application in their numerical examples (see Posner (1977, pp.123-124) and Landes and Posner (1980)), but later they change their position and criticize the independent approach from the view of the conditional-based-on-efficient-behavior approach (see Posner (1986, pp.154-155) and Landes and Posner (1987, pp.88-89)).

expenditure on precaution by the injurer would reduce the expected accident losses by more than that incremental expenditure. (In a discrete model the increment would be discrete and in a continuous model the increment would be infinitesimal.) The definition of the victim's negligence is similar.

### 3. Conditional application, contingent on the other party's *efficient* behavior

This is the usual approach of the standard modern liability rule model, since Brown (1973)<sup>4</sup>. Assume that the court is wise enough to know exactly how the accident probability  $P$  (and possibly the extent of the loss  $L$ ) depends on the choices  $c_I$  and  $c_V$ . Assume that the court is able to recognize that total expected costs to society from accidents of the type being litigated are given by the formula  $c_I + c_V + PL$ . Assume that there is a unique pair of values  $c_I^*$  and  $c_V^*$  which minimize total expected costs to society from these accidents. Then  $c_I^*$  and  $c_V^*$  are called the efficient levels of care of the injurer and victim respectively<sup>5</sup>.

Now the test injurer's  $c_I < PL$  is made contingent on  $c_V^*$ , and the victim's test  $c_V < PL$  is made contingent on  $c_I^*$ . That is, the test for each party is made *contingent on the efficient* behavior of the other party.

This would work as follows in the numerical example from the preceding subsection: First, observe that the efficient choices in the example are for injurer to spend  $c_I^* = \$60$ , and for victim to spend  $c_V^* = \$0$ . Now suppose  $c_I = c_V = \$0$ . If an accident occurs and there is litigation, the Hand test is applied to the injurer: Since  $c_V = c_V^* = \$0$ , the test works exactly as in the previous subsection, and the injurer is found negligent. Next, the Hand test is applied to the victim. But the test as applied to the victim is made contingent *not on the actual* expenditure  $c_I = \$0$  of the injurer, *but on the efficient*  $c_I^* = \$60$  of the injurer. And here we have a result that is different from the

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<sup>4</sup> Brown (1973) also examines the conditional-based-on-actual-behavior approach, which he calls the limited information approach. But his principal results are based on the conditional/efficient approach, and the limited information approach has long been ignored in the literature. In his recent article on the Learned Hand rule (Brown (1998)), he does not mention the limited information approach. For a formal analysis of the conditional/actual approach and its implications, see Kim (2002).

<sup>5</sup> To see how the case where  $c_I^*$  and  $c_V^*$  aren't unique might be treated, see Jain and Singh (2002).

previous subsection, because if the injurer were spending the efficient \$60 for care, then it would make no sense for the victim to spend \$40 for care, because that burden of \$40 would only buy a \$0 reduction in expected accident losses. Therefore the victim is found non-negligent. Next suppose  $c_I = \$0$  and  $c_V = \$40$ . Suppose an accident occurs and the Hand test is applied to the injurer. Here again we have a result contrary to the previous subsection. In the previous subsection it makes no cost-benefit sense for the injurer to spend his \$60 if the victim is already spending \$40, but now the court would judge the injurer's behavior contingent on the *efficient* behavior of the victim, namely  $c_V^* = \$0$ . Therefore, the injurer would now be found negligent by the Hand test.

Before leaving this section we will note the following: In the modern tort liability theory, where expected loss from accidents is a continuous and differentiable function of the care expenditure levels of injurer and victim  $c_I$  and  $c_V$ , and where the benevolent judicial system is calculating a unique efficient pair  $c_I^*$  and  $c_V^*$  to minimize total expected social costs  $c_I + c_V + PL$ , the Learned Hand rule becomes equivalent to the following: injurer is negligent if and only if  $c_I < c_I^*$ ; and victim is negligent if and only if  $c_V < c_V^*$ .

#### **IV. A game theoretic analysis of *United States v. Carroll Towing Co.***

##### **1. Structure of the game: dichotomous choice, partly alternative care technology**

We now return to Judge Hand's treatment of *United States v. Carroll Towing Co.* Our main interest is to see how Judge Hand's decision in that case would have motivated the various parties in that case had they known it beforehand, and how the decision would motivate similar parties in similar situations in the future. In *United States v. Carroll Towing* there are two care-taking activities that could, according to the judge, affect expected losses: careful adjustment and inspection of the Anna C's moorings by the potential injurers Grace Line and Carroll Towing Co., and having the bargee on board by the potential victim Connors Marine Co. What incentives does Judge Hand's decision provide these and future similar litigants? Would the incentives lead to efficient outcomes?

In *United States v. Carroll Towing Co* there were two defendants, Grace Line and Carroll Towing Co., who were found equally liable for collision damages, presumably



because of the inadequate care of their employees, the harbormaster and the tug captain, respectively. Grace Line wanted to be relieved from liability because its harbormaster was supposedly not authorized to judge the Anna C's faults, but Judge Hand didn't buy this claim. For our purposes, and for purposes of the subsequent law and economics literature, the dispute between the two defendants is an unimportant sideshow. Therefore, in what follows we shall pretend there was only one defendant or injurer, namely Carroll Towing Co. We use the following notation in our analysis of the case:

$L_C$ : Collision damages, that is, cost of repairs to the Anna C had she been prevented from sinking.

$L_S$ : Sinking damages, including cost of salvaging the Anna C, and value of lost cargo.

$P$ : The probability that a barge will break away from her moorings and collide, if her moorings are not carefully adjusted and inspected.

$c_I$ : The cost to the injurer, Carroll Towing Co., of careful adjustment and inspection of the barge moorings.

$c_V$ : The cost to the victim, Connors Marine Co., of having a bargee on board<sup>6</sup>.

Judge Hand's opinion clearly shows that the prevention technology is *partly alternative (redundant)* in this case, in the following senses: First, if the victim had his bargee on board the Anna C there would still have been collision damages, but sinking damages would have been prevented. Judge Hand applies his formula to the victim's behavior, and concludes that the victim Connors Marine Co. was negligent with regard to *sinking* damages because it failed the test; i.e.,  $c_V < PL_S$ . Second, although the judge is not absolutely clear about this, it appears that he believes the injurer Carroll Towing would have completely prevented both sinking damages and collision damages, if it had been sufficiently careful in adjusting and inspecting the barge moorings. Therefore we

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<sup>6</sup> The relationship between Connors Marine Co. and its employee the bargee creates other complications that we want to avoid in this paper. In economic terms, this is a principal/agent problem. Legally, Connors Marine Co. is responsible for the misdeeds of its employee. The bargee was supposed to be on board when the accident occurred, and so Connors Marine Co. was the victim in this case of its bargee's shirking behavior. It is also interesting to note one of the Judge's general observations about bargees: "...the barge must not be the bargee's prison, even though he lives aboard; he must go ashore at times."

will assume in the following that had  $c_I$  been spent, the probability of an accident would have dropped to zero, and neither type of damage would have occurred.<sup>7</sup> In fact, it appears that throughout the process of the suit, the injurer’s negligence was never an issue. While Judge Hand’s opinion takes the injurer’s negligence for granted and therefore doesn’t specifically direct the Hand test toward the injurer, we will assume, we think safely, that  $c_I < P(L_C + L_S)$  holds in this case.

In Table 1 below we show total social cost, defined as the sum of care-taking costs of each party plus the expected accident losses, for each of the four possible scenarios, in this game between the victim and the injurer.

		INJURER (Carroll Towing Co.)	
		Careful Inspection of Moorings (C)	Careless Inspection of Moorings (NC)
VICTIM (Conners Marine Co.)	Bargee Aboard (B)	Total Cost = $c_V + c_I$	Total Cost = $c_V + PL_C$
	No Bargee (NB)	Total Cost = $c_I$	Total Cost = $P(L_C + L_S)$

**Table 1. Total Social Cost Matrix.**

Note that we are adding some additional minor notation: For the victim Conners Marine Co., the choice is to have a bargee aboard (at a cost of  $c_V$ ), or to have no bargee aboard (at a cost of 0); the first action we abbreviate B, and second NB. For the injurer Carroll Towing Co., the choice is to adjust and inspect the moorings carefully (at a cost of  $c_I$ ), or to adjust and inspect carelessly (at a cost of 0); the first action we abbreviate C and the second NC. The four scenarios we examine are the four combinations of actions: (B,C), (B,NC), (NB,C), and (NB,NC). In each scenario there is a combination of actions, a resulting outcome in terms of accidents, and a resulting social cost, comprised of the sum of prevention costs and expected accident losses. For instance, at (B,NC), the victim has expended  $c_V$  to ensure the bargee is aboard, the injurer has expended 0 on adjustment

<sup>7</sup> It is also possible to assume that the accident probability would be reduced by the injurer’s careful mooring job, but not to zero. (This can be called “imperfect prevention technology”). Judge Hand’s opinion does not explicitly rule this possibility out. Assuming imperfect prevention technology would change the game’s payoff structure and makes our analysis significantly more complicated, but the main conclusion of this section – that the game may lead to inefficient equilibria – would remain intact.

and inspection, accidents occur with probability  $P$ , but when they do occur there are no sinking damages, and so total social cost is given by  $c_V + PL_C$ .

Efficiency in this model is simply a matter of total social cost minimization. The pair (B,C) is not efficient, since the victim's ensuring the bargee is aboard is wasteful given that the accident probability is 0 when the injurer spends  $c_I$  to ensure the moorings are carefully adjusted and inspected. The pair (NB, NC) is not efficient, since care-taking behavior by *either* party will reduce total social cost under our assumption that the inequalities  $c_V < PL_S$  and  $c_I < P(L_C + L_S)$  both hold. So the only possible candidates for efficiency are (B,NC) and (NB,C). A quick examination of the corresponding cells in the Table 1 matrix leads to the conclusion that if  $c_I < c_V + PL_C$ , then (NB,C) is efficient; and if  $c_I > c_V + PL_C$ , then (B,NC) is efficient<sup>8</sup>.

## **2. Is the conditional/efficient application or the independent application of the Hand rule consistent with Judge Hand's opinion in *United States v. Carroll Towing Co.*?**

If the Hand formula were applied conditionally in this case, contingent on the other party's *efficient* behavior, the standard of care for each party would be set at the efficient level: That is, in the case where  $c_I < c_V + PL_C$ , and (NB,C) is efficient, the victim's due care level would be set at 0 (no bargee need be aboard), whereas the injurer's due care level would be set at  $c_I$  (spend enough to ensure careful adjustment and inspection of the moorings). In alternative case, where  $c_I > c_V + PL_C$ , and (B,NC) is efficient, the victim's due care level would be set at  $c_I$  (spend enough to ensure that the bargee is aboard), whereas the injurer's due care would be set at 0. Because of the (partly) alternative characteristic of the parties' care-taking activities, there *would not exist* a situation in which both would be found negligent. In the (NB,NC) scenario, one of the two parties would be non-negligent, because efficiency in Table 1 requires that one of the parties take no precaution (no bargee, or careless adjustment and inspection), so one of the parties would be meeting his standard of care.

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<sup>8</sup> Note that throughout this paper we assume any inequality is a strict inequality; that is, for the sake of simplicity, and for the sake of unique equilibria, we assume away the equality cases.

We can easily show that with the use of a conditional/efficient application of the Hand formula, every type of negligence-based rule results in an efficient outcome as a unique Nash equilibrium of the game. The party whose standard of care is 0 - that is, the higher cost avoider - has no incentive to take care, because he will never be found negligent, and his behavior will never have an impact on the standard of care for the other party. Knowing this, the other party chooses to meet his standard of care, because otherwise he bears the whole accident burden.

However, note that the conditional/efficient application of the Hand rule is completely inconsistent with Judge Hand's opinion in *United States v. Carroll Towing Co.* A reading of his opinion reveals that the Judge had no interest in comparing  $c_I$  and  $c_V + PL_C$ . He did not attempt to discover which party was the higher cost avoider of accidents, and exempt that party from liability. Instead, when faced with a lawsuit resulting from the (NB,NC) scenario, Judge Hand found *both* parties negligent, and therefore equally liable for the sinking damages. The words from his opinion clearly show that he thought an otherwise non-negligent party should prepare for the possible negligent behavior of the other party, and should react to such *actual* behavior, as long as such reaction to actual behavior is cost-justified:

“At the locus in quo – especially during the short January days and in the full tide of war activity – barges were being constantly ‘drilled’ in and out. Certainly it was not beyond reasonable expectation that, with the inevitable haste and bustle, the work might not be done with adequate care. In such circumstances we hold – and it is all that we do hold – that it was a fair requirement that the Connors Company should have a bargee aboard, during the working hours of daylight.”

So we are led to the conclusion that Judge Learned Hand, in *United States v. Carroll Towing Co.*, does not apply the Hand formula in the conditional-based-on-efficient-behavior mode.

Also, we believe an independent application of the Hand formula is inconsistent with the structure of this case. Because the expected benefit from action C by the injurer, that is, careful adjustment and inspection of the Anna C's moorings, *varies* depending on whether the victim has a bargee aboard or not, a reasonable Hand test applied to the injurer must be contingent on the victim's behavior. The case Judge Hand decided was

the outcome of a (NB,NC) scenario. But suppose that a bargee had been aboard the Anna C, preventing sinking damages, and a lawsuit had followed because of the collision damages. That is, suppose the scenario had been (B,NC). If this hypothetical case had been brought to Judge Hand, we think that he would have compared  $c_I$  with  $PL_C$ , in order to determine the injurer's negligence, rather than comparing  $c_I$  with  $P(L_C + L_S)$ . That is, to determine the injurer's negligence, Judge Hand would have taken the victim's actual behavior into account. He simply could not, and would not, have abstracted from the victim's actual behavior.

### 3. Equilibrium of the game

We argued above that Judge Hand's negligence rule, as applied to the case of *United States v. Carroll Towing Co.*, could not have involved an independent application of the Hand rule, or a conditional-based-on-efficient behavior application of the Hand rule. We believe therefore that his negligence rule must have been a conditional rule contingent on the *actual* behavior of the other party. In this section we will carefully analyze the equilibria of the *United States v. Carroll Towing Co.* game, with that understanding of the Hand rule. We noted above that we are making 2 assumptions about parameter values in this paper, first, that  $c_V < PL_S$ , and second that  $c_I < P(L_C + L_S)$ . In order to proceed with the analysis, we will distinguish between 2 cases consistent with the second inequality. The two cases are: case i,  $c_I < PL_C$  and case ii,  $PL_C < c_I < P(L_C + L_S)$ . These two cases will produce different outcomes of the game<sup>9</sup>.

#### Case i: $c_I < PL_C$

Table 2 below represents the payoff structure of the game, distributed between the victim (Connors Marine Co.,) and the injurer (Carroll Towing Co.), when  $c_I < PL_C$  is satisfied, and the Hand rule, contingent on actual behavior of the

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<sup>9</sup> In *United States v. Carroll Towing Co.* it is likely that  $c_I < PL_C$ , although Judge Hand isn't clear about this. Judge Hand's opinion does include the following passage: "Applied to the situation at bar, the likelihood that a barge will break from her fasts and the damage she will do, vary with the place and time; for example, if a storm threatens, the danger is greater; so it is, if she is in a crowded harbor where moored barges are constantly being shifted about." That is,  $PL_C$  may vary, depending on factors like the weather. Thus we think the Hand test should be applicable whether the parameter values support case i or case ii.

other party, is used to determine negligence. The first entry of each cell represents the expected costs to the victim, comprised of both prevention costs and expected accident costs falling on him. The second represents expected costs to the injurer. The liability rule is the circa 1947 U.S. Admiralty law, which puts all of the damages on the victim if the injurer is not negligent, all of the damages on the injurer if the injurer is negligent and the victim is not, and allocates the damages half to each if both injurer and victim are negligent. (This is the “equal-division rule.”)

		INJURER (Carroll Towing Co.)	
		Careful Inspection of Moorings (C)	Careless Inspection of Moorings (NC)
VICTIM (Conners Marine Co.)	Bargee Aboard (B)	$c_V , c_I$	$c_V , PL_C$
	No Bargee (NB)	$0 , c_I$	$\frac{1}{2} PL_S , PL_C + \frac{1}{2} PL_S$

**Table 2. Payoff Matrix When  $c_I < PL_C$ .**

Since  $c_I < PL_C$  holds by assumption in case i, if an accident occurs and the Hand rule is applied to the injurer, he will be found negligent both when the victim is cautiously choosing B (and expected damages are  $PL_C$ ), and when the victim is carelessly choosing NB (and expected damages are greater than  $PL_C$ ). Therefore the injurer is negligent in the entire NC column. In the (B,NC) cell, the injurer is negligent and the victim, with his bargee aboard, is non-negligent. Therefore expected accident costs (i.e.,  $PL_C$ ) fall entirely on the injurer. In the (NB,NC) cell, the injurer is negligent, and, under the Hand rule based on actual behavior of the other party, the victim is also negligent since  $c_V < PL_S$ . That is, as with Judge Hand’s *United States v. Carroll Towing Co.* opinion, the victim is negligent (along with the injurer) because of a cost-benefit test applied to sinking damages only. Therefore, under the Admiralty rule, the sinking damages are split equally between the two negligent parties. The collision damages, on the other hand, fall on the injurer entirely, since the victim could not have prevented such damages.

Based on the payoff structure of this game, careful adjustment and inspection of the moorings, that is C, is a dominant strategy for the injurer. Knowing this, the victim's best choice is NB, no bargee aboard the Anna C. So we have a unique Nash equilibrium for the game at (NB,C).

Is the (NB,C) combination efficient? It is if  $c_I < c_V + PL_C$  holds. And in case i, since  $c_I < PL_C$  holds,  $c_I < c_V + PL_C$  must also hold. Therefore, in case i, the Hand rule, applied conditionally based on actual behavior of the other party, provides incentives for a game whose sole equilibrium is efficient. Surely that's a good result.

But note that in case i, at the efficient game equilibrium (NB,C), there should be no bargee aboard the Anna C. Paradoxically, this seems to fly directly in the face of Judge Hand's opinion!

**Case ii:  $PL_C < c_I < P(L_C + L_S)$**

Now we consider the other possible case, where  $PL_C < c_I < P(L_C + L_S)$  holds. Table 3 represents the payoff matrix in this case; note that it is constructed in the same manner as Table 2; that is, it is based on the conditional/actual interpretation of the Hand rule, and on the Admiralty law, equal division rule.

		INJURER (Carroll Towing Co.)	
		Careful Inspection of Moorings (C)	Careless Inspection of Moorings (NC)
VICTIM (Conners Marine Co.)	Bargee Aboard (B)	$c_V, c_I$	$c_V + PL_C, 0$
	No Bargee (NB)	$0, c_I$	$\frac{1}{2} PL_S, PL_C + \frac{1}{2} PL_S$

**Table 3. Payoff Matrix When  $PL_C < c_I < P(L_C + L_S)$ .**

Now, whether the injurer's cautious inspection of moorings is cost-justified or not depends on whether there is a bargee aboard or not. Given that the victim chooses B, the injurer's careless inspection is *not* negligent by the Hand rule because the expected benefit from careful inspection  $PL_C$ , is less than the care-taking cost  $c_I$ .

Therefore in the (B,NC) cell, the injurer is not negligent. This is a paradoxical result, because in the (NB,NC) cell the injurer *is* negligent, and so the victim's cautious action – choosing B rather than NB – has the odd consequence of making the injurer non-negligent. With respect to liability in the (B,NC) cell, under the Admiralty rule, the non-negligent injurer is not liable, whereas in the (NB,NC) cell the injurer is negligent, and liable (for the collision damages.)<sup>10</sup> Note that, the only difference between Table 2 and Table 3 arises in the (B,NC) cell.

In case i, shown in Table 2, the injurer had a dominant strategy, to adjust and inspect the moorings carefully. In case ii this is no longer true, and a careful examination of Table 3 reveals that neither the victim nor the injurer has a dominant strategy. As for Nash equilibria for the game, we have several different possibilities, depending on the relative magnitudes of the parameters. Note that the four possibilities listed below are all consistent with the two original inequalities we assumed for our various parameters, and with the inequalities defining case ii. In the following we briefly describe each possibility. The reader can refer to Table 3 to convince herself of the accuracy of our description of the game's equilibrium and its efficiency or lack thereof:

- (i) If  $c_v + PL_C > \frac{1}{2}PL_S$  and  $c_I > PL_C + \frac{1}{2}PL_S$ , then (NB,NC) is the unique Nash equilibrium. However, it is *inefficient*.
- (ii) If  $c_v + PL_C < \frac{1}{2}PL_S$  and  $c_I < PL_C + \frac{1}{2}PL_S$ , then there exist two Nash equilibria, (B,NC) and (NC,B). One is efficient and the other is not. (Note that we are restricting our attention to pure strategy equilibria only. There is also a mixed strategy equilibrium, which is inefficient.)

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<sup>10</sup> The rules of simple negligence, negligence with contributory negligence as a defense, and comparative negligence, all produce the same payoff structure. On the other hand, the rules of strict liability with a defense of contributory negligence (the reverse Hand rule) and strict liability with a defense of dual contributory negligence, would change the payoff structure in (B,NC) cell to  $(c_v, PL_C)$ . This is because under those rules, non-negligent injurers bear the accident burden for non-negligent victims. However, changing the payoff structure in this fashion would still leave us with the possibility of inefficient game equilibria.



(iii) If  $c_v + PL_C > \frac{1}{2}PL_S$  and  $c_I < PL_C + \frac{1}{2}PL_S$ , then (NB,C) is the unique Nash equilibrium. This is efficient if  $c_I < c_v + PL_C$  is also satisfied, but if  $c_I > c_v + PL_C$ , it is *inefficient*.

(iv) If  $c_v + PL_C < \frac{1}{2}PL_S$  and  $c_I > PL_C + \frac{1}{2}PL_S$ , then (B,NC) is the unique Nash equilibrium, and it is efficient, because the two above inequalities together imply  $c_I > c_v + PL_C$ .

In sum, the *United States v. Carroll Towing Co.* game has an efficient Nash equilibrium only if one of the following three conditions holds: (1)  $c_I < PL_C$ , (2)  $c_v + PL_C > \frac{1}{2}PL_S$ ,  $PL_C < c_I < PL_C + \frac{1}{2}PL_S$ , and  $c_I < c_v + PL_C$ , (3)  $c_v + PL_C < \frac{1}{2}PL_S$  and  $c_I > PL_C + \frac{1}{2}PL_S$ . If none of these three conditions holds, the game either has multiple equilibria, some of which are inefficient, or a unique inefficient equilibrium.

We find it surprising to see that the Hand formula itself, as we believe Judge Hand would have interpreted it, does not guarantee efficient outcomes in a model specifically based on *United States v. Carroll Towing Co.* Moreover, the possibility of inefficiency, which our analysis reveals, does *not* depend on the Admiralty law equal division rule that governed *United States v. Carroll Towing Co.* We can show that every kind of negligence-based rule fails to guarantee efficiency<sup>11</sup>, as long as both parties are negligent in (NB,NC) cell, which a careful reading of Judge Hand's opinion seems to demand.

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<sup>11</sup> For example, if the rule of negligence with contributory negligence as a defense is used, the payoff structure in (NB,NC) cell in Table 2 and 3 should change into  $(PL_S, PL_C)$ . The game's equilibrium is efficient only when one of the following two conditions holds: (1)  $c_I < PL_C$ , (2)  $c_v + PL_C < PL_S$  and  $c_v + PL_C < c_I$ . Otherwise, the game has a unique inefficient equilibrium.

## V. Concluding remarks

The Learned Hand rule for determining negligence has fascinated students of law and economics for many decades, partly because Judge Hand wrote an algebraic expression – an inequality with an expected value on one side – that is intuitive and attractive to economists. Although Judge Hand never used the words “efficient” or “efficiency” or “optimality” or “game” or “equilibrium” in his *United States v. Carroll Towing Co.* opinion, the law and economics literature has come to the belief that the great jurist was groping toward a basis for efficient liability rules, and that, when analyzed in a modern formal game-theoretic framework, the Hand rule serves to provide incentives to injurers and victims that lead to efficient equilibria.

In this paper we have argued that there are three distinct ways to interpret the Hand rule, and that only one of them, what we call conditional-based-on-actual-behavior approach, could reasonably represent Judge Hand’s meaning. We have shown that, based on this understanding of the Hand rule, a careful model of the facts of the *United States v. Carroll Towing Co.* case can produce (1) inefficient equilibria, and (2) odd paradoxes, such as the possibility that an efficient equilibrium might require that no bargee be aboard the Anna C. We conclude that the Hand rule, as Judge Hand meant it, is no *deus ex machina* that theoretically solves all liability problems.

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