Work effort effects in the classical labor supply model

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Abstract

This paper considers an extension of the classical static labor-leisure choice model to allow for an on-the-job leisure choice. The key result is that an income-compensated wage increase, while theoretically increasing hours worked, will likely increase on-the-job leisure.

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Introduction

The classical static labor-leisure choice model assumes that time spent at work is leisure-less (see, e.g., Ehrenberg and Smith, 2000). That is, work effort on the job is implicitly assumed to be at its maximum, which is rarely recognized in the literature. A simple extension of the classical model is considered, which allows for workers to choose time not spent at work (off-the-job leisure) as well as leisure time on-the-job. This extension has not yet been shown for the hourly wage model—it has been shown for piece-rate pay (Dickinson, 1999) and efficiency wages (Lin, 2003). Dickinson (1999) comments that the extension of his Combined Model to hourly (as opposed to piece-rate) wages leads to ambiguous results on work effort, but he does not elaborate or derive the result. This paper fills the gap by deriving the key result of this extension of the classic hourly wage model.

The Classical Labor-Leisure Model Extension

Assume a utility function that is quasiconcave, twice continuously differentiable, and increasing in consumption, c, off-the-job leisure, l, and on-the-job leisure, h_l. Therefore, we have $U=U(c,l,h_l)$ with $U_c > 0,\ U_l > 0,\ U_{h_l} > 0$ and the usual diminishing marginal utility assumption. To simplify the notation in much of the derivations, I will often refer to goods c, l, and h_l as goods 1, 2, and 3, respectively. For example, $U_c,\ U_l,\ U_{h_l}$ will be referred to as U_1, U_2, and U_3 (with similar numeric notation for higher order derivatives). In the classical labor supply model, workers are paid a wage, w, per hour of work, h, and the constraints are:

$wh+F=pc$ (the budget constraint)
$h+l=T$ (the time constraint)

where F is exogenous income, c is consumption, p is the price of consumption, and T is total time available for work or off-the-job leisure. The important new constraint introduced in the Combined Model of hourly wages is

$h_w+h_l=h$ (the work time constraint)

where h_w is productive hours spent at work. The budget constraint can be written as usual, $wT-wl+F-c=0$ (where p is normalized to $p=1$), and the Lagrangian to be maximized is now

$$\mathcal{L}=U(c,l,h_l)+\lambda(wT-wl+F-c) \tag{1}$$

The first order conditions from the constrained maximization problem are:

$$\begin{aligned} &\mathcal{L}_\lambda:\ wT-wl+F-c=0\\ &\mathcal{L}_1:\ U_1-\lambda=0\\ &\mathcal{L}_2:\ U_2-\lambda w=0\\ &\mathcal{L}_3:\ U_3=0 \end{aligned} \tag{2}$$

It is important to note that a solution will only exist at a point where the marginal utility of on-the-job leisure, h_l, is equal to zero, which follows from the assumption that workers are paid for all hours at work, whether or not they are productive hours.

From this we define the Jacobian matrix of partial derivatives from these first-order conditions

$$[J]=\begin{bmatrix} 0 & -1 & -w & 0\\ -1 & U_{11} & U_{12} & U_{13}\\ -w & U_{21} & U_{22} & U_{23}\\ 0 & U_{31} & U_{32} & U_{33} \end{bmatrix} \tag{3}$$

Because the Jacobian matrix is identical to the bordered-Hessian matrix of the constrained maximization problem is (1), we know that $|J|<0$ from the second-order conditions for utility maximization. Thus, the conditions of the Implicit Function Theorem will hold, and there exists a neighborhood in which we can define the set of implicit functions

$$\begin{aligned}\lambda &= \lambda(w,F)\\ c &= c(w,F)\\ l &= l(w,F)\\ h_l &= h_l(w,F)\end{aligned} \tag{4}$$

The comparative static analysis will focus on the wage and income effects on off- and on-the-job leisure.

Comparative Static Analysis

First, totally differentiate the first-order conditions in (2) and arrange to get

$$\begin{aligned} -dc - wdl &= ldw - Tdw - dF\\ -d\lambda + U_{11}dc + U_{12}dl + U_{13}dh_l &= 0\\ -wd\lambda + U_{21}dc + U_{22}dl + U_{23}dh_l &= \lambda dw\\ U_{31}dc + U_{32}dl + U_{33}dh_l &= 0\end{aligned} \tag{5}$$

The income effects are found by setting *dw=0* and dividing each equation by *dF*. Interpreting each ratio as a partial derivative, we have

$$\begin{aligned} \frac{-\partial c}{\partial F} - w\frac{\partial l}{\partial F} &= -1\\ \frac{-\partial\lambda}{\partial F} + U_{11}\frac{\partial c}{\partial F} + U_{12}\frac{\partial l}{\partial F} + U_{13}\frac{\partial h_l}{\partial F} &= 0\\ -w\frac{\partial\lambda}{\partial F} + U_{21}\frac{\partial c}{\partial F} + U_{22}\frac{\partial l}{\partial F} + U_{23}\frac{\partial h_l}{\partial F} &= 0\\ U_{13}\frac{\partial c}{\partial F} + U_{32}\frac{\partial l}{\partial F} + U_{33}\frac{\partial h_l}{\partial F} &= 0\end{aligned} \tag{6}$$

Or, in matrix form,

$$[J]\cdot\begin{bmatrix}\partial\lambda/\partial F\\ \partial c/\partial F\\ \partial l/\partial F\\ \partial h_l/\partial F\end{bmatrix}=\begin{bmatrix}-1\\0\\0\\0\end{bmatrix} \tag{7}$$

By Cramer's rule, we solve for the two income effects for on- and off-the-job leisure

$$\frac{\partial l}{\partial F}=\frac{-1}{|\mathrm{J}|}\begin{vmatrix}-1 & U_{11} & U_{13}\\ -w & U_{21} & U_{23}\\ 0 & U_{31} & U_{33}\end{vmatrix}\quad\text{and}\quad\frac{\partial h_l}{\partial F}=\frac{1}{|\mathrm{J}|}\begin{vmatrix}-1 & U_{11} & U_{12}\\ -w & U_{21} & U_{22}\\ 0 & U_{31} & U_{32}\end{vmatrix} \tag{8}$$

The total wage effects are found by setting *dF*=0 and dividing the equations in (5) through by *dw* to get

$$[J]\cdot\begin{bmatrix}\partial\lambda/\partial w\\ \partial c/\partial w\\ \partial l/\partial w\\ \partial h_l/\partial w\end{bmatrix}=\begin{bmatrix}-h\\0\\\lambda\\0\end{bmatrix} \tag{9}$$

We can now solve for the two total wage effects of this labor supply model

$$\frac{\partial l}{\partial w}=\frac{\lambda}{|\mathrm{J}|}\begin{vmatrix}0 & -1 & 0\\ -1 & U_{11} & U_{13}\\ 0 & U_{31} & U_{33}\end{vmatrix}-h\frac{1}{|\mathrm{J}|}\begin{vmatrix}-1 & U_{11} & U_{13}\\ -w & U_{21} & U_{23}\\ 0 & U_{31} & U_{33}\end{vmatrix}=\frac{\lambda}{|\mathrm{J}|}\begin{vmatrix}0 & -1 & 0\\ -1 & U_{11} & U_{13}\\ 0 & U_{31} & U_{33}\end{vmatrix}+h\cdot\frac{\partial l}{\partial F} \tag{10}$$

$$\frac{\partial h_l}{\partial w}=\frac{-\lambda}{|\mathrm{J}|}\begin{vmatrix}0 & -1 & -w\\ -1 & U_{11} & U_{12}\\ 0 & U_{31} & U_{32}\end{vmatrix}+h\frac{1}{|\mathrm{J}|}\begin{vmatrix}-1 & U_{11} & U_{12}\\ -w & U_{21} & U_{22}\\ 0 & U_{31} & U_{32}\end{vmatrix}=\frac{-\lambda}{|\mathrm{J}|}\begin{vmatrix}0 & -1 & -w\\ -1 & U_{11} & U_{12}\\ 0 & U_{31} & U_{32}\end{vmatrix}+h\cdot\frac{\partial h_l}{\partial F} \tag{11}$$

In each of (10) and (11), the result is the typical breakdown of the total wage effect into substitution (first term) and income effect (second term). The classical result from the static labor-leisure choice model is that the compensated wage effect on leisure (labor) is negative (positive). As can be seen, this result holds in the expanded model:

$$\left.\frac{\partial l}{\partial w}\right|_{F=\bar{F}} = \frac{\lambda}{|J|}(-U_{33}) < 0 \tag{12}$$

Absent an income effect that is strong, the hours-of-labor supply curve will slope upward, as in the classical labor-leisure choice model. However, the key result is that the compensated (i.e., substitution) effect on on-the-job leisure has an ambiguous sign.

$$\left.\frac{\partial h_l}{\partial w}\right|_{F=\bar{F}} = \frac{-\lambda}{|J|}(wU_{31} - U_{32}) \tag{13}$$

So, this compensated wage effect on on-the-job leisure in (13) is only negative if $wU_{31}<U_{32}$. Considering the variables involved, the most plausible assumption is that $U_{32}>0$. That is, there is a type of substitutability between on- and off-the-job leisure such that an increase in the consumption of one decreases the marginal utility of the other. This was also the critical assumption to produce a positive compensated wage effect on off-the-job leisure in the piece-rate model in Dickinson (1999) and the efficiency wage model of Lin (2003).

The assumption that $U_{32}>0$ is most reasonable given that on- and off-the-job leisure, by definition, cannot be consumed together. On the other hand, U_{31} is likely to be zero or positive. A utility function that is additively separable in consumption and on-the-job leisure would produce $U_{31}=0$, whereas $U_{32}>0$ would result if consumption and on-the-job leisure are complementary in the sense that an increase in consumption increases the marginal utility of on-the-job leisure (e.g., using work time to shop online). In short, the more plausible assumptions of U_{32}<0 and U_{31}≥0 would imply that $\left.\frac{\partial h_l}{\partial w}\right|_{F=\bar{F}}$ >0. Therefore, the modification of the classical labor-leisure choice model, which considers work *effort* a choice variable along with hours of work, implies that a compensated wage increase will *decrease* work effort (i.e., increase on-the-job leisure) under very plausible assumptions.

Conclusion

A criticism of the model extension presented would lie in the fact that workers are paid for all time on-the-job, regardless of whether they are working or not. While this is a valid criticism, it serves to highlight a key criticism of the classical labor-leisure choice model. Namely, time spent at work is not necessarily time spent working. Once this is made explicit, employers may then choose to monitor workers' on-the-job activities—an efficiency wage type model such as in Lin (2003) is more complete. Hammermesh (1990) examines break times at work and also concludes that, while some level of on-the-job leisure may be appropriate, managers would do well to monitor workers. His theoretical model, however, does not examine a dual choice of both hours of work and work effort.

The goal of this present paper is to show the confounding effects on actual "labor supply" that begin to appear as soon as one introduces the second leisure dimension to the theoretical model. All of the existing models in the literature that allow for on- and off-the-job leisure, whether it be an hourly, piece-rate, or efficiency wage model have key similarities. Namely, the standard wage effects must operate through an interaction of multiple leisure goods, and this can produce results opposite the conventional wisdom.

References

Dickinson, David L. 1999. "An Experimental Examination lf Labor Supply and Work Intensities." *Journal of Labor Economics*, 17(4: part 1): 638-670.

Ehrenberg, Ronald G., and Robert S. Smith. 2000. *Modern Labor Economics: Theory and Public Policy*, 7th edition. Addison-Wesley, New York.

Hammermesh, Daniel S. 1990. "Shirking or Productive Schmoozing: Wages And The Allocation Of Time At Work." *Industrial and Labor Relations Review*, 43 (Special Issue): 121s-144s.

Lin, Chung-cheng. 2004. "A Backward-Bending Labor Supply Curve Without An Income Effect." *Oxford Economics Papers*, 55: 336-343.