# Good Fences Make Good Neighbors: Endogenous Property Rights in a Game of Conflict*

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**Abstract**

This paper derives the conditions under which property rights can arise in an anarchy equilibrium. The creation of property rights requires that players devote part of their endowment to the public good of property rights protection. In the Nash equilibrium, players contribute zero to the protection of property rights. In contrast, a king who provides property rights protection paid for by a tax on endowments can completely eliminate conflict, but such a king has an incentive to take the surplus for himself. Thus players have an incentive to find a solution that keeps power in their own hands. In a social contract, players first credibly commit part of their endowments to providing property rights and then allocate the balance of their endowments between production and conflict. While property rights can arise under a social contract if the productivity of resources relative to the size of the population is sufficiently high, these property rights may be less than perfectly secure. Nevertheless, for sufficiently high productivity of resources relative to the size of the population, the social contract welfare dominates autocracy.

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# 1 Introduction

It has long been recognized that property rights are an essential ingredient to a well-functioning economy. However, relatively little is known about the conditions that determine when property rights might arise absent a state. Demsetz (1967) observed that the Montagnais bands of the Algonquians of Labrador created property rights for beaver ponds once the fur trade made the establishment of property rights sufficiently valuable. He hypothesized that exogenous increases in the value of a resource will lead to the establishment of property rights for that resource.[1] While this may be the case, it is also possible that an increase in the value or productivity of a resource may instead result in an increase in conflict. This appears to have occurred in Nigeria, where the discovery of oil raised the level of conflict rather than resulting in the formation of well-defined property rights. Similarly, the current conflict in Darfur has been attributed to a drought, which raised the marginal value of water. However, instead of property rights for water organically arising, deadly conflict has enveloped the region, with some estimates suggesting over 400,000 dead.[2] What conditions have fostered the creation of property rights in some situations and hindered their creation in others?

In the absence of property rights, there exists what Hirshleifer (1995) calls an *anarchy equilibrium*.[3] In an anarchy equilibrium, players have an endowment that they may invest in production, or they may use it to protect their own production (or steal from others), or they may consume it directly. When production has low value relative to consuming the endowment, players may simply consume their endowments, living a subsistence existence. Because theft is difficult when there is no accumulation of production (Murphy, Shleifer and Vishny, 1993), there is little need for property rights. As investing the endowment into production becomes more valuable in the sense of Demsetz, players will wish to reallocate some of their endowment from subsistence consumption to production. But if produced goods are subject to thievery, in the absence of property rights, players also have to devote some resources to appropriating their production. Thus as the value of production rises, the level of conflict also rises. However, this means that the social value of creating property rights rises as well, since conflict is socially costly as it uses resources to redistribute existing production that could have been used to create more production. Therefore, players' welfare can improve

[1] The June 2002 special issue of the *Journal of Legal Studies* (Merrill, 2002) reviews the the body of economic research testing Demsetz's hypothesis.

[2] "Hundreds Killed in Attacks in Eastern Chad," *Washington Post*, April 11, 2007, p. A.10.

[3] On conflict models more generally, see Skaperdas (1992), Hirshleifer (1995), and Grossman and Kim (1995). This growing literature is surveyed in Garfinkel and Skaperdas (2007).

if they can solve the public goods problem of providing property rights protection. The question this paper asks is under what conditions can property rights arise in an anarchy equilibrium?

Our first finding suggests that it is difficult for property rights to arise organically. If players in an anarchy equilibrium *simultaneously* allocate their endowments between production, conflict, and a contribution to the public good of property rights protection, then in the Nash equilibrium, *zero* protection for property is provided. Investing in conflict benefits players by increasing the proportion of their own production that they appropriate and by increasing the proportion of others' production that they expropriate. Investing in property rights, in contrast, increases the proportion of their own production that they appropriate, but it reduces the proportion of others' production that they expropriate. Thus property rights suffer from an extreme version of the under-provision problem with private provision of public goods.

We then consider two alternatives to private provision of public property rights. In the first, property rights are provided by an external player, a chief, a lord, or a king, who offers protection of property in exchange for the right to tax his 'citizens'. Only if the value of creating property rights is sufficiently high is a king able to create and enforce property rights. However, when that condition is satisfied, the king is able to enforce *perfectly defined property rights*, in the sense that the economy under a king has no conflict. But allowing a king to create property rights is a risky strategy, since a king who has the power to protect property may use that power to take property [e.g., Wintrobe (1990), Olson (1993), Grossman (2002), Hurwicz (2008)]. While this has distributional consequences, since a despotic king can take all of the surplus he creates, it can also have efficiency consequences. Under the form of contract between king and subjects common in the middle ages, that of a *tax on endowments*,[4] we show that a despotic king is unable to simultaneously solve the puzzle of how to create incentives to generate surplus while exploiting that surplus.

Therefore, given the distributional and efficiency risks to devolving power, it is in the interest of players to find an alternative in which property rights can be established without relinquishing their say in how those rights will be protected. In what we call a *social contract* game, players make the establishment of property rights an antecedent to their allocation between production and conflict. In the social contract stage of the game, a voluntary contributions public goods game occurs in which players simultaneously allocate part of their

[4]William the Conquerer, the Norman who became king of England after defeating the Anglo-Saxon king Harold in the Battle of Hastings in 1066, provides the starkest example of the way in which a king funded his government by a tax on the endowment. William declared all land "*terra regis*," or the king's land. This lead Bloch (1960, at p. 188) to conclude, "All land was held of a lord and this chain, which was nowhere broken, lead link by link to the king."

endowment towards the protection of property rights. Then in the conflict stage of the game, players allocate the remainder of their endowment between conflict and production. An organic and credible form of property rights arises that does not require externally provided force. This occurs because the strategic effect of property rights protection overcomes the incentive to free-ride on others' provision of property rights protection that dominates the Nash equilibrium.

This sort of Lockean ideal is, of course, not costless. Players still must allocate resources away from productive uses towards the establishment of property rights. In a rational expectations equilibrium, they do this by correctly anticipating how property rights affect their behavior in the subsequent conflict game. The subgame perfect Nash equilibrium level of property rights protection is Pareto improving, provided that the Demsetz condition is satisfied that the establishment of property rights is sufficiently valuable. The minimum level of the Demsetz parameter under which property rights can be established in the social contract game, however, is at least as great as, and as the size of the population rises, is strictly greater than that which a king (benevolent or despotic) can create property rights. This occurs because unlike a king, a social contract cannot fully eliminate the incentive to free-ride in the private provision of property rights protection. Thus, the Demsetz condition that the value of establishing property rights be sufficiently high is a *necessary* condition in order that well-defined property rights be established, but it is not a *sufficient* condition to establish these property rights.

While the security of property is endogenous in all models of conflict, there are only a small number of papers that have examined the creation of property rights explicitly. In Grossman (2002), a king provides property rights. He also finds that when a king creates property rights, he drives conflict to zero. However, Grossman does not consider either of the organic alternatives to a king that we consider. In Hafer (2006), there are two productive processes, one of which requires a resource that is subject to thievery. Players for whom ownership of the resource yields higher marginal productivity relative their outside alternative are more inclined to defend it. As other players learn this over time, conflict diminishes and a form of *de facto* property rights emerges. Kolmar (2008) studies a contest game in which one player has a prize and the other attempts to steal the prize. When allocations to defensive investments occur before allocations to expropriating investments, players are able to completely deter expropriation. Like these papers, we property rights arise in our model in a sequential game. However, neither Hafer nor Kolmar considers investment in property rights explicitly as a alternative form of investment to conflict. Property rights in our model arise as a result of private contributions to the public good of property rights protection, rather than through repeated or sequential investments in conflict. Thus in our model, not only do property rights arise, but so does a primitive *state*, one with a purpose and the means to accomplish that purpose.[5]

## 2 Examples of Property Rights Creation

We begin by discussing examples which highlight the main results of the subsequent model. The first two examples are cases where property rights for valuable resources arose organically out of a social contract. The third example illustrates how the social contract can break down due to free-riding when the number of players is large. The final example illustrates the danger of allowing a king to enforce property rights.

### 2.1 Water Rights in 15th Century Valencia, Spain

Ostrom (1990, at pp. 73-79) examined the formation of property rights to water in the Valencia region of Medieval Spain. Irrigation canals were built to enhance agriculture. However, an institution was also created to prevent common property dissipation of the rents. Water was used sequentially by the farmers along a canal. Property rights were secured by a system that simultaneously involved monitoring and sanctions. Each successive user had an incentive to monitor the previous user since only when the previous farmer's fields were irrigated, could the next farmer could begin irrigating. Monitoring, however, would have been useless without sanctions. A farmer who was accused of wasting water could be brought before a council of *all* farmers. Thus the threat of sanctions by the council provided a public good in the form of security of property rights. Ostrom finds that similar successful systems of organically derived property rights evolved in Swiss grazing fields, Japanese forests, and Nova Scotia fisheries.

### 2.2 Texas Oil Fields

Libecap and Wiggins (1985) examined the establishment of property rights to oil fields. While surface rights existed, the migratory nature of oil meant that a field was common

[5]Individuals appear to have had some say in their public affairs in some of the earliest examples of city-states. In *The Epic of Gilgamesh*, when King Gilgamesh (*c.*2700BC) wished to go to war, he first sought permission of the elders of the city to do so, and when that was not forthcoming, he appealed to an "assembly of all the men of the city of fighting age" (Saggs, 1989, at pp. 35-36). However, the Egyptians and Persians were autocracies throughout their histories. The Athenians, in the sixth to fourth centuries BC were the first to be ruled directly by an assembly of its own citizens. Thus, in Greek city-states we see the first formations of social contracts of the form we study.

property. The Slaughter field in west Texas was discovered in 1936 and is 71,000 acres in surface size. With over a hundred surface rights owners, an attempt to unitize production on the whole field failed, but twenty-eight sub-units were created.[6] Libecap and Wiggins (1985, at p. 694) found that by 1975, 427 wells had been drilled along the boundaries of the sub-units at a total cost of 156 million dollars. The purpose of these wells was to re-inject water that has been recovered along with the oil and gas to prevent migration of oil across subunits. Boyce and Nøstbakken (2008) report that as of 2006, approximately 3000 total wells had been drilled on the field and that 1.3 billion barrels of oil had been extracted from this field since 1936. Using the average production rate and real oil prices from 1936 to present, they estimated that the present value (at 4 percent real interest rate) of revenues from oil production on the field was roughly nine billion 2007 dollars. Using Libecap and Wiggins' estimate of a cost of 360,000 dollars per well, they find that total drilling costs were on the order of four billion 2007 dollars. Thus ignoring natural gas production, which was substantial, delays in the timing of some drilling, and the fact that field production generally declines over time, Boyce and Nøstbakken find that the field generated over five billion 2007 dollars in net of drilling cost revenues for its owners. Therefore, the dissipation of rents by the 427 injection wells on the subunit boundaries was less than three percent of the estimated rents earned. By prohibiting migration of oil across sub-unit boundaries, these wells provided a public good which prevented even worse dissipation of rents had the oil been allowed to migrate.

As with water rights in 15$^{th}$ Century Valencia, the Slaughter field in west Texas illustrates that it is possible for players to privately provide the public good of property rights protection. As the next example suggests, however, the ability for players to successfully provide property rights hinges on the number of players in the game.

### 2.3 The California Gold Rush 1848-1850

When gold was discovered in California in January 1848, California had just become a territory of the United States following a war with Mexico. As it was not until September 1850 that California became a state, there was no legal foundation for property rights during the gold rush (Umbeck, 1977). By the end of 1848, between 5,000 and 10,000 miners arrived in the Sierra Mountains, but the area was large enough that new arrivals simply moved elsewhere on the rivers. During the period 1848-1850, property rights were established and

[6] Unitization means that a single operator decides the rate of extraction, but the rents are distributed across the owners via a pre-determined allocation. This effectively eliminates the common property rent-dissipation.

protected by miners using informal organizations. The size of claims would be decided at a miner's meeting, usually by majority rule, and the miners would stake their claims by marking their territory and then working it. As long as the owner did this, the other miners would help keep "claim jumpers" off of each other's property. However, in 1849, an additional 40,000 people arrived, and by the end of 1852 the population of California had increased by 150,000. As the number of players increased, the incentive to free-ride on the provision of property rights by other the miners increased. Clay and Wright (2005) note that it became increasingly expensive for miners to protect their claims. Indeed California courts would later recognize claims that had been acquired by "claim jumpers" on equal par with those acquired by the first claimants. Thus, when the number of players grew too large, the incentive to free-ride on property rights protection resulted in a collapse of the organic property rights system the miners had established.

One way to overcome free-riding is to elect a king who can tax players to finance the provision of property rights protection. The next example shows the danger in that approach.

### 2.4 The Magna Carta

Prior to the Norman invasion, property rights in England were based upon the declaration of Saxon King Cuthred in 745 that, "all gifts of former kings... in country houses, and in villages and lands, and farms and mansions... shall remain firm and inviolate, as long as the revolution of the pole shall curry the lands and seas with regular movement around the starry heavens" [sic] (Barrington, 1900, at p. 35). However, upon assuming the English throne in 1066, the Norman invader, King William the Conquerer, declared all land in England as his own. The assessment of the value of all his holdings, the *Domesday Book*, estimated the value of all land in England outside of the towns and cities in 1087 as £73,000.

> "Of this sum the king and his family received £17,650; his servants and officials, the king's sergeants, £1,800; the church £19,200; and some few entrusted Englishmen £4,000. The remainder, amounting to a sum of £30,350, was apportioned out to some 170 baronies as rewards for the Normans who had shared in the enterprize of conquest" (Poole, 1955, at p. 2).

However, property granted by the king was of a tenuous nature. When in need of money, William would take lands that he had previously granted and resell them to the highest bidder, sometimes "taking them away from the purchaser, and again selling to one who would bid higher" (Barrington, 1900, at p. 55). William and his successors also imposed various taxes on their citizens.[7]

The *Magna Carta*, which King John I accepted in 1215, limited the rights of the King over his barons. The Magna Charta arose in response to John's taxes. In 1203, after losing Normandy in a war caused by John's taking the wife of a Baron named Philip as his own, John imposed a tax equal to $\frac{1}{7}^{\text{th}}$ of the value of the barons' holdings to pay for his war debts. Then in 1209, in a dispute with Pope Innocent over who should become Archbishop of Canterbury, John confiscated all church lands. In response the Pope excommunicated John and absolved all nobles of their oaths of fealty to John. In 1215 some 2000 earls, barons ad knights marched upon London to force John to accept the Magna Charta. Section 12 stated that no taxes could be imposed by the king without the consent of a council of nobles and sections 28-31 and 39 forbid the taking of property by the king without due process (Barrington (1900, at pp. 228-250), Poole (1955, at pp. 474-76)). Hence, the Magna Carta allowed the king to collect taxes, but limited his ability to capture all of the surplus.

## 3 Model Assumptions

We now present a formal model of conflict in which security of property is a public good. Consider a game in which there are $N \geq 2$ players, indexed $i = 1, \ldots, N$. Each player has an endowment of $\omega$ units of a resource. There are four different goods that the endowment may be used to produce. First, the endowment may be consumed directly. This is how a hunter-gatherer society treats its endowment. Each unit of the endowment consumed in this fashion yields one unit of utility, which we shall call "subsistence" consumption, as it corresponds to the minimum level of possible equilibrium utility. On the other hand, the endowment can be invested to produce a consumable good, "corn". An investment of $k_i$ units of endowment into corn production produces $Ak_i$ units of utility, where $A$ is the Demsetz parameter that tells how valuable corn is relative to consuming the endowment directly through subsistence consumption. The security of property is an issue because, corn, which is harvested in the autumn and stored in granaries, is easily stolen. However, since subsistence consumption occurs through the on-going processes of hunting and gathering, it is more difficult to steal. We stylize this by assuming that subsistence consumption cannot be stolen but that corn can be stolen. Clearly, if $A < 1$, planting corn produces less than could be obtained through subsistence consumption, hence, insecure property rights for corn

[7] In 1084, William imposed a tax of six shillings on every hide (a hide is approximately 60-120 acres). His successor, Rufus, in 1096 imposed taxes on the Church of 10,000 marks, which the church paid by "melting chalices and robbing their crucifixes" (Barrington, 1900, at p. 69). In 1109, Henry II, imposed another tax of six shillings per hide.

are not of economic importance. While conflict does indeed begin at this lower bound on $A$, it takes higher levels of productivity of corn in order for property rights to arise because property rights protection is itself socially costly.

A player may also invest his endowment into two goods that affect the security of property. The tool of conflict, $x_i$, is "guns". Guns serve two purposes. They are a simultaneously a tool that can be used to protect one's own property and that can be used for stealing the property of others.[8] Therefore, we say that an increase in $x_i$ increases the share of $i$'s own corn production that $i$ *appropriates* and it increases the share of the other players' corn production that $i$ *expropriates*. But guns are not the only component of conflict. The private provision of the public good of property rights protection, $y_i$, is "security" [9] An increase in security increases the share of player $i$'s own corn that $i$ appropriates but it reduces the share of other players' corn that $i$ expropriates. Thus while an increase in $i$'s guns increases both his appropriation and his expropriation, an increase in security has an asymmetric effect on appropriation and expropriation. Guns make the owner better off and others worse off (and so provide a negative externality); security is a public good that makes the provider both better off (by making their own production secure) and worse off (by making their thievery less effective).

Security can be provided either privately or by the state, if one exists. Private security, $Y \equiv \sum_{i=1}^{N} y_i$, is paid for by a contribution from the endowment of each individual. Public security, $\Psi$, is paid for through a tax, $\tau$, imposed by the state. The owner of production has an underlying natural advantage over thieves in protecting his own property, given by the parameter $\theta$. This could be due to a barrier such as a mountain or a river that divides one's property from others. The sum $\theta + \Psi + Y$ measures the advantage a player has in appropriating his own corn production relative to expropriating the corn production of others. Thus, $Y$, $\Psi$, and $\theta$ are perfect substitutes for one another.

The contest success functions used in most models of anarchy (e.g., Skaperdas, 1992) are based on the rent-seeking model of Tullock (1980). Because we are interested in analytical solutions, we simplify the contest success function in two ways. First, in the phrase of Hirshleifer (1995), the model we consider has a 'decisiveness' parameter equal to one.[10]

[8] It is possible to split the tools of conflict in to defensive (e.g., locks) and offensive (lock picks) tools, as in Grossman and Kim (1995). However, in the model we consider, the sum of offensive and defensive tools is equal to the value of $x_i$.

[9] We include in security all aspects of protection of property rights including prevention, enforcement, dispute resolution, and sanctions. In medieval England, William the Conquerer required of his subjects to build some 1200 castles, which housed his sheriffs. The miners of the California gold rush acted both as makers and enforcers of rules regarding mining claims. In the Texas oil field example, the investment in injection wells prevented migration of oil across subunit boundaries. All of these are subsumed into our use of the word 'security.'

[10] In Tullock, the probability player $i$ investing $x_i$ wins a prize worth $R$ when $N$ players compete for the prize is $p_i = x_i^m / \sum_{j=1}^{N} x_j^m$. In Hirshleifer, the proportion of one's own endowment that one ap propriates is $p_{ii} = x_i^m / \sum_{j=1}^{N} x_j^m$,

This means that in a symmetric conflict game with no security of property, each player's proportion of their own production that they appropriate is $\frac{1}{N}$. However, as we wish to emphasize the role of property rights, we allow the contest success function to be asymmetric in the sense that in a symmetric game property rights make one's appropriation of one's own output greater than $\frac{1}{N}$ and one's expropriation of the other players output less than $\frac{1}{N}$ when there is security of property. We obtain this by letting property rights enter the contest success function additively rather than multiplicatively as in Grossman and Kim (1995).[11] Therefore, the conflict technology has the following properties. The proportion of player $i$'s corn production that player $i$ appropriates is given by

$$p_{ii} = \frac{\theta + \Psi + Y + x_i}{\theta + \Psi + Y + X}, \qquad i = 1, \ldots, N, \tag{1}$$

where $X \equiv \sum_{i=1}^{N} x_i$, and the proportion of player $j$'s corn production that player $i$ gets to expropriate is

$$p_{ij} = \frac{x_i}{\theta + \Psi + Y + X}, \qquad i \neq j, i = 1, \ldots, N. \tag{2}$$

The proportion of $i$'s corn production that $i$ appropriates is increasing in $x_i$, $\theta$, $\Psi$ and $Y$, and decreasing in $X_{-i} \equiv X - x_i$. The proportion $i$ expropriates from others is increasing in $x_i$ and decreasing in $X_{-i}$, $\theta$, $\Psi$ and $Y$. Since corn production is either appropriated or expropriated, the logit condition (Dixit, 1987) must hold:

$$p_{ii} + \sum_{j \neq i}^{N} p_{ji} = 1, \text{ for all } i = 1, \ldots, N. \tag{3}$$

It is natural to think of the quality of property rights in terms of $p_{ii}$. When $p_{ii} = 1$, (3) implies that property rights are perfectly protected. When $p_{ii} < 1$, property rights are insecure. It is clear from (1) that property rights are perfectly secure if, and only if, $x_1 = x_2 = \cdots = x_N = 0$.

Absent state or privately provided protection of property rights, (1) and (2) implies that the parameter $\theta$ creates an asymmetry that favors the holder of the endowment. We assume that the natural advantage to protecting one's own property is limited:

**Assumption 1.** $\omega > \theta \geq 0$.

where $m$ is the decisiveness parameter.

[11] In Grossman and Kim (1995) the sum $\theta + \Psi + Y$ is multiplied by $x_i$ in (1) and by $x_j$ in (2). Our specification has the limitation that absent property rights, the natural advantage cannot go to the expropriator as can happen in Grossman and Kim, but it has the advantage that it easily yields closed form solutions. See Skaperdas (1992, 1996), Hirshleifer (1991b, 2000), and Grossman and Kim (1995, *n.* 6 at p. 1279) for detailed discussions of alternative specifications for the technology of conflict. Mueller (2003, at p. 379) provides a rent-seeking analog to the contest function we use.

Assumption 1 limits how much of his endowment each player devotes to guns in the conflict equilibrium. It also plays a role in determining whether or not property rights can be created.

Each player's utility is the sum of what he appropriates from his own corn production and what he expropriates from the corn production of the other players, plus his subsistence consumption:

$$u_i = p_{ii}Ak_i + \sum_{j \neq i}^{N} p_{ij}Ak_j + c_i, \qquad i = 1, \ldots, N. \tag{4}$$

Each player simultaneously maximizes his utility by choosing how he allocates his after-tax endowment, $\omega - \tau_i$, across the four possible choices: corn production, private provision of security, subsistence consumption, and guns:

$$k_i = \omega - \tau_i - x_i - y_i - c_i, \qquad i = 1, \ldots, N. \tag{5}$$

Thus $k_i$ is the residual from the choices of $c_i$, $x_i$, $y_i$ and the rate of taxation, $\tau_i$.

As we have assumed that each player's endowment is identical, we restrict our attention to symmetric equilibria in which $x_1 = x_2 = \cdots = x_N \equiv x \geq 0$, $y_1 = y_2 \cdots = y_N \equiv y \geq 0$, and $c_1 = c_2 \cdots = c_N \equiv c \geq 0$. However, all of the results presented can be derived with few alterations if we were to allow the endowments to differ.[12]

## 4 The Nash Equilibrium

Our objective in this section is to see how well property rights are protected absent a state and to characterize the Nash equilibrium in terms of the Demsetz productivity parameter, $A$, the number of competitors, $N$, and the security of property parameter, $\theta$. Since no state exists, we set $\Psi = \tau = 0$.

In the Nash equilibrium, each player simultaneously chooses $x_i$, $c_i$, and $y_i$ to maximize $u_i$, taking the other players' actions as given. The first-order-necessary-conditions for player $i$ include (5) and the following:

$$\frac{\partial u_i}{\partial y_i} = \frac{\partial p_{ii}}{\partial Y}Ak_i + \sum_{j \neq i}^{N} \frac{\partial p_{ij}}{\partial Y}Ak_j - Ap_{ii} \leq 0, i = 1, \ldots, N, \tag{6}$$

[12] When endowments differ, players with lower endowments devote a larger portion of their endowment to conflict in the Nash equilibrium (Hirshleifer, 1991a). A king of either type taxes those with a larger endowment at a higher rate, and under a social contract, those with a larger endowments devote more resources towards security.

$$\frac{\partial u_i}{\partial c_i} = 1 - Ap_{ii} \leq 0, \qquad i = 1, \ldots, N, \tag{7}$$

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial p_{ii}}{\partial x_i} Ak_i + \sum_{j \neq i}^{N} \frac{\partial p_{ij}}{\partial x_i} Ak_j - Ap_{ii} \leq 0, i = 1, \ldots, N. \tag{8}$$

From (1) and (2), the rates at which the appropriation and expropriation parameters change as $x_i$, $y_i$, and $c_i$ increase are given by

$$\begin{aligned} \frac{\partial p_{ii}}{\partial Y} = \frac{\partial p_{ii}}{\partial x_i} = \frac{X_{-i}}{\left(\theta + Y + X\right)^2} > 0, \qquad \frac{\partial p_{ii}}{\partial c_i} = \frac{\partial p_{ij}}{\partial c_i} = 0, \\ \frac{\partial p_{ij}}{\partial Y} = \frac{-x_i}{\left(\theta + Y + X\right)^2} < 0, \qquad \text{and} \qquad \frac{\partial p_{ij}}{\partial x_i} = \frac{\theta + Y + X_{-i}}{\left(\theta + Y + X\right)^2} > 0. \end{aligned} \tag{9}$$

Each unit of the endowment allocated to any of subsistence consumption, guns or security has an opportunity cost in foregone appropriated corn production of $Ap_{ii}$. From (7), the marginal benefit from an increase in subsistence consumption is simply the direct increase in the utility from the subsistence consumption, as an increase in $c_i$ has no effect upon the proportion of one's own corn production appropriated nor upon the proportion of others corn production expropriated. From (8), the marginal benefit from an increase in guns is the increase in the amount player $i$'s own corn production that player $i$ gets to appropriate, plus the increase in the amount of the other players' corn production that player $i$ gets to expropriate. From (6), the net marginal benefit from an increase the size of security is the increase in the share, $p_{ii}$, that player $i$ appropriates from his own investment in corn production, less the reduction in the share, $p_{ij}$, of the other players' corn production that player $i$ gets to expropriate. Therefore, an increase in expenditures on guns by $i$ increases both $i$'s appropriation and expropriation shares, and an increase in expenditures on security by $i$ increases the $i$'s appropriation share but decreases $i$'s expropriation share. This asymmetry implies that players will spend more on guns than on security in the Nash equilibrium. The extent of this asymmetry is given in the following proposition:

**Proposition 4.1.** *In the symmetric Nash equilibrium to the conflict game, each individual contributes zero to the public good of property rights protection.*

*Proof.* See the Appendix. □

While private provision of a public good is well known to result in under provision of the public good relative to the social optimum (Samuelson, 1954), here the problem is particularly acute. No player wishes to contribute a positive quantity to security in the symmetric

Nash equilibrium.[13] This occurs because (9) implies that $\partial p_{ii}/\partial Y = (N-1)\partial p_{ij}/\partial Y$, thus the gain to appropriation is just offset by the loss in expropriation. Therefore, Proposition 4.1 implies that insecure property rights remain insecure. Furthermore, this result is unaffected by the size of $A$, which suggests that the Demsetz hypothesis does not hold in the Nash equilibrium.

Given that $y^{NE} = 0$, absent state-provided property rights, the symmetric Nash equilibrium condition for the choice of guns, $x$, and subsistence consumption, $c$, given by (8) and (7), respectively, can be written, using (5), as

$$\frac{\partial u_i}{\partial x_i} = \frac{A(\theta + x)}{(\theta + X)^2}\big[(\omega - c - x)(N-1) - (\theta + x)\big] \leq 0, \qquad i = 1, \ldots, N, \tag{8$'$}$$

$$\frac{\partial u_i}{\partial c_i} = \frac{1}{\theta + X}\big[(\theta + X) - A(\theta + x)\big] \leq 0, \qquad i = 1, \ldots, N. \tag{7$'$}$$

The next result shows that $k^{NE} < \omega$:

**Proposition 4.2.** *Under Assumption 1, in the symmetric Nash equilibrium, no player devotes his entire endowment to production.*

*Proof.* See the Appendix. □

This result occurs because setting $c = x = 0$ results in positive net marginal utility to $x_i$ by (8′), and maybe even to $c_i$ by (7′).

Given Propositions (4.1) and (4.2), there are three types of equilibria that may arise. Equilibria where $x^{NE} > 0$, $k^{NE} > 0$ and $c^{NE} = 0$ are called the *Hobbesian conflict Nash equilibrium* (HCNE), since conflict and production are both positive in this equilibrium.[14] Equilibria of type where $x^{NE} = k^{NE} = 0$ and $c^{NE} = \omega$ are called the *Rousseauian subsistence Nash equilibrium* (RSNE), as there is neither conflict nor production in this equilibrium.[15] Finally, equilibria in which $k^{NE} > 0$, $x^{NE} \geq 0$ and $c^{NE} > 0$ are called the *Lockean subsistence-conflict Nash equilibrium* (LSCNE), as there is simultaneously conflict, production and subsistence consumption in these equilibria.[16]

[13] It can be shown that this result also holds with the Grossman and Kim (1995) conflict technology, and that this result also holds in our model with asymmetric endowments.

[14] Thomas Hobbes (1651, Chapt. 13 at p. 185) wrote that "during the time men live without a common power to keep them all in awe, they are in that condition which is called war; and such a war as is of every man against every man" from which he deduced that "the life of man [is] solitary, poor, nasty, brutish, and short" (Hobbes, 1651, Chapt. 13 at p. 186).

[15] Jean-Jacques Rousseau (1762), states "Men are not natural enemies, for the simple reason that men living in their original state of independence do not have sufficiently constant relationships among themselves to bring either a state of peace or a state of war" (Rousseau, 1762, Book 1, Chapt. 4 at p. 145).

[16] John Locke (1690), fits between Rousseau and Hobbes both temporally and in his views of the state of nature. In Locke, as in Hobbes, the state of nature involved conflict, but, like Rousseau, he believed that conflict could be overcome by means other than autocracy: "The state of nature has a law of nature to govern it, which obliges every one: and reason, which is that law, teaches all mankind, who will but consult it" (Locke, 1690, Sect. 6 at p. 5).

Let us consider the Hobbesian conflict Nash equilibrium. In this equilibrium, $c^{NE} = 0$, but $x^{NE} > 0$ and $k^{NE} > 0$. Then (8′), (5), (3) and (4) imply that

$$y^{NE} = 0, x^{NE} = \frac{\omega(N-1)-\theta}{N},\ k^{NE} = \frac{\omega+\theta}{N},\ \text{and } u^{NE} = \frac{A(\omega+\theta)}{N}. \tag{10}$$

By Assumption 1, the level of conflict in the HCNE is positive for all $N \geq 2$. Hence, property is less than perfectly secure in the HCNE:

$$p_{ii}^{NE} = \frac{\omega+\theta}{N\omega} \text{ and } p_{ij}^{NE} = \frac{(N-1)\omega-\theta}{N(N-1)\omega}.$$

A necessary condition to be in the HCNE is that $c^{NE} = 0$. Therefore, from (7′), the Demsetz parameter must satisfy the following condition:

$$c^{NE} = 0,\ x^{NE} > 0,\ \text{and } k^{NE} > 0 \text{ for all } A \text{ such that } A \geq \bar{A} \equiv \frac{N\omega}{\omega+\theta}. \tag{11}$$

The state of nature in which $A \geq \bar{A}$ is the *Hobbesian state of nature*; in it players allocate their entire endowment to either production or conflict. The minimum value of the Demsetz parameter, $\bar{A}$, such that the HCNE occurs is increasing in $N$ and $\omega$, and decreasing in $\theta$.

Next, consider the Rousseauian subsistence Nash equilibrium. In this equilibrium, $c^{NE} = \omega$ and $x^{NE} = k^{NE} = 0$. Since $x^{NE} = 0$ implies that $p_{ii}^{NE} = 1$, (7′) implies that $c^{NE} > 0$ only if $A \leq 1$. When $A < 1$, players wish to devote their entire endowment to subsistence consumption. Therefore, utility in the RSNE is equal to $u^{NE} = \omega$. Define $\hat{A} \equiv 1$ to be the upper bound on the Demsetz parameter $A$ such that the RSNE occurs. The region where $A < \hat{A}$ is the *Rousseauian state of nature*; it corresponds to a pure subsistence economy in which there is no conflict. For values of $A$ in this region,

$$y^{NE} = x^{NE} = k^{NE} = 0 \text{ and } c^{NE} = u^{NE} = \omega,\ \text{for } A < \hat{A}. \tag{12}$$

Property rights are irrelevant in the RSNE since subsistence consumption cannot be stolen and everyone devotes all of their endowment to subsistence consumption.

The intermediate case is the *Lockean state of nature*, in which $\hat{A} \leq A < \bar{A}$. The LSCNE is characterized by $c^{NE} > 0$, $x^{NE} \geq 0$ and $k^{NE} > 0$. These are the simultaneous solutions to (8′), (7′) and (5):

$$y^{NE} = 0, x^{NE} = \frac{\theta(A-1)}{N-A}, c^{NE} = \omega - \frac{\theta A}{N-A},\ \text{and } k^{NE} = \frac{\theta}{N-A},\ \text{for } \hat{A} \leq A < \bar{A}. \tag{13}$$

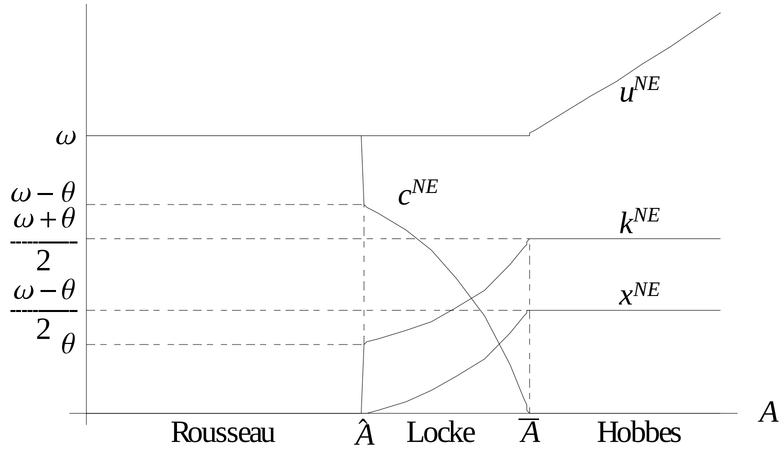


**Figure 1:** The Nash Equilibrium with $N = 2$ Players.

Two properties of the LSCNE are noteworthy. First, as $\theta$ approaches zero, $x^{NE} = k^{NE} = y^{NE} = 0$ and $c^{NE} = \omega$ in the LSCNE. Thus, $\theta > 0$ is necessary for the LSCNE to exist. Second, in the LSCNE, $u^{NE} = \omega$, which is the same as the utility in the RSNE. The marginal utility of subsistence consumption is equal to unity, and in the LSCNE, subsistence consumption coexists with production and conflict, thus the marginal product of conflict and production are also unity.

The Nash equilibrium is summarized in the following proposition:

**Proposition 4.3.** *The symmetric Nash equilibrium is characterized as follows:*

*(i)* *for* $A < \hat{A}$*, the RSNE satisfies* (12)*;*

*(ii)* *for* $\hat{A} \leq A < \bar{A}$*, the LSCNE satisfies* (13)*;*

*(iii)* *for* $\bar{A} \leq A$*, the HCNE satisfies* (11)*.*

Figure 1 displays the equilibrium choices for a range of values of the Demsetz parameter, $A$, for the case where $N = 2$ and $\theta = \frac{1}{4}\omega$. The RSNE occurs for $A < \hat{A}$. At $\hat{A}$, $k^{NE}$ jumps from zero to a positive value and $c^{NE}$ drops by the same amount when $\theta > 0$. Thus, at $\hat{A}$ production is profitable but conflict is not. As $A$ increases in the LSCNE, $c^{NE}$ decreases and $k^{NE}$ and $x^{NE}$ increase. Once $A \geq \bar{A}$, $c^{NE} = 0$ and $k^{NE}$ and $x^{NE}$ are each constant in the HCNE. Because the level of conflict is bounded, utility is increasing in $A$ in the Hobbesian state of nature. The upper boundary to the Lockean state of nature, $\bar{A}$, is increasing in $N$ and decreasing in $\theta$, but the lower boundary, $\hat{A}$, is invariant to $N$ and $\theta$.

Perhaps the starkest example of how the Demsetz parameter determines the nature of the

Nash equilibrium is provided by Diamond (1997). Around 1000AD, Polynesians settled both New Zealand and the Chatham Islands, located some 500 miles southeast of New Zealand. The rich environment of New Zealand allowed the Maori population to prosper. In contrast, the Moriori who settled the Chatham Islands found a cold climate unsuited to the Polynesian agriculture. While the Maori grew to a rich and warlike society, the Moriori society reverted to an unstructured hunter-gatherer society. In 1835, upon learning of the existence of the Chatham Islands, 900 Maori sailed there where they encountered some 2000 Moriori, whom the Maori declared to be their slaves. The Moriori, who "had a tradition of resolving disputes peacefully," intended to share their resources with the Maori, but before an offer could be made the Maori attacked. A Moriori survivor described the ensuing slaughter: "[The Maori] commenced to kill us like sheep. . . [We] were terrified, fled to the bush, concealed ourselves in holes underground and in any place to escape our enemies. It was of no avail; we were discovered and killed—men, women, and children indiscriminately" (p. 53). Hence the Moriori, who had existed for over 800 years in a RSNE, were ill suited for surviving in the HCNE to which the Maori had become accustomed.

## 5 Property Rights by Social Contract

Now we consider a variation in the game in which property rights may arise through private provision. We continue to assume that there is no state. Hence $\Psi = \tau = 0$.

Suppose that players break the game into two stages. First, in the social contract stage each player simultaneously and voluntarily contributes an allocation of $y_i$ from their endowment for the provision of property rights. Then, after the size of security, $Y$, has been realized, in the conflict stage each player simultaneously allocates his remaining endowment between subsistence consumption, guns, and corn production. We call this a *social contract*, since players commit to the allocation of $y_i$ prior to making each of the other economic decisions. As the equilibrium is subgame perfect, the allocations to $Y$ are credible.[17]

Subgame perfection requires that players solve the game using backwards induction. Suppose that $A$ is sufficiently large such that conflict occurs in the conflict stage of the game.

[17]In order for the social contract to be effective, however, it must be that players cannot renege on their investment in the public good protection of property rights. Reneging forces players back into the Nash equilibrium. In the California mining example, the investment in property rights protection was an agreement by miners to jointly protect one another's claim. Clay and Wright (2005) give an example where a group of miners who, when approached by a larger group of claim jumpers, decided to renege upon their earlier agreement and let the claim jumpers have a share of their claims. There are ways in which this outcome can be avoided. In the Texas oil field example, the commitment of using the boundary injection wells is enforced by the cost of removing that well from its present purpose of water injection and using it to extract oil. Thus, commitment is solved by the putty-clay nature of the investment in the public good. Even when investment is of a putty-putty nature, commitment can be achieved in some instances. In the Valencia irrigation example, monitoring other player's actions was rational in the subgame because each player had an incentive to ensure that he got his turn at using the water.

From the analysis in Section 4, the Nash equilibrium to the conflict stage game involves zero subsistence consumption when $A \geq \bar{A}$ and zero provision of security for all $A$. However, here we allow $y_1, y_2, \ldots, y_N$ to take arbitrary values when we consider the choices in the conflict stage of the game, although we continue to assume that $c_1 = c_2 = \cdots = c_N = 0$. (We later find under what condition the latter assumption is valid.) The choices of $x_i$ and $k_i$ depend upon the values of $y_1, y_2, \ldots, y_N$ from the social contract stage decisions.

As the intermediate steps to finding the subgame perfect Nash equilibrium are somewhat opaque, they are relegated to the appendix. However, to gain some intuition, we consider the case where there are only two players. In this case, given that the conflict stage choices of $x_1^*(y_1, y_2)$ and $x_2^*(y_1, y_2)$ depend upon the investment in security by each player, the equilibrium choice of security is given by

$$\begin{aligned} \frac{du_i}{dy_i} &= \frac{\partial u_i}{\partial y_i} + \frac{\partial u_i}{\partial x_i}\frac{dx_i^*}{dy_i} + \frac{\partial u_i}{\partial x_j}\frac{dx_j^*}{dy_i} \\ &= \underbrace{\frac{\partial u_i}{\partial y_i}}_{\text{direct effect}} + \underbrace{\frac{\partial u_i}{\partial x_i}\frac{dx_i^*}{dy_i}}_{\text{zero by envelope theorem}} + \underbrace{\frac{\partial u_i}{\partial x_j}\frac{\partial x_j^*}{\partial x_i}\frac{\partial x_i^*}{\partial y_i}}_{\text{own effect}} + \underbrace{\frac{\partial u_i}{\partial x_j}\frac{\partial x_j^*}{\partial y_i}}_{\text{rival effect}} = 0, \qquad i = 1, 2. \end{aligned} \tag{14}$$

In the Nash equilibrium, only the direct effect (which is negative) occurs. In the strategic game, there are two strategic effects, denoted as the "own effect" and the "rival effect", which together make up the rate of change in $i$'s utility as conflict by $j$ changes.[18] Since an increase in investment in property rights by player $i$ makes player $i$ less inclined to invest in conflict and player $i$'s utility is decreasing in the level of conflict chosen by player $j$, the sign of the own effect (Fudenberg and Tirole, 1984) depends on whether conflict is a strategic substitute or a strategic complement. The rival effect (Church and Moldovan, 2008) unambiguously makes player $i$ better off as an increase in property rights makes player $j$ less inclined to invest in conflict and player $i$'s utility is decreasing in the level of conflict chosen by player $j$.

We plot the best-response curves to the conflict stage of the game in Figure 2 for the case

[18] The own and rival effects come from expanding the term $\frac{\partial u_i}{\partial x_j}\frac{dx_j^*}{dy_i}$. The total differential is found using Cramer's rule on the system of first-order conditions for the choices of conflict (Church and Moldovan, 2008). This is given by

$$\frac{dx_j^*}{dy_i} = \frac{-\frac{\partial^2 u_i}{\partial x_i^2}\frac{\partial^2 u_j}{\partial x_j \partial y_i} + \frac{\partial^2 u_j}{\partial x_j^2}\frac{\partial^2 u_i}{\partial x_i \partial y_i}}{\frac{\partial^2 u_i}{\partial x_i^2}\frac{\partial^2 u_j}{\partial x_j^2} - \frac{\partial^2 u_i}{\partial x_j \partial x_i}\frac{\partial^2 u_j}{\partial x_i \partial x_j}}.$$

The denominator is positive if the best-response correspondences are stable. The rival effect occurs because $\frac{\partial^2 u_j}{\partial x_j \partial y_i} \neq 0$.

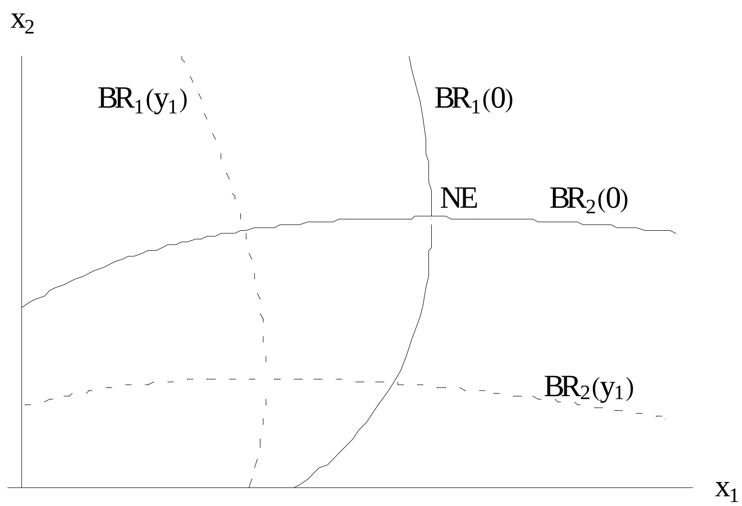


**Figure 2:** The Effect of Property Rights on Conflict, when $N = 2$ Players.

where $N = 2$ and where $A$ is strictly greater than $\bar{A}$. The Nash equilibrium corresponds to the intersection of the two solid best-response curves labeled $BR_1(0)$ and $BR_2(0)$, respectively, where the 0 refers to the level of investment in security by the two players. These plot the best-response correspondences implicitly given by (A.2) in the appendix. The best-response correspondences are non-monotonic and at the Nash equilibrium conflict is neither a strategic substitute nor strategic complement. The intercept on these best-response correspondences is non-zero because of the presence of a positive $\theta$ parameter. It is also easy to see by inspection that these correspondences are stable.[19]

An investment of $y_1$ by player 1 in security, has two effects upon the best-response curves. The own effect shifts his own best-response curve inwards to the dashed curve labeled $BR_1(y_1)$ from $BR_1(0)$. In this region, conflict is a strategic complement to player 2. The own effect is therefore positive in total. The rival effect shifts the other player's best-response curve inward from $BR_2(0)$ to $BR_2(y_1)$; this also makes player $i$ better off. Therefore, the total strategic effect is positive. Thus positive investment in property rights can occur in the social contract game where credible commitments to property rights are made prior to an investment in conflict. The level of conflict is also lower in this game. Both of these properties are shown to hold more generally in the next proposition.

**Proposition 5.1.** *For all* $A \geq A_{SC} \equiv \frac{N(N+1)\omega}{2(N\omega+\theta)}$, *the symmetric social contract subgame*

[19] A plot of the best-response correspondences in $y_1$ and $y_2$ space, using (A.9), reveals that those best response curves are also non-monotonic and that at the SCSPNE, investments in security are also neither strategic complements nor strategic substitutes. However, the shapes of the best-response curves are inverted relative to Figure 2 as utility of each player is increasing in the investment in property rights by the other player.

*perfect Nash equilibrium (SCSPNE) satisfies*

$$c^{SC} = 0, x^{SC} = \frac{(N\omega - \theta)(N-2)}{(N+1)N}, y^{SC} = \frac{\omega - \theta}{N+1}, k^{SC} = \frac{2(N\omega + \theta)}{N(N+1)},$$
$$p_{ii}^{SC} = \frac{2}{N}, \qquad p_{ij}^{SC} = \frac{N-2}{N}, \text{ and } u^{SC} = \frac{2A(N\omega + \theta)}{N(N+1)}. \quad (15)$$

*For $A < A_{SC}$, the SCSPNE is equal to the LSCNE or the RSNE.*

*Proof.* See the Appendix. □

Thus unlike the Nash equilibrium in which zero security is chosen, when the allocation to security occurs prior to the allocation of the remainder of the endowment, in the subgame perfect Nash equilibrium there is positive private provision to the public good. This occurs because there is a strategic effect of reduced conflict by other players when the amount of security increases. It is this effect that is absent in the Nash equilibrium.

When $N = 2$, conflict is completely eliminated since (15) implies that $x^{SC} = 0$ and $p_{ii}^{SC} = 1$ when $N = 2$.[20] Furthermore, when $N = 2$, total expenditures on private provision of security is less than the Hobbesian Conflict Nash equilibrium expenditures on guns, i.e., $N = 2$ implies that $y^{SC} = \frac{\omega-\theta}{3} < \frac{\omega-\theta}{2} = x^{NE}$. Thus, the total resources spent on security of property are less than the resources spent on conflict in the Nash equilibrium. This means that production is higher in the SCSPNE than in the HCNE, i.e., $k^{SC} = \frac{2(N\omega+\theta)}{N(N+1)} > \frac{\omega+\theta}{N} = k^{NE}$. It follows that utility is also higher in the SCSPNE than in the HCNE.

However, for $N > 2$, the SCSPNE does not fully eliminate conflict, since $x^{SC} > 0$ for $N > 2$. As with the HCNE, the limit as $N \to \infty$ is that $k^{SC} = y^{SC} = u^{SC} = 0$ and $x^{SC} = \omega$. This occurs because as $N$ grows the effect any player can have upon influencing the behavior of the balance of the population diminishes. Thus for $N$ sufficiently large, there is little strategic effect from investing in security. From (15), the size of the aggregate provision to security is $Y^{SC} = \frac{N(\omega-\theta)}{N+1}$. This is increasing in $N$, but is bounded from above by $Y^{SC} \leq \omega - \theta$. In contrast, total guns expenditure is increasing roughly linearly in $N$. The result is that the proportion of one's own corn production that one appropriates is $p_{ii}^{SC} = 2/N$, which is diminishing towards zero as $N$ increases. In contrast, the share of the other $N-1$ competitors corn production that $i$ gets to expropriate is $p_{ij}^{SC} = (N-2)/N$, which tends towards one as $N$ grows large. Thus as $N$ grows, players have increasing success in steal the dwindling corn production of others.

[20] In Figure 2, we have only shown the effect of investment by player 1. Player 2 has similar incentives.

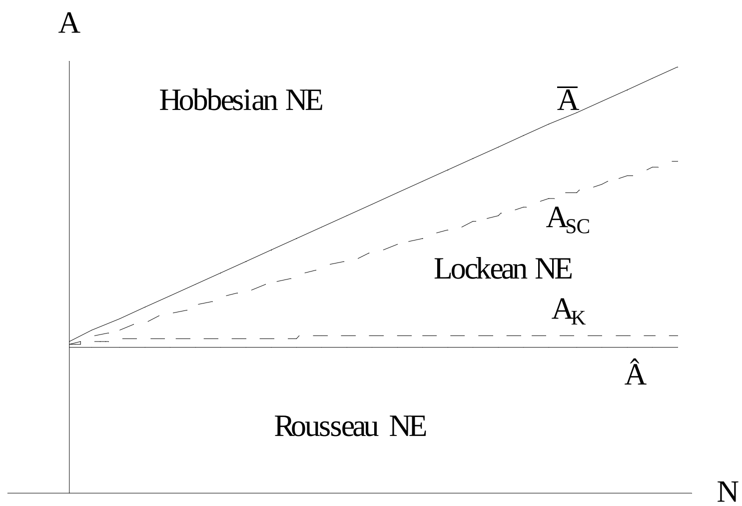


**Figure 3:** The Conflict Equilibrium Boundaries.

In order for the social contract to arise as an equilibrium arrangement, as $N$ increases, an ever greater value of the Demsetz parameter is required. Indeed, $A_{SC}$ is unbounded as $N \to \infty$. Nevertheless, the value of $A_{SC}$ is always less than the value of $\bar{A}$ by Assumption 1. Figure 3 shows the boundaries of the Hobbesian, Lockean and Rousseauian Nash equilibrium areas in $(A, N)$ space with the boundary of the social contract superimposed upon it. Above the boundary $A_{SC}$, the social contract is possible.

As $\theta$ varies between zero and $\omega$, $A_{SC}$ is bounded between $\frac{N+1}{2}$ and $\frac{N}{2}$, respectively. Since $A_{SC}$ is decreasing in $\theta$, an increase in $\theta$ increases the range of values $\{A, N\}$ such that the social contract is possible. In addition, by (16) an increase in $\theta$ decreases $x^{SC}$ and $y^{SC}$ and increases $k^{SC}$ and $u^{SC}$. This is because $\theta$ and $Y$ are perfect substitutes, but only $Y$ is socially costly.

The surplus created under a social contract is the difference between utility under the Nash equilibrium and under the social contract:

$$\Delta S^*_{SC} = N(u^{SC} - u^{NE}) = \begin{cases} \frac{(N-1)A(\omega-\theta)}{N(N+1)} & \text{if } A \geq \bar{A} \\ \\ \frac{\omega N[2A-(N+1)]+2A\theta}{N+1} & \text{if } A_{SC} \leq A < \bar{A} \end{cases} \tag{16}$$

When $A \geq \bar{A}$, the welfare gain to a social contract is positive by Assumption 1. It is also positive for all values of $A_{SC} < A < \bar{A}$. For $A < \bar{A}$, a player can always guarantee utility equal to $\omega$ by reverting to subsistence consumption. Thus, $u^{SC} \geq \omega$. However, it is this condition that defines $A_{SC}$. Thus, we have proved:

**Proposition 5.2.** *Welfare is higher under the social contract than under the Nash equilibrium for all values of $A > A_{SC}$.*

Hence, in the Slaughter oil field in west Texas, with over a hundred separate operators, less than three percent of the rents were dissipated by the use of boundary injection wells. However, Clay and Wright (2005) argue that gold mining claims were much less secure than had been claimed by Umbeck (1977), noting that with the influx of around 150,000 people into California during the gold rush, "every two or three claims supported at least one lawyer" (at p. 170). Thus in large societies, conflict dominates the SCSPNE, just as it does the Nash equilibrium.

## 6 Autocratically Provided Property Rights

Next, we consider two benchmarks to compare with the social contract equilibrium. In both, we suppose that a third party, whom we call a *king*, offers to create property rights by providing state-sponsored security of size $\Psi$ to supplement the existing natural property rights of size $\theta$. He does this in exchange for the right to impose a tax of $\tau$ on the endowment of each player.

We consider two cases: $(i)$ A benevolent king, the Aristotelian ideal, devotes all of the tax revenues to supplement security and chooses the size of state-sponsored security to maximize social welfare. $(ii)$ A despotic king keeps any surplus tax revenues above the costs of supplying the security for himself, and chooses the level of security to maximize the surplus he is able to grab from his citizens. In each case, the king funds his activities using a tax on the endowment.

The medieval system of vassal homage which led to autocratic rule in Europe arose in response to the anarchy following the collapse of the Roman Empire. Bloch (1960) describes its origin

> "Neither the State nor the family any longer provided adequate protection. The village community was barely strong enough to maintain order within its own boundaries; the urban community scarcely existed. Everywhere, the weak man felt the need to be sheltered by someone more powerful." (1960, at p. 148).

If a king can impose a lump sum tax, then he can achieve the first-best. Because of the free-riding problem, a social contract can never hope to replicate the first-best. But the social contract may do as well as a king if the king is uses an inefficient method of taxation. The tax used by William the Conquerer and his successors was a lump sum tax, but it was a tax on the endowment, not upon the output.[21] Bloch describes the method of taxation of a lord upon his subject:

> "The powerful individual who forced his weaker neighbor to submit to him was apt to require the surrender of his property as well as his person. The lesser men, therefore, in offering themselves to the chief, also offered their lands. The lord, once the bond of personal subordination had been sealed, restored to his new dependent the property thus temporarily surrendered, but subject now to his superior right, expressed by the various obligations imposed upon it. This great movement of land surrender went on at every social level during the Frankish period and the first feudal age" (1960, at p. 171).

In the Nash equilibrium, players take the tax rate $\tau$ and the king's choice of $\Psi$ as given when choosing how to allocate their after-tax endowment between corn production, guns, subsistence consumption and private provision of the public good of property rights protection. Therefore, in the symmetric Nash equilibrium, the first-order-necessary conditions for the choices of $y$, $c$, and $x$, respectively, satisfy

$$\frac{\partial u_i}{\partial y_i} = -A\left[\frac{\theta + \Psi + Y + x}{\theta + \Psi + Y + X}\right] \leq 0, \qquad i = 1, \ldots, N, \tag{17}$$

$$\frac{\partial u_i}{\partial c_i} = 1 - A\left[\frac{\theta + \Psi + Y + x}{\theta + \Psi + Y + X}\right] \leq 0, \qquad i = 1, \ldots, N, \tag{18}$$

$$\frac{\partial u_i}{\partial x_i} = \frac{A\left[(\omega - x - c - y - \tau)(N-1) - (\theta + \Psi + Y + x)\right]}{\theta + \Psi + Y + X} \leq 0,\ i = 1, \ldots, N. \tag{19}$$

As in the case where no king exists to create property rights, the first-order condition (17) for $y_i$ is negative for all feasible values of $y$, $x$, and $c$. Hence:

**Proposition 6.1.** *In the symmetric Nash equilibrium under a king, each player sets $y^* = 0$.*

Therefore, when a king exists, only the king provides property rights protection. However, there is no crowding out of private provision, as $Y^{NE} = 0$ by Proposition 4.1.

In the Nash equilibrium, utility equals $\omega$ if any of the endowment is consumed directly as subsistence consumption. This result occurs with a king as well, which implies that a king

[21] A tax on the endowment was probably common in the contracts between a king and his subjects in the feudal system in medieval Europe because it does not subject the king to the moral hazard problem that occurs when output is taxed.

cannot improve welfare if his citizens have an incentive to devote part of their endowment to subsistence consumption. Thus the interesting outcomes occur in the Hobbesian state of nature. Absent a king, this occurs when $A \geq \bar{A}$. From (19), (4) and (5), for any feasible values of $\Psi$ and $\tau$, when $A \geq \bar{A}$ the level of investment in guns and corn production and the corresponding symmetric equilibrium utility satisfy the following:

$$x^*(\Psi, \tau) = \frac{(N-1)(\omega - \tau) - \theta - \Psi}{N}, k^*(\Psi, \tau) = \frac{\omega + \theta + \Psi - \tau}{N} \\ \text{and } u^*(\tau, \Psi) = \frac{A(\omega + \theta + \Psi - \tau)}{N}. \tag{20}$$

These are simply the HCNE values with $\omega$ replaced by $\omega - \tau$ and $\theta$ replaced by $\theta + \Psi$.

### 6.1 A Benevolent King: The Second-Best

A benevolent king solves for the second-best tax on the endowment that provides the level of property rights that maximizes welfare. Since state-sponsored security costs $\Psi$ units of endowment, to balance the budget the benevolent king chooses the tax rate such that

$$\Psi = N\tau. \tag{21}$$

From (20), the equilibrium values of $x_B^*(\tau)$, $k_B^*(\tau)$ and $u_B^*(\tau)$ depend upon the tax rate $\tau$:

$$x_B^*(\tau) = \frac{\omega(N-1) - \theta - \tau(2N-1)}{N}, k_B^*(\tau) = \frac{\omega + \theta + (N-1)\tau}{N}, \\ \text{and } u_B^*(\tau) = \frac{A\big[\omega + \theta + (N-1)\tau\big]}{N}. \tag{22}$$

$u_B^*(\tau)$ and $k_B^*(\tau)$ are each increasing in $\tau$ and $x_B^*(\tau)$ is decreasing in $\tau$. Therefore, a benevolent king maximizes welfare by setting $\tau$ just large enough to drive $x_B^*(\tau)$ to zero. Thus from (21) and (22),

$$\tau_B^* = \frac{(N-1)\omega - \theta}{2N-1}, \Psi_B^* = \frac{(N-1)N\omega - N\theta}{2N-1}, \\ x_B^* = c_B^* = y_B^* = 0, k_B^* = \frac{N\omega + \theta}{2N-1}, \text{ and } u_B^* = \frac{A(N\omega + \theta)}{2N-1}. \tag{23}$$

Each citizen devotes $k_B^*$ of his endowment to corn production and the remainder to paying taxes for the provision of the security. Property rights are perfectly enforced, since $x_B^* = 0$ implies that $p_{ii}^* = 1$ and $p_{ij}^* = 0$. Furthermore, evaluated at $x_B^* = 0$, we see from (18) that $\frac{\partial u_i}{\partial c_i} = 1 - A$, which is negative for all $A > \hat{A} = 1$. Thus at the optimal tax rate, $\tau_B^*$, no subject wishes to switch to subsistence consumption to avoid the tax for any $A > \hat{A}$. As $u^{NE} = \omega$ in the LSCNE, we may use (23) to solve for the minimum level of the Demsetz parameter under which a king can create property rights:

**Proposition 6.2.** *Under Assumption 1, a benevolent king improves welfare for all values of $A > A_K$, where*

$$A_K \equiv \frac{\omega(2N-1)}{N\omega + \theta} \subset (\hat{A}, \bar{A}), \tag{24}$$

*Proof.* See the Appendix. □

Thus the minimum Demsetz parameter under which a benevolent king can create property rights is $A_K > \hat{A}$. The wedge between $A_K$ and $\hat{A}$ is caused by property rights being socially costly even with a benevolent king. A benevolent king is able to create and perfectly enforce property rights only if there is conflict, since $x^{NE} > 0$ for all $A > \hat{A}$. However, the presence of conflict is not sufficient to ensure that a benevolent king is able to create property rights, since a king cannot exist when $\hat{A} < A < A_K$, even though conflict occurs in the Nash equilibrium.

The minimum value of the Demsetz parameter such that a benevolent king can arise is increasing in $N$ at a decreasing rate; the limit of $A_K$ as $N \to \infty$ is $A_K = 2$. Recall that the minimum level under which a social contract can arise is also increasing in $N$, but as $N \to \infty$, $A_{SC}$ is unbounded. This suggests that property rights under a benevolent king can arise for lower values of $A$ than under a social contract. This is made exact by the following:

**Proposition 6.3.** *The minimum Demsetz parameter under which a social contract can create property rights is equal to the minimum Demsetz parameter under which a king can create property rights when $N = 2$, but is greater than the minimum Demsetz parameter under which a king can create property rights when $N > 2$. That is $A_{SC} \geq A_K$ as $N \geq 2$.*

*Proof.* See the Appendix. □

This result is shown in Figure 3 as the dashed locus labeled $A_K$, which, for $N > 2$, lies below the $A_{SC}$ locus. When $N$ gets large, players under a social contract have difficulty creating perfect property rights because the free-riding problem overwhelms the strategic incentive to provide property rights protection. In contrast, a benevolent king can coerce his citizens to contribute at the appropriate level.

As with the social contract, an increase in $\theta$ increases the range of values $\{A, N\}$ such that a benevolent king can improve welfare relative to the Nash equilibrium, since $A_K$ is decreasing in $\theta$. Varying $\theta$ reveals that $A_K$ is bounded between $\frac{2N-1}{N}$ and $\frac{2N-1}{N+1}$. Also, we see from (23) that while $c_B^*$, $y_B^*$, and $x_B^*$ are independent of $\theta$, $\tau_B^*$ and $\Psi_B^*$ are each decreasing in $\theta$ and $k_B^*$ and $u_B^*$ are each increasing in $\theta$. Hence, an increase in $\theta$ allows a benevolent king to lower his provision of property rights as $\theta$ and $\Psi$ are perfect substitutes.

The net gain to society under a benevolent king is the difference in aggregate utility relative to the Nash equilibrium:

$$\Delta S_B^* \equiv N(u_B^* - u^{NE}) = \begin{cases} \frac{A(N-1)}{2N-1}\big[\omega(N-1)-\theta\big], & \text{for } A \geq \bar{A} \\ \\ \frac{N}{2N-1}\big\{\omega[N(A-2)+1]+\theta A\big\} & \text{for } A_K \leq A < \bar{A}. \end{cases} \tag{25}$$

Next, we compare the utility of citizens under a social contract with that under a benevolent king.

**Proposition 6.4.** *A benevolent king produces at least as great of surplus as a social contract for all $N \geq 2$ and for all $A \geq A_K$:*

$$\Delta S_{SC} \left\{ \begin{array}{c} = \\ < \end{array} \right\} \Delta S_B^* \text{ if, and only if, } N \left\{ \begin{array}{c} = \\ > \end{array} \right\} 2.$$

*Proof.* See the Appendix. □

Thus, a social contract can do as well as a benevolent king only when $N = 2$. For all $N > 2$, the incentive to free-ride in the social contract exceeds the strategic incentive of providing protection of property rights.

Together these two propositions show that not only is a benevolent king able to create property rights over a greater range of values of the Demsetz parameter than can players who attempt to write a social contract, but also that when the king creates property rights, he does so more efficiently than can be done under the social contract. This occurs because the king does not face the free-riding problems that plague the social contract. However, we have assumed that the king is benevolent. Let us now turn to the case where the king is a despot who attempts to maximize his own welfare at the expense of his citizens.

## 6.2 A Despotic King

The despotic king we have in mind uses his power of taxation to expropriate wealth for his own consumption. This corresponds the the "tinpot" form of dictatorship in Wintrobe (1990).[22] The potential for despotism by the king is especially dangerous since even a

[22] Mueller (2003, at p. 409) gives examples of several such kings. The Roman emperor Nero composed and sang in public, bribed his way to winning in Olympic games, and was alleged to have played his lyre while Rome burned. French King Louis

benevolent king chooses $\Psi_B^*$ at just high enough level so that citizens give up their guns. However, we show that the problem is more serious than simply having an inequitable distribution of wealth—the surplus under a despotic king is, under some conditions, less than the surplus generated by a benevolent king, and may even be less than that which occurs under a social contract. This occurs because the despotic king takes too much of the endowment for his own consumption, which means that the amount left for corn production is too low relative to either the benevolent king or the social contract.

The surplus, $R_D$, the despotic king earns is the difference between his tax revenues and his costs of providing state-sponsored security:

$$R_D = N\tau - \Psi. \tag{26}$$

The despot sets the tax rate, $\tau_D^*$ such that his citizens are indifferent between the Nash equilibrium outcome without a king and the Nash equilibrium outcome in which the king taxes them at rate $\tau$ and provides security equal to $\Psi$.[23] Thus, the surplus gain is entirely captured by the despot, which means that $R_D$ measures the welfare gain under a despot. There are two cases to consider, depending upon whether $A \geq \bar{A}$ or $A < \bar{A}$.

### 6.2.1 Equilibrium with a Despotic King when $A \geq \bar{A}$

When $A \geq \bar{A}$, the equilibrium payoffs absent a king are the payoffs in the HCNE. From, (10), that utility is $u^{NE} = \frac{A}{N}(\omega+\theta)$. When each citizen takes $\Psi$ and $\tau$ as given when choosing $x_i$ and $k_i$, the utility with a king is given by (20). Thus the participation constraint faced by a despot is

$$u_D^*(\tau, \Psi) = \frac{A(\omega + \theta + \Psi - \tau)}{N} \geq \frac{A}{N}(\omega + \theta) \equiv u^{NE} \text{ for } A \geq \bar{A}. \tag{27}$$

Taking the tax rate as given and solving for the level of security that just makes each player indifferent between having a king who provides property rights protection of $\Psi$ and doing without a king and experiencing the HCNE utility yields $\Psi_D^*(\tau) = \tau$. Substituting this into the surplus function (26) yields $R_D^*(\tau) = (N-1)\tau$. Thus the surplus the despotic king earns is strictly increasing in $\tau$. From (20), the $\tau_D^*$ that maximizes the despot's surplus, given the

XIV and English King Henry VIII are other examples of kings whose consumption (houses and wives, respectively) was deemed extravagant. In modern times, Imelda Marcos, wife of a Philippines dictator, became famous for her 3,000 pairs of shoes. In contrast, totalitarian dictators wish to control the lives of their subjects. Hitler, for example, lived quite simply.

[23] Bloch (1960, at p. 146) notes that the vassal was often referred to in medieval times as the 'man of mouth and hands,' to his lord. The reference to 'mouth' is taken to imply that the lord is responsible for providing the vassal with the means to provide for himself. We take this to imply that the king cannot reduce welfare beyond the Nash equilibrium welfare.

behavior of his citizens, is the value of $\tau_D^*$ such that $x^*(\tau_D^*) = 0$. Like a benevolent king, a despotic king eliminates conflict by setting the level the security sufficiently high so as to prevent any investment in guns by his citizens. Therefore, under a despotic king, when $A > \bar{A}$, the equilibrium is given by

$$y_D^* = c_D^* = x_D^* = 0, k_D^* = \frac{\omega + \theta}{N}, \tau_D^* = \Psi_D^* = \frac{(N-1)\omega - \theta}{N},$$
$$\text{and } R_D^* = \frac{(N-1)[(N-1)\omega - \theta]}{N}, \text{ for } A \geq \bar{A}. \quad (28)$$

The values in (28) are familiar. The $y_D^*$, $c_D^*$, and $k_D^*$ are identical to the HCNE allocations to private provision of security, subsistence consumption, and corn production, respectively, given in (10). Furthermore, the tax charged by the despotic king, $\tau_D^*$, is identical to the HCNE allocation to guns, $x^{NE}$, and is independent of the Demsetz parameter, $A$. By Assumption 1, the despot improves welfare since $\Delta S_D^* = R_D^* > 0$ for all $A \geq \bar{A}$ and for all $N \geq 2$. But he is unable to induce his citizens to increase their investment in corn relative to the HCNE, as he simply replaces the conflict between individual citizens with exploitation by the king. As a result, while the despotic king eliminates conflict, he is not able to improve efficiency in production. The effect of this is that the surplus gain is independent of $A$. This is unlike either the social contract nor the benevolent king, where the surplus gain is increasing in $A$.

An increase in $\theta$ forces the despotic king to decrease the tax rate and provision of property rights, which means that more is available for production and less is available for expropriation by the despot. This suggests that a country like Switzerland, with good natural protection, will suffer less from a despot, and the despot will thereby prosper less, than in a country in which it is more difficult to protect one's own property.

#### 6.2.2 Equilibrium with a Despotic King when $A < \bar{A}$

Next, consider the case where $A < \bar{A}$. The utility each citizen earns in the LSCNE absent a king is $u^{NE} = \omega$. Solving for the size of state-sponsored security, $\Psi_D^*(\tau)$, that equates the left side of (27) to utility in the LSCNE yields

$$\Psi_D^*(\tau) = \frac{N\omega}{A} - \omega - \theta + \tau.$$

Therefore, the despotic king's surplus is

$$R_D^*(\tau) = (N-1)\tau - \frac{N\omega}{A} + \omega + \theta.$$

Again, $R_D^*$ is strictly increasing in $\tau$, which implies that $\tau_D^*$ is chosen to set $x_D^* = 0$. Thus, both a benevolent and despotic king always drive conflict to zero (*cf.* Grossman, 2002).[24] From 18, $x_D^*(\tau_D^*) = 0$ implies that $c_D^* = 0$ for all $A > \hat{A}$. Therefore, the despotic king chooses

$$\tau_D^* = \frac{(A-1)\omega}{A}, \text{ and } \Psi_D^* = \frac{(N-1)\omega}{A} - \theta, \text{ for } A_K \leq A < \bar{A}. \tag{29}$$

Hence,

$$y_D^* = c_D^* = x_D^* = 0, k_D^* = \frac{\omega}{A}, \text{ and } R_D^* = \frac{\omega[N(A-2)+1] + \theta A}{A}, \text{ for } A_K \leq A < \bar{A}. \tag{30}$$

At $A = A_K$, $R_D^* = 0$. Thus a despotic king can arise at the same minimum level of the Demsetz parameter that a benevolent king can arise. By Proposition 6.3 this implies that a despotic king can arise at lower levels of the Demsetz parameter than can the social contract whenever $N > 2$.

The tax rate (which corresponds to the level of conflict in the Nash equlibrium) and the investment in corn production under the despotic king are each greater than the corresponding LSCNE levels.[25] These results occur because there is no subsistence consumption with a despotic king.

When $A_K \leq A < \bar{A}$, $k_D^*$ is independent of $\theta$ by (30) and an increase in $\theta$ is exactly offset by a decrease in $\Psi_D^*$ by (29). Thus, an increase in $\theta$ is fully expropriated by the despotic king when $A_K \leq A < \bar{A}$. This is the opposite of what happens when $A \geq \bar{A}$. The reason is that when $A < \bar{A}$, the utility in the Nash equilibrium is independent of $\theta$. Thus, the despot expropriates all of the surplus from an increase in $\theta$ in this region. However, for $A \geq \bar{A}$, the Nash equilibrium utility is increasing in $\theta$. Hence, the participation constraint forces the despot to leave that surplus with his citizens.

#### 6.2.3 Comparison of Equilibria under a Despotic King and a Social Contract

Now we can compare the equilibrium welfare under a social contract with that in which property rights are provided by a despotic king. For all values of $A$ between $A_K$ and $A_{SC}$, the social contract is dominated by a despotic king because the despot is able to create surplus relative to the Nash equilibrium (which he takes), while the social contract can do no better

[24] Describing the order which William the Conquerer brought to England during his reign of 1066-1087, it was remarked that "It was such than any man, who was himself aught, might travel over the kingdom with bosom full of gold unmolested; and no man durst kill another however great the injury he might have received from him" [sic] (Barrington, 1900, at p. 57).

[25] From (30), $k_D^* = \frac{\omega}{A} > \frac{\theta}{N-A} = k^{NE}$ in the Lockean subsistence-conflict Nash equilibrium (see (13)). Rearranging this inequality yields $A < \bar{A}$, which must hold in the LSCNE. Similarly, $A < \bar{A}$ implies that $\tau_D^* = \frac{(A-1)(\omega)}{A} > \frac{\theta(A-1)}{N-A} = x^{NE}$.

than the Nash equilibrium. However, for $A \geq \bar{A}$, the welfare gain under the social contract is linearly increasing in $A$ (see (16)), while the welfare gain to a despot is independent of $A$ (see (28)). Therefore, we may state the following:

**Proposition 6.5.** *For $A \leq A_{SC}$, a despot creates greater welfare than the social contract, but for $A$ sufficiently high, the social contract welfare dominates a despotic king.*

*Proof.* See the Appendix. □

Both the despot and the social contract face the constraint that in order to exist, utility of the citizens must be at least as large as the Nash equilibrium utility. Because of the free-riding problem, between $A_K$ and $A_{SC}$, only an autocrat (despotic or not) is even capable of creating surplus. However, while a despot's surplus in increasing in $A$ for $A_K \leq A < \bar{A}$ where individual utility is fixed at the subsistence level in the Nash equilibrium, for $A \geq \bar{A}$ the net surplus the despot creates is fixed at $R_D^* = (N-1)x^{NE}$, which is independent of $A$. This bounds the social gain relative to the Nash equilibrium under a despot. In contrast, under a social contract, the welfare gain relative to the Nash equilibrium is linearly increasing in $A$ for all $A \geq A_{SC}$. Thus, for some $A$ sufficiently larger than $A_{SC}$, the social contract dominates the despot.

To illustrate this, suppose that $A \geq \bar{A}$. Then the surplus created under a social contract exceeds the surplus created under a despotic king if $A \geq A^* \geq \bar{A}$, where[26]

$$A^* = \frac{[(N-1)\omega - \theta](N+1)}{\omega - \theta}. \tag{31}$$

Thus a social contract is able to do better than a despotic king so long as the Demsetz parameter is sufficiently large. However, as $A^*$ is increasing in $N$ at an increasing rate, holding $A$ constant and increasing $N$ implies that a despotic king does better than a social contract for large societies. This occurs because a king is able to overcome the free-riding problem that overwhelms the social contract equilibrium as $N$ grows large.

Figure 4 illustrates the regions in which each form of property rights protection may arise.[27] Below the locus $A_K$, neither a social contract nor a despot, nor a benevolent king may arise. Thus, there is no social investment in property rights, and the Nash equilibrium is the RSNE with only subsistence consumption if $A < \hat{A}$ and is the LSCNE with simultaneous investments in subsistence consumption, corn production, and conflict if $\hat{A} \leq A < A_K$. In the region where $A_K \leq A < A_{SC}$, either a despotic or benevolent king may arise but no

[26]It is also possible that the social contract welfare dominates the despotic king for some $A_{SC} \leq A < \bar{A}$.
[27]The locus $A^*$ in Figure 4 is scaled by a factor of 1/2 in order that the area between $A_K$ and $A_{SC}$ may be distinguished.

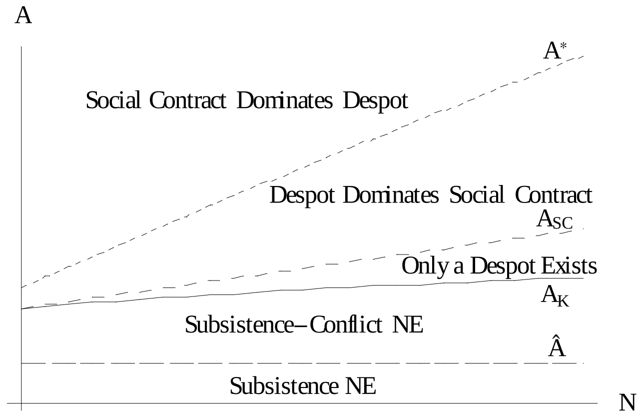


**Figure 4:** Welfare Maximizing Outcomes in $A$-$N$ Space ($\theta = \omega/5$).

social contract is possible. With either a benevolent or a despotic king, there is positive social investment in property rights, and the both stamp out conflict. In the region $A_{SC} \leq A < A^*$, property rights may be provided by either a social contract or by a despot, but the despot welfare dominates the social contract. In the region where $A \geq A^*$, property rights may be provided by either a social contract or by a despot, but the social contract welfare dominates the despot. (The benevolent king welfare dominates the social contract for all $N > 2$ and $A \geq A_{SC}$.[28])

Assuming that all autocrats become despotic and that the welfare dominating method of social organization is chosen, Figure 4 summarizes the equilibrium types of outcomes that can be sustained as a function of the size of the population and the Demsetz parameter. Holding $N$ constant and raising $A$ results in moving from a subsistence economy with no conflict to a Lockean Subsistence-Conflict Nash Equilibrium in which conflict and production both occur, but utility is held at the same low level as in the subsistence economy. In the region $A_K \leq A < A^*$ a despotic king is able to offer a contract that rational citizens are indifferent between accepting and not that allows him to increase aggregate welfare, and to keep the surplus he creates. In this region, conflict is zero. However, above the $A^*$ locus, the

[28]There also exists values of $A$ and $N$ such that a despotic king does better in terms of aggregate welfare created than does a benevolent king. This can be seen by noting that welfare under a despotic king is greater than welfare under a benevolent king whenever

$$\Delta S_B^* < R_D^* \text{ if, and only if, } A_K \equiv \frac{\omega(2N-1)}{N\omega + \theta} \geq A < \frac{2N-1}{N} \equiv \tilde{A}.$$

The benevolent king is maximizing the sum of utilities, which are concave in $\tau$ and $k$, while the despotic king is maximizing a linear function in $\tau$. This advantage is squandered for high values of $A$ because the despot cannot increase the proportion put into production, so at high values of $A$ this leaves society worse off under a despot.

gains from production are sufficiently high that aggregate welfare is improved by adopting a social contract, even though the social contract is inefficient relative to a benevolent king, and even though for $N > 2$ conflict is again positive.[29] Thus conflict is non-monotonically changing as $A$ increases. Below $\hat{A}$, there is no conflict. Conflict is increasing in $A$ (and $N$) in the region between $\hat{A}$ and $A_K$, then is zero in the region of autocracy, but rises again once the social contract comes into effect.

The fragility of the social contract in terms of $N$ is also evident in Figure 4. Holding $A$ constant, an increase in $N$ makes it possible that a despotic king can increase aggregate welfare relative to an existing social contract. This could occur because a benevolent king successfully argues (correctly) that he can increase welfare relative to the social contract because he can eliminate conflict, but either he or his successor recognizes that he can capture that surplus. Once this occurs it is impossible for a social contract to successfully increase the aggregate pie.

## 7 Conclusions

This paper has examined whether the Demsetz hypothesis that property rights arise when the value of creating them rises holds in an anarchy equilibrium. We considered a game in which players may allocate their endowment across subsistence consumption, investment in productive activities, investment in conflict, and investment in the public good of property rights protection. The Demsetz parameter is the value of the marginal product of investment in production relative to the value of subsistence consumption. We evaluated how equilibrium behavior changes as the Demsetz parameter, the number of players, and the inherent security of property are varied.

Property rights are not be provided in the Nash equilibrium in a game in which players simultaneously choose to allocate their endowment among production, conflict, and property rights. This occurs because an investment in conflict raises the proportion of a player's own production that he appropriates and the proportion of other players' production that he expropriates. In contrast, while an investment in property rights raises the proportion of his own production appropriated it reduces the proportion of others' production he expropriates. Thus players prefer to invest in conflict rather than the public good of property rights protection. Because the proportion of the endowment invested in conflict is bounded from

[29]A social contract may also arise in the region $A_{SC} \leq A < A^*$, simply because citizens recognize that there is a surplus gain to themselves by redistributing some of the surplus of the despot among the citizens, but a rational despot who is able to return enough of the surplus to make his citizens indifferent between the social contract and the side-payments of the despot may successfully stay in power as autocracy welfare dominates the social contract in this region.

above, as the Demsetz parameter rises society becomes richer, all else equal. But larger societies are characterized by higher levels of conflict and hence lower levels of utility.

A king who taxes the endowments of his citizens to provide property rights and who keeps all of the surplus above the Nash equilibrium level of utility of his citizens is able to provide property rights in a conflict society so long as the Demsetz parameter large enough to pay for the provision of property rights. Indeed, such a king creates perfectly enforced property rights, driving conflict to zero. But when the value of the Demsetz parameter is large, a despotic king merely replaces the Nash equilibrium level of conflict with expropriating taxes. As the level of conflict is bounded, so is the level of taxation. Thus the amount of surplus a despotic king can create relative to the Nash equilibrium is limited.

In a social contract, players first simultaneously allocate part of their endowment to property rights protection and then allocate the remaining endowment between conflict and production. The resulting subgame-perfect Nash equilibrium creates perfectly protected property rights at the same cost as a despotic king only when the number of players is two. As the number of players rises, the minimum Demsetz parameter necessary to allow a social contract to arise is always higher than the minimum Demsetz parameter under which a despotic king can create property rights. Furthermore, property rights created in this fashion are imperfect, as players under-invest in property rights protection (relative to the social optimum) in an attempt to free-ride on the provision of property rights protection by others. Nevertheless, the social contract welfare dominates a despotic king when the Demsetz parameter is sufficiently high, as the surplus the despot creates is bounded in the Demsetz parameter while the surplus under a social contract is unbounded in the Demsetz parameter. However, the level of the Demsetz parameter that is sufficiently high to allow a social contract to welfare dominate a despotic king is increasing in the size of the population at an increasing rate. Thus a social contract is most likely to occur in small populations with high levels of potential productivity of investment. However, autocracy can reestablish itself if the size of the population rises, as this increases the incentive to free-ride on property rights protection in a social contract.

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# A Mathematical Appendix

## A.1 Proof of Proposition 4.1

*Proof.* In the symmetric Nash equilibrium, (9) implies $\partial p_{ii}/\partial Y = -(N-1)\partial p_{ij}/\partial Y$. Therefore, (6) can be written as

$$\frac{\partial u_i}{\partial y_i} = -p_{ii}A < 0, \qquad i = 1, \ldots, N.$$

Thus in the symmetric Nash equilibrium, each player sets $y_i = 0$. □

## A.2 Proof of Proposition 4.2

*Proof.* Suppose not. Suppose that players devote their entire endowment to production, then $x^{NE} = c^{NE} = 0$. However, $x^{NE} = c^{NE} = 0$ implies that (8′) can be written as

$$\frac{\partial u_i}{\partial x_i} = \frac{A}{\theta}\big[(N-1)\omega - \theta\big],$$

which is positive by Assumption 1. This contradicts $x^{NE} = 0$. □

## A.3 Proof of Proposition 5.1

*Proof.* Taking $y_1, y_2, \ldots, y_N$ as given, but holding $c_1 = c_2 = \cdots = c_N = 0$, we may write the utility of the $i^{th}$ player as

$$u_i = \frac{A}{\theta + Y + X}\Big[(\theta + Y + x_i)(\omega - x_i - y_i) + x_i \sum_{j \neq i}(\omega - x_j - y_j)\Big], \qquad i = 1, \ldots, N. \tag{A.1}$$

### A.3.1 The Conflict Stage

The first-order-necessary conditions for the choice of $x_1, x_2, \ldots, x_N$ satisfy

$$\frac{\partial u_i}{\partial x_i} = \frac{A}{(\theta + Y + X)^2}\Big[X_{-i}(\omega - x_i - y_i) + \sum_{j \neq i}(\theta + Y + X_{-i})(\omega - x_j - y_j) - (\theta + Y + x_i)(\theta + Y + X)\Big] = 0, \qquad i = 1, \ldots, N. \tag{A.2}$$

Solving the joint system of (A.2) for $x_i(Y, y_i)$ yields

$$x_i(Y, y_i) = \frac{(N-1)(N\omega - Y)^2 - N^2(\theta + Y)(\omega - y_i)}{N^2(N\omega - Y)}, \qquad i = 1, \ldots, N. \tag{A.3}$$

Therefore, substituting (A.3) into (1) and (2) yields

$$p_{ii}(Y, y_i) = \frac{(N-1)(N\omega - Y)^2 + N^2(Y + \theta)[(N-1)\omega - Y + y_i]}{N^2(N\omega - Y)^2}, \qquad i = 1, \ldots, N, \tag{A.4}$$

and

$$p_{ij}(Y, y_i) = \frac{(N-1)(N\omega - Y)^2 - N^2(Y + \theta)(\omega - y_i)}{N(N-1)(N\omega - Y)^2}, \qquad i = 1, \ldots, N. \tag{A.5}$$

The amount invested in corn production, $k_i(Y, y_i)$, is found from the resource constraint (5):

$$k_i(Y, y_i) = \frac{N^2(N\omega + \theta)(\omega - y_i) - (N-1)(N\omega - Y)^2}{N^2(N\omega - Y)}, \qquad i = 1, \ldots, N. \tag{A.6}$$

Summing of over the $j \neq i$ of the $k_j$ yields:

$$\sum_{j\neq i}^{N} k_j(Y, y_i) = \frac{[(N-1)\omega - Y + y_i](N\omega+\theta) - (N-1)^2(N\omega - Y)^2}{N^2(N\omega - Y)}, \qquad i = 1, \ldots, N. \tag{A.7}$$

Substituting (A.6),(A.4)-(A.7) into the utility function (A.1) yields, after some simplification, the value function in terms of $y_i$ and $Y$:

$$u_i(Y, y_i) = \frac{A}{N^2}\left[\frac{N^2\omega(\omega+\theta) + N(N-2)\omega Y + Y^2 - N^2 y_i(Y+\theta)}{N\omega - Y}\right], \qquad i = 1, \ldots, N. \tag{A.8}$$

### A.3.2 The Social Contract Stage

Given the utility functions (A.8), each player in the public goods provision stage chooses $y_i$, taking the $y_{-i}$ as given. Hence, the first order condition in the choice of $y_i$ is

$$\frac{\partial u_i}{\partial y_i} = \frac{A}{N^2(N\omega - Y)^2}\Big\{(N^2-1)Y^2 + NY[N\theta - (N-2)\omega] - N^2[y_i(N\omega+\theta) - (N-1)(\omega-\theta)\omega]\Big\} = 0, \qquad i = 1, \ldots, N. \tag{A.9}$$

Imposing symmetry on (A.9), so that $y_1 = y_2 = \cdots = y_N \equiv y$, yields

$$\frac{\partial u_i}{\partial y_i} = \frac{A(N-1)[\omega - \theta - (N+1)y]}{N^2(\omega - y)} = 0. \tag{A.10}$$

Solving this for $y$ yields the subgame perfect level of private provision to security, and substituting these results back into (A.3)-(A.8) yields the results in (15). □

## A.4 Proof of Proposition 6.2

*Proof.* Let us first show that this is true when $A \geq \bar{A}$. Relative to the HCNE in which there is no king, the gain to aggregate welfare is given by the expression on the first line in (25). By Assumption 1, this is positive for all $N \geq 2$.

Second, when $\hat{A} \leq A < \bar{A}$, the gain in welfare relative to the LSCNE utility of $u^{NE} = \omega$ when there is no king is given by the expression on the second line in (25). Evaluated at $A = \hat{A}$, this expression equals $\Delta S_B^* = -\frac{\omega(N-1)-\theta}{2N-1}$, which is negative by Assumption 1. Evaluated at $A = \bar{A} = \frac{N\omega}{\omega+\theta}$, this expression equals $\Delta S_B^* = \frac{\omega(N-1)[\omega(N-1)-\theta]}{(2N-1)(\omega+\theta)}$, which is positive by Assumption 1. Given that $\Delta S_B^*$ is increasing in $A$, and that $A_K$ solves $\Delta S_B^* = 0$, $\Delta S_B^* > 0$ for all $A > A_K$.

Next, we show that $\hat{A} < A_K < \bar{A}$. By Assumption 1, $(N-1)\omega > \theta$. Thus $(2N-1)\omega > N\omega + \theta$, so that $A_K > 1 \equiv \hat{A}$ To show that $A_K < \bar{A}$, note that by Assumption 1,

$$\begin{aligned}
(N-1)\omega &> \theta \\
(N-1)^2\omega &> (N-1)\theta \\
(N^2 - 2N + 1)\omega &> (N-1)\theta \\
N^2\omega + N\theta &> (\omega+\theta)(2N-1) \\
\bar{A} \equiv \frac{N\omega}{\omega+\theta} &> \frac{\omega(2N-1)}{N\omega+\theta} \equiv A_K.
\end{aligned}$$

This completes the proof. □

## A.5 Proof of Proposition 6.3

*Proof.* We saw above that a king (benevolent or despotic) is able to create property rights only if $A \geq A_K = \frac{\omega(2N-1)}{N\omega+\theta}$. For all $N \geq 2$,

$$\begin{aligned}
0 &\leq (N-2)(N-1) \\
0 &\leq N^2 - 3N + 2 \\
2(2N-1) &\leq N(N+1) \\
A_K = \frac{(2N-1)\omega}{N\omega+\theta} &\leq \frac{N(N+1)\omega}{2(N\omega+\theta)} = A_{SC}.
\end{aligned}$$

When $N = 2$, the minimum value of the Demsetz parameter under which a social contract can exist is the same as when a king can exist. However, for $N > 2$, the inequalities hold strictly. □

## A.6 Proof of Proposition 6.4

*Proof.* Since a social contract cannot arise for values of $A_K \leq A < A_{SC}$, we restrict our attention to the case where $A > A_{SC}$. From (23), the after-tax utility each citizen earns under a benevolent king is $u_B^* = \frac{A(N\omega+\theta)}{2N-1}$. From (15), the utility each citizen earns under a social contract is $u^{SC} = \frac{A(N\omega+\theta)}{N(N+1)/2}$. The difference in equilibrium utilities is

$$u_B^* - u^{SC} = \frac{A(N-2)(N-1)(N\omega-\theta)}{2N^2+N-1} \geq 0 \text{ for all } A \geq A_{SC}.$$

This difference is zero when $N = 2$, but is strictly positive for all $N > 2$. □

## A.7 Proof of Proposition 6.5

*Proof.* When $A = A_{SC}$, the welfare gain to the social contract relative to the Nash equilibrium is zero by (16). However, under a despotic king the welfare gain relative to the Nash equilibrium is

$$R_D^*(A_{SC}) = \frac{(N-2)(N-1)(N\omega+\theta)}{N(N+1)}.$$

This is positive for all $N > 2$ and equal to zero for $N = 2$. For $A \geq \bar{A}$, the surplus gain relative to the Nash equilibrium to the social contract is linearly increasing in $A$ by (16). However, the surplus gain relative to the Nash equilibrium to the despot is given by (28), which is positive, but independent of $A$. Thus, for $A$ sufficiently large, $\Delta S_{SC}(A) > R_D^*(A)$. □