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ABSTRACT

Understanding the allocation of labor between collective and private activities within cooperatives has been an issue of interest for economists and policy makers. This paper extends existing literature by incorporating income uncertainty from both private and collective activities, and by assuming that members are risk averse. The analysis suggests a member's labor response to policy parameters can be decomposed into three components: the mean effect, reflecting the labor response under certainty or risk neutrality; the variance effect, reflecting the response to changes in risk; and the wealth effect, reflecting the response to changes in risk aversion associated with changes in wealth. The analysis demonstrates the labor response may be reversed from the certainty or risk neutral case, due to a stronger, opposing variance effect.

COOPERATIVE LABOR ALLOCATION UNDER UNCERTAINTY

I. INTRODUCTION

Over the past 40 years, governments and international organizations have made significant efforts to organize and support production cooperatives, mainly in agriculture. As Dorner (1977) suggested, this organizational form promotes both efficiency and equity. Nevertheless, as Laidlaw (1977) points out, the performance of agricultural production cooperatives has been disappointing.

The term production cooperative applies to a wide variety of organizations from fully collectivized farms where members produce all commodities jointly to groups of independent farmers that combine purchase of inputs and sale of output. Most agricultural production cooperatives, however, have both private and collective plots. A key problem that hampers the performance of such cooperatives is insufficient contribution of work to the collective plot.

A potential aid to increasing member's labor allocation and, thereby, production would be an examination of factors affecting a member's allocation of labor. Rather than analyzing incentives provided by government policy instruments, this paper will focus on parameters cooperative members can alter. The purpose is to determine the work incentives internal to the cooperative which can increase participation.

This research builds on earlier models of Sen (1966), Bonin (1977), Chinn (1980), Israelson (1980), and Putterman (1980) with respect to a member's choice of labor allocation between collective and private production. In the tradition of these papers, member interaction and income distribution rules are included as factors affecting labor allocation. Past studies, however, have either ignored or limited the effects of uncertainty on labor allocation. The model developed in this paper provides a comprehensive characterization of effects of uncertainty on labor allocation. By modeling uncertainty with respect to returns, the source of uncertainty in either the private or collective sector can be price or yield. The model specification also allows private and cooperative returns to be correlated. The analysis will indicate how uncertainty combined with risk alters the certainty case results reported by previous authors.

Early analysis of cooperative labor supply assumed cooperatives were centralized decision-making firms with the objective of maximizing net returns to members (Ward, 1958; Domar, 1966; and Oi and Clayton, 1968). Sen (1966) was the first to establish members rather than managers as the labor allocation decision-makers by modeling cooperatives as a group of utility-maximizing individuals.

The literature has focused generally on the effects of output price, fixed charges, and production quotas on labor allocation. However, income distribution systems, member interaction, and uncertainty have also been identified as affecting labor allocation. Israelson (1980) demonstrates that cooperatives with income distributed according to labor shares provide more incentives to cooperative labor than do cooperatives based on equal income distribution. Chinn (1979) demonstrates that members' interdependence affects the labor allocation responses when members are not identical, and both Chinn (1980) and Putterman (1980 and 1981b) have addressed the game theoretic aspects of member interaction. Bonin (1977) in his model incorporates behavioral interdependence and production uncertainty on the collective plot and price uncertainty on the private plot. He demonstrates that the labor allocation response to government policies under uncertainty differs from the response in a certain world. Bonin's (1977) work is expanded here by not limiting the source of uncertainty in each mode of production and by allowing members to vary in their behavioral assumptions about other members' labor response.

The approach of this research is to treat labor allocation as a portfolio selection problem. Members must allocate their labor between production processes with correlated returns and differential uncertainties. Private and collective incomes are assumed to be correlated because both forms of production face the same weather and market conditions. The level of uncertainty will vary for private and cooperative income, however, due to differences in product and technique.

Changes in four variables used to increase cooperative labor supply are analyzed in this paper. These four variables are: cooperative income variability, private and cooperative income correlation, behavioral interdependence, and income distribution rules. The next section describes the model in which these four parameters operate.

II. COOPERATIVE LABOR SUPPLY MODEL

Consider a producer cooperative consisting of N members. Each member, i, allocates time between work on the collective plot, h_i^c, and work on his/her private plot, h_i^p. If the total work time is normalized to one, work allocation is specified as:

$$h_i^c + h_i^p = 1. \tag{1}$$

Member i's net income consists of two parts: net income from the private plot, y_i^p, and income from the cooperative activity, $y_i^c = A_i Y^c$. The total cooperative net income is Y^c; A_i is member i's share, which is assumed to be a linear combination of a portion based on member i's relative supply of cooperative labor and a portion based on equal shares. With H^c representing the total labor allocation to the cooperative and α representing the weight given to equal shares, the formula for member i's share of cooperative net income is:

$$A_i = (1 - \alpha) \frac{h_i^c}{H^c} + \alpha \frac{1}{N} \qquad 0 \leq \alpha \leq 1. \tag{2}$$

If $\alpha = 0$, a member's cooperative income is proportional to his/her labor contribution to the cooperative; if $\alpha = 1$, each member receives an equal share.

When member i determines labor input to the cooperative plot, his/her perception of other members' behavior is used in the decision. Therefore, the values of A_i and H^c used in member i's decision reflect ex anti perceptions. The sum of the members' labor allocation, H^c, can be decomposed into the contribution of member i, h_i^c, and the other members' cooperative labor supply, H^c_{-i}. Each member takes into account the effect of his/her labor supply on the other members' labor supply. Thus, for member i, the other members' cooperative labor supply, H^c_{-i}, is a function of member i's cooperative labor:

$$H^c = \sum_i h_i^c = h_i^c + H^c_{-i}(h_i^c). \tag{3}$$

Bonin (1977) incorporated member interaction into a labor elasticity measure,

$$\eta = \frac{\delta H^c}{\delta h_i^c} \frac{h_i^c}{H^c}.$$

To stress the subjective aspect of this measure, η will be called a member's cohesion conjecture. The value of the cohesion conjecture is assumed between h_i^c/H^c and 1. If a member acts as if his marginal behavior has no affect on other members or if no one follows, the cohesion conjecture takes the lower bound; if there exists perfect cohesion or total emulation, the conjecture equals 1. The cohesion conjecture, η, varies among members, is subjective, and is assumed positive.

Assuming income uncertainty from both private and cooperative production and using the Just and Pope (1978) formulation, private and cooperative net income are represented by:

$$\tilde{y}_i^p = y_i^p(h_i^p) + g_i^p(h_i^p)\ e^p \qquad i = 1, 2, \ldots, N \tag{4}$$

$$\tilde{Y}^c = Y^c[h_i^c + H_{-i}^c\ (h_i^c)] + g^c[h_i^c + H_{-i}^c\ (h_i^c)]\ e^c. \tag{5}$$

This income specification allows for differential mean and variance effects for input factors. The deterministic sections of private and cooperative net income are, respectively, y_i^p and Y^c; the stochastic portions are $g^p e^p$ and $g^c e^c$. The stochastic structure encompasses two elements. The first element reflects exogenous factors which are not affected by members actions such as weather or market conditions. This stochastic element of income is represented by the pure random terms e^p and e^c. These pure random terms have zero mean and covariance of $\sigma_{pc} = \rho\sigma_p \sigma_c$ where ρ is the correlation coefficient between cooperative and private incomes; and σ_c^2 and σ_p^2 are cooperative and private income variability. The second stochastic element of income reflects the affect of the member's labor allocation on income variability and is represented by the g_i^p and g^c functions. For the effect of the labor inputs on income variability, we assume without loss of generality that $g_i^p \geq 0$ and $g^c \geq 0$.[1] With this specification, the marginal effect of labor on income variability, $g^{c\prime}$ and $g^{p\prime}$, may increase, decrease, or be constant with respect to their argument.

Let

$$\tilde{y}_i = y_i^p + g_i^p e_i^p + A_i(Y^c + g^c e^c) \tag{6}$$

denote the income of member i, $\tilde{y}_i$ is a random variable and, since the time constraint is assume to be binding ($h_i^p = 1 - h_i^c$), $\tilde{y}_i$'s distribution is a function of h_i^c.

Let a member's wealth at the end of the season be denoted by

$$\tilde{w}_i = w_i^0 + \tilde{y}_i$$

where w_i^0 is the initial wealth of member i. Now assume that member i is risk averse with utility function $U(\cdot)$ defined on wealth; $U' > 0$, $U'' < 0$. Thus, member i selects an expected utility-maximizing, time-allocation scheme which is derived solving

$$\max_{h_i^c} E\{U(w_i^0 + \tilde{y}_i)\} \tag{7}$$

subject to

$$0 \leq h_i^c \leq 1.$$

Assuming an internal solution, the first-order condition is:[2]

$$\frac{\delta EU}{\delta h^c} = E\left(U_w \frac{\delta \tilde{y}}{\delta h^c}\right) = 0 \tag{8}$$

where U_w is the marginal utility of wealth. The associated second-order condition holds under concavity of the utility and income functions in w and h^c respectively.

To examine the implications of (8), consider a first order Taylor's series approximation of marginal utility about expected wealth (Newbery and Stiglitz, 1979; Just and Zilberman, 1983),

$$U_w = U_w + U_{ww}(\tilde{w} - \bar{w}) \tag{9}$$

where $\bar{w} = w^0 + \bar{y}$ is expected wealth while $\bar{y} = y^p + AY^c$ is expected profit. The $\bar{U}_w$ and $\bar{U}_{ww}$ are the first and second derivitives of U at $\bar{w}$. Since $\tilde{w} - \bar{w} = \tilde{y} - \bar{y}$, condition (9) can be rewritten:

$$U_w = U_w + U_{ww}(\tilde{y} - \bar{y}). \tag{10}$$

Introducing (10) into (8) yields an approximation of the first-order condition:

$$\frac{1}{\overline{U}_w}\frac{\delta EU}{\delta h^c} = \frac{\delta \overline{y}}{\delta h^c} - \frac{\overline{R}}{2}\frac{\delta V}{\delta h^c} = 0$$

or
$$\delta \overline{y}/\delta h^c = \frac{1}{2}\overline{R}\ \delta V/\delta h^c \tag{11}$$

where $\overline{R} = -\overline{U}_{ww}/\overline{U}_w$ is the Arrow-Pratt measure of absolute risk aversion at mean wealth, and $V = E(g^p e^p + Ag^c e^c)^2$ is the variance of profit (and wealth). This approximated first-order condition suggests the optimal of labor allocated for cooperative activities occurs when the marginal mean effect, $\delta\overline{y}/\delta h^c$, is equal to the marginal risk effect, $\frac{1}{2}\overline{R}\ \delta V/\delta h^c$. The marginal risk effect consists of the marginal effect of cooperative labor on the variance of income, $\delta V/\delta h^c$, weighted by the measure of absolute risk aversion.

Using $\overline{y} = y^p + AY^c$, the marginal impact of h^c on mean income is derived to be

$$\frac{\delta \overline{y}}{\delta h^c} = (1 - \eta)(1 - \alpha)\frac{Y^c}{H^c} + \eta Y^{c\prime}\left[1 + \alpha\left(\frac{\hat{h}^c}{h^c} - 1\right)\right] - y^{p\prime} \tag{12}$$

where $\hat{h}^c$ represents the average cooperative labor supply as perceived by members since $\hat{h}^c = H^c/N$. The value of $\hat{h}^c/h^c_i$ categorizes members into types of workers. Member i is an individualist, average worker, or cooperativist depending on whether $\hat{h}^c/h^c_i$ is greater than, equal to, or less than one, respectively.[3]

Using $V = g^{p^2}\ \sigma_p^2 + A^2 g^{c^2}\ \sigma_c^2 + 2\ A\rho\sigma_p\sigma_c g^p g^c$, the marginal impact of h^c on the variance of income is:

$$\frac{\delta V}{\delta h^c} = (g^p \sigma_{pc} + Ag^c \sigma_c^2) \left\{(1 - \alpha)(1 - \eta) \frac{g^c}{H^c} + \left[1 + \alpha\left(\frac{\hat{h}^c}{h^c} - 1\right)\right] \eta g^{c'}\right\}$$

$$- g^{p'} (g^p \sigma_p^2 + Ag^c \sigma_{pc}). \qquad (13)$$

Under certainty, at the optimum, the marginal mean income with respect to h^c is equal to zero ($\frac{\delta \bar{y}}{\delta h^c} = 0$). The introduction of uncertainty results in an optimal cooperative labor allocation that equates marginal mean effect to marginal variance effect. Assuming $\bar{y}$ is concave in h^c, the introduction of uncertainty increases (decreases) labor allocation to the cooperative activity when, at the optimum, the marginal risk effect is negative (positive). If the optimal h^c is greater with uncertainty incorporated in the model than without uncertainty, then the cooperative activity serves as the safer activity. The risk averse decision maker gives up expected profit to reduce risk by increasing h^c beyond the point where $\delta\bar{y}/\delta h^c = 0$. For the same reason when, at the optimum, $\frac{\delta \bar{y}}{\delta h^c} > 0$, the collective activity is perceived as the riskier activity.

The introduction of uncertainty into the model also extends the set of parameters influencing member's decisions. The additional parameters are the behavioral parameters of the risk aversion coefficient and its derivatives with respect to wealth, and the technical parameters of the covariances of private and cooperative incomes and their risk response functions g^p and g^c.

The impact of parameter x (x may be σ_c^2, ρ, η, α) on cooperative labor supply is obtained by differentiating (11) with respect to h^c and x. Such differentiation will yield the following generic expressions for cooperative analysis:

$$\frac{dh^c}{dx} = \frac{1}{D}\left(\frac{\delta^2\bar{y}}{\delta h^c \delta x} - \frac{\bar{R}}{2}\frac{\delta^2 V}{\delta h^c \delta x} - \frac{1}{2}\frac{\delta V}{\delta h^c}\frac{\delta \bar{R}}{\delta x}\right) \tag{14}$$

where

$$D = -\frac{\delta}{\delta h^c}\left(\frac{\delta \bar{Y}}{\delta h^c} - \frac{\bar{R}}{2}\frac{\delta V}{\delta h^c}\right)$$

is assumed to be positive since -D is an approximated second-order condition derived from (11). Let $\gamma = -\delta\bar{R}/\delta\bar{w} \cdot \bar{w}/\bar{R}$ be the elasticity of absolute risk aversion with respect to wealth. Note that γ is equal to 0 when the member has constant absolute risk aversion and is equal to 1 when the member has constant relative risk aversion. Following Arrow and assuming decreasing absolute and increasing relative risk aversion, it is reasonable to assume $0 < \gamma < 1$. Using this definition and (11), the generic comparative static results can be rewritten as:

$$\frac{dh^c}{dx} = \frac{1}{D}\left(\frac{\delta^2\bar{y}}{\delta h^c \delta x} - \frac{\bar{R}}{2}\frac{\delta^2 V}{\delta h^c \delta x} - \frac{\gamma}{\bar{w}}\frac{\delta \bar{y}}{\delta x}\frac{\delta \bar{y}}{\delta h^c}\right). \tag{15}$$

A change in parameter x may have three effects on cooperative labor supply. First is the mean effect resulting from the impact of x on the marginal effect of cooperative labor on expected income. The second effect is the impact of x on the marginal effect of cooperative labor on the variance of income. This variance effect increases with the level of absolute risk aversion at average wealth. The third effect is the wealth effect on risk aversion resulting from the impact of a change in expected wealth associated with the change in x. This wealth effect represents a reduction in risk aversion associated with increased average wealth. This effect strengthens the mean effect relative to the variance effect. The wealth effect does not exist when absolute risk aversion is constant ($\gamma = 0$).

In the following section, changes in model parameters are analyzed. Each of the four parameter changes seems to be a plausible incentive for cooperative labor. For example, a cooperative may attempt to increase the supply of cooperative labor by increasing the proportion of cooperative income distributed based on relative labor contribution. Alternatively, a cooperative may attempt to increase the amount of cooperative labor by choosing to produce products which either have low-income variability or less-income correlation with the privately produced products. The fourth parameter option would be to foster cohesiveness among members in the belief that greater cooperative participation would follow. However, the comparative static analysis of a change in each of these parameters demonstrates cooperative labor supply may not increase.

III. COMPARATIVE STATIC RESULTS OF POLICY OPTIONS

This section outlines the impacts on cooperative labor supply of changes in the following four variables: cohesion conjecture; income distribution rule; income correlation; and cooperative income variability. The comparative static equations are found in the Appendix.

A. The Impact of an Increase in the Cohesion Conjecture, η

An increase in the responsiveness of other members' cooperative labor supply has both an income effect and a relative labor share effect. The income gain due to increased production is offset by a decline in a member's relative labor share. Thus, the labor allocation response to an increase in the cohesion conjecture is strongly related to the income distribution rule.

1. Mean Effect

The following results hold for an increase in the cohesion conjecture if the mean income effect dominates or if a member is risk neutral. These certainty case results match the findings of Putterman (1980).

a. An increase in the cohesion conjecture increases cooperative labor if cooperative income is divided equally, $\alpha = 1$.

With equal sharing, everyone benefits from the increasing production because the decline in relative shares does not affect the distribution of the increased income.

b. An increase in the cohesion conjecture decreases cooperative labor if cooperative income is distributed only according to relative labor shares, $\alpha = 0$.

The decline in a member's relative labor share caused by the increase in the cohesion conjecture outweighs the potential increase in income if the cooperative is operating in the efficient zone where average product is greater than marginal product.

c. For values of α between 0 and 1, the cooperative labor response to an increase in the cohesion conjecture depends on worker type.

Individualists are more likely than cooperativists to increase cooperative labor for an increase in the cohesion conjecture. Individualists free ride if any portion of the net cooperative income is divided equally because they contribute less than average. Thus, the larger the value of α, the greater the benefits to individualists for increases in the cohesion conjecture.

2. Variance Effect

The certainty case results, however, can be overpowered by the variance and wealth effects of an increase in the cohesion conjecture. If the variance effect dominates, the following results hold for an increase in the cohesion conjecture.

a. An increase in the cohesion conjecture increases cooperative labor if cooperative labor is risk reducing at the margin, $g^{c\prime} \leq 0$.

When the variability effect dominates, the possible loss in relative labor shares is overshadowed by the desire to reduce variability. A member will always increase cooperative labor supply if more members will follow since the increased participation reduces variability.

b. An increase in the cohesion conjecture decreases cooperative labor if marginal cooperative labor increases risk, $g^{c\prime} > 0$, and $g^{c\prime} > (1 - \alpha)/[1 + \alpha(\hat{h}^c/h^c) - 1]\ g^c/H^c$.

Because a member's relative labor share is reduced by an increase in the cohesion conjecture, a member can spread risk by increasing cooperative labor. However, for values of $g^{c\prime}$ above the specified threshold, the risk spreading potential is overwhelmed by the increase in risk. Note, if income is divided equally, risk cannot be spread so members will always decrease cooperative labor.

3. Wealth Effect

The third effect of a change in the cohesion conjecture is through the impact on wealth. This effect consists of (1) the elasticity of absolute risk aversion divided by expected wealth, $\gamma/\bar{w}$; (2) the effect on wealth due to an increase in the cohesion conjecture, $\delta\bar{y}/\delta\eta$; and (3) the effect on net income of an increase in cooperative labor, $\delta\bar{y}/\delta h^c$. We have assumed the elasticity of absolute risk aversion is between 0 and 1, and the magnitude and direction of the other two elements varies among members. An increase in the cohesion conjecture can either increase or decrease a member's wealth, depending on how income is divided and what type of worker the member is, and an increase in cooperative labor can either increase or decrease a member's income depending on which activity is more profitable at the margin. The following results indicate the wealth effect impacts on cooperative labor supply when an increase in the cohesion conjecture increases wealth.

a. An increase in the cohesion conjecture increases cooperative labor if cooperative labor is the more risky activity.

b. An increase in the cohesion conjecture decreases cooperative labor if cooperative labor is the less risky activity.

Risk is reduced when an increase in the cohesion conjecture increases wealth, $\delta\bar{y}/\delta\eta > 0$. A member's labor response will depend on which activity is riskier at the margin. If $\delta\bar{y}/\delta h^c > 0$, a member will increase cooperative labor; if $\delta\bar{y}/\delta h^c < 0$, a member will reduce cooperative labor.

In summary, a member's cooperative labor response to an increase in the cohesion conjecture is determined by the sum of the mean, variance, and wealth effects. The direction of the response is indeterminate without knowing the relative magnitudes of the three effects. Risk aversion considerations can override the certainty case responses indicated by the mean effect.

B. The Impact of an Increase in the Proportion of Cooperative Income Distributed According to Work Supplied, $(1 - \alpha)$

The labor response depends strongly on worker type. Members whose supply to the cooperative is less than average generally do not respond to greater weight being placed on relative shares. In addition, the increase in direct rewards for increased cooperative labor does not always offset the possible increased risk due to increased labor:

1. Mean Effect

If the mean effect dominates or if a member is risk neutral, the following certainty case result holds.

a. An increase in the weight given to relative participation increases cooperative labor allocation among nonindividualist members.

An increase in $(1 - \alpha)$ increases the marginal income of members contributing at least the average thereby enhancing their willingness to participate in the collective activity.

2. Variance Effect

If the variance effect dominates, the following results hold.

a. Cooperativists will reduce cooperative labor if cooperative labor is risk increasing at the margin and private labor is risk reducing at the margin.

The potential income benefits due to increased weight given to cooperative labor participation are overridden by risk considerations. It is possible individualists may increase cooperative labor in this case because they do not carry their share of the increased risk burden for increases in cooperative labor.

b. Average workers will decrease cooperative labor when their cooperative labor reduces risk.

3. Wealth Effect

The impact of the third effect or wealth effect on cooperative labor depends on worker type and which activity is riskier and more profitable at the margin. (Recall cooperative labor is the risky, more profitable activity if, at the optimal labor allocation, $\delta\bar{y}/\delta h^c > 0$.) An increase in rewards based on relative participation decreases wealth for an individualist and increases wealth for a cooperativist. Therefore, the wealth effect will encourage individualists to increase the insurance activity and for cooperativists to increase the risky activity.

In summary, a members' cooperative labor response to an increase in $(1 - \alpha)$ is generally indeterminate. Without information on parameter values, only risk-neutral members contributing at least the average can be identified as increasing cooperative labor for increases in payment based on relative share.

The impact of a change in income correlation and cooperative income variability refer to changes in the variance and correlation of e^p and e^c, the pure errors of private and cooperative income. A change in these parameters only affects the labor allocation decision through the variance effect. There is no mean and wealth effect in the comparative static equation.

C. The Impact of a Decrease in Income Correlation, ρ

A decrease in income correlation reduces risk. A member will increase labor to the risky, more profitable activity when risk is reduced.

1. A decrease in income correlation decreases cooperative labor if the income variability elasticities, $\varphi^c = \delta g^c/\delta H^c \cdot H^c/g$ and $\varphi^p = \delta\varphi^p/\delta H^c \cdot h^p/g^p$, are equal and risk increasing.

An hour contributed to private production increases risk more than an hour contributed to cooperative production because the increased risk in the cooperative activity is shared by other members.

2. A decrease in income correlation increases cooperative labor if the income variability elasticities are equal and risk decreasing.

An increase in cooperative labor reduces risk less than an equivalent increase in private production because in the cooperative activity the decrease in risk is diluted through sharing.

3. A decrease in income correlation increases the labor activity which increases variability if the income variability elasticities have the opposite effect.
4. When both labor activities have the same effect on risk but are unequal, the labor allocation response depends on the relative magnitudes of the income variability elasticities.

If both labor activities increase risk, members will reduce cooperative labor for decreases in correlation as long as marginal cooperative labor is relatively less risk increasing. This condition is represented by:

$$\varphi^c < \frac{1}{\eta\ [1 + \alpha\ (\hat{h}^c/h^c) - 1]} \left[\varphi^p \frac{H^c}{1 - h^c} - (1 - \alpha)\ (1 - \eta)\right] .$$

Similarly, if both labor activities decrease risk, members will increase cooperative labor for decreases in correlation as long as private labor has the relative advantage in reducing risk. This condition is represented by:

$$|\varphi^c| < \frac{1}{\eta\ [1 + \alpha\ (\hat{h}^c/h^c) - 1]} \left[|\varphi^p| \frac{H^c}{1 - h^c} - (1 - \alpha)\ (1 - \eta)\right] .$$

D. The Impact of a Decrease in Cooperative Income Variability, σ_c^2

With positive income correlation, a decrease in the cooperative variability decreases the riskiness of both private and cooperative activity. Therefore, the labor allocation response to a decrease in cooperative income variability will be to increase labor in the riskier activity. (We assume the correlation of private and cooperative agricultural income is nonnegative.)

1. If cooperative and private labor have the opposite effect on variability at the margin, a member will increase labor in the activity which increases variability.
2. If both labor activities decrease risk, members will increase cooperative labor as long as private labor has the relative advantage in reducing risk.

This condition is represented by:

$$|\varphi^c| < \frac{1}{\eta\ [1 + \alpha(\hat{h}^c/h^c) - 1]} \left[(1 - \alpha)\ (1 - \eta) + \frac{g^p\ \sigma_p\ \rho H^c |\varphi^p|}{(g^p\ \sigma_p\ \rho + 2Ag^c\ \sigma_c)\ (1 - h^c)} \right].$$

3. If both labor activities increase risk, members will decrease cooperative labor as long as cooperative labor is relatively less risk increasing than private labor.

This condition is represented by:

$$\varphi^c < \frac{1}{\eta\ [1 + \alpha(\hat{h}^c/h^c) - 1]} \left[\frac{g^p\ \sigma_p\ \rho H^c\ \varphi^p}{(g^p\ \sigma_p\ \rho + 2Ag^c\ \sigma_c)\ (1 - h^c)}\ (1 - \alpha)\ (1 - \eta) \right].$$

The impact on cooperative labor allocation for decreases in income correlation and cooperative income variability indicate that policies aimed at reducing cooperative income uncertainty do not guarantee risk averse members will increase cooperative participation. Moreover, the results identify the conditions under which members will decrease cooperative participation for decreases in cooperative income uncertainty.

IV. CONCLUSION

The paper incorporated uncertainty and risk aversion into the decision process of a cooperative member allocated labor between private and collective activities. It is found that responses to policy changes can be decomposed into three components: mean effect, the response under risk neutrality (or certainty); variance effect, responses resulting from policy impacts on the variance of income; and wealth effect, the response resulting from changes in risk aversion associates with changes in wealth.

The incorporation of uncertainty and risk aversion extends the set of parameters affecting labor allocation decisions to include variance-covariance parameters of private and collective incomes. This enables us to analyze effects on cooperative labor supply of policies aimed at reducing risk. Moreover, it is demonstrated that results obtained under certainty may be reversed due to dominating variance effects.

Although the theoretical framework presented here expands the existing literature, two further extensions are especially pertinent. In the model presented here, total labor time is assumed constant which ignores the labor-leisure choice. Future analysis should allow variability among members' labor time to reflect taste and endowment differences. If leisure is included in the present general model, none of the results are signable. To obtain unambiguous results, a specific functional form will have to be assumed.

Issues of existence and stability of equilibrium positions within cooperatives need to be analyzed. General equilibrium conditions are especially important when member interaction is included. As the model presented here is a partial equilibrium model, it does not explicitly consider equilibrating forces within the cooperative. Here the behavior of the individual members is depicted under a given assumption regarding other members' responses. Future research should identify conditions under which equilibrium can be maintained and the conditions under which utility-maximizing members will have positive cohesion conjecture.

APPENDIX

1. The directional effect of a change in η on cooperative labor supply is obtained from the following comparative static equation:[6]

$$\frac{dh^c}{d\eta} \overset{s}{=} -(1-\alpha)\frac{Y^c}{H^c} + \left[1+\alpha\left(\frac{\hat{h}^c}{h^c}-1\right)\right] Y^{c\prime}$$

$$-\frac{1}{2}R(g^p\sigma_{pc} + Ag^c\sigma_c^2)\left\{\left[1+\alpha\left(\frac{\hat{h}^c}{h^c}-1\right)\right]g^{c\prime} - (1-\alpha)\frac{g^c}{h^c}\right\}$$

$$+\frac{\gamma}{w}\left\{(1-\eta)(1-\alpha)\frac{Y^c}{H^c} + \eta Y^{c\prime}\left[1+\alpha\left(\frac{\hat{h}^c}{h^c}-1\right)\right] - y^{\rho\prime}\right\}$$

$$\left(\frac{\delta H^c}{\delta\eta}\left\{\left[(1-\alpha)\frac{h^c}{H^c}+\frac{\alpha}{N}\right]Y^{c\prime} - (1-\alpha)\frac{h^c}{H^{c2}}Y^c\right\}\right).$$

If the mean income effect dominates the variability effect, the direction of the cooperative labor response to an increase in η is strongly influenced by the rules that govern the distribution of cooperative income. With equal income distribution, $\alpha = 1$, the positive marginal income term dominates, and members increase their cooperative participation (result A.1.a.). When α equals 0, the negative average income term dominates since the cooperative is assumed to be operating in the efficient production zone where $Y^c/H^c > Y^{c\prime}$ (result A.1.b.). Each member has a critical value for α above which he/she increases cooperative labor for increases in η. This critical value is:[4]

$$\alpha = \frac{(Y^c/H^c + Y^{c'})}{(Y^c/H^c) + Y^{c'} - Y^{c'}(\hat{h}^c/h^c)}.$$

The critical α value is greater for cooperativists ($\hat{h}^c/h^c < 1$) than for individualists ($\hat{h}^c/h^c > 1$). Thus, individualists are more likely to increase cooperative labor for increases in the cohesion conjecture for any given value of $\alpha < 1$ (result A.1.c.).

If the variability effect dominates the mean income effect, the direction of the cooperative labor response to an increase in η is strongly influenced by the marginal effect to labor on income variability, $g^{c'}$. If $g^{c'}$ is negative, the variability effect is positive, and members will increase cooperative labor (result A.2.a.). If $g^{c'}$ is positive and $\alpha < 1$, the cooperative labor response depends on the relative size of the marginal and average effect of labor on variability (result A.2.b.). If $\alpha = 1$ and $g^{c'}$ is positive, members will decrease labor.

2. The directional effect of a change in $(1 - \alpha)$ on cooperative labor supply is obtained from the following comparative static equation:

$$\frac{dh^c}{d\alpha} \underline{\underline{s}} -(1 - \eta)\frac{Y^c}{H^c} + \left(\frac{\hat{h}^c}{h^c} - 1\right)\eta Y^{c'}$$

$$- \frac{1}{2}\bar{R}\left((g^p \sigma_{pc} + Ag^c \sigma^2_c)\left[-\frac{g^c}{H^c}(1 - \eta) + \left(\frac{\hat{h}^c}{h^c} - 1\right)\right]\eta g^{c'}\right\}\Bigg)$$

$$+ g^c \sigma_{pc}\left(\frac{1}{N} - \frac{h^c}{H^c}\right)\left\{(1 - \alpha)(1 - \eta)\frac{g^c}{H^c} + \left[1 + \alpha\left(\frac{\hat{h}^c}{h^c} - 1\right)\right]\eta g^{c'} - g^{p'}\right\}\Bigg)$$

$$+\frac{\gamma}{\bar{W}}\left\{(1-\alpha)(1-\eta)\frac{Y^c}{H^c}+\eta Y^{c\prime}\left[1+\alpha\left(\frac{\hat{h}^c}{h^c}-1\right)\right]-y^{\rho\prime}\right\}\left(\frac{1}{N}-\frac{h^c}{H^c}\right)Y^c.$$

If the mean effect dominates, cooperativists and average workers will increase cooperative labor because the mean effect is negative for an increase α, or positive for an increase in $(1-\alpha)$ (result B.1.a.). For individualists, the coefficient on the marginal income term is negative, and, therefore, their labor response is indeterminate.

For the variance effects, the term premultiplied by $-\frac{1}{2}\bar{R}$ is considered. If $g^{c\prime}$ is positive and $g^{p\prime}$ is negative, the variance effect of an increase in α is positive for cooperativists because $(\hat{h}^c/h^c) - 1$ and $(1/N - h^c/H^c)$ are negative. Therefore, for <u>decreases</u> in α, cooperativists will decrease cooperative labor (result B.2.a.). For average members, the variance effect is always negative for a decrease in α (result B.2.b.).

3. The directional effect of a change in correlation on cooperative labor is obtained from the following comparative static equation:

$$\frac{dh^c}{d\rho} \underset{=}{\leq} -\frac{1}{2}\bar{R}g^p\, g^c\, \sigma_c\, \sigma_p\, \frac{1}{H^c}\left\{(1-\alpha)(1-\eta)+\left[1+\alpha\left(\frac{\hat{h}^c}{h^c}-1\right)\right]\eta\varphi^c-\frac{H_c}{1-h^c}\varphi^p\right\}.$$

The coefficient on φ^p is $\geq H^c$ and is much larger than the coefficient of φ^c. Therefore, if $\varphi^c = \varphi^p$, the third term within the bracket dominates the impact of a change in correlation (result C.1 and C.2). If $\varphi^c \leq 0$ and $\varphi^p \geq 0$ or $\varphi^c > 0$ and $\varphi^p < 0$, the bracketed term is negative, and members will decrease cooperative labor when correlation is reduced (result C.3). Result C.4 is obtained by setting the bracketed term to 0 and solving for φ^c when φ^c, $\varphi^p > 0$ and φ^c, $\varphi^p < 0$.

4. The directional effect of a change in cooperative income variability on cooperative labor is obtained from the following comparative static equation:

$$\frac{dh^c}{d\sigma_c} \stackrel{s}{=} -\frac{1}{2}R\,\frac{g^c}{H^c}\left((g^p\,\sigma_p\,\rho + 2Ag^c\,\sigma_c)\left\{(1-\alpha)(1-\eta) + \left[1 + \alpha\left(\frac{\hat{h}^c}{h^c} - 1\right)\right]\eta\varphi^c\right\}\right.$$

$$\left. - \varphi^p\,g^p\,\sigma_p\,\rho\,\frac{H^c}{1-h^c}\right).$$

The results in Section D are obtained in a manner similar to the results in Section C.

FOOTNOTES

[1]Let $\tilde{e} = g^k e^k$ with $k = c, p$. Then, $E(\tilde{e}) = 0$ and $\mathrm{var}(\tilde{e}) = g^2\sigma^2$. The sign of g does not affect the mean or variance.

[2]The subscript i is dropped for convenience.

[3]Various names have been given to these types of workers. Chinn (1980) refers to them as lazy and industrious, and Putterman (1981a) refers to them as shirkers and zealots. As our labels indicate the members' cooperative labor supply relative to the average, we refrain from applying pejorative labels.

[4]We assume the cooperative is operating in the efficient zone where $Y^c/H^c > Y^{c\prime}$.

[5]As can be easily verified, equation (11) is a result of:

$$E\left\{\frac{\delta\tilde{y}}{\delta h^c}(\tilde{y} - \bar{y})\right\} = \frac{1}{2}\frac{\delta V}{\delta h^c}$$

where $\tilde{y} = y^p + g^p e^p + Ay^c + Ag^c e^c = \bar{y} + g^p e^p + Ag^c e^c$

$$V = g^{p^2}\sigma_p{}^2 + A^2 g^{c^2}\sigma_c{}^2 + 2g^p A g^c \sigma_{pc}.$$

[6]The symbol $\underline{\underline{s}}$ means equal in sign.

REFERENCES

Bonin, John, "Work Incentives and Uncertainty on a Collective Farm." J. Comparative Econ. 1, 1:77-97, March 1977.

Chinn, Dennis, "Team Cohesion and Collective-Labor Supply in Chinese Agriculture." J. Comparative Econ. 3, 4:375-394, Dec. 1979.

Chinn, Dennis, "Diligence and Laziness in Chinese Agriculture Production Teams." J. Develop. Econ. 7, 3:331-344, Sept. 1980.

Domar, Evsey D., "The Soviet Collective Farm as a Producer's Cooperative." Amer. Econ. Rev. 56, 3:734-757, Sept. 1966.

Dorner, Peter, and Kanal, Don, "Introduction: Some Economic and Administrative Issues in Group Farming." In Cooperative and Commune, pp. 3-11. Madison: The University of Wisconsin Press, 1977.

Israelson, Dwight L., "Collectives, Communes, and Incentives." J. Comparative Economics 4, 2:99-124, June 1980.

Just, R. E., and Pope, R. D., "Stochastic Specification of Production Functions and Economic Implications." J. Econometrics 7, 1:67-86, Feb. 1978.

Just. R. E., and Zilberman, David, "Stochastic Structure, Farm Size, and Technological Adoption in Developing Agriculture." Oxford Econ. Papers 35, 4:307-328, 1983.

Laidlaw, Alexander Fraser, Cooperatives and the Poor: A Review from Within the Cooperative Movement. Ottawa: International Cooperative Alliance and Canadian International Development Agency, 1977.

Newbery, D. M. G., and Stiglitz, J. E., "The Theory of Commodity Price Stabilization Rules: Welfare Impact and Supply Responses." Econ. J. 89, 356:799-817, Dec. 1979.

Oi, Walter, and Clayton, Elizabeth, "A Peasant's View of a Soviet Collective Farm." Amer. Econ. Rev. 58, 1:37-59, March 1968.

Putterman, Louis, "Voluntary Collectivization: A Model of Producer's Institutional Choice." J. Comparative Econ. 4, 2:125-157, June 1980.

Putterman, Louis, "Incentives and the Kibbutz: Toward an Economics of Communal Work Motivation." Brown University, Department of Economics, Providence, RI, Aug. 1981a.

Putterman, Louis, "Is a Democratic Collective Agriculture Possible? Theoretical Considerations and Evidence from Tanzania." J. Development Econ. 9, 2:375-403, Dec. 1981b.

Sen, Amarata K., "Labor Allocation in a Cooperative Enterprise." Rev. Econ. Stud. 33, 4:361-371, Oct. 1966.

Ward, Benjamin, "The Firm in Illyria: Market Syndicalism." Amer. Econ. Rev. 48, 4:566-589, Sept. 1958.