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On the Nonneutrality of Neutral Technological Change in Agriculture

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1. Introduction

The purpose of this paper is to show that under some general conditions of agricultural production, new technologies which are technically neutral to farm scale, are unlikely to be neutral in their impact on marginal farmers. A marginal farmer is one who covers all costs including land and labor opportunity costs but earns zero managerial rent. More specifically it will be shown that a technological change which causes the same proportionate increase in output across all farms irrespective of their size, is likely to either enhance or detract from the chances of survival of marginal farm firms. A corollary of this argument is that small-scale biased technologies are neither necessary nor sufficient to enhance the economic viability of marginal farms.

Occasionally social scientists concerned with the plight of small farmers, both in developed and developing countries, advocate that R and D funds for agriculture should be directed towards discovering technologies which are biased in favor of small scale farming operations. While this concern is obviously well intentioned, unless other factors are considered in the allocation of agricultural research funds, in particular the characteristics of the farm output markets, the resulting technological advances apparently favoring smaller farmers, may be detrimental to their chances of survival. On the other hand, given the appropriate market conditions, it is quite possible that large-scale biased technologies, if inexpensive and easy to develop and adopt, may be the most appropriate way to assist marginal farmers.

The model developed to support these propositions is presented below. It is a "sector equilibrium" model in the sense that it includes considerations of the equilibrium conditions in farm output and input markets. Most analyses of the impact of new technologies on farm firms have considered responses at the farm level with particular attention to potential changes in the shape of the long run average cost curve. Alternative and approximately equivalent[1/] analyses have studied "shifts" in production surfaces, as shown, for example, by changes in the sum of Cobb-Douglas production function input elasticities.

While the model developed below and the subsequent analysis appears somewhat mathematical, the general elements of the model, the results and the policy implications are fairly straight foreward. On the assumption that some readers may wish to skip the rigorous mathematical analysis, the paper will proceed as follows. The next section will contain a "rough" verbal description of the model and the most important results obtained will be outlined. An attempt will be made to provide an "intuitive feel" for these results. In Section 3, the model will be rigorously developed, and the mathematical proofs of specific statements are outlined. Some mathematical footnotes to these proofs are relegated to appendices. In the fifth and final section, the results of the analysis and their policy implications will be discussed.

2. A Description of the Model and the Intuition of the Results

The farm sector or sub-sector of interest is viewed as consisting of a large number one-man or one-family farm units. Each family supplies its labor and farm managerial talent to the farm firm inelastically. The two other farm inputs are land and purchased inputs. Irrespective of whether land is owned or rented, it must earn its opportunity cost when used within the farm firm otherwise it will be allocated to its best alternate use. Purchased inputs are supplied at a constant price from outside the farm sector. Farm management talent is assumed to have no opportunity cost outside the sector of interest. On the other hand, the opportunity cost of labor used on the farm is the going wage rate in the non-agricultural sectors of the economy. This wage is assumed to be the same for all farm labor units. The output of each farm is determined by the quantities of land and purchased inputs and the level of managerial talent. It is assumed that larger farm units are more difficult to manage and operate so that there is a direct correspondence between the scale of farm operations and the level of management ability. This "diseconomy of scale" may also be seen as a reflection of the inelastic family labor supply on each farm.

Viewing the population of the economy as a whole, it is assumed that there exists a spectrum or distribution of farm management talent within the population. Those with relatively low levels of farm management talent will find employment outside agriculture while those who have a high level of farming ability will be farmers and will earn positive profits (i.e. managerial rents) as farmers. A marginal farmer is defined as one who just breaks even; in other words after paying for the purchased inputs, the amount remaining out of gross farm receipts is just sufficient to cover the opportunity cost of the land he uses and his labor. If gross revenue falls then losses accrue so long as he remains in farming. Such farmers will tend to leave farming for better paying jobs elsewhere and they will release their land to alternative more profitable uses. On the other hand if gross revenue of a marginal farmer increases, then he will earn positive managerial rents (i.e. profits) and he will no longer be marginal. He may try to expand his farming operations but in doing so he is likely to find that land rents (in the Ricardian sense) are tending to increase as new farmers are attracted by the higher profits being earned in farming. At least some of these new entrants will become the new marginal farmers. The model incorporates an explicit market for land wherein the sector of interest must compete for land with other land users. This implies the assumption that the sector of interest is a significant user of land. The market for labor generates a constant alternative wage for farm labor. This implies that the farm sector in the model is a small user of labor relative to the total labor supply to the economy.

The aggregate output of the farm sector is sold in a market characterized by downward sloping demand. In other words, if output of the sector increases, the output price will fall.

These are the essential elements of the model. Without giving any specific form to the distribution of farming talent and only giving very general forms to the production technology, output demand and land supply conditions, it is possible to obtain some very definite and somewhat interesting results. Of particular interest in this case is the fate of the marginal farmer as a consequence of some change in the production technology. The model facilitates an investigation of the impact on the marginal farmer of some change in the production environment because the talent level of farm managers is explicitly incorporated into the model. Since a marginal manager is one who earns zero profits, the fate of the marginal farmer can be inferred simply by observing how the talent level corresponding to zero profits changes as a consequence of some primary change, such as an improvement in technology.

The simplest type of technological change to study is one which is neutral to scale; that is, a technology which increases the output of all farms by the same proportion. To maintain simplicity, it is assumed that the technological change is adopted by all farmers instantaneously and at zero cost.

Using the mathematical model below, it can be shown that the only case in which the impact on marginal farmers is neutral, is when the market demand for the output of the sector has unit elasticity. That is, marginal farmers will observe no change in their zero profit status when a neutral technological advance occurs only if the elasticity of market demand for their output is unitary. Moreover, if market demand for the sector's output is inelastic, then marginal farmers will begin making losses and so they will move out of the sector when an apparently scale neutral change in technology occurs. On the other hand, if market demand is elastic, the same scale neutral technological change will cause previously marginal farmers to move into a positive profit situation and new entrants will be attracted to the sector.

This result is intuitively appealing. Consider an industry which experiences a costless productivity increase such that the output of all firms, regardless of size, increases by the same proportion. For a given level of conventional inputs, the industry output will expand by that proportion. If the market demand for that output has unit elasticity, then the output price will fall just enough to maintain the gross revenues of all firms at existing levels. If output demand is inelastic, price will fall more than proportionally, gross revenues will fall, marginal farmers will be "squeezed out", and land rents will fall. Average farm size will increase. Conversely, if output demand is elastic, output price will fall less than proportionally, gross revenues will rise, new entrants will be attracted, and land rents will rise. Average farm size will fall.

There are some extensions to this argument which will be presented in the final section.

3. A Model of an Agricultural Sector or Sub-Sector

The approach used here is derived from the model of business firm size proposed by Lucas (1978). Consider a farm sector or sub-sector which is a relatively small part of the economy so that the non-agricultural wage rate and the price of purchased inputs can be treated parametrically. All farm firms are assumed to be single man (one family) units with all labor provided inelastically by the farm family to the farm firm. Off-farm (part-time) employment for farm labor is assumed to be zero. The sector has to compete for land with other sectors of the economy. Purchased inputs and land are assumed to be homogeneous.

Let the farm production technology, $F(a,b)$, be freely available to all, where a is the quantity of land, and b is the quantity of the purchased input. $F(\cdot)$ is C_2, increasing and strictly concave in both arguments. That is, $F_a > 0$, $F_b > 0$, $F_{aa} < 0$, $F_{bb} < 0$, $F_{ab} > 0$. While it does not appear necessary to the result, it is convenient to assume that $F(\cdot)$ exhibits constant returns to scale so that we can write: $F(a,b) = af(b/a)$. Also $F_a = f(r) - rf'(r)$ and $F_b = f'(r)$ where $r = b/a$. Moreover $F_{aa} = \frac{r^2 f''}{a} < 0$, $F_{bb} = \frac{f''}{a} < 0$, and $F_{ab} = \frac{-rf''}{a} > 0$, and so $f(\cdot)$ is C_2, increasing and strictly concave.

Let there be a managerial technology reflecting two elements: farm management talent and "a span of control" limitation. Let the farm management talent of an individual be x, drawn from a fixed distribution $T:R^+ \to [0,1]$. The "span of control" limitation reflects an assumption that large scale operations are more difficult to manage to the extent that an actual output increase is less than that which is technically feasible. Let the "span of control" limitation be given by $g(\cdot)$ such that $g:R^+ \to R^+$ is C_2, increasing, strictly concave and satisfies $g(0) = 0$. This means that if an attempt is made to double output by some appropriate increase in inputs, actual output will less than double. For instance, if $F(a,b)$ exhibits CRS then $g[F(\lambda a, \lambda b)] = g[\lambda \cdot F(a,b)] < \lambda \cdot g[F(a,b)]$. It is assumed that $g(\cdot)$ has the form: $g(y) = \alpha y^\beta$, where $\alpha > 0$ and $0 < \beta < 1$. Lucas shows that if the basic production function is linear homogenous and if Gibrat's law[2/] applies, then the assumed form of $g(\cdot)$ is necessary. Accordingly the assumed managerial technology has the form $x\alpha[F(a,b)]^\beta$ where x is the managerial talent level of the farmer,

$\alpha > 0$, $0 < \beta < 1$ and a and b are the levels of land and purchased inputs respectively. If $F(\cdot)$ is linear homogeneous the the output of manager can be written as: $x\alpha[af(b/a)]^{\beta}$. If z is the managerial talent level of the marginal manager (= the cut-off management level = COML) then the total output of the sector is given by: $N\int_{z}^{\infty} x\alpha[F(a,b)]^{\beta}\, dT(x)$ where N is the total population size. It should be realized that a and b will vary directly with x so that more correctly, the output of manager x should be written as: $x\alpha[F(a(x),b(x))]^{\beta}$.

The necessary and sufficient condition for the existence of a farm firm is that:

$$\pi_x = px\alpha[F(a,b)]^{\beta} - bv - au - w \geq 0 \qquad (1)$$

where p = price of the farm output
v = price of the purchased input
u = rental value of land
w = non-agricultural wage rate
x = the managerial talent level
a and b are the input levels of land and purchased inputs
π_x = profit (= managerial rent) of a farmer with talent level x.

The supply of land to the sector is given by the inverse supply function S(L), where $0 < S'(L) < \infty$. The market demand for the output of the sector is given by the inverse demand function P(Q), where $-\infty < P'(Q) \leq 0$.

Resource allocation in the farm sector of interest, is defined as a number z (the COML), and a pair of functions a(x) and b(x): $R^{+} \rightarrow R^{+}$ such that $a(x) = b(x) = 0$ for all $x < z$, and $a(x), b(x) > 0$ for all $x \geq z$. An allocation is feasible if it does not use more than the available resources. It is assumed that the farm sector of interest competes with other sectors for land, and the number of farm managers is an insignificant fraction of the total number of agents in the economy. Also purchased inputs are assumed to be available under conditions of perfect supply elasticity.

An efficient resource allocation in the sector maximizes the net social value of the output of the sector. It is assumed that there are no externalities so that a competitive equilibrium yields an efficient feasible allocation. A competitive equilibrium is given by:

$$\underset{\text{wrt}:z,a,b,Q^*,L^*}{\text{Maximize}} \quad Y = \int_0^{Q^*} P(Q)dQ - N\int_z^{\infty}(vb(x) + w)dT(x) - \int_0^{L^*} S(L)dL \quad (2)$$

$$\text{subject to:} \quad L^* - N\int_z^{\infty} a(x)dT(x) \geq 0 \quad (3)$$

$$\text{and } N\int_z^{\infty} x\alpha[F(a,b)]^{\beta}\, dT(x) - Q^* \geq 0 \quad (4)$$

where Q^* is the total output of the sector
L^* is the total land use of the sector

The Lagrangian of this problem is:

$$M = \int_0^{Q^*} P(Q)dQ - N\int_z^{\infty}(vb(x) + w)dT(x) - \int_0^{L^*} S(L)dL$$

$$+ u[L^* - N\int_z^{\infty} a(x)dT(x)] + p[N\int_z^{\infty} x\alpha[F(a,b)]^{\beta} dT(x) - Q^*] \quad (5)$$

where u and p are the shadow (market) prices of land and the sector output respectively. Unless otherwise stated, the upper and lower integral limits are ∞ and z respectively. $F[a(x), b(x)]$ will usually be abbreviated to F henceforth. The first order necessary conditions (FONC) for an interior maximum are:

$$\frac{\partial M}{\partial z} \equiv N\{vb(z) + w + ua(z) - pz\alpha[F(a(z), b(z))]^{\beta}\} = 0 \quad (6)$$

$$\frac{\partial M}{\partial a} \equiv N\{-u\int dT + p\int x\alpha\beta F^{\beta} F_a dT(x)\} = 0 \quad (7)$$

$$\frac{\partial M}{\partial b} \equiv N\{-v\int dT + p\int x\alpha\beta F^{\beta} F_b dT(x)\} = 0 \quad (8)$$

$$\frac{\partial M}{\partial Q^*} \equiv P(Q^*) - p = 0 \quad (9)$$

$$\frac{\partial M}{\partial L^*} \equiv -S(L^*) + u = 0 \tag{10}$$

$$\frac{\partial M}{\partial u} \equiv L^* - N\int a(n)dT(x) = 0 \tag{11}$$

$$\frac{\partial M}{\partial p} \equiv N\int x\alpha F^{\beta} dT(x) - Q^* = 0 \tag{12}$$

The first FONC (equation (6)) gives the break even condition for the marginal firm (i.e. the COML). The second and third FONC give the marginal conditions for the use of land and the purchased input. The fourth and fifth FONC give the shadow (market) prices of the sector output and land respectively. Finally, the last two FONC ensure feasibility in the use of land and the sale of output.

It is assumed that the second order conditions sufficient for the existence of a maximum solution are satisfied. That is, if $\overline{H}$ is the appropriate bordered Hessian matrix then $|\overline{H}_2| > 0$, $|\overline{H}_3| < 0$, $|\overline{H}_4| > 0$ and $|\overline{H}_5| < 0$. If F(a,b) is assumed to be linear homogeneous then it can be shown that the second order conditions are satisfied.[3]

The bordered Hessian matrix from the above FONC is:

$$\overline{H} = N \begin{bmatrix} \frac{-vb-w-ua}{z} & 0 & 0 & 0 & 0 & a & \frac{-vb-w-ua}{p} \\ 0 & p\alpha\beta\underline{A} & p\alpha\beta\underline{D} & 0 & 0 & -(1-T) & \frac{u}{p}(1-T) \\ 0 & p\alpha\beta\underline{D} & p\alpha\beta\underline{B} & 0 & 0 & 0 & \frac{v}{p}(1-T) \\ 0 & 0 & 0 & \frac{P'}{N} & 0 & 0 & -\frac{1}{N} \\ 0 & 0 & 0 & 0 & -\frac{S'}{N} & \frac{1}{N} & 0 \\ a & -(1-T) & 0 & 0 & \frac{1}{N} & 0 & 0 \\ \frac{-vb-w-ua}{p} & \frac{u}{p}(1-T) & \frac{v}{p}(1-T) & \frac{-1}{N} & 0 & 0 & 0 \end{bmatrix}$$

where simplifying substitutions from the first order conditions have been made and where the following abbreviations are used:

$-vb-w-ua \equiv -vb(z)-w-ua(z)$

$(1-T) \equiv (1-T(z))$

$\underline{A} \equiv \int_z^\infty x[(\beta-1)F^{\beta-2}F_a^2 + F^{\beta-1}F_{aa}]dT(x)$

$\underline{B} \equiv \int_z^\infty x[(\beta-1)F^{\beta-2}F_b^2 + F^{\beta-1}F_{bb}]dT(x)$

$\underline{D} \equiv \int_z^\infty x[(\beta-1)F^{\beta-2}F_aF_b + F^{\beta-1}F_{ab}]dT(x)$

If F (a,b) is linear homogeneous then it can be shown, using (7) and (8) that:

$$\frac{F_a}{F_b} = \frac{f(r) - rf'(r)}{f'(r)} = \frac{u}{v}$$

and so $r = \frac{b(x)}{a(x)}$ is independent of x. Accordingly:

$$\underline{A} = \int_z^\infty xF^{\beta-2}[(\beta-1)F_a^2 + FF_{aa}]dT$$

$$= \int xF^{\beta-2}[(\beta-1)\ [f(r) - rf'(r)]^2 + r^2f(r)f'']\ dT(x)$$

$$= \{(\beta-1)[(f(r)-rf'(r)]^2 + r^2f(r)f''\}\int xF^{\beta-2}\ dT(x) \quad (13)$$

Similarly:

$$\underline{B} = \{(\beta-1)[f'(r)]^2 + f(r)\ f''\}\int xF^{\beta-2}dT(x) \quad (14)$$

and

$$\underline{D} = \{(\beta-1)[f(r) - rf'(r)]f'(r) - rf(r)f''\}\int xF^{\beta-2}dT(x) \quad (15)$$

$$= (1-\beta)(rf(r)f'')\ (\sigma - \frac{1}{1-\beta})\int xF^{\beta-2}dT(x) \quad (16)$$

where σ = elasticity of substitution of F(a,b). Clearly $\underline{A}$ and $\underline{B}$ are negative while the sign of $\underline{D}$ is indeterminate. However, it can be shown that:

$$(\underline{A}\ \underline{B} - \underline{D}^2) = (\beta-1)\ f(r)^3\ f''[\int xF^{\beta-2}dT(x)]^2 \qquad (17)$$

and so $(\underline{A}\ \underline{B} - \underline{D}^2) > 0$. The sign of D depends on the sign of $(\sigma - \frac{1}{1-\beta})$.

Using these relationships it can be shown that the SOC are satisfied. If the SOC for an interior maximum are satisfied, then $|\bar{H}| < 0$.

The derivative of interest in the current investigation is $\frac{dz}{d\alpha}$ because an increase in α corresponds to a proportional increase in output across all farm firms. Applying the implicit function theorem and Cramer's rule we get:

$$\frac{dz}{d\alpha} = \frac{|J|}{|\bar{H}|} \qquad (18)$$

where:

$$|J| = N^7(1-T)^4 \begin{vmatrix} pzF^\beta & 0 & 0 & 0 & 0 & a & -z\alpha F^\beta \\ -\frac{u}{\alpha} & \lambda\underline{A} & \lambda\underline{D} & 0 & 0 & -1 & \frac{u}{p} \\ -\frac{v}{\alpha} & \lambda\underline{D} & \lambda\underline{B} & 0 & 0 & 0 & \frac{v}{p} \\ 0 & 0 & 0 & \frac{P'}{N} & 0 & 0 & -\frac{1}{N} \\ 0 & 0 & 0 & 0 & -\frac{S'}{N} & \frac{1}{N} & 0 \\ 0 & -1 & 0 & 0 & \frac{1}{N} & 0 & 0 \\ \frac{-Q^*}{\alpha N} & \frac{u}{p} & \frac{v}{p} & -\frac{1}{N} & 0 & 0 & 0 \end{vmatrix}$$

where $\lambda = p\alpha\beta/[1-T(z)]^2$. Evaluating this determinant:

$$|J| = N^4(P'Q^*+p)\left[\frac{zF^\beta}{N}(p\alpha\beta)^2(\underline{A}\ \underline{B}-\underline{D}^2)-S'(1-T)^2\beta[(vb+w)\underline{B}+av\underline{D}]\right] \qquad (19)$$

$(\underline{A}\ \underline{B} - \underline{D}^2)$ is positive by the assumption that the SOC are satisfied. $[(vb + w)\underline{B} + av\underline{D}]$ can be proven negative if F(a,b) is linear homogeneous:

$$(vb + w)\underline{B} + av\underline{D} = \{(vb + w)\ [(\beta-1)f'^2 + ff'']$$

$$+ av[(\beta-1)[f - rf']f' - rff'']\} \int xF^{\beta-2}dT(x)$$

$$= w\underline{B} + (\beta-1)avff' \int xF^{\beta-2}dT(x) < 0$$

Even if F(a,b) is not linear homogeneous, it is likely that $[(vb + w)\underline{B} + av\underline{D}]$ is negative because all but one term in D is negative. Therefore, even though:

$$\{\frac{zF^\beta}{N}(p\alpha\beta)^2\ (\underline{A}\ \underline{B} - \underline{D}^2) - S'(1-T)^2\beta[(vb + w)\underline{B} + av\underline{D}]\}$$

cannot be proven positive without assuming a specific form for F(a,b), it seems reasonable to assume that it is.

Accordingly, because $|\overline{H}| < 0$ then:

$$\frac{dz}{d\alpha} = \frac{|J|}{\overline{H}} \left(\begin{matrix}>\\=\\<\end{matrix}\right) 0 \text{ if } (P'Q^* + p) \left(\begin{matrix}<\\=\\>\end{matrix}\right) 0$$

That is:

$$\frac{dz}{d\alpha} = \left(\begin{matrix}>\\=\\<\end{matrix}\right) 0 \text{ if } E = \left[\frac{P'Q^*}{p}\right]^{-1} \left(\begin{matrix}>\\=\\<\end{matrix}\right) -1$$

where: E = elasticity of market demand for the farm output.

Therefore, a neutral technological change will only be neutral in its impact $(\frac{dz}{d\alpha} = 0)$ if output demand has unitary elasticity. A corollary of this is that if demand is inelastic then it is possible that a small-scale biased technological change could be detrimental to the continued existence of farm firms which are marginal prior to the change (i.e. small farms). Similarly a large-scale biased technology might improve the rents of marginal farmers provided that output demand is sufficiently elastic.

It should be noted that linear homogeneity of F(a,b) is sufficient, but not necessary, for the result obtained above.

4. Discussion

The argument here is not that scale bias of technological change is irrelevant to the fortunes of marginal farmers but rather that considerations of scale bias alone are insufficient to predict the directional impact of technology change on small farmers. Also, the above analysis is consistent with the proposition that, other things equal, technologies biased in favor of small scale are more likely to benefit marginal farmers than are technologies less biased in that direction. The analysis warns us that no matter what the scale bias of a new technology, benefits to marginal farmers (or any farmer) are not guaranteed. On the other hand, if output demand is sufficiently elastic, even a technology change biased in favor of large farms could benefit marginal farmers by increasing their managerial rents.

In addition, the above analysis tells us that provided output demand is elastic, a neutral change in technology will confer benefits on marginal farmers; in fact all existing farmers will experience an increase in managerial rents. Also the model predicts that this will be accompanied by new entrants, some of whom will become the new marginal producers; that is, their managerial rents will be zero. Unless there are barriers to entry, the model predicts that no matter how much technical progress occurs, marginal farmers will always exist.

There are a considerable number of applications for this model. For instance an increase in β would represent a large-scale biased technological change. It can be shown that $\frac{dz}{d\beta} < 0$ is possible although $\frac{dz}{d\beta} > 0$ is most likely.[4/]

The implications of the above discussion are important for the process of allocating agricultural research and extension funds. If one of the goals of such research is to develop and extend technologies favorable to marginal farmers, then elasticity of demand for the final output is an important consideration. A technology which may benefit marginal farmers when farm output price is guaranteed may hurt marginal farmers when market forces are allowed to operate. There are also implications for agricultural trade policies. An agricultural sector selling to a protected domestic market will have an output demand that is less elastic than would occur if the sector is internationally competitive. A technological change may be detrimental to marginal farmers in the first case but not in the second.

Finally, if aggregate agricultural output demand is price inelastic, long term technological change which may appear neutral to scale, will be detrimental to marginal farmers. The long run consequences will be fewer, larger farmers. In such cases consumers are the beneficiaries. It is appropriate that agricultural research should be funded from general taxes if aggregate demand for farm output is inelastic and if the resulting technologies are neutral to scale.

FOOTNOTES

1. The relationship between the two different concepts of return to scale is discussed in Hanoch (1975). One concept is defined in terms of proportional input increases and the other is defined in in terms of the least cost expansion path (which underlies the long run cost curve).
2. Gibrat's law states that firm growth is independent of firm size. See Lucas (1978)
3. See the appendix for the proof that the SOC are satisfied when F(a,b) is linear homogeneous.
4. See the appendix for a proof that $dz/d\beta<0$ is a possibility.

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MATHEMATICAL APPENDIX

Assume the farm production technology is $F(a,b)$, and that it is completely and costlessly available to all who wish to use it. $F(a,b)$ is C_2, increasing and strictly concave in both arguments; that is: $F_a>0$, $F_b>0$, $F_{aa}<0$, $F_{bb}<0$ and $F_{ab}>0$. Let $F(a,b)$ exhibit constant returns to scale so that we can write: $F(a,b) = af(b/a) = af(r)$. $f(r)$ is also C_2, increasing and strictly concave. Constant returns to scale of the production technology is a sufficient but not a necessary condition for the results obtained below.

Let there be a managerial technology such that the actual output of a given farm is:

$$x\,\alpha\,[F(a,b)]^{\beta}$$

where $x \geq 0$ is an index of the farm management ability of the farmer, and $\alpha>0$ and $0<\beta<1$ are parameters reflecting a 'span of control' limitation common to all farmers. These span of control parameters represent the assumption that larger production units are more difficult to manage than smaller units so that the fixed managerial input into the farm firm causes decreasing returns to scale despite the fact that the basic production technology exhibits constant returns to scale.

It is assumed that each agent is endowed with a managerial talent level x, drawn from a fixed distribution $T:R^{+}\to[0,1]$. The variable inputs (a=land and b=purchased inputs) are assumed homogenous. If we assume a continuum of agents, so that the entire distribution of talent is always present, then we can envisage some allocation of resources, described by two functions, $a(x)$ and $b(x)$, which give the quantities of land and purchased inputs managed by agent x. Obviously not everyone will be a farmer. It is assumed that only the best farm managers will be farmers while most agents in the economy will work in the nonagricultural sector for a wage rate w. Accordingly there will be some cutoff talent level, z, which separates farmers from nonfarmers. That is, $a(x)=b(x)=0$ for $x<z$ and $a(x)$, $b(x)>0$ for $x \geq z$. It is further assumed that the structure of the economy is such that only a small fraction of agents are farmers.

Total sector output is given by:

$$N\int_{z}^{\infty} x\,\alpha\,[\,F\,(\,a(x),b(x)\,)\,]^{\beta}\,dT(x) \qquad (1)$$

where N is the size of the total population.

It is assumed that the demand by the economy for farm output is described by the inverse demand function $P(Q,w,N)$ where $P_Q<0$, $P_w>0$, $P_N>0$. In the present analysis w and N are assumed constant and so we will consider $P(Q)$ only.

Further it is assumed that the farm sector of interest must compete with other sectors of the economy for land. All other sectors are assumed to be in equilibrium so that the supply of land to the farm sector can be completely described by the inverse supply function S(L), such that $S_L > 0$, where L is the amount of land used by the farm sector. In the absence of externalities, S(L) will reflect the social opportunity cost of land used by the farm sector.

Finally it is assumed that the purchased input is perfectly elastically supplied by the rest of the economy at price v.

In the absence of externalities a competitive equilibrium will involve an efficient allocation. A competitive equilibrium will be the solution of:

$$\underset{z,a,b,Q^*,L^*}{\text{Maximize wrt}}\ Y = \int_0^{Q^*} P(Q)\,dQ - N\int_z^{\infty} (vb(x)+w)\,dT(x) - \int_0^{L^*} S(L)\,dL \tag{2}$$

subject to:

$$L^* - N\int_z^{\infty} a(x)\,dT(x) \geq 0 \tag{3}$$

and

$$N\int_z^{\infty} x\,\alpha\,[\,F(a(x),b(x))\,]^{\beta}\,dT(x) - Q^* \geq 0 \tag{4}$$

The FONC for an interior maximum solution to this problem are:

$$\frac{\partial \mathcal{L}}{\partial z} \equiv N\{vb(z)+w+ua(z)-pz\alpha[F(a(z),b(z))]^{\beta}\} = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial a} \equiv N\{-u\int_z^{\infty} dT(x) + p\int_z^{\infty} x\alpha\beta F^{\beta} F_a\,dT(x) = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial b} \equiv N\{-v\int_z^{\infty} dT(x) + p\int_z^{\infty} x\alpha\beta F^{\beta} F_b\,dT(x) = 0 \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial Q^*} \equiv P(Q^*) - p = 0 \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial L^*} \equiv -S(L^*) + u = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial u} \equiv L^* - N\int_z^{\infty} a(x)\,dT(x) = 0 \tag{10}$$

$$\frac{\partial \pounds}{\partial p} \equiv N \int_z^{\infty} x \alpha F^{\beta} \, dT(x) - Q^* = 0 \qquad (11)$$

u is the shadow price of land and p is the shadow price of farm output.

The SOC are $|\bar{H}_2| \geq 0$, $|\bar{H}_3| \leq 0$, $|\bar{H}_4| \geq 0$, $|\bar{H}_5| \leq 0$ where:

$$\bar{H} = N \begin{bmatrix} -p\alpha F^{\beta} & 0 & 0 & 0 & 0 & a & -z\alpha F^{\beta} \\ 0 & p\alpha\beta\underline{A} & p\alpha\beta\underline{D} & 0 & 0 & -[1-T] & u[1-T]/p \\ 0 & p\alpha\beta\underline{D} & p\alpha\beta\underline{B} & 0 & 0 & 0 & v[1-T]/p \\ 0 & 0 & 0 & P'/N & 0 & 0 & -1/N \\ 0 & 0 & 0 & 0 & -S'/N & 1/N & 0 \\ a & -1-T] & 0 & 0 & 1/N & 0 & 0 \\ -z\alpha F^{\beta} & u[1-T]/p & v[1-T]/p & -1/N & 0 & 0 & 0 \end{bmatrix}$$

$$\text{where } \underline{A} = \int_z^{\infty} x \, [(\beta-1) \, F^{\beta-2} F_a^2 + F^{\beta-1} F_{aa}] \, dT(x) \qquad (12)$$

$$\underline{B} = \int_z^{\infty} x \, [(\beta-1) \, F^{\beta-2} F_b^2 + F^{\beta-1} F_{bb}] \, dT(x) \qquad (13)$$

$$\underline{D} = \int_z^{\infty} x \, [(\beta-1) \, F^{\beta-2} F_a F_b + F^{\beta-1} F_{ab}] \, dT(x) \qquad (14)$$

Also [1-T] is an abbreviation of [1-T(z)] and will be used henceforth.

To obtain $\bar{H}$ in the form given above several substitutions from the FONC have been made. The SOC involve:

$$|\bar{H}_2| = 1 > 0 \qquad (15)$$

$$|\bar{H}_3| = N \, p \, \alpha\beta \, \underline{B} + N^2 \, P' \, \{v(1-T)/p\}^2 \qquad (16)$$

≤ 0 because $\underline{B} \leq 0$ and $P' \leq 0$

$$|\bar{H}_4| = -N^4 P' \{v[1-T]/p\}^2 - N^4 S' [1-T]\, p\alpha\beta\, \underline{B}$$

$$+ N^3 P' [1-T] \{v^2\underline{A} + u^2\underline{B} - 2uv\underline{D}\}\, \alpha\beta/p$$

$$+ N^2 (p\alpha\beta)^2 (\underline{A}\underline{B}-\underline{D}^2)$$

$$\geq 0 \text{ if } (v^2\underline{A} + u^2\underline{B} - 2uv\underline{D}) \leq 0 \qquad (17)$$

$$\text{and } (\underline{A}\underline{B}-\underline{D}^2) \geq 0$$

Recall that $P' \leq 0$, $\underline{B} \leq 0$

$$|\bar{H}_5| = - N^5 P'S'[1-T]^2 \left(a^2\{v^2\underline{A}+u^2\underline{B}-2uv\underline{D}\} + 2a\{vb+w+au\}\{v\underline{B}-u\underline{D}\} + \{vb+w+au\}^2\underline{B} \right)$$

$$- N^5 P' \alpha F^{\beta} [1-T]^2 \alpha\beta (v^2\underline{A}+u^2\underline{B}-2uv\underline{D})$$

$$+ N^5 P'S' p\alpha F^{\beta} \{v[1-T]^2/p\}^2$$

$$+ N^4 S' p\alpha F^{\beta} [1-T]^2 p\alpha\beta\, \underline{B}$$

$$+ (p\alpha\beta)^2 (\underline{A}\underline{B}-\underline{D}^2) [N^4P'(z\alpha F^{\beta})^2 - N^4S'a^2 - N^3p\alpha F^{\beta}] \qquad (18)$$

≤ 0 given $P' \leq 0$ and $\underline{B} \leq 0$ if the following conditions hold:

$$(\underline{A}\underline{B}-\underline{D}^2) \geq 0$$

$$(v^2\underline{A} + u^2\underline{B} - 2uv\underline{D}) = \underline{E} \leq 0$$

and $$(a^2\underline{E} + 2a\underline{C}(v\underline{D}-u\underline{B}) + \underline{C}^2\underline{B}) \leq 0$$

where $\underline{C} = (vb+w+ua)$

In summary, the sufficient conditions for the SOC to be satisfied are:

(i) $\underline{A}\underline{B}-\underline{D}^2 \geq 0$ ($\underline{A}$, $\underline{B}$ and $\underline{D}$ are defined above)

(ii) $v^2\underline{A} + u^2\underline{B} - 2uv\underline{D} = \underline{E} \leq 0$

(iii) $a^2\underline{E} + 2a\underline{C}(v\underline{D}-u\underline{B}) + \underline{C}^2\underline{B} \leq 0$ where $\underline{C} = vb+w+ua$

The proofs that (i), (ii) and (iii) hold when F(a,b) is linear homogeneous are now given.

If F(a,b) is linear homogeneous then af(r) = F(a,b) where r=b/a. Therefore $F_a = f(r)-rf'(r)$, and $F_b = f'(r)$. From the FONC, $[f(r)-rf'(r)/f'(r)] = u/v$ and so r=b/a is independent of x. Therefore we can write:

$$\underline{A} = \int_z^\infty x F^{\beta-1} \{(\beta-1)\, F_a^2 + F\, F_{aa}\}\, dT(x) \tag{19}$$

$$= \{(\beta-1)\, [f(r)-rf'(r)]^2 + r^2 f(r) f''(r)\} \int_z^\infty x\, F^{\beta-2}\, dT(x) \tag{20}$$

Similarly

$$\underline{B} = \{(\beta-1)\, [f'(r)]^2 + f(r) f''(r)\} \int_z^\infty x\, F^{\beta-2}\, dT(x) \tag{21}$$

$$\underline{D} = \{(\beta-1)\, [f(r)-rf'(r)] f'(r) - rf(r) f''(r)\} \int_z^\infty x\, F^{\beta-2}\, dT(x) \tag{22}$$

Accordingly:

$$[\underline{AB}-\underline{D}^2] = [\, \{(\beta-1)(f-rf')^2+r^2ff''\}\{(\beta-1)f'^2+ff''\}$$

$$-\{(\beta-1)(f-rf')f'-rff''\}^2\,]\ \ [\int_z^\infty xF^{\beta-1}\, dT(x)\,]^2 \tag{23}$$

$$= (\beta-1)\, f^3\, f''\, [\int_z^\infty xF^{\beta-2}\, dT(x)\,]^2 \tag{24}$$

> 0 because $(\beta-1)<0$ and $f''(r)<0$

qed

and

$$v^2\underline{A}+u^2\underline{B}-2uv\underline{D} = v^2[\underline{A}+(u/v)^2\underline{B}-2(u/v)\underline{D}] \tag{25}$$

$$= v^2 \int_z^\infty xF^{\beta-2}\, dT(x)\ \{[(\beta-1)(f-rf')^2+r^2ff'']$$

$$+[(f-rf')/f']^2[(\beta-1)f'^2+ff'']$$

$$-2[(f-rf')/f'][(\beta-1)(f-rf')f'-rff'']\} \tag{26}$$

$$= v^2 \int_z^\infty xF^{\beta-2}\, dT(x)\ \{r^2ff''+(u/v)^2ff''+2(u/v)rff''\} \tag{27}$$

< 0 because $f''<0$

qed

Finally:

$$a^2\underline{E} + 2a\underline{C}\,(v\underline{D}-u\underline{B}) + \underline{C}^2\underline{B}$$

$$=a^2(v^2\underline{A}+u^2\underline{B}-2uv\underline{D})+2a(vb+w+ua)(v\underline{D}-u\underline{B})+(vb+w+ua)^2\underline{B} \quad (28)$$

$$=v^2(a^2\underline{A}+b^2\underline{B}+2ab\underline{D})+2wv(b\underline{B}+a\underline{D})+w^2\underline{B} \quad (29)$$

$$= \int_z^\infty xF^{\beta-2}\, dT(x)\ \{(\beta-1)(vaf(r)+wf'(r))^2+w^2f(r)f''(r)\} \quad (30)$$

< 0 because $\beta-1<0$ and $f''(r)<0$

qed

Therefore linear homogeneity of F(a,b) is sufficient for the SOC to be satis fied.

The value of current interest is: $\frac{dz}{d\alpha} = \frac{|J|}{|H|}$ where:

$$\underline{J}=N \begin{bmatrix} pzF^{\beta} & 0 & 0 & 0 & 0 & a & -z\alpha F^{\beta} \\ \frac{-u(1-T)}{\alpha} & p\alpha\beta\underline{A} & p\alpha\beta\underline{D} & 0 & 0 & -(1-T) & \frac{u(1-T)}{p} \\ \frac{-v(1-T)}{\alpha} & p\alpha\beta\underline{D} & p\alpha\beta\underline{B} & 0 & 0 & 0 & \frac{v(1-T)}{p} \\ 0 & 0 & 0 & P'/N & 0 & 0 & -1/N \\ 0 & 0 & 0 & 0 & -S'/N & 1/N & 0 \\ 0 & -(1-T) & 0 & 0 & 1/N & 0 & 0 \\ -Q^*/\alpha N & \frac{u(1-T)}{p} & \frac{v(1-T)}{p} & -1/N & 0 & 0 & 0 \end{bmatrix}$$

Therefore:

$$\frac{dz}{d\alpha} = \frac{(P'Q^{*}+p)}{|\underline{H}|} \{N^{3}zF^{\beta}(p\alpha\beta)^{2}(\underline{A}\underline{B}-\underline{D}^{2}) - N^{4}S'(1-T)^{2}\beta[(vb+w)\underline{B}+av\underline{D}]\} \tag{31}$$

From the SOC we have $\underline{A}\underline{B}-\underline{D}^{2} > 0$ and so if $(vb+w)\underline{B}+av\underline{D} < 0$ then:

$$\frac{dz}{d\alpha} \begin{pmatrix} > \\ = \\ < \end{pmatrix} 0 \quad \text{if} \quad (P'Q^{*}+p) \begin{pmatrix} < \\ = \\ > \end{pmatrix} 0$$

That is:

$$\frac{dz}{d\alpha} \begin{pmatrix} > \\ = \\ < \end{pmatrix} 0 \quad \text{if} \quad \varepsilon = \frac{p}{P'Q^{*}} \begin{pmatrix} > \\ = \\ < \end{pmatrix} -1$$

where ε is the output demand elasticity.

If $F(a,b)$ is linear homogeneous then:

$$(vb+w)\underline{B}+av\underline{D} =$$

$$w\underline{B} + \int_{z}^{\infty} xF^{\beta-2}\, dT(x)\ \{vb[(\beta-1)f'^{2}+ff'']+av[(\beta-1)(f-rf')f'-rff'']\} \tag{32}$$

$$= w\underline{B} + \int_{z}^{\infty} xF^{\beta-2}\, dT(x)\ \{av(\beta-1)ff'\} \tag{33}$$

< 0 because $\underline{B}<0$ and $\beta-1<0$

qed

Therefore the result $dz/d\alpha = 0$ if $\varepsilon = -1$ is established. Some insight into the forces operating as a result of a technological change can be obtained by considering some more restricted cases.

Consider the simple case in which the sector of interest is the only user of land and the output demand is perfectly elastic. This situation is consistent with an open, small economy with one major agricultural crop. A competitive equilibrium solves the problem:

$$\underset{\text{wrt. } z,a,b}{\text{Maximize}}\ \psi = N\int_z^{\infty} \{px\alpha\ [F(a,b)]^{\beta}\ -vb-w\}\ dT(x) \qquad (34)$$

$$\text{subject to:}\quad L - N\int_z^{\infty} a\ dT(x) \geq 0 \qquad (35)$$

The FONC for an interior solution are:

$$\frac{\delta \pounds}{\delta z} \equiv -zp\alpha F^{\beta} + vb + + w + ua = 0 \qquad (36)$$

$$\frac{\delta \pounds}{\delta a} \equiv N\ [\int_z^{\infty} xp\alpha\beta\ F^{\beta-1}\ F_a\ dT(x) - u\int_z^{\infty} dT(x)] = 0 \qquad (37)$$

$$\frac{\delta \pounds}{\delta b} \equiv N\int_z^{\infty} [xp\alpha\beta\ F^{\beta-1}\ F_b - v]\ dT(x) = 0 \qquad (38)$$

$$\frac{\delta \pounds}{\delta u} \equiv L - N\int_z^{\infty} a\ dT(x) = 0 \qquad (39)$$

The borbered Hessian matrix of interest is:

$$\bar{K} = N \begin{bmatrix} -p\alpha F & 0 & 0 & a \\ 0 & p\alpha\beta\underline{A} & p\alpha\beta\underline{D} & -[1-T] \\ 0 & p\alpha\beta\underline{D} & p\alpha\beta\underline{B} & 0 \\ a & -[1=T] & 0 & 0 \end{bmatrix}$$

where $\underline{A},\underline{B},\underline{D}$ and T are as previously defined.

$$|\bar{K}| = N^4\ \{[1-T]^2 p\alpha F\ p\alpha\beta\underline{B} - a^2(p\alpha\beta)^2(\underline{AB}-\underline{D}^2)\} \qquad (40)$$

The SOC for a maximum require:

(i) $|\bar{K}_2| = -N^3\ [1-T]^2\ p\alpha\beta\ \underline{B} \geq 0$

This is satisfied because $\underline{B} \leq 0$.

(ii) $|\bar{K}_3| = |\bar{K}| \leq 0$

This requires $\underline{B} \leq 0$ (which is satisfied) and $(\underline{AB}-\underline{D}^2) \geq 0$

A sufficient condition for $(\underline{A}\underline{B}-\underline{D}^2)\geq 0$ is F(a,b) being linear homogeneous (see proof above)

The derivative of current interest is: $\frac{dz}{d\beta} = \frac{|G|}{|K|}$ where:

$$\underline{G} = N \begin{bmatrix} zp\alpha F^{\beta}(\ln F) & 0 & 0 & a \\ -\int_z^{\infty} xp\alpha F_a F^{\beta-1}(1+\beta\ln F)\, dT(x) & p\alpha\beta\underline{A} & p\alpha\beta\underline{D} & -[1-T] \\ -\int_z^{\infty} xp\alpha F_b F^{\beta-1}(1+\beta\ln F)\, dT(x) & p\alpha\beta\underline{D} & p\alpha\beta\underline{B} & 0 \\ 0 & -[1-T] & 0 & 0 \end{bmatrix}$$

Therefore assuming F(a,b) is linear homogeneous

$$|G| = N^4 p^2\alpha^2\beta F\,[1-T] \int_z^{\infty} x\, F^{\beta-2}\, dT(x)$$
$$\{\, ff'' \int_z^{\infty} xF^{\beta-1}(1+\beta\ln F)\, dT(x)$$
$$- zF^{\beta-1}(\ln F)[1-T][(\beta-1)f'^2+ff'']\} \qquad (41)$$

The sign of |G| depends on the sign of the term in brackets, which could be positive or negative. Therefore the derivative of interest $\frac{dz}{d\beta}$ could be positive or negative. The significance of this is that an increase in the parameter β represents a technological change which is biased in favor of larger scale producers. If the possibility of: $\frac{dz}{d\beta} < 0$ is admitted, it means that a large-scale biased technological change could confer benefits on the managers of marginal firms in the sense that their managerial rents would become positive.

Now consider the most simple (and obvious) case where the sector of interest is an insignificant user of land so that the supply of land to the sector (or subsector) can be assumed perfectly elastic. Also assume that output demand is perfectly elastic as before. A competitive equilibrium is the solution to:

$$\underset{z,a,b}{\text{Maximize}}\ \pi = N \int_z^{\infty} \{px\alpha[F(a,b)]^{\beta} - vb - w - ua\}\, dT(x) \tag{43}$$

wrt.
z,a,b

The FONC are:

$$\frac{\partial \pi}{\partial z} \equiv N\,\{-pz\alpha[F(a(z),(b))]^{\beta} + vb + w + ua\} = 0 \tag{44}$$

$$\frac{\partial \pi}{\partial a} \equiv N \int_z^{\infty} \{xp\alpha\beta\, [F(a,b)]^{\beta-1}\, F_a - u\}\, dT(x) = 0 \tag{45}$$

$$\frac{\partial \pi}{\partial b} \equiv N \int_z^{\infty} \{xp\alpha\beta\, [F(a,b)]^{\beta-1}\, F_b - v\}\, dT(x) = 0 \tag{46}$$

The Hessian matrix of interest is:

$$\underline{R} = N \begin{bmatrix} -p\alpha F^{\beta} & 0 & 0 \\ 0 & p\alpha\beta\underline{A} & p\alpha\beta\underline{D} \\ 0 & p\alpha\beta\underline{D} & p\alpha\beta\underline{B} \end{bmatrix}$$

$$|R| = -N^3\, p\alpha F^{\beta}\, (p\alpha\beta)^2\, (\underline{A}\underline{B} - \underline{D}^2) \tag{47}$$

The SOC for a maximum are satisfied if $(\underline{A}\underline{B} - \underline{D}^2) > 0$. A sufficient condition for this is F(a,b) being linear homogeneous.

The derivative of interest here is: $\frac{dz}{d\beta} = \frac{|T|}{|R|}$ where:

$$\underline{T} = N \begin{bmatrix} zp\alpha F^{\beta} \ln F & 0 & 0 \\ -\int_z^{\infty} xp\alpha F_a F^{\beta-1}\, (1+\beta \ln F)\, dT(x) & p\alpha\beta\underline{A} & p\alpha\beta\underline{D} \\ -\int_z^{\infty} xp\alpha F_b F^{\beta-1}\, (1+\beta \ln F)\, dT(x) & p\alpha\beta\underline{D} & p\alpha\beta\underline{B} \end{bmatrix}$$

and so $\frac{dz}{d\beta} = -z \ln F < 0$ which implies that an increase in β causes the cutoff management level to fall. In other words, in this simple case, a technological change favoring larger, nonmarginal farmers must also improve the position of marginal farmers because new entrants are attracted to the sector which implies that positive managerial rents are being received by new entrants. The intuitive reasoning behind this result lies in the fact that even though ultramarginal farmers receive greater gains from the new technology than marginal farmers (gains are directly proportional to the talent level), their response, which involves expansion, has no impact on lower talent farmers because the supply of land in this case has been assumed perfectly elastic. This throws some light on the penultimate case, where $dz/d\beta$ could not be signed. It would seem that the result depends on the relative strengths of the β change and the resulting land rent increase, which in turn depends upon the elasticity of substitution of F(a,b).

The general conclusion from the above analysis is that an apparently neutral change in technology may be detrimental or beneficial to the continued existence of marginal firms. Moreover a technolgical improvement which is biased in favor of larger, ultramarginal farm firms may not necessarily be detrimental to marginal farm firms because under certain conditions the managerial rents of marginal farmers may increase as a result of such a change. The factors which govern the actual outcome include the elasticities of supply in input markets and demand in output markets.

THE END OF THE MATHEMATICAL APPENDIX