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AN ECONOMIC ANALYSIS OF STABILIZING SCHEMES

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I. Introduction

Recently, market stabilization has become an important issue in the controversy over establishing reserve stocks of grains. W. W. Cochrane [1] has strongly urged the establishment of buffer stocks of agricultural products to help stabilize prices. However, Cochrane and many others who advocate the establishment of buffer stocks for stabilizing prices do not justify their positions from a theoretical point of view. It seems that their objective function is the stabilization of prices as a goal in itself. On the other hand, from a theoretical point of view and using conventional consumer and producer surplus analysis, B. F. Massell [2] integrated some previous results obtained by F. V. Waugh [6] [7] and by W. Oi [4] to show that, from the social point of view, a buffer stock which stabilizes prices is beneficial when compared to a free market situation. Similar results are obtained by Turnovsky [5] who analyzes the welfare implications of price stabilization under different assumptions concerning the behavior of supply.

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Therefore, it may be concluded that the advocates for the establishment of buffer stocks which stabilize prices can justify their positions from a theoretical point of view. But if price stabilization is justified in terms of social welfare, might it also be the case that stabilizing other market variables will be even more beneficial than stable prices? In other words, if maximizing social welfare is the objective of the policy maker, there is no *a-priori* reason why stabilizing the quantities demanded or the quantity supplied should not be considered as possible alternatives to price stabilization.

The main purpose of this paper is to analyze the welfare implications of stabilizing consumption and production and to compare it with the already-known welfare implications of stabilizing prices. Two sets of assumptions regarding supply behavior will be considered: (1) supply reacts instantaneously to a change in market prices, (2) producers react to changes in expected prices and expectations are "rational" within the context developed by J. F. Muth. [3] As a by-product of the analysis, the relation between the gains of stabilization and the size of the buffer stocks necessary to achieve the stabilizing goal at given probability levels also will be shown. Finally, the analysis will be extended to cover instability due to fluctuations in export demand.

II. Method of Analysis

The basic behavioral model which will be analyzed for the different stabilizing schemes is:

$$(1)\qquad \begin{aligned} D(P_t) &= \alpha_o - \alpha_1 P_t + u_t \\ S(P_t) &= \beta_o + \beta_1 P_t + v_t \end{aligned}$$

where D and S are the quantities demanded and supplied, P is the market price and u and v are the random elements affecting demand and supply. It is assumed that the error terms are independent and have finite variances, σ_u^2 and σ_v^2. In the absence of any stabilizing intervention, the market is assumed to behave competitively.

The free market solution of system (1) is:

$$(2) \qquad P_e = \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} + \frac{u_t - v_t}{\alpha_1 + \beta_1}$$

$$Q_e = \frac{\alpha_o\beta_1 + \alpha_1\beta_o}{\alpha_1 + \beta_1} + \frac{\beta_1 u_t + \alpha_1 v_t}{\alpha_1 + \beta_1}$$

The effects of any stabilizing scheme on consumers and producers will be measured in terms of consumers and producers surplus as compared to free market behavior. The drawbacks of measuring welfare gain or loss by means of various areas associated with demand and supply functions are well known and have been dealt with extensively elsewhere. Those arguments will not be repeated here but should be kept in mind in any application of these measures.

The areas representing the consumers and producers surplus (gains or losses) will be obtained as follows: Let P_e be the free market equilibrium price and let P_s be the market price under any of the stabilization schemes considered. Similarly, let D_e and S_e be the free market equilibrium demand and supply respectively ($D_e = S_e = Q_e$); also, let $D(P_s)$ and $S(P_s)$ be the demand and supply respectively for any of the stabilization schemes considered. It follows that the consumer gains (or losses) due to any stabilizing scheme will be measured as:

$$(3) \qquad G_c = \int_{P_s}^{P_e} D(P)dP = \int_{P_s}^{P_e} (\alpha_o - \alpha_1 P)dP = \tfrac{1}{2}(P_e - P_s)(D_e + D_s)$$

Similarly, the producers gains (or losses) will be measured as:

$$(4) \qquad G_s = \int_{P_e}^{P_s} S(P)dP = \int_{P_e}^{P_s} (\beta_o + \beta_1 P)dP = \tfrac{1}{2}(P_s - P_e)(S_s + S_e)$$

The total gain or losses from any stabilizing scheme will be:

$$(5) \qquad G = G_c + G_s .$$

The levels at which the various market variables (price, quantity demanded, and quantity supplied) are stabilized correspond to their expected values which would have been obtained from the free market system. In other words, stabilization is aimed at eliminating the free market randomness in the variable which is being stabilized.

Since the welfare gains (or losses) as measured in (3), (4) and (5) relate stabilization to free market situations which are affected by random elements, these measured gains (or losses) are themselves random. Therefore, we will be comparing the expected values of the welfare gains (or losses) derived from the different stabilization schemes.

The reason for dealing with a static model as in (1) is only a matter of convenience. The addition of demand and supply shifters will not change the nature of most of the results obtained. Whenever a changing stabilized variable through time will have an impact on the derived results, this possibility will be mentioned and analyzed.

III. Stabilizing Schemes When Supply Reacts Instantaneously to a Change In Market Price

Price Stabilization

The welfare implications of price stabilization in this case have been analyzed by Massell and Turnovsky. [2] [5] Nevertheless, their analyses are restated here to provide a basis for comparison between the different stabilization schemes.

Price is stabilized at the expected free market price, namely:

$$(6) \qquad P_s = \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} = \overline{P}$$

At that price, the quantities demanded and supplied are:

$$(7) \qquad D(P_s)_t = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} + u_t$$

$$S(P_s)_t = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} + v_t$$

It follows that when price is stabilized at $\overline{P}$, government will be buying or selling stocks according to:

$$(8) \qquad \Delta G_t(\overline{P}) = S(P_s)_t - D(P_s)_t = v_t - u_t$$

This scheme may be analyzed graphically, figure 1.

When demand is unstable and it is equally likely that D will shift to either D_1 or D_2, the expected gain in consumers welfare is positive and measured by the area of ABCD-DJFG. Similarly, the expected loss in producers surplus is measured by the area of ABJD-DJFG. It follows that the social gains is measured by the area of BCJ + EJF.

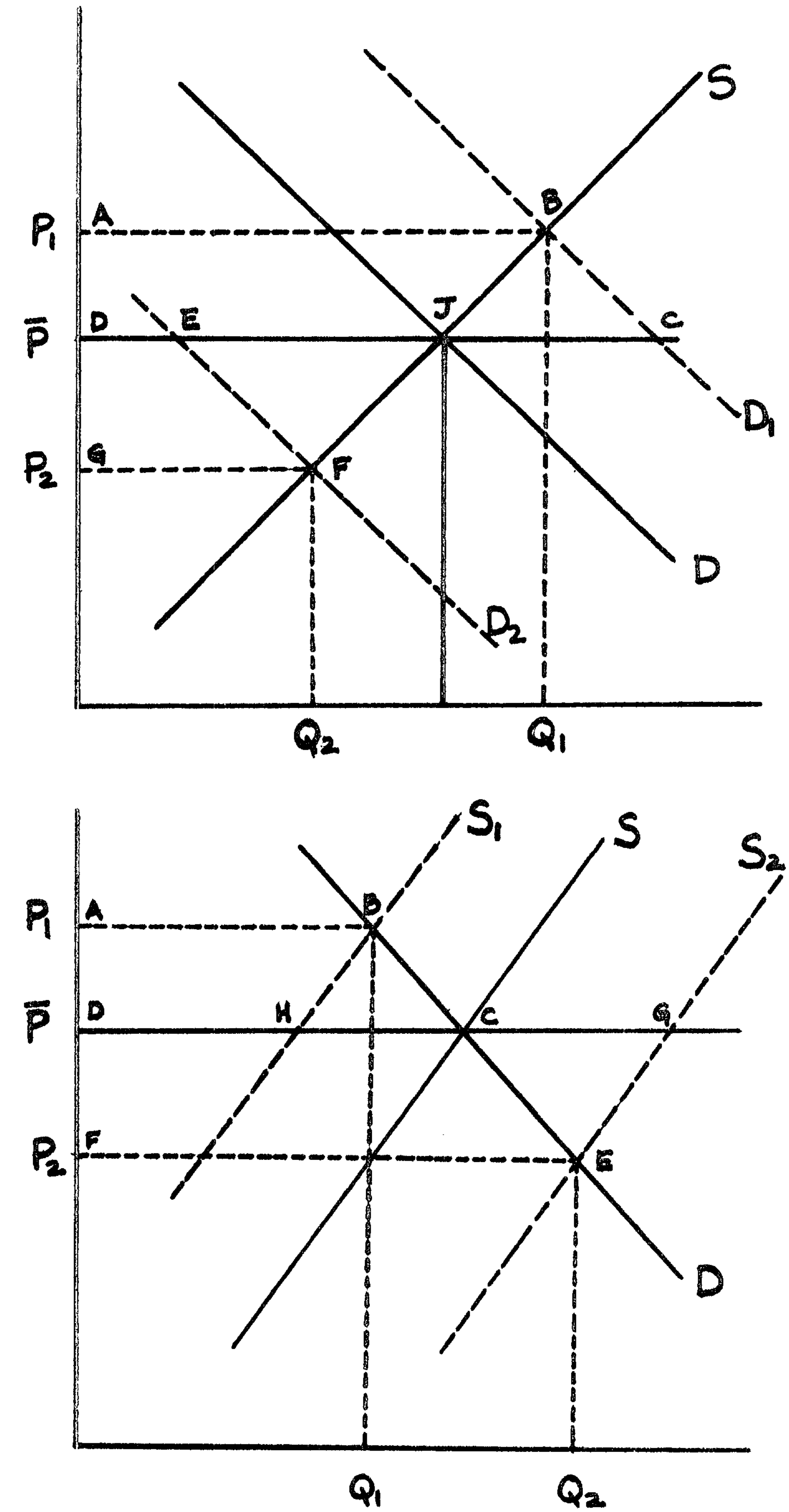
If supply is unstable and it is equally likely that S will shift to either S_1 or S_2, the loss in consumer welfare is measured by the area of DCEF-ABCD. The gain in producers surplus will be measured as the area of DGEF-ABDH. Social gains will be measured as the area of CGE + HBC.

Using the analytical approach introduced in section II, we obtain the following results involving the expected gains (or losses) derived by consumers, producers, and society when market prices are stabilized:

FIGURE 1:

a) UNSTABLE DEMAND

b) UNSTABLE SUPPLY



(9) $$EG_c(\overline{P}) = \frac{1}{2(\alpha_1 + \beta_1)^2} \left[(\alpha_1 + 2\beta_1)\sigma_u^2 - \alpha_1 \sigma_v^2\right]$$

(10) $$EG_s(\overline{P}) = \frac{1}{2(\alpha_1 + \beta_1)^2} \left[(2\alpha_1 + \beta_1)\sigma_v^2 - \beta_1 \sigma_u^2\right]$$

(11) $$EG(\overline{P}) = \frac{1}{2(\alpha_1 + \beta_1)} \left[\sigma_u^2 + \sigma_v^2\right]$$

These results have already appeared and been discussed in the cited literature. We will return to these results in a latter stage when they will be compared to alternative stabilization schemes.

However, there is one further result which is worth mentioning for future reference. Let $\sigma^2(\overline{P})$ be the variance of the changes in government stocks due to a price stabilizing scheme. From (8) we obtain:

$$\sigma^2(\overline{P}) = \sigma_v^2 + \sigma_u^2$$

It follows that the relation between the variance of the change in government stocks and the expected social benefits from a price stabilizing scheme is:

$$EG(\overline{P}) = \frac{1}{2(\alpha_1 + \beta_1)} \sigma^2(\overline{P})$$

Stabilization of Consumer Demand

To carry out this scheme, the quantity demanded is stabilized by government intervention at:

(12) $$D_s = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1}$$

If the quantity demanded is stabilized at D_s, then the quantity supplied, the market price and the change in government stocks become:

$$(13) \qquad S(D_s)_t = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} + \frac{\beta_1}{\alpha_1} (u_t) + v_t$$

$$(14) \qquad P(D_s)_t = \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} + \frac{1}{\alpha_1} (u_t)$$

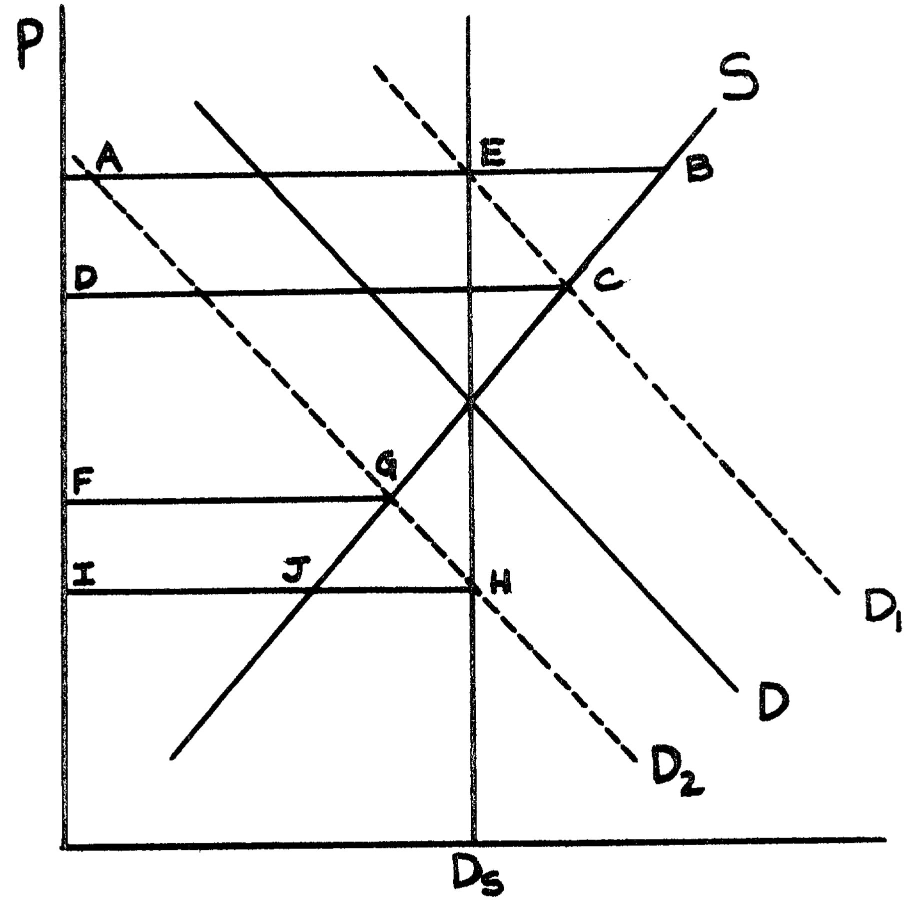
$$(15) \qquad \Delta G(D_s)_t = \frac{\beta_1}{\alpha_1} (u_t) + v_t$$

In considering this scheme, it is immaterial whether the target level of consumption is announced ahead of time or not. All that is needed is for the government to behave as in (15). We may get a clearer notion of the workings of this scheme by using some graphic analysis, figure 2.

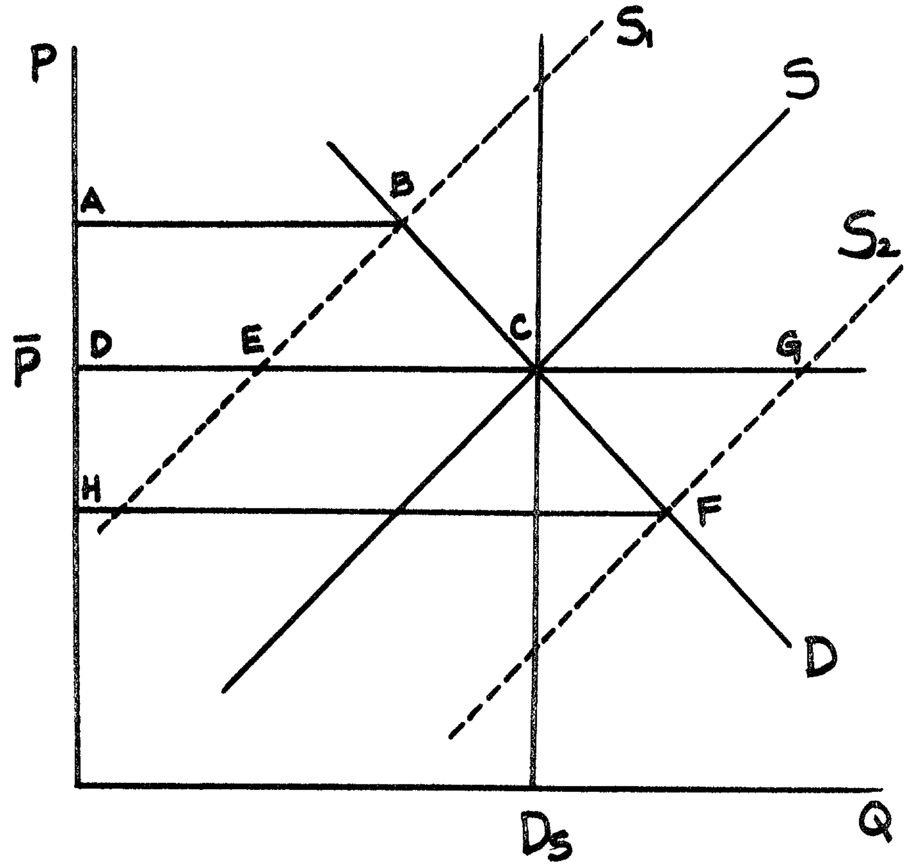
Let demand be unstable and assume that a shift to D_1 is as likely to happen as a shift to D_2. If D_1 occurs, the free market solution would be at C, figure 2(a). Here the quantity demanded is bigger than the target quantity, D_s. The government would buy EB of stocks, driving the market price up to E. Similarly, if D_2 occurred, the free market solution would be at G, figure 2(a). Here the quantity demanded is smaller than the target. The government would sell the amount of JH, driving the market price down to H. Therefore, if the source of instability is in demand, the stabilization of the quantity demanded will destabilize market price. Clearly this should be detrimental to consumers as a group since they are not allowed to adjust consumption when the range of price variability increases. In terms of figure 2(a), consumers expected losses are measured by the area AECD-FGHI. Similarly, since producers are allowed to adjust production, they will take advantage of the increased price variability and benefit from consumption stabilization. In terms of figure 2(a), these expected benefits are measured by the area ABCD-FGIJ. The expected total social benefits when only demand is unstable are measured by the area ECB + JGH.

FIGURE 2:

a) UNSTABLE DEMAND



b) UNSTABLE SUPPLY



In figure 2(b) we can see the workings of a consumption stabilization scheme when the source of instability is in supply. If S_1 occurs, the free market solution would be at B where the quantity demanded is smaller than the target, D_s. The government then will sell EC stocks, driving the price down to $\overline{P}$. Notice that $\overline{P}$ is the stabilized price when a price stabilization scheme is in effect. If S_2 occurs, the free market solution would be at F. The government will buy CG stocks in order to decrease consumption to the target level. The additional government purchases will raise prices to $\overline{P}$. As a result, the expected welfare loss by consumers is measured as the area of DCFH-ABCD while the expected gains by producers are measured as the area of DGFH-ABDE. In total, when supply is unstable, expected social gains are measured by the area of CGF + BCE.

Using the methodology of Section II, the algebraic expressions for the expected welfare implications of stabilizing consumption are:

$$(16) \qquad EG_c\ (D_s) = \frac{-1}{2\alpha_1(\alpha_1 + \beta_1)^2} \quad (\beta_1^{\ 2}\ \sigma_u^{\ 2} + \alpha_1^{\ 2}\ \sigma_v^{\ 2})$$

$$(17) \qquad EG_s\ (D_s) = \frac{2\alpha_1 + \beta_1}{2\alpha_1^{\ 2}\ (\alpha_1 + \beta_1)^2} \quad (\beta_1^{\ 2}\ \sigma_u^{\ 2} + \alpha_1^{\ 2}\ \sigma_v^{\ 2})$$

$$(18) \qquad EG\ (D_s) = \frac{1}{2\alpha_1^{\ 2}\ (\alpha_1 + \beta_1)} \quad (\beta_1^{\ 2}\ \sigma_u^{\ 2} + \alpha_1^{\ 2}\ \sigma_v^{\ 2})$$

As compared to the welfare effects for consumers when price is stabilized, consumers do not benefit from demand instability when the quantity demanded is stabilized. On the other hand, demand instability is beneficial to producers when the quantity demanded is stabilized, but is detrimental to producers when price is stabilized.

The reason that consumers do not benefit from demand instability when consumption is stabilized as compared to when price is stabilized can be explained as follows. When demand is unstable, a relatively high free market equilibrium price will be the result of a relatively large quantity demanded. On the other hand, a relatively low free market equilibrium price will be the result of a relatively small quantity demanded. When the free market equilibrium price is relatively high but price is stabilized, the government sells stocks to drive the market price down. This induces an increase in the quantity demanded. Since the quantity demanded was large to begin with, consumer gains will be relatively large. Alternatively, when free market prices are relatively low, price stabilization will involve an increase in prices and a decline in the quantity demanded. This yields a welfare loss to consumers. But since consumption was small to begin with, the loss also will be relatively small.

When consumption is stabilized, just the opposite happens. At a high free market equilibrium price and a relatively large quantity demanded, the government drives the market price up further by buying stocks inducing a decline in demand and a loss for consumers. This loss is relatively big since the quantity demanded was large to begin with. On the other hand, when the free market equilibrium price is relatively low and the quantity demanded small, by stabilizing consumption government will sell stocks to stabilize consumption, driving the price down. As a result, consumers gain, but this gain will be relatively small.

Producers gain from demand instability when consumption is stabilized, but they lose when price is stabilized. This occurs because producers are better off when allowed to produce at random along a given supply function than to produce the expected quantity at the expected price. When supply is the only source of instability, the stabilization of consumption gives the same result as the stabilization of prices.

It should be noted that when demand is unstable, the government will have an expected deficit. It can be shown that the extent of this expected deficit is measured by $\frac{\beta_1}{\alpha_1^2} \sigma_u^2$. It would be tempting to subtract this deficit from the expected social gains to arrive at a measure of net expected gains. In our view, this procedure would be too restrictive for any real life application in which equilibrium prices and quantities change through time because of systematic shifts in demand and supply. The reason is that, while the expected budget loss at any given time is related to the objective conditions at that time, the budget situation, unlike the benefits and losses to society, will depend on previous constellations of equilibrium prices and quantities. Without all this previous data, it is impossible to integrate the budgetary aspects of the stabilization schemes. It should also be mentioned that, because of the above reasons, even a price stabilization scheme only seems to be budgetarily neutral in any real life application. We therefore confine this discussion only to the welfare implications of the different stabilizing schemes without considering their budgetary implications.

Before turning to the next scheme, we shall relate the expected social benefits from consumption stabilization to the variance of the changes in government stocks. Let $\sigma^2(D_s)$ be the variance of the changes in government stocks when consumption is stabilized. Then from (15)

$$\sigma^2(D_s) = \frac{\beta_1^2}{\alpha_1^2} \left(\sigma_u^2\right) + \sigma_v^2 = \frac{\beta_1^2 \sigma_u^2 + \alpha_1^2 \sigma_v^2}{\alpha_1^2}$$

combining this with (18) we get:

$$EG(D_s) = \frac{1}{2(\alpha_1 + \beta_1)} \sigma^2(D_s)$$

Supply Stabilization

For this scheme, stabilization is achieved at the expected free market equilibrium supply. Government buys and sells stocks in such a way that the target supply will be forthcoming to the market. The target supply is:

(19) $$S_s = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1}$$

At that level of supply, the quantity demanded, the market price and the change in government stocks are:

(20) $$D(S_s)_t = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} + \frac{\alpha_1}{\beta_1} v_t + u_t$$

(21) $$P(S_s)_t = \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} - \frac{1}{\beta_1} v_t$$

(22) $$\Delta G(S_s)_t = -\left(\frac{\alpha_1}{\beta_1} v_t + u_t\right)$$

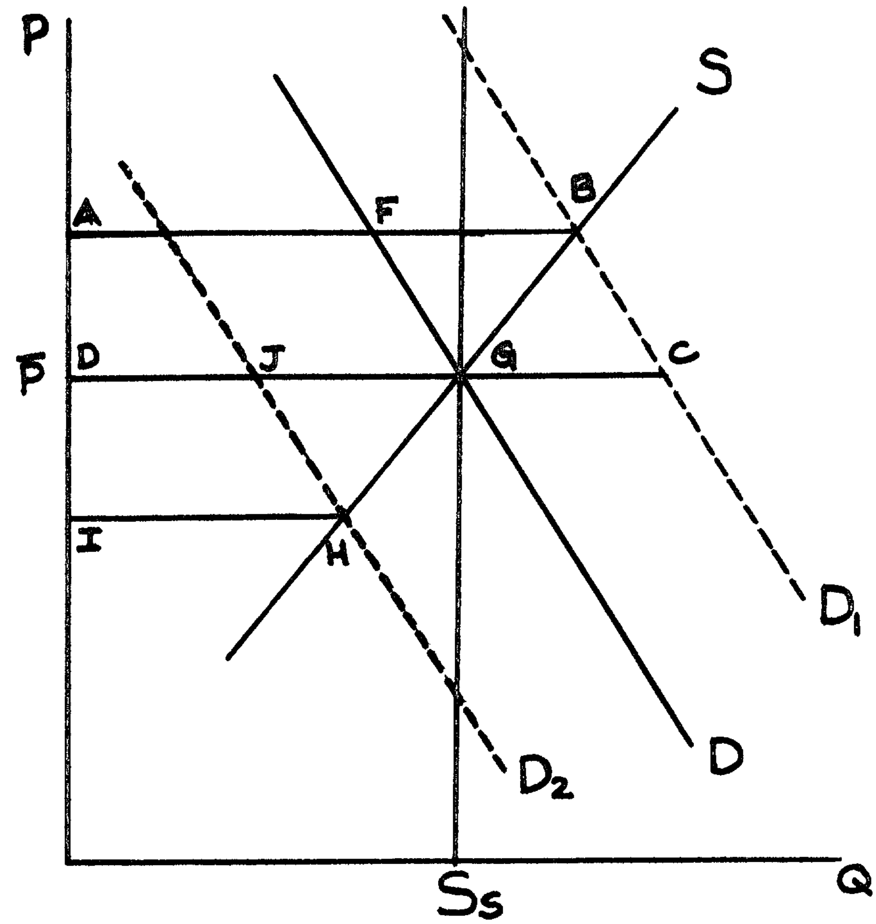
In a real world application of this scheme, it is enough that government behaves as in (22) in order to achieve supply stabilization.

Before we present the analytical and welfare results for this scheme, a graphical presentation may be helpful, figure 3.

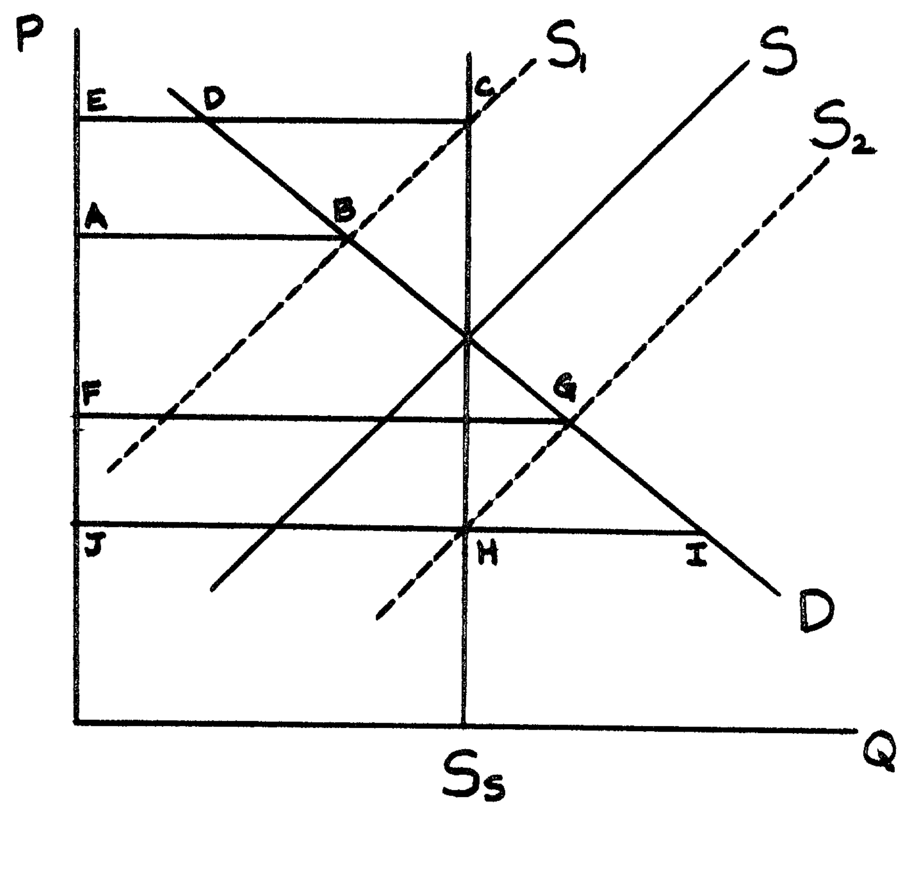
When only demand is unstable and a shift to either D_1 or D_2 is equally likely to happen, stabilizing supply has the same effects as stabilizing price; compare figure 1(a) to figure 3(a). In figure 3(a) we

FIGURE 3:

a) UNSTABLE DEMAND



b) UNSTABLE SUPPLY



observe that if D_1 occurs, the free market solution is at B. Here supply is larger than the target S_s. Government will sell stocks thereby reducing the market price in order to decrease production to the target level. Similarly, the free market solution with D_2 would be at H where production is smaller than the target. Government will buy stocks in order to increase prices so as to induce producers to increase production up to the desired level. The result is an expected loss of producers surplus measured by the area of ABDG-DGHI and a gain in expected consumer surplus measured by the area ABCD-DJIH. When only demand is unstable, the overall expected social gains are measured by BCG + GHJ.

The workings of supply stabilization when there is a random element affecting the supply function is shown in figure 3(b). The free market solution with S_1 would be at B where supply is too small. The government would then buy DC stocks to bring about a price increase to C which in turn stimulates an increase in supply to the desired target level.

Similarly, the free market solution with S_2 would be at G where supply is larger then the target. The government then would sell HI stocks, reducing price to H. At H producers will supply the target quantity S_s. As a result producers will suffer an expected loss measured by the area FGHJ-ECAB while consumers will gain an expected surplus measured by the area FGIJ-ABED. In total, when only supply is unstable, the expected social gains are measured by the area BCD + GHI.

The expected gains and losses resulting from a supply stabilizing scheme are summarized as follows:

$$(23) \qquad EG_c(S) = \frac{(2\beta_1 + \alpha_1)}{2\beta_1^2 (\alpha_1 + \beta_1)^2} \left[\alpha_1^2 \sigma_v^2 + \beta_1^2 \sigma_u^2 \right]$$

$$(24) \qquad EG_s(S) = \frac{-1}{2\beta_1(\alpha_1 + \beta_1)^2} \left[\alpha_1^2 \sigma_v^2 + \beta_1^2 \sigma_u^2\right]$$

$$(25) \qquad EG(S) = \frac{1}{2\beta_1^2 (\alpha_1 + \beta_1)} \left[\alpha_1^2 \sigma_v^2 + \beta_1^2 \sigma_u^2\right]$$

Unlike the price stabilizing scheme, supply instability is beneficial for consumers when the amount supplied is stabilized. The reason is that consumers are better off if allowed to move freely on a given demand function than if they are forced to consume the expected quantity at the expected price.

Also, unlike the price stabilizing scheme, supply schedule instability is detrimental to producers when the amount supplied is stabilized. In this case, government actions destabilized market prices as compared with the free market prices. In other words, when free market prices would have been relatively high, production stabilization makes them even higher, causing an increase in production. If free market prices would have been low, production stabilization makes them lower still, inducing a decline in production. Yet when prices are relatively high, free market production is relatively low and when prices are relatively low, free market production is relatively high. Therefore, any gains derived from an increased production at relatively high prices would be smaller than the losses incurred by decreasing production at relatively low prices.

Contrast this with price stabilization. When prices are stabilized and supply is unstable, just the opposite happens. If free market equilibrium prices are high, government sells stocks inducing a decline in price and a decreased production. But since equilibrium production is small, at the relatively high prices, the loss incurred by decreasing production is relatively small. By the same reasoning, when the free market equilibrium prices are relatively low, government buys stocks to increase prices and thereby production. Since this occurs when production is relatively high, the gains derived by the increased production are high.

It also should be noticed that if the only source of instability is in demand, stabilizing the quantity supplied will result in the same behavior as when prices are stabilized.

Similar to the consumption stabilization scheme, a supply stabilization scheme seems to indicate an expected budgetary deficit measured as $\frac{\alpha_1}{\beta_1^2} \sigma_v^2$. For the reasons already stated, we will abstract from this aspect of the stabilizing schemes.

For future reference, we show the relation between the expected gains from consumption stabilization and the variance of the changes in government stocks $[\sigma^2(S_s)]$. From (22) we get:

$$\sigma^2(S_s) = \frac{\alpha_1^2 \sigma_v^2 + \beta_1^2 \sigma_u^2}{\beta_1^2} \tag{26}$$

Then

$$EG(S_s) = \frac{1}{2(\alpha_1 + \beta_1)} \sigma^2(S_s) \tag{27}$$

<u>Summary of Stabilization Schemes</u>

Three stabilization schemes have been analyzed under the assumption that supply reacts instantaneously to a change in price. The welfare implications of the different schemes for consumers, producers, and society as a whole have been shown. These welfare implications do not depend on whether the system is static or whether equilibrium solutions change through time.

The expected welfare gains at every point of time are independent. On the other hand the budgetary implications of the different schemes cannot be analyzed in the context of static models. They depend on the time path of the equilibrium values of prices and quantities. Consequently, we abstract from any budgetary considerations when we compare the stabilizing schemes.

In terms of the expected social benefits of the schemes that were analyzed, the following conclusions can be drawn:

(a) If $\alpha_1 > \beta_1$:

$$EG(S_s) > EG(P_s) > EG(D_s)$$

(b) If $\beta_1 > \alpha_1$:

$$EG(D_s) > EG(P_s) > EG(S_s)$$

These results have a cost counterpart measured by the stocks needed by the government in order to carry out the different stabilizing schemes. Using the Central Limit Theorem of statistics, we may assume that the needed changes in government stocks are approximately normally distributed. (This assumption becomes more realistic, the larger the model and the more sources of instability are built into the system.) It follows that in order to be effective in 95% of all possible cases, the government would need to hold 1.96 times the variance of stocks indicated under any particular scheme. If a cost function for carrying stocks is known, then a price tag could be attached to each of the stabilizing schemes. But even if such cost functions are not known, our analysis at least enables us to rank the different schemes according to their relative costs. These relative costs are directly related to the size of the variance in government stocks associated with the different schemes. We have shown that for every stabilizing scheme the variance of the change in government stocks is proportional to expected social gains. Moreover, the proportionality coefficient is equal for all of the stabilizing schemes. So it follows that:

(a) If $\alpha_1 > \beta_1$:

$$\sigma^2(S_s) > \sigma^2(P_s) > \sigma^2(D_s)$$

(b) If $\beta_1 > \alpha_1$:

$$\sigma^2(D_s) > \sigma^2(P_s) > \sigma^2(S_s)$$

In other words, the more beneficial the scheme, the higher the costs involved in affecting that scheme.

IV. Stabilizing Schemes Which Assume Rational Expectations

The assumption that supply reacts instantaneously to price changes is not a realistic assumption for many agricultural production processes. Characteristically once the level of production has been decided, changing market conditions will have little if any bearing on the actual quantity supplied.

Therefore, the quantity supplied in any marketing period will not depend on current price but on expected price as visualized by producers at the time their production decision is made. It follows that the basic model which now will be analyzed for the different stabilizing schemes is:

$$(28) \quad \begin{aligned} D(P_t) &= \alpha_o - \alpha_1 P_t + u_t \\ S(P_t^*) &= \beta_o + \beta_1 P_t^* + v_t \end{aligned}$$

where P_t^* is the expected price as visualized by producers at the time when the production decision for the marketing period t is made.

It will be assumed that price expectations are "rational" as Muth defines the term. [3] This means that producers decide on output according to the expected equilibrium price that would be obtained from the solution of the structural model. It follows that the expected price is:

$$P^* = \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} \tag{29}$$

In the absence of any stabilizing intervention, the competitive market solutions when producers expectations are rational is as follows:

$$(30) \quad \begin{aligned} P_e &= \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} + \frac{u_t - v_t}{\alpha_1} \\ Q_e &= \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} + v_t \end{aligned}$$

The expected gains and losses from the different stabilizing schemes will then be obtained following the same procedure as used in Section II. In the following discussion of the different stabilizing schemes, attention will be given only to those results which are qualitatively different from those already obtained.

Price Stabilization

Price is stabilized at:

$$P_s = \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} \tag{31}$$

As price is stabilized at P_s, the market quantities demanded and supplied, and the change in government stocks are:

$$D(P_s)_t = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} + u_t$$

$$(32) \qquad S(P_s)_t = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} + v_t$$

$$\Delta G_t(P_s) = v_t - u_t$$

When these solutions are compared to the free market solution, the expected gains from price stabilization by consumers, producers and in total are obtained as follows:

$$(33) \qquad EG_c(P_s) = \frac{1}{2\alpha_1} (\sigma_u^2 - \sigma_v^2)$$

$$(34) \qquad EG_s(P_s) = \frac{1}{\alpha_1} \sigma_v^2$$

$$(35) \qquad EG(P_s) = \frac{1}{2\alpha_1} (\sigma_u^2 + \sigma_v^2)$$

Technically these results differ from those presented in (9), (10) and (11) only to the extent that $\beta_1 = 0$. Quantitatively, the difference is because now producers do not prefer to produce at random along any given supply function because of random shifts in demand as was the case when the amount supply reacted instantaneously to a price change. Graphically, this can be shown in figure 4.

When only demand is unstable and D_1 and D_2 are equally likely, then the free market solutions would be at B and E. With no price stabilization, the producers would either lose the amount measured by ABCD or gain the amount

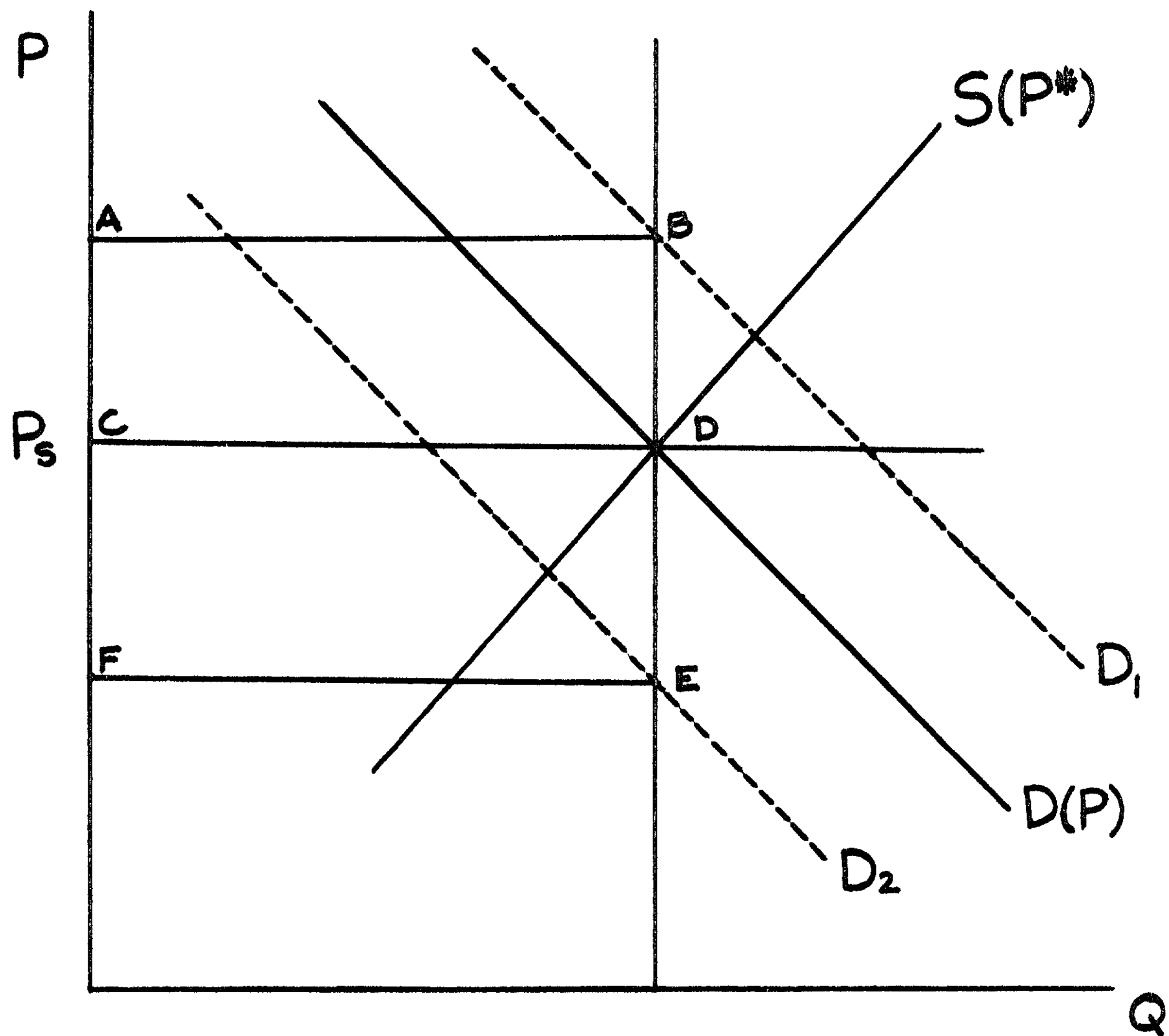


FIGURE 4.

CDEF. Since on the average they do not lose or gain with a free market situation, it follows that by stabilizing price at P_s they also do not lose or gain.

As in the previous case, there is/a simple relation between the expected social gains and the variance of the changes in government stocks, $\sigma^2(P_s)$. Since $\sigma^2(P_s) = \sigma_u^2 + \sigma_v^2$

it follows that:

$$EG(P_s) = \frac{1}{2\alpha_1} \sigma^2(P_s) \quad (36)$$

Stabilization of Consumer Demand

The quantity demanded is stabilized at the expected free market equilibrium as follows:

$$D_s = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} \quad (37)$$

The stabilization of consumer demand implies that the market price, the quantity supplied, and the changes in government stocks will be as follows:

$$(38) \quad \begin{aligned} P(D_s)_t &= \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} + \frac{1}{\alpha_1} u_t \\ S(D_s)_t &= \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} + v_t \\ \Delta G(D_s)_t &= v_t \end{aligned}$$

Notice that the government will buy or sell stocks to stabilize consumer demand only if supply is unstable.

The stabilization of consumer demand will generate the following expected gains:

$$(39) \qquad EG_c\ (D_s) = \frac{-1}{2\alpha_1}\ \sigma_v^2$$

$$(40) \qquad EG_s\ (D_s) = \frac{1}{\alpha_1}\ \sigma_v^2$$

$$(41) \qquad EG\ (D_s) = \frac{1}{2\alpha_1}\ \sigma_v^2$$

Clearly, these results are the same as those in (16), (17) and (18) when $\beta_1 = 0$. The gains now are independent of demand instability. We have already discussed the reason for the independence of producers' gains from demand instability. The reason for the independence of consumer gains from demand instability remains to be shown.

In figure 5, A and B are free market equilibrium solutions for D_1 and D_2 respectively when rational expectations apply. Such solutions do not require the intervention of the government in order to satisfy the consumption target. Therefore, when consumption is stablized and only demand is unstable, the free market solution will be the same as the stabilized solution. Since the solutions are the same, it follows that demand instablity will have no effect on consumer welfare.

Unlike the case in which demand was stabilized and supply reacted instantaneously to a change in price, now there will be no expected deficit in the government budget.

The variance of the change in government stocks is:

$$(42) \qquad \sigma^2(D_s) = \sigma_v^2$$

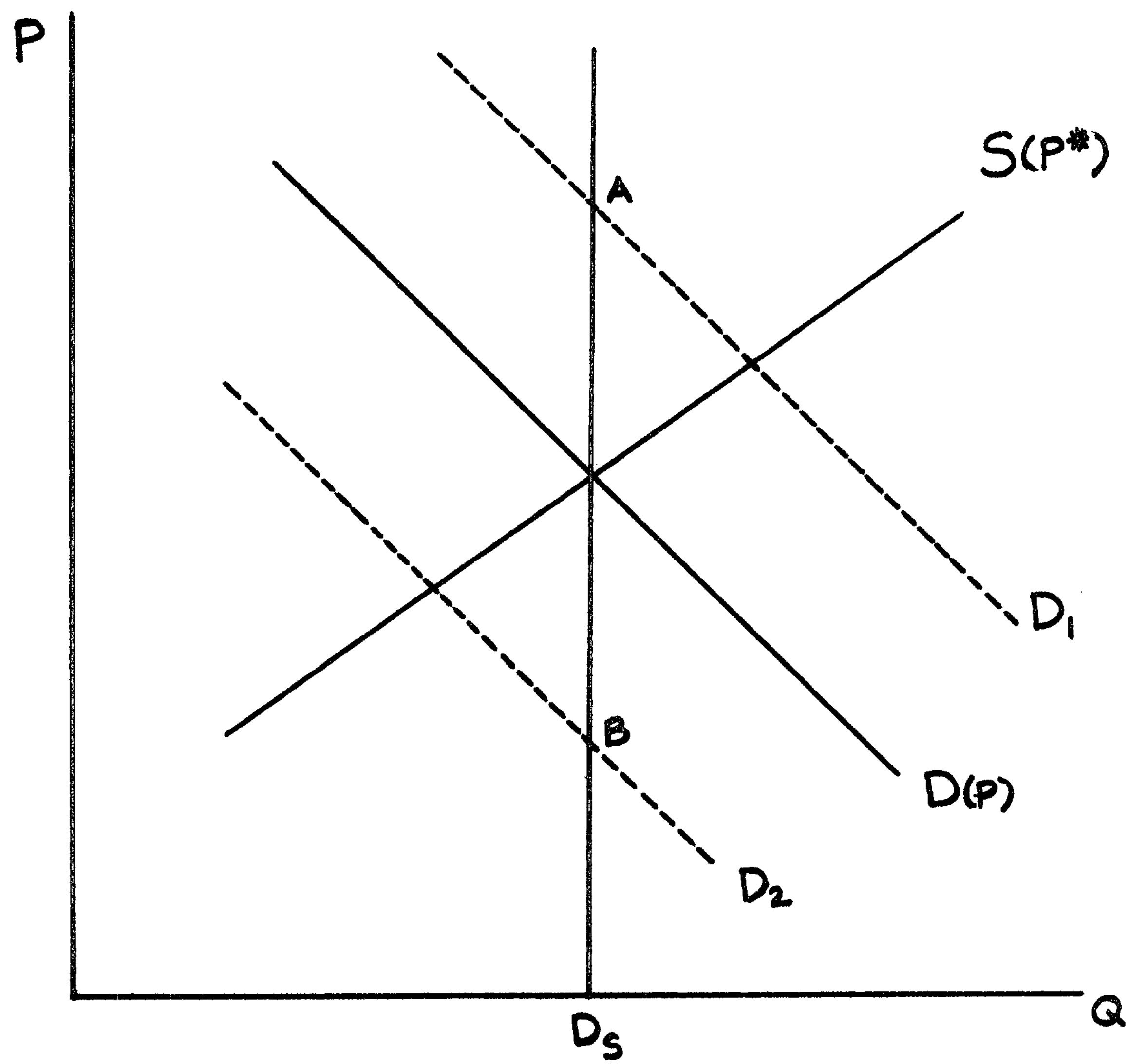


FIGURE 5.

It follows that:

(43) $$EG(D_s) = \frac{1}{2\alpha_1} \sigma^2(D_s).$$

Supply Stabilization

The supply target is set at:

(44) $$S_s = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1}$$

Clearly, once producers have decided the level of production, supply will depend only on that level of production and on the random elements affecting supply. It follows that when supply is stabilized and expectations are rational, government will have to buy and sell stocks according to total quantity supplied. But the price at which purchases and sales are made will depend on the supply target. We will return to this point later when a graphical analysis of this stabilizing scheme is presented.

When the supply target is set at S_s, the market quantities demanded and supplied, the market price, and the change in government stocks are as follows:

(45)
$$D(S_s)_t = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} + \frac{\alpha_1}{\beta_1} v_t + u_t$$

$$S(S_s)_t = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} + v_t$$

$$P(S_s)_t = \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} - \frac{1}{\beta_1} v_t$$

$$\Delta G(S_s)_t = (1 - \frac{\alpha_1}{\beta_1}) v_t - u_t$$

The expected gains generated by this scheme are:

$$(46) \qquad EG_c(S_s) = \frac{1}{2\alpha_1} \left(\sigma_u^2 + \frac{\alpha_1^2 - \beta_1^2}{\beta_1^2} \sigma_v^2\right)$$

$$(47) \qquad EG_s(S_s) = \frac{\beta_1 - \alpha_1}{\alpha_1 \beta_1} \sigma_v^2$$

$$(48) \qquad EG(S_s) = \frac{1}{2\alpha_1} \left[\sigma_u^2 + \left(1 - \frac{\alpha_1}{\beta_1}\right)^2 \sigma_v^2\right]$$

A better understanding of these analytical results may be gained by tracing out geometrically the effects of demand and supply instability on the measures of social welfare.

In figure 6 we examine only the effects of unstable demand. The supply target is satisfied at S_s. If demand shifts to D_1, the free market equilibrium is at B since S_s is supplied under the rational expectations assumption of producers' behavior. It is only at C where the price paid by consumers is the same as the one expected by producers and upon which they planned their output. It is in this sense that supply stabilization has to be understood when expectations are rational.

To achieve a solution as C the government has to sell stocks, driving the price down to P_e. Similarly, if the free market solution would have been as F with D_2, the government would buy stocks driving the price up to P_e. If D_1 and D_2 are equally likely, the expected gains by consumers will be increased by the area of ABCD-DEFG which is clearly positive. On the other hand, it can easily be shown that the expected producers' gains are ABDH-DHFG = 0.

The effects of supply instability are more complex. While the effect of supply instability is beneficial for the total expected gains, its

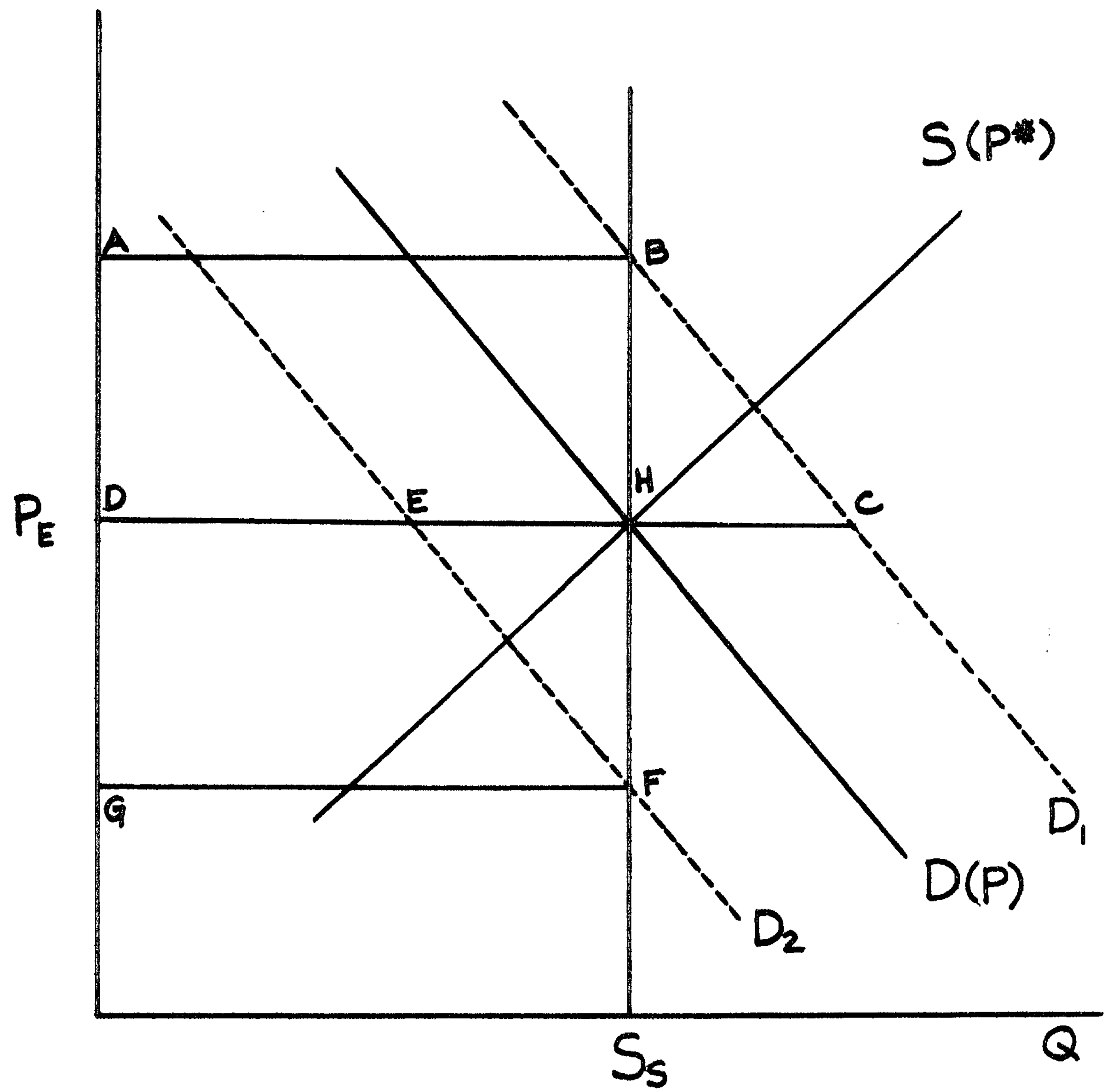


FIGURE 6.

differential effect on consumers and on producers depends on the relative size of α_1 and β_1. When $\alpha_1 > \beta_1$, supply instability is beneficial for expected consumer gains and detrimental for expected producer gains. The opposite holds for $\beta_1 > \alpha_1$.

In figure 7 we illustrate the effects of supply instability when $\alpha_1 > \beta_1$. The supply curve $S(P^*)$ remains constant, the random effects only change the quantity supplied after the production decision is made. We assume that a shift to either S_1 or to S_2 is equally likely. When S_1 is supplied, government buys BE of stocks driving the price up to E. This is because E is the price which producers would have needed to produce S_s if they could adjust their production given the random disturbance which actually occurred. Similarly if S_2 is supplied, government will sell JH of stocks driving down the price to J. As a result, the consumers' expected gains are FGHI-ABCD. Producers' expected losses are FGIJ-AEDC. The total expected gains are BEC + GJH.

Figure 8 illustrates the effects of supply instability when $\beta_1 > \alpha_1$. Assume S_1 and S_2 are equally likely shifts of the quantity supplied. With S_1, the price which would induce producers to produce S_s (if they would be able to adjust given the S_1 disturbance) is smaller than the free equilibrium price. Similarly with S_2 where the price which would induce producers to adjust production to S_s is higher than the free market equilibrium price.

It follows that if S_1 happens, government will sell EC stocks driving the price down to C. If S_2 happens, government will buy GJ stocks driving the price up to G. As a result, consumers' expected losses are HGFI-ABCD, producers' expected gains are FJHI-ABDE, and total expected net gains are measured by BCE + GJH.

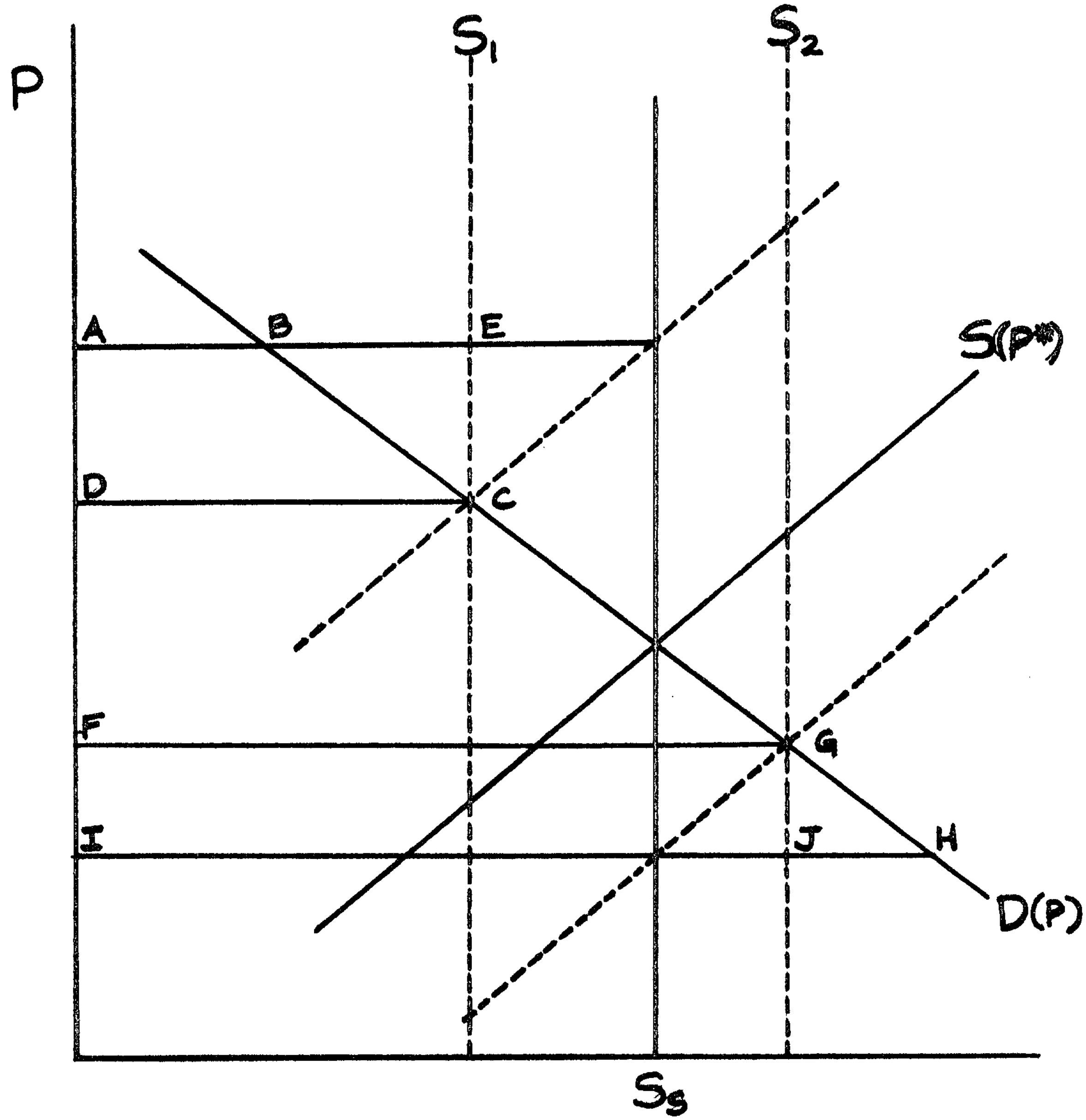


FIGURE 7.

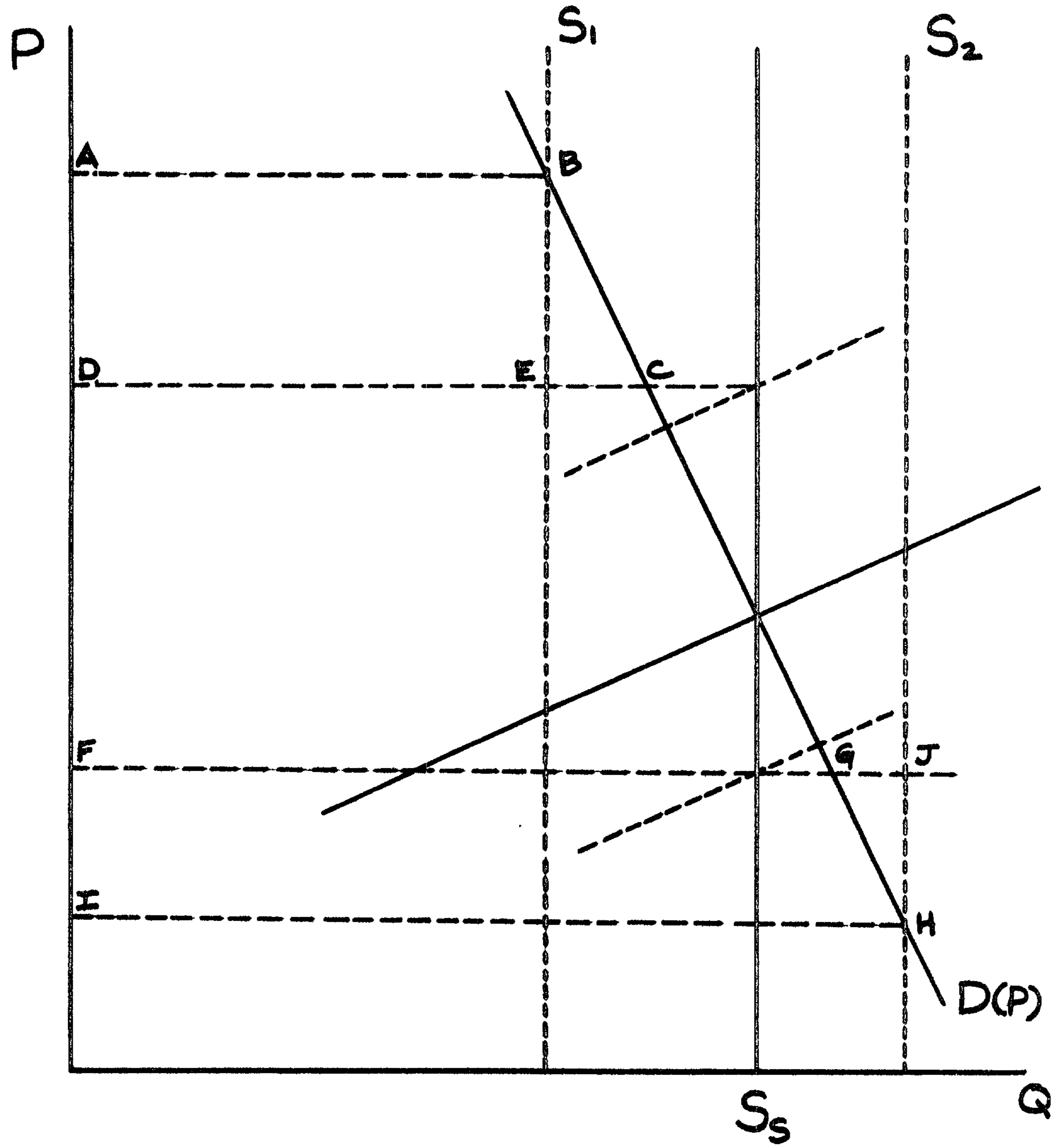


FIGURE 8.

Since government intervention in stabilizing supply may result in either increasing price variability or decreasing it, (depending on the relative sizes of α_1 and β_1) we may expect either a deficit or a surplus in government expenditures when considering a static model. In such a case the expected deficit is $\frac{\sigma^2}{\beta_1^2}(\alpha_1 - \beta_1)$. When $\alpha_1 > \beta_1$, prices are destabilized and a deficit occurs. When $\beta_1 > \alpha_1$, prices are more stable, and there will be a surplus. At any rate, as already mentioned, it is difficult to incorporate any clear ideas about government costs without knowing the equilibrium path of prices and quantities.

Once again we derive the variance of the changes in government stocks as:

$$(49) \qquad \sigma^2(S_s) = (1 - \frac{\alpha_1}{\beta_1})^2 \sigma_v^2 + \sigma_u^2.$$

This variance is related to the expected social benefits as follows:

$$(50) \qquad EG(S_s) = \frac{1}{2\alpha_1} \sigma^2(S_s)$$

Summary

Considering only the expected social gains and abstracting from budgetary considerations, the following can be easily shown:

(a) If $\alpha_1 \geq 2\beta_1$

$$EG(S_s) \geq EG(P_s) > EG(D_s)$$

(b) If $\alpha_1 < 2\beta_1$

$$EG(P_s) > EG(S_s) > EG(D_s)$$

If we assume that the changes in government stocks are normally distributed, it will be necessary to hold 1.96 times the variance of stocks in every period to achieve a given stabilizing target 95% of the time. The storage costs involved will be ranked in the same order as the expected gains from the different stabilizing schemes. This follows from the proportionality relation between expected social costs and the variance of the change in government stocks. We therefore conclude that:

(a) If $\alpha_1 \geq 2\beta_1$:

$$\sigma^2(S_s) \geq \sigma^2(P_s) > \sigma^2(D_s)$$

(b) If $\alpha_1 < 2\beta_1$:

$$\sigma^2(P_s) > \sigma^2(S_s) > \sigma^2(D_s).$$

V. Stabilization Schemes When Foreign Demand is Unstable

Our previous results may be easily extended to cover situations in which foreign trade may add an additional source of instability to prices and quantities in the internal market. Hueth and Shmitz analyzed the welfare implications of a price stabilization scheme when foreign markets are unstable and when internal supply reacts instantaneously to a change in price. [8]

In this section the welfare implications of the different stabilizing schemes will be analyzed when foreign markets are unstable and when supply either reacts instantaneously to a change in price or reacts to a change in rational expectations. In all cases, it will be assumed that the demand for exports is inelastic. This assumption facilitates the introduction of instability in the foreign markets market without depriving generality from our results. Therefore, the level of exports will be reflected by

(50) $$x_t = \bar{x}_t + \varepsilon_t$$

where $\bar{x}_t$ = expected level of exports at time t (known ahead of time)

ε_t = random component of exports.

It will be assumed that ε_t is independently distributed of u_t and v_t and that its variance is finite at σ_ε^2 . In addition, we assume that the country facing this fluctuating demand is the only source of supply.

Supply Reacts Instantaneously to Price

The basic model becomes

(51) $$D(P_t) = \alpha_o - \alpha_1 P_t + u_t$$
$$S(P_t) = \beta_o + \beta_1 P_t + v_t$$
$$D(P_t) + x_t = S(P_t) .$$

The free market solutions of domestic consumption, world price and supply are

(52) $$D_E = \frac{\alpha_o\beta_1 + \alpha_1\beta_o}{\alpha_1 + \beta_1} - \frac{\alpha_1}{\alpha_1 + \beta_1}(\bar{x}_t) + \frac{\beta_1 u_t + \alpha_1(v_t - \varepsilon_t)}{\alpha_1 + \beta_1}$$

(53) $$P_E = \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} + \frac{1}{\alpha_1 + \beta_1}(\bar{x}_t) + \frac{u_t - v_t + \varepsilon_t}{\alpha_1 + \beta_1}$$

(54) $$S_E = \frac{\alpha_o\beta_1 + \alpha_1\beta_o}{\alpha_1 + \beta_1} + \frac{\beta_1}{\alpha_1 + \beta_1}(\bar{x}_t) + \frac{\beta_1(u_t + \varepsilon_t) + \alpha_1 v_t}{\alpha_1 + \beta_1}$$

By assumption, all sources of instability are independent. Since we have already dealt with demand and supply instability and since these effects are additive we may now focus attention on export instability while abstracting from any other source of instability.

We shall presently show that for all the stabilizing schemes analyzed in this paper (stabilizing prices, stabilizing domestic consumption and stabilizing production), export instability has the same effect. This can be shown by a graph, figure 9. In figure 9, let

$$ED_E = \bar{D}$$

$$EP_E = \bar{P}$$

$$ES_E = \bar{S} \ .$$

Namely, the barred variables, which are the expected values of the endogeneous variables, are the levels at which these variables are stabilized.

Let $(D + \bar{x})_1$ be as likely to happen as $(D + \bar{x})_2$. If price is stabilized, it will be set at $\bar{P}$. It follows that if $(D + \bar{x})_1$ occurs, government will sell the amount of AB stocks and drive the price down to $\bar{P}$. At the stabilized price, domestic consumption will be at $\bar{D}$ and production, at $\bar{S}$. Similarly, if total demand happens to shift to $(D + \bar{x})_2$, government will buy AC stocks and prices will settle at $\bar{P}$. Notice that at $\bar{P}$, domestic consumption and production remain at $\bar{D}$ and $\bar{S}$ respectively.

When production is stabilized, it is set at $\bar{S}$. If a random shift to $(D + \bar{x})_1$ happens, production can be brought back to $\bar{S}$ only if market price is made to decline to P. This will be achieved after government sells AB stocks. Domestic demand settles at $\bar{D}$. By the same token, if the random shift is towards $(D + \bar{x})_2$, supply will be increased to $\bar{S}$ only if the price is raised to $\bar{P}$. This will be achieved if government buys AC stocks. Again domestic demand settles at $\bar{D}$.

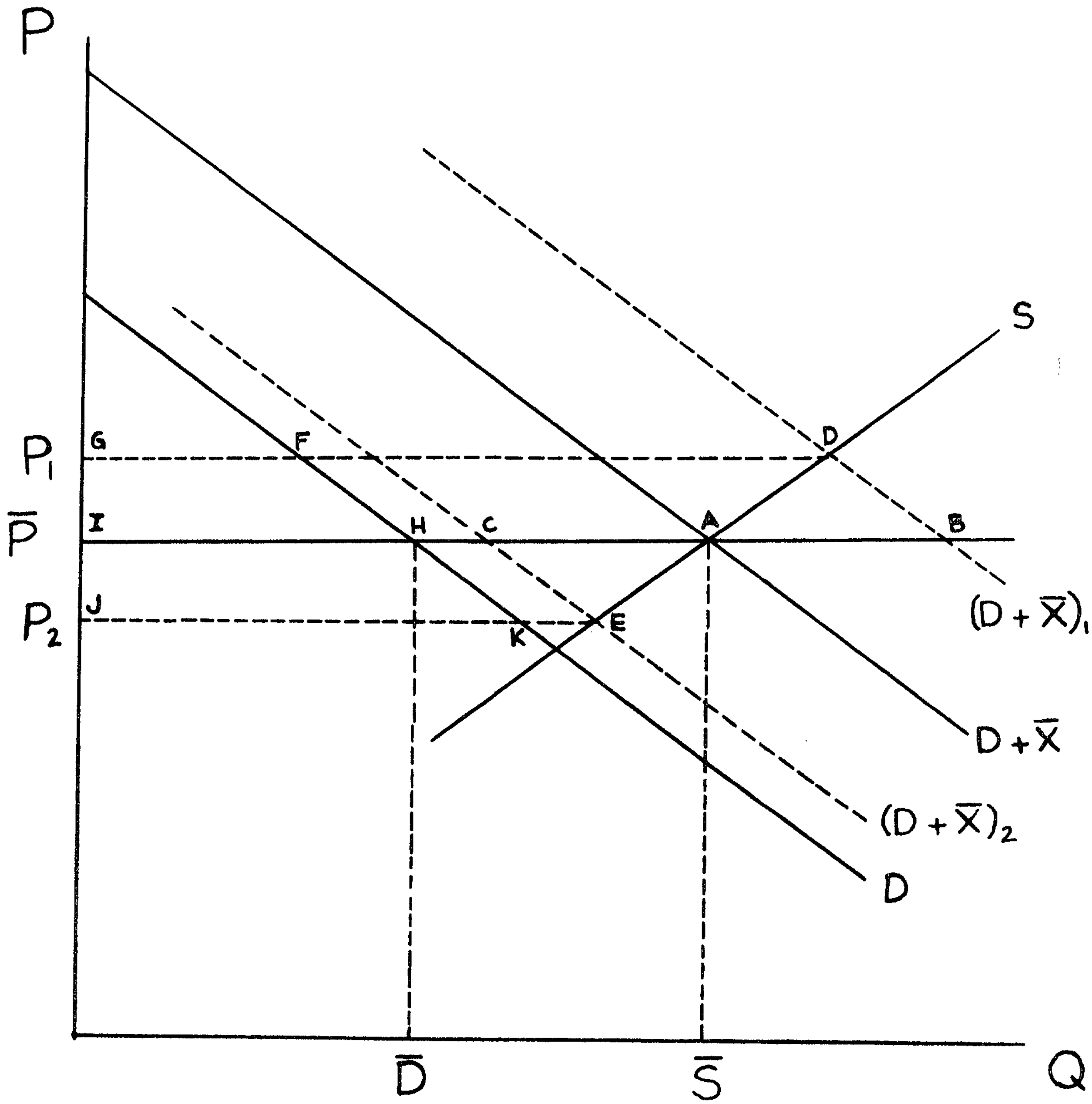


FIGURE 9.

When domestic consumption is stabilized, its level is set at $\bar{D}$. By the same procedure as outlined above, domestic consumption will be stabilized at $\bar{D}$ only if the price is $\bar{P}$ and supply is $\bar{S}$. This will be achieved only if government will counteract by either selling AB stocks or buying AC stocks.

We therefore conclude that when exports are unstable all stabilization schemes analyzed converge into the same behavior and have the same effects on the expected gains of producers and domestic consumers.

In terms of the graphical analysis the expected gains in consumer welfare is negative and measured by the area of HIJK-FGIH. Similarly, the expected gains in producers surplus is negative and measured by the area of AIGD-AEJI.

It can be shown that for all the stabilizing schemes analyzed, the expected gains of producers, domestic consumers and for the stabilizing country as a whole when exports are unstable, are

(56) $$EG_C = - \frac{\alpha_1}{2(\alpha_1 + \beta_1)^2} \sigma_\varepsilon^2$$

(57) $$EG_S = - \frac{\beta_1}{2(\alpha_1 + \beta_1)^2} \sigma_\varepsilon^2$$

(58) $$EG = - \frac{1}{2(\alpha_1 + \beta_1)} \sigma_\varepsilon^2$$

We conclude that in the face of export instability (when exports are the only source of instability in the system) no stabilizing scheme will improve social welfare. This result stems from the fact that when exports are unstable, all stabilizing schemes will be detrimental to domestic consumers as well as to domestic producers. That export instability is detrimental to producers gains is no surprise since all sources of demand instability are detrimental to these gains. The reason that domestic consumers will also suffer from any stabilization scheme when exports are unstable stems from the fact that by stabilization the gains obtained by avoiding higher prices is smaller than the loss derived from not being able to buy at the lower prices.

Clearly, if we include the welfare effects of stabilization on the importing country, the above results will change. Returning to the graph in figure 9, it can be seen that in this broader case, the expected consumer gains (domestic and abroad) become positive and are measured by the area of GDIB-ICEJ. Expected producer gains do not change. Analytically, it can be shown that the total expected gains (domestic and abroad) from stabilization policies at home becomes

$$(59) \qquad EG^T = \frac{1}{2(\alpha_1 + \beta_1)} \sigma_\varepsilon^2$$

We find that in total, the world as a whole benefits from any stabilization scheme in a single exporting nation when exports demand is unstable. The difference between the world expected gains and the domestic expected gains are the expected gains which the importing country derives from stabilizing the domestic market

$$(60) \qquad EG^T - EG = \frac{\sigma_\varepsilon^2}{\alpha_1 + \beta_1}$$

This expected benefit can be shown to be equal to the expected savings in import outlays due to stabilization by the exporter. This can be shown as follows

$$(61) \qquad M = (\bar{P} + \eta)(\bar{x} + \varepsilon)$$

where

$$M = \text{Import outlay}$$

$$\eta = \frac{\varepsilon}{\alpha_1 + \beta_1}$$

When the internal market is not stabilized, the expected outlay becomes

$$EM = \overline{PX} + \frac{\sigma_\varepsilon^2}{\alpha_1 + \beta_1} \tag{62}$$

On the other hand, when the internal market is stabilized, the expected outlay becomes

$$EM^* = \overline{PX} \tag{63}$$

Therefore

$$EM - EM^* = \frac{\sigma_\varepsilon^2}{\alpha_1 + \beta_1} \tag{64}$$

are the additional expected import outlays when the domestic market is not stabilized. As such it represents expected savings to the importing country due to market stabilization at home.

These results indicate some of the trade policy issues which become relevant when stabilization schemes are considered. For example, the export nation might wish to eliminate the source of export instability by contracting in advance with foreign countries the amounts which they will import. This might require the administration of some kind of a buffer stock scheme in the importing countries. Alternatively, if importing countries are not willing to contract in advance, the exporter might stipulate export quotas in advance. If no quotas or advanced contracting are possible, the domestic market may be stabilized and importing countries may be levied an amount equal to $\frac{\sigma_v^2}{\alpha_1 + \beta_1}$. This result will be beneficial to the exporter and lead to free market behavior in world markets.

Finally, since export instability identically affects the expected welfare gains derived from each of the stabilization schemes, its occurance does not affect the relative ranking of the schemes in terms of welfare gains or storage costs.

- Supply Determined By Rational Expectations

Following the earlier formulation of ration expectations, the model now becomes

$$D(P_t) = \alpha_o - \alpha_1 P_t + u_t$$

$$(65) \qquad S(P_t) = \beta_o + \beta_1 P_t^* + v_t$$

$$D(P_t) + x_t = S(P_t^*)$$

where because of rational expectations

$$(66) \qquad P_t^* = \frac{\alpha_o - \beta_o}{\alpha_1 + \beta_1} + \frac{1}{\alpha_1 + \beta_1} (\bar{x}_t)$$

It follows that the free market solutions are

$$(67) \qquad D_E = \frac{\alpha_o \beta_1 + \alpha_1 \beta_o}{\alpha_1 + \beta_1} - \frac{\alpha_1}{\alpha_1 + \beta_1} (\bar{x}_t) - \varepsilon_t + v_t$$

$$(68) \qquad P_E = \frac{\alpha_o - \beta_o}{\alpha + \beta} + \frac{1}{\alpha + \beta} (\bar{x}_t) + \frac{u_t - v_t + \varepsilon_t}{\alpha}$$

the importing countries. Alternatively, if importing countries are not willing to contract in advance, the exporter might stipulate export quotas in advance. If no quotas or advanced contracting are possible, the

As with the previous case, export instability in the presence of rational expectations identically affects the expected welfare gains derived from each of the stabilization schemes. Its occurance therefore does not affect the relative ranking of the schemes in terms of welfare gains or storage costs.

Just as in the previous case, it can be shown that export instability has the same effects on all the stabilization schemes analyzed.

Notice that if the only source of instability is in exports, the previous graphical analysis carries over to the present model with the difference that supply becomes completely inelastic at its expected equilibrium level. This implies that, in the absence of any other source of instability, export instability will have no effect on the expected gains of producers.

Analytically, it can be shown that

$$EG_C = EG = -\frac{1}{2\alpha_1}\sigma_\varepsilon^2$$

In this case, stabilization policies at home when exports are unstable become a burden only on domestic consumers. This result stems from the fact that, by stabilizing the internal market, the gains derived from avoiding higher prices are smaller than the losses derived from not being able to buy at the lower prices.

Following the same procedure as outlined in the previous section it can be shown that stabilization by the exporter brings about expected savings in outlays by importers equal to $\frac{\sigma_\varepsilon^2}{\alpha_1}$. These are the gains which the importing country obtains from stabilization by the exporter.

VI. Summary

This paper has presented analysis of welfare implications of the deliberate stabilization of either consumption or production and then compared those results with the already-known welfare implications of stabilized prices. Two sets of assumptions regarding supply behavior were considered, and unstable export demand also was added to the plausible, but abstract, theoretical models.

The ordinal welfare ranking of these three schemes depends upon the relative size of the demand and supply price response coefficients. In most cases, price stabilization is intermediate in its welfare implications between consumption stabilization or production stabilization. To the extent that the relative costs of operating various stabilization schemes are directly related to the variance in government stocks associated with them, the ordinal ranking from high to low on welfare grounds is the same as the ordinal ranking of high to low on cost grounds.

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