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ON FARMERS WHO SOLVE EQUATIONS

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> This sense of the word (abstractness) is important, and the logicians are quite right to stress it, since it embodies a truism which a good many people who ought to know better are apt to forget. It is quite common, for example, for an astronomer or a physicist to claim that he has found a 'mathematical proof' that the physical universe must behave in a particular way. All such claims, if interpreted literally, are strictly nonsense. It cannot be possible to prove mathematically that there will be an eclipse to-morrow, because eclipses, and other physical phenomena, do not form part of the abstract world of mathematics; and this, I suppose, all astronomers would admit when pressed, however many eclipses they may have predicted correctly.
>
> G. H. Hardy (p. 47)

In 1987, the American Journal of Agricultural Economics printed what is surely the most radical analysis of farmer behavior in the history of human thought:

"Farm households, therefore, solve

$$(1) \quad \max_{c,L_1,L_2} \int_0^{\infty} u(c(t),\ H - L_1(t) - L_2(t))e^{-\delta t}\, dt,$$

subject to

$$(i) \quad \dot{E} = \rho(E(t),\ L_1(t),\ v) + wL_2(t) + y(t) - c(t)$$

$$(ii) \quad E(0) = \bar{E}." \quad \text{(Chambers and Lopez, p. 370)}$$

At least I thought it was a radical analysis. I later discovered that it is a rather common notion among certain researchers that farmers

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routinely tackle even the most intractable of equations. For example, the 1988 AAEA Outstanding Journal Article award went to the developer of an econometric model which required data "assumed to be generated by farmers solving a single-period maximization problem". (Antle, p. 510)

Nor do only farmers reach for their trusty calculus books in times of crisis. When milk was found to be contaminated in Hawaii, the hapless citizens of Oahu found their problem to be one of solving this beauty:

"max $L = U(X_1(Z_1(N)),X_2) + \lambda(I - P_1X_1 - P_2X_2 - CN)$"

(Smith et al., p. 513)

Even though many agricultural economists have assumed farmers solve equations, none have reported the names and addresses of those farmers. To make matters worse, the farmers I work with are either not of the equation solving variety or too modest to admit to being so. This being the case, I am left with little choice but to adopt the working hypothesis that farmers do not really solve these equations. I offer my apologies to those who claim they do, however, and encourage them to keep a sharp eye out for these most interesting of life forms. My task here will be the more modest one of investigating the wisdom of assuming that farmers finish up a hard day in the fields with a bout of equation solving.

I suspect that most would, with a quick tip of the hat to Milton Friedman's defense of "positive economics", say that farmers merely act "as if" they solved equations. Most economists have at one time or another spent some time with Friedman in his world where leaves on a tree act "as if each leaf deliberately sought to maximize the amount of sunlight it receives, given the position of its neighbors, as if it knew

the physical laws determining the amount of sunlight that would be received in various positions and could move rapidly and instantaneously from any one position to any other desired and unoccupied position". (Friedman, p. 19) This world, too, is one in which an expert billiard player acts "as if he knew the complicated mathematical formulas that would give the optimum directions of travel, could estimate accurately by eye the angles, etc., describing the location of the balls, could make lightning calculations from the formulas, and could then make the balls travel in the direction indicated by the formulas." (Friedman, p. 21).

"As if" turns out to be a powerful concept, indeed, in the hands of a positive economist. It, in effect, frees one from mundane concerns over the truth of assumptions. Only how the results of analysis beginning with "as if" coincide with observations of reality need matter. We therefore need not worry about whether farmers solve equations, so long as they act as if they do.

Let me add one more "as if" to Friedman's collection: "The gunfire pierced the night as if it were thundering". This type of statement, the simple simile, has been around for a long time. It is often descriptive to compare two unlike things such as gunfire and thunder. Properly applied, similes add richness in normal conversation and beauty in poetry.

The simile belongs to the world of individual perception. While I might think gunfire sounds like thunder, you may disagree. If so, the value of my simile in communication with you is clearly limited. Whether gunfire really sounds like thunder may even become a point of contention and thereby seriously hamper our originally-intended conversation. We

must therefore choose our similes to reflect the common experience of many if clarity of conversation is our goal.

To say that leaves know physics, that billiard players mimic high speed computers, or that farmers solve equations are also similes; no more, no less. They relate what is, in the eye of the beholder, a common property shared by two dissimilar worlds. These particular similes are, in my opinion, so poorly chosen that their fantastic nature detracts from clear conversation. This in no way, of course, implies that their authors feel otherwise.

While authors may choose similes as they see fit, they may not use them beyond their conventional descriptive limits. The simile provides no basis whatsoever for logical analysis. The mathematician G. H. Hardy was moved by a particularly fine piece of poetic simile to comment: "Could lines be better, and could ideas be at once more trite and more false?" (p. 24) It makes good sense to say that gunfire sounds like thunder, but no sense whatsoever to further infer that the presence of gunfire means rainfall is imminent.

Let me consider more closely the argument from gunfire to thunder to rainfall. I will write it as follows:

(1) Gunfire sounds as if it is thundering.

(2) Thunder is associated with rainfall.

(3) Gunfire sounds as if it is raining.

In (1), I apply simile. Then, in (2), I use thunder to introduce rainfall. I conclude in (3) that gunfire sounds like rain.

What went wrong? The key step in the argument is that the initial simile is used to introduce a theory (meteorology) which has nothing to

do with gunfire. All sorts of weather-related similes about gunfire can then be generated. Most will be absurd, even though a few like "gunfire looks as if it were lightning" may be appealing. But none of these conclusions, absurd or otherwise, can claim validity because meteorology was used. Nor is the argument that gunfire looks like lightning because gunfire sounds like thunder any more "rigorous" than if the lightning simile had been simply stated as an observation. Meteorology has nothing to say about gunfire, and no simile is going to change that.

The digression into the sound of gunfire now complete, I return to the connection "as if" provides between mathematics and farmers who solve equations. A typical argument might go like this:

(1) Farmers act as if they solve a particular equation.

(2) We can derive some result A from the equation.

(3) Farmers act as if result A holds.

In a particularly striking example of this type of reasoning, Chambers and Lopez state that one of the equations they have derived "implies that farmers work both on and off the farm." (p. 371) That farmers work on and off the farm is obvious; that mathematics has anything to say about where farmers work is far less obvious.

The farmer syllogism has exactly the same structure as that concerning gunfire and thunder. It begins with a simile relating two very different worlds. Then, in its second step, a theory appropriate for one part of the simile (equation solving) is assumed to apply to the second part of the simile (farmer behavior). Mathematics, rather than meteorology, is then used to derive new results from the equation farmers

"solve." The argument ends by concluding one of the new mathematical results also applies to farmers.

The implications of using mathematics in the farmer example are the same as those of using meteorology in the gunfire example. The conclusion in the farmer syllogism may or may not be true. The fact that it was derived mathematically tells us nothing more than we should be suspicious because the conclusion was drawn inappropriately. Furthermore, should we find the conclusion appealing, it would still be preferable to simply approach it directly as an "as if". At least then it would not be cloaked in false claims of "rigor", "proof", or "deduction".

In short, the mathematics in the farmer argument, no matter how sophisticated, contributes nothing. This conclusion, while perhaps a bit unsettling for research in agricultural economics, would not bother most mathematicians. To quote Bertrand Russell:

> We are prepared to say that one and one are two, but not that Socrates and Plato are two, because, in our capacity of logicians or pure mathematicians, we have never heard of Socrates and Plato. A world in which there were no such individuals would still be a world in which one and one are two. It is not open to us, as pure mathematicians or logicians, to mention anything at all, because, if we do so, we introduce something irrelevant and not formal. (pp. 196-7)

I now turn to this question: "If Bertrand Russell was unwilling to use mathematics in mentioning anything at all, why is our profession so hell-bent on using it to mention virtually everything?"

One often hears that mathematics adds rigor to arguments. But I have shown that our use of mathematics depends on the least rigorous of all claims, the simile, the "as if". Anything, no matter how absurd, can be shown with the "as if" con game. We can "rigorously" show that gunfire sounds like rainfall or that farmers buy infinitely divisible tractors.

Then, too, mathematics is at times said to somehow "quantify" things. Granted, mathematics is comfortably at home when farmers and their products are being counted, when interest rates are being calculated, and when budgets are being prepared. But this is not what is being done in modelling. In modelling, statements about how the world works are made in the language of mathematics. Nothing is quantified in this process; one simply substitutes one language for another. And, as we have seen, the choice of mathematics as a language to describe farmer behavior is no better than that of meteorology as a language to describe gunfire.

We also hear claims that using mathematics somehow simplifies our analyses. For example, Chambers and Lopez assume that farmers "derive utility not only from consumption during their current lives but also from future descendants future consumption" (p.370) on into infinity. Why make buying a candy bar such a complicated decision? The answer: "for analytic simplicity". (p. 370). And, too, why does Antle want to assume all farmers are solving single-period maximization problems? Again, it is "to simplify the presentation". (p. 510).

Claiming that framing discussions of the farm economy in mathematical terms so complicated that only a very few can participate adds simplicity is curious, to say the least. There is one sense, however, in which simplicity does arise from the passion for mathematics: the subject matter content of arguments and ensuing journal articles becomes very simple, indeed. Complex mathematics do not make for complex statements about reality; in fact, Russell seems to be saying the opposite. The more rigor we demand from mathematics, the more we

sacrifice any connection with reality, and the fewer observations of a non-trivial nature we are able to make.

Take, for example, the work of Lee and Chambers. After leading the reader through many pages bristling with equations, they come to the following important (their term) conclusion: "Farmers do not face a perfectly elastic supply of funds or credit upon which they can effortlessly draw to finance their production decisions". (p. 865) We are somehow, one supposes, to now feel more comfortable in holding what is perfectly obvious to everyone. But maybe not, because the authors finally admit that "a conclusive resolution of the issue awaits a more thorough empirical study". (p. 865)

As another example, Just and Zilberman challenge the venerable "law of supply" by pointing out that higher product prices may also bring about more price risk. This can cause risk averse producers to diversify into other crops in spite of the higher prices. Particularly since the authors only claim "may" for their statement, the results appear to need no further defense (save for a possible remark or two on why such trivial matters need to be in the literature at all). But instead of ending their article at the end of the second paragraph, the authors take the reader on a ten-page mathematical steeplechase, only to conclude what was already stated quite nicely in the introduction.

Is there an alternative to mathematics? One that is sometimes overlooked, but nonetheless a good candidate, is natural language--plain English, if you will. We often see even the most enthusiastic of math purveyors resort to an occasional "intuitive explanation" of their models. These lapses into natural language are somehow intended, one supposes, to

clarify what the equations are saying anyway. What they are in fact doing, however, is clarifying what the authors would be saying if they weren't using the mathematics. In short, they are trying to communicate with natural language.

The attempt at introducing natural language through the back door of intuitive explanations has its problems. The main one is that the intuitive explanation and the mathematics are not related in any formal way. What, for example, is intuitive about farm profits which are "twice continuously differentiable and convex in v, nondecreasing in output prices, nonincreasing in input prices, positively linearly homogeneous in v, and nondecreasing in L_1 and K"? (Chambers and Lopez, p. 371) Later, these same authors provide us with "for any given level of wealth, farmers maximize their net farm income by choosing an optimal combination of outputs, inputs, and investment." (p. 371) To claim that this explanation of farmer behavior is true is one thing; to claim it is somehow inherent in the equations and their properties is quite another.

But why bother with the equations, anyway? The so-called intuitive explanation contains all that is necessary to reach the practical conclusions of most articles. Natural language and a little elementary logic can serve our purposes quite nicely. If we as a profession would only accept natural language as a proper means of discourse on important matters, we would be freed to address complex issues without so many "simplifying assumptions": that is, in non-trivial ways.

Farmers don't solve equations. Perhaps we shouldn't, either.

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