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A Simple Procedure to Evaluate Ex-ante Producer Welfare Under Price Uncertainty

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Abstract

We propose a simple and tractable procedure for evaluating producer welfare under price uncertainty. These properties are achieved at the cost of assuming constant absolute risk aversion, where risk attitude depends on the stock of wealth but not on the flow of income. Numerical examples corroborate the procedure's properties; the validity of the constant absolute risk aversion case as an approximation is discussed.

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1. Introduction

The welfare consequences of choices made by risk averse producers under conditions of uncertainty are basic components of benefit-cost evaluations of policies that are intended to change the uncertainty conditions, e.g., price stabilization, and have therefore attracted research. The (small but growing) literature in this vein deals primarily with (i) extending welfare measures to situations of risk aversion and uncertainty and (ii) developing practical means to evaluate these measures (Newbery and Stiglitz, Chavas and Pope, Just *et al.*, Pope *et al.*). The first task has been explored in considerable detail; if anything, too much may have been accomplished as the literature offers a multiplicity of measures: the Compensating Variation (CV), the Equivalent Variation (EV) and the Certainty (money) Equivalent (CE). CV and EV are borrowed from demand analysis, whereas CE is unique to situations involving uncertainty.

The second task — the evaluation of these welfare measures — has received less attention. The traditional approach is to approximate CV and EV by the producer surplus, calculated as the area to the left of the *ex-ante* output supply function (Pope and Chavas have shown that this is a legitimate approximation in most cases). Another procedure, proposed by Larson, produces an exact evaluation of CV and EV. These procedures rely, in one way or another, on the *ex-ante* output supply and/or input demand functions. Without some restrictions on risk preferences, these functions tend to be quite complicated and often are unmanageable for empirical work; as a result, applications are scarce. The complexity of the evaluation task is further exacerbated by the multiplicity of indices, as the three indices may have different values.

The present paper attempts to develop a simple and practical mean to evaluate producer welfare under price uncertainty. With this goal in mind, we consider a particular specification of risk preferences, namely, constant absolute risk aversion where the farmers' risk attitude depends on the stock of wealth but not on the flow of income. This leads to a considerable simplification of the evaluation task: the "multiplicity problem", mentioned above, is avoided, as the three welfare indices are equal in this case, and the *ex-ante* functions are simplified considerably, which facilitates their utilization in empirical applications.

We begin, in Section 2, by specifying the sources of uncertainty and defining changes in uncertainty. Section 3 summarizes the producer decision model and the associated welfare measures. The evaluation procedure is described in Section 4 and is implemented numerically in Section 5. The validity of the constant absolute risk aversion case as an approximation of a general risk preferences structure is discussed in the closing section, which also comments on the use of the analysis in empirical works and suggests extensions.

2. Uncertainty Sources and Changes in Uncertainty

We present the analysis in the simple case of a single output, where only the output price is uncertain; the more general case of multiple outputs with uncertain input, as well as output, prices is discussed in the Appendix.

The uncertainty is represented by the distribution of the output price, which is assumed to be completely characterized by the vector $\theta = (\mu, \sigma^2, \ldots)$ of the mean, variance, and higher moments. A change in uncertainty, i.e., in the output price distribution, is thus represented by a change in θ. Each such change consists of (i) a distribution-preserving shift (DPS) and (ii) a mean-preserving spread (MPS) or shrink (MPSH). A DPS affects only the mean and is represented by the parameter m. A MPS (MPSH) involves changes in the variance and higher moments; its effect on the variance is represented by the parameter s.

A random price P′ is a DPS of P if the distribution of P′ is identical to that of P + m for some scalar m. The concept of MPS is defined in Rothschild and Stiglitz; P is a MPSH of P′ if P′ is a MPS of P. Intuitively speaking, P′ is a MPS of P if both P′ and P have the same mean but the distribution of P′ has more weights in the tails. Alternatively, P′ is a MPS of P if both have the same mean and P′ is riskier than P in the sense that risk averse individuals prefer P to P′ and are willing to pay a positive amount to move from P′ to P. Rothschild and Stiglitz have shown that these two definitions are equivalent.

We consider compound DPS/MPS shifts of the form

$$P(m,s) \overset{d}{=} P + m + (Z-\mu_z), \tag{1}$$

where P(m,s) is a compound DPS/MPS of P, m is the DPS parameter, Z is a random variable distributed independently of P with mean μ_z and variance s^2, and $\overset{d}{=}$ denotes "has the same distribution as".

Example 1: Let P and Z be two independent gamma variates with parameters (α,β) and (γ,β), respectively. The distribution of P is characterized by $\theta = (\mu,\sigma^2)$, where $\mu = \alpha\beta$ and $\sigma^2 = \alpha\beta^2$. The variable P + Z is distributed as $\text{Gamma}(\alpha+\gamma,\beta)$, and $P(m,s) = P + m + (Z-\mu_z)$ is a compound DPS/MPS for which the distribution is characterized by $\theta(m,s) = (\mu+m,\sigma^2+s^2)$ where $\mu_z = \gamma\beta$ and $s^2 = (\gamma+\alpha)\beta^2$.

Example 2: Let P be a $N(\alpha,\tau^2)$ variate truncated from below at $d \geq 0$, and let Z be $N(\mu_z,s^2)$ distributed independently of P. The distribution of P is characterized by $\theta = (\mu,\sigma^2,\ldots)$, with $\mu \equiv E\{P\} = \tau\dfrac{\phi(\tilde{d})}{1-\Phi(\tilde{d})} + \alpha$, $\sigma^2 \equiv Var\{P\} = \tau^2\left\{1 + \tilde{d}\dfrac{\phi(\tilde{d})}{1-\Phi(\tilde{d})} - \left(\dfrac{\phi(\tilde{d})}{1-\Phi(\tilde{d})}\right)^2\right\}$, and $\tilde{d} = (d-\alpha)/\tau$ (see Appendix). Thus, $P(m,s) = P + m + (Z-\mu_z)$ is a compound DPS/MPS with the moment vector $\theta(m,s) = (\mu+m,\sigma^2+s^2,\ldots)$.

3. Firm's Decisions and Welfare Measures

We consider a supplier of a single product who faces uncertain product price P (see Appendix for the general case). The conservatively minded risk-averse producer follows the time-honored tradition of maximizing expected utility of wealth (not having been informed yet of some sophisticated, non-expected utility modes of decision-making). Furthermore, the producer's risk attitude, as represented by the absolute risk coefficient, depends solely on the endowed wealth, W_o. The profit, to be realized at the end of the period, is given by $\Pi = PY - C(Y)$, where Y is the (planned) output and C(Y) is the variable cost of producing Y, as determined by the production technology. Because the absolute risk coefficient, A, depends only on W_o and not on profit, the utility of wealth can be specified, without further loss of generality, as

$$U(\Pi) = 1 - e^{-A(W_o)\Pi}. \tag{2}$$

The firm is a taker of a price distribution, $\theta(m,s)$. The *ex-ante* supply function, $Y(\theta(m,s),W_o)$, is the output level that maximizes $E\{U(P(m,s)Y-C(Y))\}$ and satisfies

$$E\left\{U'(\Pi(\theta(m,s),W_o))\cdot[P(m,s)-C'(Y(\theta(m,s),W_o))]\right\} = 0, \tag{3}$$

where $E\{\cdot\}$ denotes expectation with respect to the price distribution $\theta(m,s)$, $U'(x) \equiv \partial U(x)/\partial x$, and

$$\Pi(\theta(m,s),W_o) = P(m,s)Y(\theta(m,s),W_o) - C(Y(\theta(m,s),W_o))$$

is the *ex-ante* profit. Due to fixed supplies of some production inputs, such as land, output cannot exceed the upper bound $\bar{Y}$. Thus Condition (3) holds only if $0 < Y(\theta(m,s),W_o) < \bar{Y}$; otherwise, $Y(\theta(m,s),W_o) = 0$ or $\bar{Y}$ as the solution of (3) is non-positive or exceeds $\bar{Y}$, respectively.

The indirect expected utility of wealth

$$V(\theta(m,s),W_o) = E\left\{1 - e^{-A(W_o)\Pi(\theta(m,s),W_o)}\right\} \tag{4}$$

constitutes a non-monetary measure of the well-being of a producer endowed with W_o who operates under output price uncertainty characterized by $\theta(m,s)$. A corresponding monetary measure is the certainty equivalent profit, $\hat{\Pi}(\theta(m,s),W_o)$, defined by

$$U(\hat{\Pi}(\theta(m,s),W_o)) = V(\theta(m,s),W_o), \tag{5}$$

which is the income level that leaves the producer indifferent between receiving it with certainty and earning the random profit $\Pi(\theta(m,s),W_o)$.

A change in the output price distribution from θ^1 to θ^2 causes the welfare change $V(\theta^2,W_o)-V(\theta^1,W_o)$. In view of (5), a monetary measure of this welfare change is the Certainty Equivalent (CE) index

$$CE = \hat{\Pi}(\theta^2,W_o) - \hat{\Pi}(\theta^1,W_o).$$

The compensating variation (CV) and the equivalent variation (EV) indices associated with this change are the income levels satisfying respectively $V(\theta^1,W_o+EV) = V(\theta^2,W_o)$ and $V(\theta^2,W_o-CV) = V(\theta^1,W_o)$. The three welfare indices are equal in the present case of constant absolute risk aversion (see, e.g., Pope *et al.*).

By differentiating both sides of (5) with respect to m, recalling (3) and (1), we obtain

$$\partial\hat{\Pi}(\theta(m,s),W_o)/\partial m = y(\theta(m,s),W_o)\cdot h(\theta(m,s),W_o),$$

where

$$h(\theta(m,s),W_o) = \frac{E\{U'(\theta(m,s),W_o)\}}{U'(\hat{\Pi}(\theta(m,s),W_o))} .$$

This result, which does not depend on specification (2), is the uncertainty analog of Hotelling's Lemma. For the constant absolute risk aversion utility (2), $E\{U'(\cdot)\} = AE\{e^{-A(W_o)\Pi(m,s,W_o)}\}$, and $U'(\hat{\Pi}(m,s,W_o)) = Ae^{-A\hat{\Pi}(m,s,W_o)}$; since (5) implies $E\{e^{-A(W_o)\Pi(m,s,W_o)}\} = e^{-A\hat{\Pi}(m,s,W_o)}$, it follows that $h(\theta(m,s),W_o) = 1$ identically for all values of θ and W_o. This leads to the following (well-known) result:

Property 1: Under specification (2),

$$\partial\hat{\Pi}(\theta(m,s),W_o)/\partial m = y(\theta(m,s),W_o). \tag{6}$$

4. The Evaluation Procedure

To facilitate notation, we suppress the argument W_o and write m,s instead of $\theta(m,s)$; e.g., A stands for $A(W_o)$ and $\Pi(m,s)$ is the short-hand notation of $\Pi(\theta(m,s),W_o)$. As $\Pi(m,s) = P(m,s)Y(m,s) - C(Y(m,s)) \overset{d}{=} (P+m+Z-\mu_z)Y(m,s) - C(Y(m,s))$, we obtain, recalling (2) and after some algebraic manipulations:

$$E(U(\Pi(m,s))) = 1 - \exp\left(-A[(m-\mu_z)Y - C(Y) - \log M(-AY)/A]\right), \tag{7}$$

where $M(\cdot)$ is the moment generating function of P+Z (assumed to exist). Evaluating (7) at the *ex-ante* supply, Y(m,s), and using (5), gives

$$\hat{\Pi}(m,s) = (m-\mu_z)Y(m,s) - C(Y(m,s)) - \log M(-AY(m,s))/A. \tag{8}$$

Differentiating both sides of (8) with respect to m and using (6) yields

$$Y(m,s) = Y(m,s) + \left(m - \mu_z - C'(Y(m,s)) + \partial \log M(-AY(m,s))/\partial(-AY)\right) \frac{\partial Y(m,s)}{\partial m}.$$

Since $\frac{\partial Y(m,s)}{\partial m} > 0$ for $0 < Y(m,s) < \bar{Y}$ (see, e.g., Sandmo), we can conclude:

Property 2: Under specification (2) and provided the solution of (9) lies between 0 and $\bar{Y}$, the *ex-ante* supply Y(m,s) satisfies:

$$\frac{\partial \log M(-AY(m,s))}{\partial(-AY)} + m - \mu_z - C'(Y(m,s)) = 0, \tag{9}$$

where $C'(Y) = \partial C(Y)/\partial Y$ is the marginal cost function.

Property 2 provides a practical means to evaluate the *ex-ante* supply Y(m,s) under various uncertainty conditions, $\theta(m,s)$. It requires only information on the marginal cost function, which is a technological relation independent of the uncertainty, and on the distribution of P+Z. With these data, the *ex-ante* supply of a grower with a risk coefficient A is obtained as the value Y(m,s) that satisfies (9). Given the solutions Y(m,s) under $\theta(m_1,s_1)$ and $\theta(m_2,s_2)$, Eq. (8) is applied to evaluate $\hat{\Pi}(m_1,s_1)$ and $\hat{\Pi}(m_2,s_2)$, and thereby to obtain $CE = \hat{\Pi}(m_2,s_2) - \hat{\Pi}(m_1,s_1)$.

5. Examples

Assuming a constant return to scale production technology, i.e., $C(Y) = c\cdot Y$, where the constant unit production cost c depends only on input prices, we evaluate the CE measure associated with the price distributions specified in Section 2.

Example 1: P and Z are independent gamma variates with parameters (α,β) and (γ,β), respectively, and P + Z is distributed as Gamma$(\alpha+\gamma,\beta)$. The moment generating function of P + Z is given by

$$M(-AY) = [1 + AY\beta]^{-(\alpha+\gamma)}.$$

The compound DPS/MPS price $P(m,s) \stackrel{d}{=} P + m + (Z-\mu_z)$ has a gamma distribution characterized by $\theta(m,s) = (\mu+m, \sigma^2+s^2)$, where $\mu = \alpha\beta$, $\sigma^2 = \alpha\beta^2$, $\mu_z = \gamma\beta$ and $s^2 = \gamma\beta^2$. Using Property 2 and noting that $\partial \log M(-AY)/\partial Y = \frac{(\alpha + \gamma)\beta}{1 + AY\beta}$, we obtain, for all $\mu_z + c > m > c - \mu$:

$$Y(m,s) = \frac{\mu + m - c}{A\beta(\mu_z + c - m)} = \frac{\mu}{A\sigma^2} \cdot \frac{\mu + m - c}{\mu_z + c - m}, \tag{10}$$

provided the right-hand side of (10) does not exceed the production capacity $\bar{Y}$; otherwise, $Y(m,s) = \bar{Y}$. To understand the restriction $\mu_z + c > m$, note that $m - \mu_z$ is the lower support of the distribution of P(m,s). Thus, if this condition is violated, i.e., $m - \mu_z \geq c$, then $P(m,s) - c \geq 0$ under all possible realizations of P(m,s). This guarantees a non-negative profit regardless of the actual realization of output price and pushes production to the maximum level $\bar{Y}$. If the other restriction, $m > c - \mu$, is violated, i.e., if $\mu + m \leq c$, then the average output price cannot exceed the unit production cost, i.e., $E\{P(m,s)\} \leq c$, implying that the expected profit is negative; a risk averse producer will prefer not to produce under such unfavorable conditions.

Note that s affects Y(m,s) via μ_z, since $\mu_z = \gamma\beta = s^2/\beta = \mu s^2/\sigma^2$. A change in μ_z, leaving μ and σ^2 unchanged, implies a change in s^2 and vice versa.

Applying (8) gives, for $\mu_z + c > m > c - \mu$ and provided $Y(m,s) < \bar{Y}$:

$$\hat{\Pi}(m,s) = \frac{\mu}{A\sigma^2}\left((\mu + \mu_z)\log\frac{\mu + \mu_z}{\mu_z + c - m} - (\mu + m - c)\right). \tag{11}$$

The $\hat{\Pi}$-levels associated with the boundary cases $Y(m,s) = \bar{Y}$ or 0 are $\hat{\Pi}(m,s) = (m-\mu_z-c)\bar{Y} + \frac{\mu}{A\sigma^2}\log(1+A\bar{Y})$ or 0, respectively. The welfare change associated with the uncertainty change $\theta(m_1,s_1) \rightarrow \theta(m_2,s_2)$ is evaluated by CE = $\hat{\Pi}(m_2,s_2) - \hat{\Pi}(m_1,s_1)$. Table 1 presents values of $\hat{\Pi}(m,s)$ and $Y(m,s)$ for various combinations of s and A.

Table 1

Example 2: P is a $N(\alpha,\tau^2)$ variate, truncated from below at $d \geq 0$, and Z is $N(\mu_z,s^2)$ distributed independently of P. It is verified in the Appendix that the moment generating function of P + Z takes the form:

$$M(-AY) = \frac{1 - \Phi(\tilde{d}+\tau AY)}{1 - \Phi(\tilde{d})}\exp\left((\tau^2 + s^2)A^2Y^2/2 - (\alpha + \mu_z)AY\right),$$

where $\Phi(\cdot)$ is the standard normal distribution function and $\tilde{d} = (d-\alpha)/\tau$.

Applying (8) gives

$$\hat{\Pi}(m,s) = (m+\alpha-c)Y(m,s) - (\tau^2+s^2)Y(m,s)^2A/2 - \frac{1}{A}\log\left(\frac{1 - \Phi(\tilde{d}+\tau AY(m,s))}{1 - \Phi(\tilde{d})}\right). \tag{12}$$

Using Property 2 and noting that

$$\frac{\partial\log M(-AY(m,s))}{\partial(-AY)} = \tau\frac{\phi(\tilde{d}+\tau AY(m,s))}{1 - \Phi(\tilde{d}+\tau AY(m,s))} - (\tau^2+s^2)AY(m,s) + \alpha + \mu_z,$$

we define Y(m,s) as the solution of

$$\tau\frac{\phi(\tilde{d}+\tau AY(m,s))}{1 - \Phi(\tilde{d}+\tau AY(m,s))} - (\tau^2+s^2)AY(m,s) + \alpha = cY(m,s)) - m, \tag{13}$$

provided this solution lies between 0 and $\bar{Y}$. Plugging Y(m,s) back into (12) yields $\hat{\Pi}(m,s)$. Table 2 presents values of Y(m,s) and $\hat{\Pi}(m,s)$ for m = 0 and various MPS changes s and risk coefficients A.

Table 2

Commodity programs, when they exist, usually involve support prices that truncate the commodity price distribution from below. It is interesting to evaluate effects of changes in the support price on the planned output supply and on growers welfare. A change in the support price affects both the mean and the variance (as well as higher moments) of the output price and can be approximated by a compound DPS/MPS shift.

In the present example, the support price is represented by d. Recalling that $\mu = E\{P\} = \tau \frac{\phi(\tilde{d})}{1-\Phi(\tilde{d})} + \alpha$ and $\sigma^2 = Var\{P\} = \tau^2\left\{1 + \tilde{d}\frac{\phi(\tilde{d})}{1-\Phi(\tilde{d})} - \left(\frac{\phi(\tilde{d})}{1-\Phi(\tilde{d})}\right)^2\right\}$, it is straightforward to calculate the changes $\Delta\mu$ and $\Delta\sigma^2$ in μ and σ^2 caused by a change Δd in d. One then sets $m = \Delta\mu$ and $s^2 = \Delta\sigma^2$ and applies (13) and (12) to calculate $Y(m,s)$ and $\hat{\Pi}(m,s)$. The changes in *ex-ante* output supply and in welfare that occur due to the shift in the support price are $Y(m,s)-Y(0,0)$ and $\hat{\Pi}(m,s)-\hat{\Pi}(0,0)$, respectively.

6. Concluding Comments

This paper develops a simple and tractable procedure to evaluate *ex-ante* producer welfare under price uncertainty. Implementation of the procedure requires knowledge of the variable cost function, the output price distribution and the absolute risk coefficient. The cost function is a technological relation independent of the uncertainty, and can be estimated using input/output data (or some other engineering technique). Information on the price distribution can be obtained from observed past prices.

The absolute risk coefficient can be estimated empirically. Property 2 is conducive for such a task, as it can be used to specify a regression model of output supply (the dependent variable) as a function of the absolute risk coefficient and the parameters of the output price distribution (the independent variables); (see Eq. (9) and its specializations to Eqs. (10) and (13) in Examples 1 and 2). The formulation of this regression model is completed by specifying the absolute risk coefficient as a parametric function of wealth and other socio-economic characteristics of the farmer and by adding error terms that account for data measurement errors and noisy output. Alternatively, for a group of producers whose risk coefficients are bounded between two values, the procedure provides the corresponding bounds on welfare changes that result from a change in uncertainty, e.g., due to a change in the support price of a commodity program.

The simplicity and tractability properties are not costless, as they require the assumption that risk aversion depends on endowed wealth and not on one-period profit. Where farming is capital intensive, a considerable amount of capital is accumulated by growers over the years (in addition to savings) and one-year profit comprises a fairly small portion of the farm's net worth. It is reasonable to assume that in this case the psychological motives determining the farmers' attitude toward risk are affected more by the stock of wealth than by the flow of income. The assumption that risk attitude is dependent on wealth and not on income therefore can serve as a plausible approximation. For growers whose wealth accumulation is negligible and their ability to continue farming (or to maintain a basic existence level) depends on their year-to-year income realizations, this assumption may not be valid.

Extensions of the analysis to situations involving input price uncertainty and multiple outputs are outlined in the Appendix. The numerical examples reveal that output and welfare are quite sensitive to the form of the price distribution. Thus, in an empirical context, it is important to relax the assumption regarding the price distribution; a possible approach entails using empirical distribution functions based on price data. We leave this task for the future. Other future extensions will incorporate yield uncertainty and intertemporal considerations.

Appendix

(a) *Input Price Uncertainty and Multiple Outputs:* Let $x = (x_1, x_2, \ldots, x_n)$ be the n-dimensional vector of input quantities and $r = (r_1, r_2, \ldots, r_n)$ the n-vector of input prices. Let $Y_k(x)$ be the production function of the k-th output and P_k the corresponding output price, $k=1,2,\ldots,K$ (additive separability of inputs and outputs is assumed). The uncertainty is represented by the joint distribution of (K+n)-dimensional price vector $(P_1, P_2, \ldots, P_K, r_1, r_2, \ldots, r_n)$. A change in uncertainty is represented by compound DPS/MPS shifts of the form

$$\begin{aligned} P_k(m_k, s_k) &\overset{d}{=} P_k + m_k + z_k - \mu_k, \quad k=1,2,\ldots,K, \\ r_i(m_{K+i}, s_{K+i}) &\overset{d}{=} r_i + m_{K+i} + (z_{K+i} - \mu_{K+i}), \quad i=1,2,\ldots,n, \end{aligned} \tag{A1}$$

where m_k (resp. m_{K+i}) are scalars and z_k (resp. z_{K+i}) are random variables with mean μ_k (resp. μ_{K+i}) and variance s_k^2 (resp. s_{K+i}^2), $k=1,2,\ldots,K$ (resp. $i=1,2,\ldots,n$).

The *ex-ante* input demand functions, $x_i(m,s)$, $i=1,2,\ldots,n$, are determined so as to maximize expected utility of profit and satisfy

$$E\left\{e^{-A\Pi(m,s)}\left(\sum_{k=1}^{K} P_k(m_k,s_k)\partial Y_k(x(m,s))/\partial x_i - r_i(m_{K+i},s_{K+i})\right)\right\} = 0, \quad i=1,2,..n, \tag{A2}$$

where $\Pi(m,s) = \sum_{k=1}^{K} P_k(m_k,s_k)Y_k(x(m,s)) - \sum_{i=1}^{n} r_i(m_{K+i},s_{K+i})x_i(m,s)$. The certainty equivalent profit, $\hat{\Pi}(m,s)$, is defined by

$$e^{-A\hat{\Pi}(m,s)} = E\{\, e^{-A\Pi(m,s)}\}. \tag{A3}$$

By differentiating both sides of (A3) with respect to m_k and m_{K+i}, and recalling (A1-2), we obtain

$$\begin{aligned} \partial\hat{\Pi}(m,s)/\partial m_k &= Y_k(x(m,s)), \quad k=1,2,\ldots,K. \\ \partial\hat{\Pi}(m,s)/\partial m_{K+i} &= -x_i(m,s), \quad i=1,2,\ldots,n. \end{aligned} \tag{A4}$$

From (A1), $\Pi(m,s) \overset{d}{=} \sum_{k=1}^{K}(m_k-\mu_k)Y_k(x(m,s)) + \sum_{k=1}^{K}(p_k+z_k)Y_k(x(m,s)) - \sum_{i=1}^{n}(m_{K+i}-\mu_{K+i})x_i(m,s) - \sum_{i=1}^{n}(r_i+z_{K+i})x_i(m,s)$. Thus,

$$E\{e^{-A\Pi(m,s)}\} = \exp\left(-A\left[\sum_{k=1}^{K}(m_k-\mu_k)Y_k(x(m,s)) - \sum_{i=1}^{n}(m_{K+i}-\mu_{K+i})x_i(m,s)\right]\right)\times$$
$$E\left\{\exp\left(-A\left[\sum_{k=1}^{K}(p_k+z_k)Y_k(x(m,s)) - \sum_{i=1}^{n}(r_i+z_{K+i})x_i(m,s)\right]\right)\right\}.$$
$$=\exp\left(-A\left[\sum_{k=1}^{K}(m_k-\mu_k)Y_k(x(m,s)) - \sum_{i=1}^{n}(m_{K+i}-\mu_{K+i})x_i(m,s) - \log M(-Ag)/A\right]\right),$$

where M is the moment generating function of the (K+n)-dimensional random vector $(P_1+z_1,\ldots,P_K+z_K,r_1+z_{K+1},\ldots,r_n+z_{K+n})$ and $g = (Y(x(m,s)),-x(m,s))$ is the (K+n)-dimensional netput vector. Thus, (A3) implies:

$$\hat{\Pi}(m,s) = \sum_{k=1}^{K}(m_k-\mu_k)Y_k(x(m,s)) - \sum_{i=1}^{n}(m_{K+i}-\mu_{K+i})x_i(m,s) - \log M(-Ag)/A. \quad (A5)$$

Differentiating (A5) with respect to m_j and using (A4), we obtain

$$0 = \sum_{i=1}^{n}(\partial x_i/\partial m_j)\left\{\sum_{k=1}^{K}(m_k-\mu_k)\partial Y_k(x(m,s)/\partial x_i - (m_{K+i}-\mu_{K+i}) + \right.$$
$$\left. + \left(\sum_{k=1}^{K}(\partial\log M/\partial(-AY_k))\partial Y_k(x(m,s)/\partial x_i - \partial\log M/\partial(Ax_i)\right)\right\},\ j=1,2,\ldots,K+n, \quad (A6)$$

where the derivatives of logM are evaluated at $-A(Y(x(m,s)),-x(m,s))$. As (A6) must hold for all $j=1,2,\ldots,K+n$, and $\partial x_i/\partial m_j$ changes with j, the term inside the curly braces must vanish for each i. Thus, the *ex-ante* input demands must satisfy:

$$0 = \sum_{k=1}^{K}(m_k-\mu_k)\partial Y_k(x(m,s)/\partial x_i - (m_{K+i}-\mu_{K+i}) +$$
$$\left(\sum_{k=1}^{K}(\partial\log M/\partial(-AY_k))\partial Y_k(x(m,s)/\partial x_i - \partial\log M/\partial(Ax_i)\right),\ i=1,2,\ldots,n. \quad (A7)$$

Given the production technology, as represented by the production functions Y_k, $k=1,2,\ldots,K$, and the price distribution, as represented by M, the *ex-ante* input demands, $x_i(m,s)$, $i=1,2,\ldots,n$, can be found as the solutions of Eqs. (A7). These are then plugged back into (A5) to determine $\hat{\Pi}(m,s)$.

If only the output prices are uncertain, then $M = E\{\exp(-AY\cdot(P+z) + Ax\cdot r)\} = e^{Ax\cdot r}M_{P+z}(-AY)$, hence $\log(M) = Ar\cdot x + M_{P+z}(-AY)$; also $m_{K+i} = \mu_{K+i} = 0$, all i. Thus, (A5) changes to:

$$\hat{\Pi}(m,s) = \sum_{k=1}^{K}(m_k-\mu_k)Y_k(x(m,s)) - \sum_{i=1}^{n}r_ix_i(m,s) - \log M_{P+z}(-AY(x(m,s)))/A$$
$$= \sum_{k=1}^{K}(m_k-\mu_k)Y_k(x(m,s)) - C(Y(x(m,s))) - \log M_{P+z}(-AY(x(m,s)))/A, \quad (A8)$$

where $C(\cdot)$ is the cost function defined by: $C(q) = \text{Min } r\cdot x$, subject to $Y(x) \le q$. Differentiating with respect to m_j and using (A4), we obtain

$$0 = \sum_{k=1}^{K}\left(m_k - \mu_k - \partial C(Y(m,s))/\partial Y_k + \partial \log M_{P+Z}(-AY(m,s))/\partial(-AY_k)\right)\partial Y_k(m,s)/\partial m_j,$$
$$j=1,2,\ldots,K, \quad \text{(A9)}$$

where $Y(m,s) = Y(x(m,s))$. As this relation must hold for all j and $\partial Y_k(m,s)/\partial m_j$ changes with j, the bracketed terms must vanish for all k, i.e.,

$$m_k - \mu_k - \partial C(Y(m,s))/\partial Y_k + \partial \log M_{P+Z}(-AY(m,s))/\partial(-AY_k), \ k=1,2,\ldots,K. \quad \text{(A10)}$$

Given the production technology, as summarized by the cost function $C(\cdot)$, and the output price distribution, as represented by M_{P+Z}, the *ex-ante* supplies $Y_k(m,s)$, $k=1,2,\ldots,K$, are found as the K roots of Eqs. (A10). These are plugged back into (A8) and determine $\hat{\Pi}(m,s)$. Note that in the single output case, where K=1, Eq. (A10) specializes to Eq. (9).

(b) *Truncated Normal Distribution:* Let X be a standard normal variate truncated from below at $\tilde{d}$, with the density function $f(x) = \phi(x)/\Phi(\tilde{d})$ for $x \geq \tilde{d}$ and $f(x) = 0$ otherwise (ϕ and Φ are respectively the standard normal density and distribution functions). Then

$$M_X(t) = E\left\{e^{tX}\right\} = \int_{\tilde{d}}^{\infty} \frac{1}{1-\Phi(\tilde{d})} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} e^{tx} dx$$
$$= \frac{1}{1-\Phi(\tilde{d})} e^{t^2/2} \int_{\tilde{d}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(x-t)^2/2} = e^{t^2/2} \frac{1-\Phi(\tilde{d}-t)}{1-\Phi(\tilde{d})}.$$

The mean and variance of X are given by: $E\{X\} = \partial M_X(0)/\partial t = \dfrac{\phi(\tilde{d})}{1-\Phi(\tilde{d})}$ and $\mathrm{Var}\{X\} = \partial^2 M_X(0)/\partial t^2 - (E\{X\})^2 = 1 + \tilde{d}\dfrac{\phi(\tilde{d})}{1-\Phi(\tilde{d})} - \left(\dfrac{\phi(\tilde{d})}{1-\Phi(\tilde{d})}\right)^2$.

If P is $N(\alpha,\tau^2)$ truncated from below at d then $P \overset{d}{=} \tau X + \alpha$, with $\tilde{d} = (d-\alpha)/\tau$. Thus, $M_P(t) = E\left\{e^{t(\tau X+\alpha)}\right\} = e^{t\alpha} M_X(t\tau) = \exp(t\alpha + t^2\tau^2/2)\dfrac{1-\Phi(\tilde{d}-\tau t)}{1-\Phi(\tilde{d})}$.

If $Z \sim N(\mu_z, s^2)$ and is independent of P, then

$$M_{P+Z}(t) = M_P(t)\cdot M_Z(t) = \exp(t(\alpha+\mu_z)+t^2(\tau^2+s^2)/2)\frac{1-\Phi(\tilde{d}-\tau t)}{1-\Phi(\tilde{d})}.$$

Evaluating at $t = -AY$ provides the moment generating function of Example 2.

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Table 1

Welfare measures $\hat{\Pi}(m,s)$ and *ex-ante* supply $Y(m,s)$ (in parentheses) for various combinations of MPS (s^2) and risk aversion (A), with $m = 0$, $c = 2$, $P \sim$ Gamma(4,1) and $Z \sim$ Gamma(γ,1).

A \ s^{2*}	0	1	2	3	4
0.001	2770.59 (1000.00)	2552.13 (666.67)	2430.79 (500.00)	2353.30 (400.00)	2299.46 (333.33)
0.01	275.26 (100.00)	253.41 (66.67)	241.28 (50.00)	233.53 (40.00)	228.14 (33.33)
0.1	25.72 (10.00)	23.54 (6.67)	22.327 (5.00)	21.55 (4.00)	21.01 (3.33)
1.0	0.77 (1.00)	0.55 (0.67)	0.43 (0.50)	0.35 (0.40)	0.30 (0.33)

*s^2 = Var(Z) = γ, since β=1.

Table 2

Welfare measures $\hat{\Pi}(m,s)$ and *ex-ante* supply Y(m,s) (in parentheses) for various combinations of MPS (s^2) and risk aversion (A), with m = 0, c =2, P ~ N(4,4) truncated from below at 0, and Z ~ N(0,s^2).

A \ s^2	0	1	2	3	4
0.001	612.72 (589.69)	453.22 (450.47)	265.75 (268.42)	160.01 (161.74)	103.14 (104.16)
0.01	61.27 (58.97)	45.32 (45.05)	26.57 (26.84)	16.00 (16.17)	10.31 (10.42)
0.1	6.13 (5.90)	4.53 (4.50)	2.66 (2.68)	1.60 (1.61)	1.03 (1.04)
1.0	0.61 (0.59)	0.45 (0.45)	0.27 (0.27)	0.16 (0.16)	0.10 (0.10)