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GROUNDWATER CONTAMINATION AND THE MANAGEMENT OF A CONJUNCTIVE GROUND AND SURFACE WATER IRRIGATION SYSTEM

Yacov Tsur



Department of Agricultural and Applied Economics

University of Minnesota
Institute of Agriculture, Forestry and Home Economics
St. Paul, Minnesota 55108

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Groundwater contamination and the management of a conjunctive
ground and surface water irrigation system*

By

Yacov Tsur

Department of Agricultural and Applied Economics, University of Minnesota

Irrigation water (including rainfall) that infiltrates the subsurface carries salts, pesticide and fertilizer residues, and other trace elements, thus causing a contamination of aquifers and soils. A similar situation occurs when irrigating with saline groundwater (aquifers containing saline water often are found in arid and semi-arid regions, where agricultural production depends critically on groundwater irrigation). Evaporation of the irrigation water increases salt concentration, causing salinization of soils and aquifers. Although not immediately noticeable, these quality deterioration processes will have long-term effects and therefore require careful management. The paper describes a general framework for the intertemporal management of a conjunctive ground and surface water irrigation system, taking into account the quality deterioration processes. Policy implications are discussed and the results are compared with those that come from a model which neglects quality effects.

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**Groundwater contamination and the management of a conjunctive
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1. Introduction

The conjunctive use of groundwater and surface water for irrigation is pervasive and has attracted much research, starting with the early work of Burt (1964a-b) followed by Brown and McGuire (1967), Cummings and Burt (1969), Burt and Cummings (1970), Cummings and Winkelman (1970), Domenico *et al.* (1970), Young and Bredehoeft (1972), Bredehoeft and Young [1983], Tsur (1990), and Tsur and Graham-Tomasi (1990) among others. The problem, in general terms, is that of allocating groundwater over time when the demand for groundwater varies according to available supply of surface water.

The term "conjunctive ground and surface water system" is applied to a number of systems; they differ according to the ground and surface water sources. The source of surface water may consist solely of stream flows emanating from the aquifer, it may be independent of the groundwater source (e.g., rainfall) or it may be a combination of the two. The groundwater aquifer may be confined (see examples in Margat and Saad [1985] and Issar (1985)) or replenishable, deep or shallow. The surface water source may be stable or it may stochastically fluctuate over time. Depending on the particular situation one wishes to study, the management problem of a conjunctive ground and surface water system can become quite involved.

Here we consider a situation in which the supply of surface water is stable and groundwater is derived from shallow aquifers. Groundwater quality can affect yield directly, if groundwater invades the root zone, and indirectly through irrigation. We shall focus attention on the first, direct effect. This effect is controlled via drainage activities.

We describe a framework for the management of an irrigation and drainage

system, where irrigation is derived both from surface and groundwater sources. We begin, in Section 2, by laying out the basic principles underlying the management of a conjunctive ground and surface water system. After deriving the optimal rules for managing such a system we argue that, due to the open-access and/or common-property nature of groundwater resources, market forces are unlikely to generate water use patterns which satisfy these rules. Possible policies to restore the optimal management rules are then discussed. In Section 3 quality considerations are introduced. In Section 4 we derive the rules governing desirable irrigation/drainage management and extend the policy discussion of Section 2 to that context. In Section 5 we distinguish between policies designed to enforce the optimal irrigation/drainage rules and those aimed at affecting the environment within which the management problem rests. Some examples of the second type of policy are discussed.

2. Basic principles of the management of a conjunctive ground and surface water system

A conjunctive ground and surface water system consists of a surface water source (stream flows, rainfall, reservoirs), a groundwater source (aquifer) and an agriculture production process which requires water as an input. Figure 1 gives a schematic representation of such a system.

Figure 1.

Let $F(x)$ denote the water response function, measured in dollar per hectare ($\$/ha$), and x indicate the level of water input, measured in cubic

meter per hectare (m^3/ha)¹. The marginal water productivity is the change in $F(x)$ resulting from a small (marginal) change in water input x and is indicated by $F'_x = \partial F/\partial x$. It plays a central role in determining the management rules. In most cases $F(x)$ increases in x at a diminishing rate, thus $F'_x(x)$ is positive and decreasing in x (on different ways to estimate this function see Howitt *et al.* (1980) and Paris and Knapp (1989)).

The quantities of surface and groundwater applied for irrigation at time t are denoted by S_t and g_t , respectively; total water input is thus $x_t = S_t + g_t$. The amount of rainfall relevant for irrigation (during the growing season) is assumed stable at the level R and is included in S_t , thus $S_t \geq R$. The stock on hand of groundwater at time t , denoted by G_t , changes over time as extraction takes place and as some of the water input (irrigation) infiltrates the aquifer:

$$dG_t/dt = \dot{G}_t = -(1-\delta)g_t + \delta S_t, \quad (1)$$

where δ is a permeability parameter indicating the fraction of the water applied for irrigation that permeates into the aquifer (when the aquifer reaches its capacity level, \dot{G}_t equals the minimum between the right-hand side of (1) and zero)

The cost of pumping groundwater at a rate g is given by $z(G)g$, where $z(G)$ is the unit cost of groundwater extraction when the groundwater stock is at the level G . $z(G)$ is non-increasing in G (a larger G means a higher

¹ $F(x)$ is derived in the following manner. Let $f(x,k)$ be an agricultural production function whose arguments are a water input, x , and a vector of other inputs, k . Given the prices of output, p , and of all inputs other than water, v , and given the level of water input, $k^*(x,p,v)$ represents the value of k that maximizes $pf(x,k) - vk$. The water response function is given by

$$F(x) = pf(x, k^*(x,p,r)) - r \cdot k^*(x,p,r).$$

where the fixed prices p and v are suppress from the notation.

groundwater table, a shorter distance to the surface and hence lower extraction costs). The unit cost of surface water irrigation (except for rainfall) is denoted by w . The instantaneous profit generated by S_t and g_t is thus given by

$$F(g_t + S_t) - z(G_t)g_t - w(S_t - R).$$

The amount of irrigation water may be subject to capacity constraints. We let C and B indicate these capacity limits, thus $g_t \leq C$ and $S_t \leq B$ for all $t \geq 0$.

A water management policy entails setting S_t and g_t for all time periods $t \geq 0$; it generates the benefit (the present value of the profit stream)

$$\int_0^{\infty} [F(g_t + S_t) - z(G_t)g_t - w(S_t - R)]e^{-rt} dt,$$

where r is the time rate of discount. We seek the policy that maximizes this benefit.

Let $V(G)$ be the maximum feasible benefit when the current stock of groundwater is G :

$$V(G) = \text{MAX} \int_0^{\infty} [F(g_t + S_t) - z(G_t)g_t - w(S_t - R)]e^{-rt} dt$$

$$\text{subject to: Eq. (1), } 0 \leq g_t \leq C, R \leq S_t \leq B, G_t \geq 0 \text{ and } G_0 = G. \quad (2)$$

The change in $V(G)$ caused by a marginal (small) change in G is the unit value of the groundwater stock and is denoted by $V_G(G)$. It represents the future benefit forgone as a result of pumping a unit groundwater today and is referred to as the shadow price or the royalty value of the aquifer.

Using a dynamic programming approach, we obtain for each time period (see appendix) the following relation:

$$rV(G_t) = \text{MAX}_{g_t, S_t} \left\{ F(g_t + S_t) - [z(G_t) + V_G(G_t)(1 - \delta)]g_t - [w - V_G(G_t)\delta]S_t + wR \right\}. \quad (3)$$

In words, the optimal conjunctive ground and surface water policy $(S_t^*, g_t^*, t \geq 0)$ is the one under which the right-hand side of (3) is maximized in each time period (subject, of course, to the constraints given in (2)). The object of maximization on the right-hand side of (3) is the instantaneous profit

corrected to account for intertemporal effects. The intertemporal effects are effects of current decisions on future profits and are represented by the shadow prices $V_g(G_t)$. Thus the cost associated with one cubic meter of groundwater applied for irrigation today consists of (a) the pumping and distribution costs as given by $z(G_t)$, and (b) the effect on future profits resulting from the drop in the stock of groundwater, which occurs due to higher pumping costs in the future and increased scarcity of groundwater. This second cost component is represented by $V_g(G_t)[1-\delta]$ (the factor $1-\delta$ accounts for the fact that only $(1-\delta) m^3$ of each $1 m^3$ pumped is lost, as δm^3 leaches back into the aquifer). The economic cost of groundwater is therefore given by $z(G_t)+V_g(G_t)[1-\delta]$, which is the coefficient of g_t on the right-hand side of (3). Similarly, the economic cost of surface water is $w-V_g(G_t)\delta$, which consists of the engineering cost, w , minus the contribution of surface water to future profits via its effect on the groundwater stock derived from the fraction δ of the surface water irrigation that leaches into the aquifer.

In view of (3) the characterization of the optimal policy becomes a straightforward exercise. Disregarding for a while the capacity limits (i.e., assuming they are not binding) and without rainfall (i.e., $R=0$) the following management rules apply:

(i) As long as the economic cost of groundwater exceeds that of surface water, i.e., $z(G_t)+V_g(G_t) > w$, only surface water is used for irrigation at a level that equates the marginal productivity of water to its cost:

$$F_x(S_t^*) = w - \delta V_g(G_t).$$

(ii) As long as the economic cost of groundwater falls below that of surface water, i.e., $z(G_t)+V_g(G_t) < w$, only groundwater is used for irrigation at a level that equates the marginal productivity of water to its cost:

$$F_x(g_t^*) = z(G_t)+V_g(G_t)(1-\delta).$$

(iii) When the economic costs of ground and surface water are equal, i.e.,

$z(G_t) + V_G(G_t) = w$, irrigation water is derived from both sources at a level that satisfies

$$F_x(g_t^* + S_t^*) = w - V_G(G_t)\delta$$

and at the mix $g_t^*/S_t^* = \delta/(1-\delta)$ such that the groundwater stock remains constant ($\dot{G}_t = 0$).

With the above interpretation of the economic costs of ground and surface water, these management rules make perfect intuitive sense. Some modifications, however, are needed in the presence of binding capacity limits and with positive rainfall; they are outlined in the appendix.

The dynamic behavior of the system is depicted in Figure 2. At all stock levels G for which $z(G) + V_G(G)$ lies above w , groundwater is more expensive than surface water, thus only the latter is applied for irrigation (cf. (i)). This causes the groundwater stock to increase, which in turn diminishes the pumping cost $z(G)$ and the shadow price $V_G(G)$ of groundwater, as represented by the declining curve labeled $z(G) + V_G(G)$. When the groundwater stock reaches the level \hat{G} , the cost of groundwater coincides with that of surface water and surface water is applied conjunctively with groundwater so as to retain the aquifer at this stock level (cf. (iii)). For stock levels above \hat{G} , groundwater is cheaper than surface water and irrigation water is derived solely from the aquifer (cf. (ii)). This causes the groundwater stock to decline toward \hat{G} . The groundwater stock level \hat{G} is called the steady state; the period in which the system moves toward \hat{G} is called the transition period (stage); the period in which $G = \hat{G}$ is called the steady period (stage).

Policy intervention

The management rules (i)-(iii) differ from the myopic rules under which the instantaneous profit is maximized in each time period. The myopic rules are derived from (i)-(iii) by setting the shadow prices $V_G(G_t)$ equal to zero. A question then arises as to whether the individual growers are motivated to

follow the intertemporal rules (i)-(iii) or whether they behave myopically? Unfortunately, the second possibility is more likely to prevail. The problem is similar to that of a "common property" situation (see Dasgupta (1982), Negri (1989)) in which the effect of each individual's extraction on the aquifer is negligible but is not at all negligible with respect to his or her own profits. Following the intertemporal rules entails giving up some present profits in return for future profits. But the future gains will materialize only if *all* (or most) growers follow the intertemporal rules. Now, if most growers follow the intertemporal rules, it is in the interest of the individual farmer to behave myopically because his or her effect on the aquifer is negligible and he can enjoy larger profits both in the present and in the future. On the other hand, if all other growers behave myopically then the grower should do the same, since otherwise there will be no future gains to compensate for the present losses. Realizing that this line of reasoning is not exclusive to any particular individual, the grower has good reasons to suspect that others will not follow the intertemporal rules, in which case he should not obey them either (this is, in a nutshell, the free rider problem). Clearly, some regulatory policies (quota, taxes) or market mechanism (water rights) to restore intertemporal considerations are in order. We shall briefly discuss the tax and quota options (on water rights see Gisser and Sanchez (1980), Gisser (1984) and Anderson, Burt and Fractor (1983), among others).

Optimal tax schedule: The engineering costs of ground and surface water ($z(G)$ and w , respectively) do not reflect their economic costs ($z(G)+V_G(G)[1-\delta]$ and $w-V_G(G)\delta$, respectively). A tax schedule to correct for this discrepancy consists of taxing each cubic meter of groundwater by the amount $V_G(G_t)[1-\delta]$ and subsidizing each cubic meter of surface water by the amount $V_G(G_t)\delta$. The problem with such a tax schedule is that it depends on

the stock of groundwater and thus must be adjusted constantly during the transition period. This might be hard to administer, since it requires constantly monitoring the aquifer level. Furthermore, it is likely to be objected by farmer who prefer stable water prices. An alternative scheme is therefore to impose the steady state tax schedule: a fixed tax of $V_g(\hat{G})[1-\delta]$ on groundwater and a fixed subsidy of $V_g(\hat{G})\delta$ on surface water. Such a tax schedule ensures a smooth transition to the steady state (though it may lengthen the transition period relative to that under the schedule described above), is easy (hence cheap) to administer, and is stable thereby facilitating compliance by growers.

Optimal water quotas: The management rules (i)-(iii) determine also the desirable quantities of ground and surface water to be applied for irrigation. During the transition period, if the aquifer stock lies below (above) its steady state level \hat{G} , the optimal policy is to prevent the use of ground (surface) water altogether; as a result only surface (ground) water is applied for irrigation and the aquifer stock increases (decreases) until it reaches the steady level \hat{G} , at which point the quota on ground and surface water is changed so as to retain the steady state, as described in (iii). The problem with this policy is that it entails a discrete jump in water policy as the system moves from the transition period to the steady stage, a jump that may require a change in the agricultural structure (e.g., crop mix) of the region. Furthermore, the option of banning the use of a particular source of water may simply be (legally) impossible. Such a policy, however, should be fairly simple to administer and is ensured to achieve the desirable water allocation.

A combined tax and quota schedule: A third option to be considered by water policy-makers is that of a combined quota/tax schedule. Such a policy consists of setting the prices of ground and surface water at their steady levels $z(\hat{G})+V_g(\hat{G})[1-\delta]$ and $w-V_g(\hat{G})\delta$, respectively, and at the same time

regulating the quantities of the more expensive water source in order to expedite the transition to the steady stage. The tax part of such the policy ensures smooth transition to the steady stage whereas the quantity regulation can be used to shorten the undesirably long transition period associated with the pure tax policy.

Policy implementation

The minimum information required to implement a tax policy contains the steady state level of the aquifer \hat{G} and the shadow price $V_G(\hat{G})$ at that level. To obtain this shadow price one needs to solve Problem (2), along the line of (3), which requires knowledge of the water response function $F(x)$ and of the permeability parameter δ . A solution of Problem (2) consists of the series S_t^* and g_t^* and the associated stock and shadow price processes G_t and $V_G(G_t)$, $t \geq 0$, and is in principle attainable (perhaps only numerically). While this is fairly easy to achieve in the simple case represented by Problem (2), it is more complicated in the realistic case described in the next section. For such cases there exist methods that provide approximates to the optimal management rules. Such a method, which approximates the steady state solution by solving a properly defined equivalent static problem, was proposed by Burt and Cummings (1977).

Closing remarks

This completes our account of the basic principles of the conjunctive management of ground and surface water for irrigation. Reality, of course, is more complicated than the simple situation considered above. Thus, numerous authors have extended and applied this framework to particular real world situations. Young and Bredehoeft (1972), for example, considered a situation in which the only source of surface water is stream flows emanating from aquifers. Cummings and Winkelman (1970), on the other hand, analyzed a system in which surface water is independent of groundwater sources.

Tsur (1990) introduced elements of uncertainty to surface water supplies and argued that groundwater, in addition to its role of increasing the supply of irrigation water, serves also as a buffer that mitigates the undesirable fluctuations in the water supply. Tsur (1990) calculated the value associated with the buffer role (the buffer value) of groundwater for wheat growers in the Israeli Negev region and found it to exceed the value associated with the increase in the water supply (the latter is the benefit that would be obtained from the groundwater had surface water supplies been stable at the mean). Tsur's (1990) analysis lacks some elements of dynamics since it considers the huge fossil water aquifer underlying the Negev to be effectively unlimited. While this may be justifiable in the particular case of the Negev, it is not so in general. Thus, Tsur and Graham-Tomasi (1990) extended this framework to the case of a finite aquifer.

We proceed now to incorporate the groundwater quality effects, leaving out the consideration of the above mentioned extensions.

3. Groundwater quality

The groundwater quality comes into effect when two distinct processes which affect agricultural yield occur as irrigation water infiltrates the shallow aquifer. The first is the rise in the groundwater table toward the root zone as the groundwater stock G increases. The second is the deterioration in the quality of the groundwater as salts and other trace elements are washed into the aquifer. Incorporating quality effects requires allowing the water revenue function to depend also on the groundwater stock G , which represents the groundwater table, and on a groundwater quality index Q , representing the groundwater salinity level. We avoid, for the time being, salinity effects via the groundwater applied for irrigation (For more on salinity control in groundwater management problems see Cummings (1971) and Cummings and McFarland (1974)). Figure 3 provides a schematic presentation of

such a system.

Figure 3.

The water response function F takes the form

$$F(x_t, G_t, Q_t).$$

As above, F is assumed to increase in a diminishing rate with the quantity of irrigation water ($F_x > 0$ and $F_{xx} < 0$). Both G and Q , on their own, do not contribute to yield and may even cause harm ($F_G \leq 0$ and $F_Q \leq 0$). The negative effect of the one is enhanced by an increase in the quantity of the other, i.e., their interaction is non-positive ($F_{GQ} \leq 0$). Thus, as the groundwater quality deteriorates (Q increases) the negative effect of the ground waterlogging is magnified (F_G decreases); likewise, as the groundwater table rises (G increases) the negative effect of Q is exacerbated (F_Q decreases).

Allowing for the application of drainage activities, which involves tiles to remove water to a drainage canal (see Figure 3), the change in the aquifer stock is represented by

$$dG_t/dt = \dot{G}_t = \delta S_t - (1-\delta)g_t - d_t, \quad (4)$$

where S_t , g_t and δ are as defined in the previous section and d_t indicates the amount of drainage (m^3/ha).

The groundwater quality index Q_t changes as salts and other trace elements are washed into the aquifer by the permeating irrigation water. This change, which is an outcome of quite complicated hydrological processes, may be represented implicitly as:

$$dQ_t/dt = \dot{Q}_t = H(\delta x_t, G_t, Q_t).$$

The larger the amount of permeating water (δx), the greater the quantities of salts washed into the aquifer, so that H increases in δx . On the other hand, we expect that H decreases in G_t (the same amount of salt changes the salinity level of a small bucket more than that of a large one). For the sake of concreteness, we assume that H is of the form

$$H(\delta x_t, G_t, Q_t) = q(G_t, Q_t) \delta x$$

where the nonnegative function $q(G, Q)$ translates quantities of permeating water (or of accumulated salts) into changes in the aquifer salinity level. The change in groundwater quality is thus given by

$$\dot{Q}_t = q(G_t, Q_t) \delta [S_t + g_t]. \quad (5)$$

A water management policy entails setting S_t , g_t and d_t for all time periods $t \geq 0$ and generates the payoff (the present value of the profit stream):

$$\int_0^{\infty} [F(S_t + g_t, G_t, Q_t) - z(G_t)g_t - md_t - w(S_t - R)] e^{-rt} dt,$$

where $z(G_t)$, w and r are as defined in Section 2 and m is the unit cost of drainage activities (m is fixed and independent of the groundwater table). We seek the policy that yields the highest payoff.

4. Irrigation and drainage management

Let $V(G, Q)$ represent the maximum available payoff when the current stock and quality of groundwater are G and Q , respectively. Formally

$$V(G, Q) = \text{MAX} \int_0^{\infty} [F(S_t + g_t, G_t, Q_t) - z(G_t)g_t - md_t - w(S_t - R)] e^{-rt} dt$$

$$\text{subject to: Eqs. (4)-(5), } 0 \leq g_t \leq C, R \leq S_t \leq B, 0 \leq d_t \leq D, G_0 = G \text{ and } Q_0 = Q, \quad (6)$$

where, as above, the parameters C and B represent respectively the capacity limits on ground and surface water supplies and D is a capacity limit on drainage activities.

The changes in $V(G, Q)$ associated with a marginal (small) change in G or Q (i.e., the derivatives of V with respect to G or Q) are denoted by $V_G(G, Q)$ and $V_Q(G, Q)$, respectively. These quantities represent the unit value of G or Q and are thus referred to as the shadow prices of G or Q . We expect that V_Q is negative (one would be willing to pay a positive amount to have Q reduced and the groundwater quality improved), while V_G may be positive or negative. At low levels of G , where the groundwater table is well below the root zone, V_G

will be positive since the finite stock of the aquifer entails a positive royalty value (the forgone benefit of not being able to use in the future the unit of groundwater pumped today). On the other hand, at high G levels where groundwater has invaded the root zone, the damage to yield may outweigh the benefit of additional water, causing V_g to become negative.

The Dynamic Programming equation of the present system is (see appendix):

$$rV(G_t, Q_t) = \text{MAX}_{S_t, g_t, d_t} \left\{ F(S_t + g_t, G_t, Q_t) - [z_t + V_{gt} - \delta(V_{gt} + V_{qt}q_t)]g_t - [w - \delta(V_{gt} + V_{qt}q_t)]S_t - (m + V_{gt})d_t + wR \right\}, \quad (7)$$

where $z_t = z(G_t)$, $V_{gt} = V_g(G_t, Q_t)$, $V_{qt} = V_q(G_t, Q_t)$ and $q_t = q(G_t, Q_t)$. Analogous to the simpler case of Section 2, the coefficients of g_t , S_t and d_t on the right-hand side of (6) represent the respective economic costs of these activities. These costs consist of the engineering costs plus terms containing the shadow prices V_g and V_q , which represent intertemporal effects. We see that the economic costs of ground and surface water irrigation, compared to those of Section 2, contain also the term $-\delta V_{qt}q_t$, which accounts for the salinity effect. Since V_{qt} is negative and q_t is positive (see discussion above) this term is positive, implying that the salinization process of groundwater increases the (economic) cost of irrigation.

The conjunctive ground and surface water management rules of Section 2 must be changed to incorporate effects of salinization of groundwater and the drainage activities. In view of (7), and with no binding capacity limits on irrigation, it is straightforward to derive the following management rules:

(i') As long as the economic cost of groundwater irrigation exceeds that of surface water, i.e., $z_t + V_{gt} > w$, irrigation water is derived only from surface sources at a quantity that equates the marginal productivity of water to the economic cost:

$$F_x(S_t^*, G_t, Q_t) = w - \delta(V_{gt} + V_{qt}q_t).$$

(ii') As long as the economic cost of surface water irrigation exceeds that of groundwater, i.e., $z_t + V_{gt} < w$, irrigation water is derived only from the aquifer at a quantity that equates the marginal productivity of water to its economic cost:

$$F_x(S_t^*, G_t, Q_t) = z_t + V_{gt} - \delta(V_{gt} + V_{qt}q_t).$$

(iii') When the economic cost of surface water irrigation equals that of groundwater irrigation, i.e., $z_t + V_{gt} = w$, irrigation water is derived from both sources at a quantity that equates the marginal water productivity to the economic cost:

$$\begin{aligned} F_x(S_t^* + g_t^*, G_t, Q_t) &= z_t + V_{gt} - \delta(V_{gt} + V_{qt}q_t) \\ &= w - \delta(V_{gt} + V_{qt}q_t); \end{aligned}$$

and the mix of ground and surface water is determined so as to preserve the condition $z_t + V_{gt} = w$.²

(iv) Drainage activities are either applied to a full extent or not applied at all as $m + V_{gt}$ is negative or positive, respectively:

$$d_t^* = \begin{cases} D & \text{if } V_{gt} + m < 0 \\ 0 & \text{otherwise} \end{cases}$$

²This mix rule is self-enforced. Suppose a non-optimal mix is applied with too much surface water (though the quantity of irrigation water is chosen optimally). This would increase G above the level required to maintain $z_t + V_{gt} = w$. As a result, $z_t + V_{gt}$ falls below w so that water irrigation is derived only from the aquifer (Rule (ii')). As a result, G decreases and $z_t + V_{gt}$ increases back toward w . Likewise, if the irrigation mix uses too much groundwater, G reduces and $z_t + V_{gt}$ rises above w , which, in turn, prompts irrigation from surface water only (Rule (i')), causing G to increase and $z_t + V_{gt}$ to diminish back toward w .

Rules (i'), (ii') and (iii') are similar in nature to their counterparts of Section 2. The main difference is in the levels of the irrigation activities, which in the present case are influenced also by the (shadow price of) salinity level of groundwater. The fourth rule concerns the drainage policy. It states that drainage activities are applied only when V_{gt} falls below $-m$.

In view of (iii'), a steady state in this problem is characterized by the condition $z_t + V_{gt} = w$, i.e., $z_t + V_{gt}$ remains constant:

$$d[z(G_t) + V_G(G_t, Q_t)]/dt = z'(G_t)\dot{G}_t + V_{GG}\dot{G}_t + V_{GQ}\dot{Q}_t = 0$$

($z'(G) = dz(G)/dG$). As long as the salinity level Q affects V_G (see discussion in Section 3), G will not remain constant in the steady state. For suppose that the mix of ground and surface water irrigation is such that $\dot{G}_t = 0$ [which can be achieved by the mix $g_t^*/S_t^* = \delta/(1-\delta)$]. Then, the irrigation water that leaches into the aquifer increases Q which, in turn, reduces V_{gt} . $z(G_t)$ is unchanged (since G_t is constant), thus $z_t + V_{gt}$ falls below w . As a result, groundwater irrigation is substituted for surface water irrigation (cf. (ii')), which causes G_t to fall. A similar argument can be used to rule out the possibility that G_t increases. Thus, as long as $V_G(G, Q)$ decreases with Q , preserving the equality $z_t + V_{gt} = w$ requires that the groundwater stock decreases at the appropriate rate so as to counter-balance the salinity effect on V_{gt} . A constant stock level will prevail in a steady state only when the groundwater table lies well below the root zone so that changes in the salinity level cannot harm yield, i.e., when V_G is independent of Q ($V_{GQ} = 0$).

Typically, $z(G) + V_G(G, Q)$ decreases in G . The situation $z(G) + V_G(G, Q) > w$ is therefore likely to occur at low G levels, where the groundwater table lies below the root zone. In such cases, the economic cost of groundwater exceeds that of surface water and groundwater salinity is not yet harmful; hence it is plausible that irrigation utilizes only surface water sources (cf. (i')).

As water permeates into the aquifer, the groundwater table raises toward the root zone and its quality deteriorates. This causes both the extraction cost, $z(G)$, and the groundwater shadow price $V_g(G, Q)$ to fall. Eventually, the equality $z(G) + V_g(G, Q) = w$ holds, extraction begins and irrigation water is derived both from the aquifer and from surface sources at just the right mix so as to preserve the equality $z(G) + V_g(G, Q) = w$ (cf. (iii')).

What happens if surface water irrigation is implemented above its optimal level (say, because growers behave myopically)? Then the groundwater table and salinity continue to rise (as the stock increases and its quality deteriorates) and V_{gt} diminishes (both because groundwater is less scarce and of lesser quality). As long as $z_t + V_{gt} < w$ and $V_{gt} > -m$, drainage activities are not required, but the situation is severe enough to warrant irrigation with groundwater only and the ceasing of surface water irrigation. The situation becomes drastic when the groundwater stock achieves a level in which its shadow price, V_{gt} , falls below $-m$; in such a case drainage activities are in order (cf. (iv)).

The dynamics of the system are characterized in Figure 4. The level \hat{G} is the maximum stock for which groundwater salinity does not affect the shadow price V_g (at stock levels below \hat{G} , the groundwater table is below the root zone and its salinity cannot affect yield, i.e., $V_{gq}(G, Q) = 0$ for all $G \leq \hat{G}$). The different curves represent the function $z(G) + V_g(G, Q)$ at different Q levels. They coincide over the interval $0 \leq G \leq \hat{G}$ (since Q is irrelevant in this interval), and for $G > \hat{G}$ they tilt clockwise as Q increases. The curves abc, abd and abe correspond respectively to quality levels Q_1 , Q_2 and Q_3 with $Q_1 < Q_2 < Q_3$. The curve ab \hat{G} corresponds to the maximum possible level of groundwater salinity.

Suppose the initial stock and quality of groundwater are G_1 and Q_1 , respectively (point α of Fig. 4). Since $z(G_1) + V_g(G_1, Q_1) < w$, irrigation water

is derived solely from the aquifer. As a result G decreases, Q increases and the system moves along the line $\alpha\beta$ until it reaches the point β where $z(G)+V_G(G,Q) = w$ holds. From there on the system progresses along the line $\beta\gamma$ toward the point γ (cf. (iii')) as Q increases and G diminishes at just the appropriate rate so as to preserve the equality $z(G)+V_G(G,Q) = w$. Eventually (perhaps after a very long time), the system comes to a rest at the point γ .

When the initial groundwater stock is smaller than \hat{G} , say at G_2 (point φ of Fig. 4), and $z(G_2)+V_G(G_2,Q) > w$, then it pays to irrigate only with surface water (cf. (i')). As a result, G increases until it reaches the level \hat{G} (point b of Fig. 4). At this stage it is still profitable to use only surface water for irrigation, so that both G and Q increase. The system progresses along the line $b\xi$ until it reaches point ξ , at which stage $z(G)+V_G(G,Q) = w$ holds. From there on the system progresses along the line $\xi\gamma$ toward the point γ as Q increases and G is reduced just at the appropriate rate to retain the condition $z(G)+V_G(G,Q) = w$.

Policy intervention

The above management rules differ from the myopic rules under which the instantaneous profit is maximized in each time period. The myopic rules are obtained by setting the shadow prices V_{gt} and V_{qt} equal to zero. It is clear from (iv) that, as long as drainage activities are costly (i.e., $m \geq 0$), no drainage activities are justified by the myopic rules. For reasons discussed in Section 2, with no policy intervention, the individual growers are likely to behave myopically. The available policy tools include taxes and/or quotas on irrigation water as well as drainage activities. The tax and quota policies are similar in nature to those discussed in Section 2; they will differ of course in the magnitudes of the taxes or quotas imposed (according to the difference between Rules (i)-(iii) and their primed counterparts). The drainage policy is unique to the present case; its implementation is

characterized in (iv).

Implementing these policies requires knowledge of the shadow prices $V_G(G,Q)$ and $V_Q(G,Q)$, which can be obtained by solving Problem (6), along the line of (7). The task of solving this dynamic programming problem may turn out to be quite formidable; approximate solutions, such as the one proposed by Burt and Cummings (1977), should thus be considered.

5. Investment policies

It may be of interest to find out how the irrigation/drainage management rules and the associated benefit change as some of the system parameters, such as the capacity limits C , B and D , or the water response function $F(\cdot)$ vary. A policy aimed at changing these parameters is regarded as an *investment policy*. We shall briefly discuss a few such policies which appear to be of general interest.

Extraction and drainage capacities

The capacity limits on groundwater extraction, C , and on drainage, D , are important components in the irrigation/drainage management rules. At the one extreme, no extraction or drainage facilities (wells, pumps, tiles) are installed, i.e., $C = D = 0$, so that only surface water irrigation can be applied and the region is doomed to reach a point where no agricultural production is feasible. At the other extreme, these capacities are unlimited and drainage activities can be carried out so as to instantly reduce the groundwater stock to any desirable level. Obviously, from the irrigation/drainage management point of view, unlimited capacity is preferred. However, extraction and drainage capacities entail investment costs and the benefits associated with unlimited capacities may not justify the investment.

To determine the optimal level of the extraction and drainage capacities, let $V(G,Q;C,D)$ be the benefit of an irrigation/drainage policy when the levels of groundwater stock and salinity are G and Q , respectively, and given that

extraction and drainage capacities are at the levels C and D , respectively. Let $E_c(C)$ and $E_d(D)$ be the investment costs required to achieve the capacities C and D , respectively (these technological relations depend, inter alia, on the hydrology, geology and topography of the region). Then the desirable capacity levels are those that maximize $V(G,Q;C,D) - E_c(C) - E_d(D)$.

Drainage Alternatives

It may be the case that more than one drainage alternative can be made available. Each drainage alternative entails operational costs (in the notation of Sections 3 and 4) and the investment cost of making it available. The latter contains direct investment costs (canals, tiles, reservoirs) and possibly indirect environmental costs associated with its operation.

Suppose there are M drainage alternatives with the unit drainage cost m_i , $i=1,2,\dots,M$. Denote the investment and environmental costs of the i 'th drainage alternative by ID_i , $i=1,2,\dots,M$. Let $V(G,Q;m_i)$, $i=1,2,\dots,M$, be the benefit of an irrigation/drainage policy when the unit cost of drainage is m_i . The desirable choice of drainage alternative is the one that generates the highest $V(G,Q;m_i) - ID_i$. If a particular alternative generates prohibitive environmental effects, then the associated investment cost will be so high that it will not be selected.

Variety or crop choice

Different crops, or different variety of the same crop, respond differently to water salinity. Those which are more resistant will be affected to a lesser extent by the saline groundwater. Changing the crop mix or the level of salt resistance of a particular crop entails changing the water response function $F(\cdot)$ and thereby the irrigation/drainage policy. In general, higher levels of salt resistance require smaller levels of drainage activities and thus facilitate the management problem.

Appendix

A. Derivation of the Dynamic Programming equations

In deriving Eq. (3), we write

$$V(G) = \text{MAX} \int_0^{\infty} [F(g_t + S_t) - z(G_t)g_t - w(S_t - R)]e^{-rt} dt$$

as

$$\begin{aligned} V(G) &= \text{MAX} \left\{ \int_0^{\tau} [F(g_t + S_t) - z(G_t)g_t - w(S_t - R)]e^{-rt} dt + \right. \\ &\quad \left. \int_{\tau}^{\infty} [F(g_t + S_t) - z(G_t)g_t - w(S_t - R)]e^{-rt} dt \right\} \\ &= \text{MAX} \left\{ [F(g_0 + S_0) - z(G_0)g_0 - w(S_0 - R)]\tau + o(\tau) + \right. \\ &\quad \left. \text{MAX} e^{-r\tau} \int_0^{\infty} [F(g_t + S_t) - z(G_t)g_t - w(S_t - R)]e^{-rt} dt \right\} \\ &= \text{MAX} \left\{ [F(g_0 + S_0) - z(G_0)g_0 - w(S_0 - R)]\tau + o(\tau) + e^{-r\tau} V(G_{\tau}) \right\}, \end{aligned}$$

where $o(\tau)$ is such that $o(\tau)/\tau \rightarrow 0$ as $\tau \rightarrow 0$. Writing $e^{-r\tau} = 1 - r\tau + o(\tau)$ and $V(G_{\tau}) = V(G) + V_G(G)\dot{G}\tau + o(\tau)$, collecting terms, dividing by τ , letting $\tau \rightarrow 0$, and using Eq. (2) yields Eq. (3).

Eq. (7) is derived in a similar manner using $F(g_t + S_t, G_t, Q_t)$ instead of $F(g_t + S_t)$, noting that $V(G_{\tau}, Q_{\tau}) = V(G, Q) + [V_G(G, Q)\dot{G} + V_Q(G, Q)\dot{Q}]\tau + o(\tau)$ and using Eqs. (4) and (5).

B. The management rules of problem (2) in the presence of capacity limits and positive rainfall.

The parameters B, C, and D represent respectively the capacity limits on surface water, groundwater and drainage; R denotes rainfall.

(i) If $z(G_t) + V_G(G_t) > w$ then:

(a) S_t^* is determined from

$$F_x(S_t^*) = w - \delta V_G(G_t),$$

provided a solution S_t^* exists such that $R \leq S_t^* \leq B$; otherwise $S_t^* = R$ or B as $F_x(R) \leq w - V_G(G_t)\delta$ or $F_x(B) \geq w - V_G(G_t)\delta$, respectively.

(b) $g_t^* = 0$ if $F_x(B) \leq z(G_t) + V_g(G_t)(1-\delta)$; otherwise g_t^* is the minimum between the solution of $F_x(B+g_t^*) = z(G_t) + V_g(G_t)(1-\delta)$ and C .

(ii) If $z(G_t) + V_g(G_t) < w$ then:

(a) g_t^* is determined from

$$F_x(g_t^*+R) = z(G_t) + V_g(G_t)(1-\delta),$$

provided a solution g_t^* exists such that $0 \leq g_t^* \leq C$; otherwise $g_t^* = 0$ or C as $F_x(R) \leq z(G_t) + V_g(G_t)(1-\delta)$ or $F_x(C+R) \geq z(G_t) + V_g(G_t)(1-\delta)$, respectively.

(b) $S_t^* = R$ (its lower bound) if $F_x(C+R) \leq w - \delta V_g(G_t)$; otherwise S_t^* is the minimum between the solution of $F_x(C+S_t^*) = w - \delta V_g(G_t)$ and B .

(iii) If $z(G_t) + V_g(G_t) = w$ then:

(a) Total irrigation $x_t^* = g_t^* + S_t^*$ is determined from

$$F_x(x_t^*) = w - V_g(G_t)\delta,$$

provided a solution x_t^* exists such that $R \leq x_t^* \leq C+B$; otherwise $x_t^* = R$ or $C+B$ as $F_x(R) \leq w - V_g(G_t)\delta$ or $F_x(C+B) \geq w - V_g(G_t)\delta$, respectively.

(b) If feasible, the desirable mix of ground and surface water satisfies $g_t^*/S_t^* = \delta/(1-\delta)$ such that $\dot{G}_t = 0$.

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Figure 1

Schematic representation of a conjunctive Ground and surface water system.

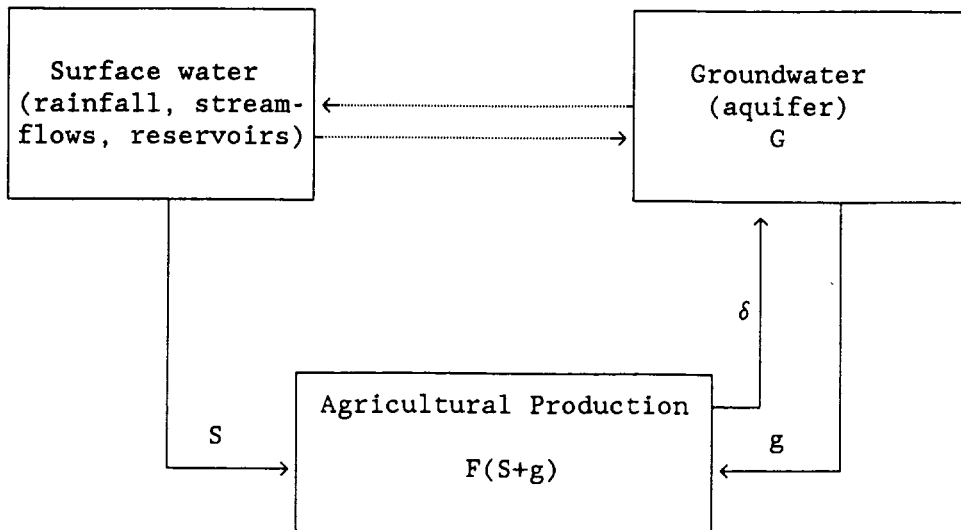


Figure 2.

Dynamic behavior of the solution of Section 2.

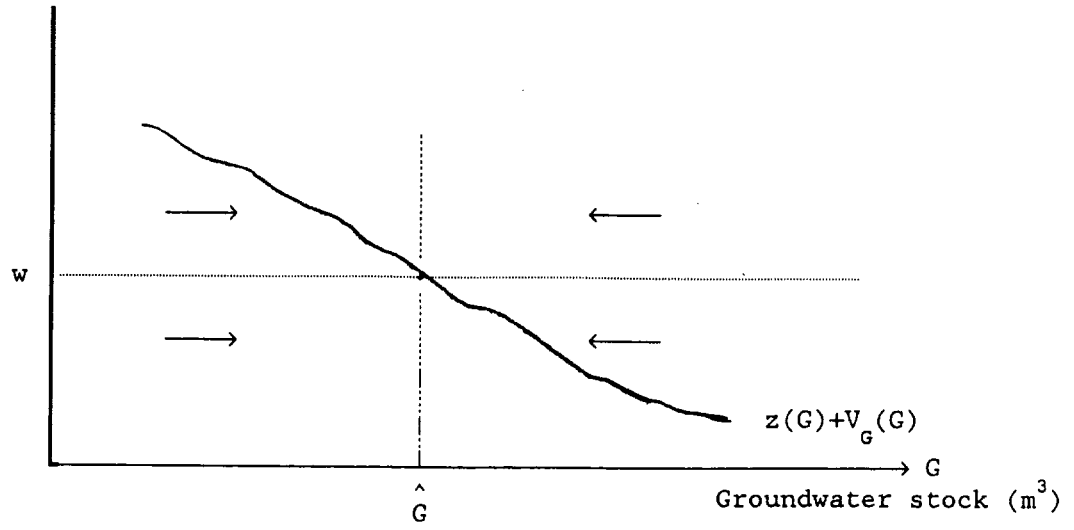
water cost ($\$/\text{m}^3$)

Figure 3.

A conjunctive Ground and surface water system with drainage.

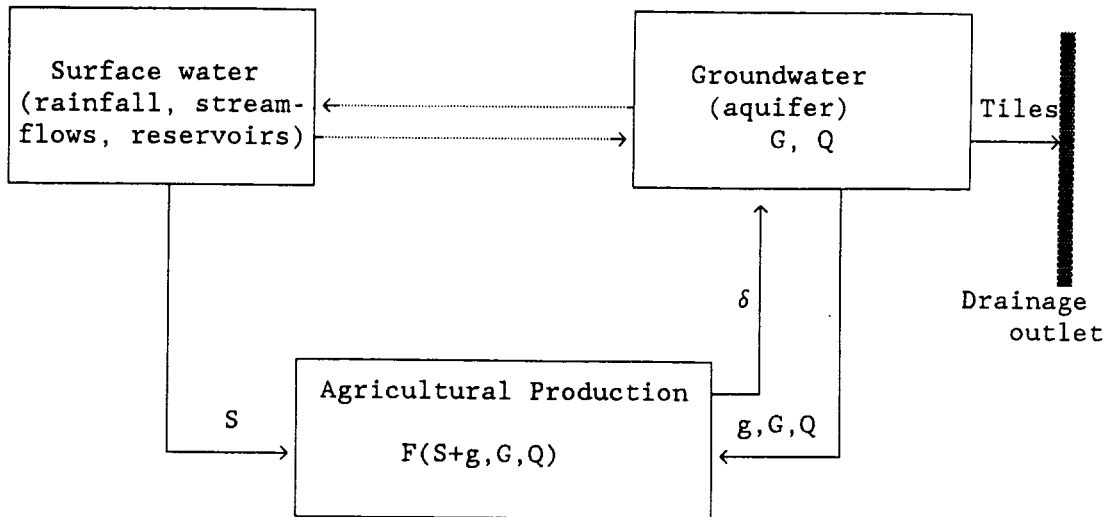


Figure 4.

Dynamic behavior of the solution of Section 4.

