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By:
Norman E. Fox
Research Assistant
Department of Agricultural and Applied Economics
University of Minnesota

Department of Agricultural and Applied Economics

University of Minnesota
Institute of Agriculture, Forestry, and Home Economics
St. Paul, MN 55108

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Norman E. Fox

The purpose of this paper is report on the construction of the thirty-five sector input-output table which represents the interindustry flows in a four-city area.[1/] The incorporated areas of the cities of Duluth, Two Harbors, and Cloquet in Minnesota, and Superior in Wisconsin are included in the study.

Two groupings of data are utilized. One grouping consists of primary data, collected and compiled by Richard Lichty of the Department of Economics, University of Minnesota, Duluth.[2/] The second grouping consists of secondary data, primarily from federal sources, but also includes a thirty-five sector input-output transaction table for Northeast Minnesota and Douglas County, Wisconsin, developed by Wilbur Maki and associates at the Department of Agricultural and Applied Economics of the University of Minnesota, St. Paul.[3/]

This methodological discussion begins with a more detailed description of the data used in the construction of the transaction matrix. Following this is a discussion of the methodology used to construct the 35-sector transaction matrix.

Economic Data Base

The Primary Data

The primary data used in this study was obtained by means of

questionnaires sent to firms in the four cities.[3/] Data collected from 189 firms was used in the construction of the transaction matrix. The data was organized into five data sets.

Data Set Number 1. This data consists of information obtained from individual firms. It includes:

1. The Standard Industrial Classification Code of each firm
2. The annual average employment of each firm in 1970
3. The total receipts of each firm in 1970
4. The value of each firm's inventory at the beginning of 1970
5. The value of each firm's inventory at the end of 1970
6. The total value of materials, parts, and supplies purchased by each firm from other firms within and outside the four cities in 1970.

Data Set Number 2. The firms from which usable data were obtained could most expediently be allocated among 14 broad industrial groupings or "sectors." These sectors are identified in Table 1.

Because of deficiencies with the primary data, the 14 sectors do not include the federal, and state and local government enterprise sectors. Coefficients in the transactions matrix relevant to these sectors had to be developed entirely from secondary sources.

Data set number 2 consists of the values of goods and services sold locally by each of the fourteen sectors to each of the other thirteen sectors.[5/] Because of deficiencies with these data, sales of the commercial sector and of the service sector are omitted. The

Table 1. Sector Breakdown of Four-City Data.

Sector Number	Standard Industrial Classification Code Sector Title
1	Construction
2	Food and Kindred Products
3	Publishing and Allied Industries
4	Primary Metals
5	Paper and Allied
6	Machinery Manufacturing
7	Other Manufacturing
8	Railroad Transportation
9	Other Transportation
10	Communications
11	Utilities
12	Commercial
13	Finance, Insurance, and Real Estate
14	Services

number "999.999" appears in all cells for which data was unavailable (Table 2). The data are presented in the matrix,

$$S = S(K, L)_{14,14},$$

where K = 1,14 and L = 1,14.

Data Set Number 3. This data consists of the values of goods and services purchased locally by each of the 14 sectors from each of the other 13 sectors (Table 3). The data are presented in the matrix,

$$P = P(K, L)_{14,14},$$

where K = 1,14 and L = 1,14.

Again, a value of 999.999 means that the particular values are not available.

Data Set Number 4. This data consists of purchases of intermediate goods by four-city industries from outside the four cities (Table 4). This data includes the sector sums of total purchases (compiled using data set 3) and the ratio of local material purchases to total material purchases (RLMP(K)).

Data Set Number 5. This data consists of 14 sector table of final demands (Table 5). Each number in the table is represented by the symbol, F(K, L). The index K represents the industrial sector and takes on the values 1 to 14. L, which takes on the values 1 to 5, represents each of five demand sectors: local government, state government, federal government, households, and inventory change.

Table 2. Sample Transactions Matrix of Interindustry Sales, Four-City Study, 1970.

Selling Sectors	Purchasing Sectors 1	2	3	4	5	6	7	8	9	10	11	12	13	14
	(thousand dollars)													
1	0	165.478	0	0	0	0	906.019	100.930	1.173	999.999	999.999	999.999	999.999	999 999
2	0	6000.000	0	0	0	0	0	0	0	0	0	0	232.237	328.903
3	0	0	241.299	0	0	0	380.410	0	0	0	0	999.999	16.480	127.641
4	0	0	0	358.340	0	0	55.000	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1349.310	0	0	0	0	0	0	0
6	0	113.380	0	0	10.000	0	0	30.000	0	0	0	0	0	0
7	3362.515	999.999	999.999	999.999	999.999	999.999	4828.775	999.999	999.999	999.999	999.999	999.999	999.999	222.403
8	0	0	0	0	563.040	0	938.400	360.050	0	0	0	0	0	0
9	0	126.800	0	32.100	187.895	5.750	2357.950	0	2650.958	1.278	0	0	9.252	30.585
10	22.411	35.464	8.684	.336	1.905	4.706	4.986	9.950	19.472	2.062	58.177	999.999	22.411	.500
11	0	0	0	1000.000	2522.000	0	2858.626	120.272	900.000	0	0	0	0	1520.000
12	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999
13	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999	999.999
14	113.430	0	.574	0	0	9.600	0	0	0	5.950	0	0	6128.186	52.322

999.999 indicates that the value for this cell is unknown

Table 3. Sample Transactions Matrix of Interindustry Purchases, Four-City Study, 1970.

Selling Sectors	Purchasing Sectors 1	2	3	4	5	6	7	8	9	10	11	12	13	14
	(thousand dollars)													
1	0	400.000	0	0	0	0	2185.350	244.264	1.100	20.000	0	999.999	0	136.506
2	0	4599.000	0	0	0	0	0	0	0	0	0	999.999	0	0
3	3.395	36.000	18.955	0	0	0	2.233	0	10.667	0	75.000	999.999	62.215	60.733
4	0	0	.180	17.000	0	131.215	0	0	1.717	0	0	999.999	0	0
5	0	535.470	0	0	0	0	9.107	0	0	0	0	999.999	0	.436
6	337.579	62.278	1.000	0	0	0	47.700	0	11.225	0	0	999.999	0	253.370
7	358.000	0	0	68.200	900.000	0	5620.627	42.271	85.587	0	0	999.999	0	20.540
8	0	0	0	0	0	0	0	0	0	0	0	999.999	0	0
9	3.297	0	19.272	0	0	19.500	10.000	0	80.000	1.104	0	999.999	82.037	216.475
10	8.772	225.500	55.082	2.200	12.000	29.948	31.763	47.100	91.976	9.656	275.000	999.999	173.045	248.437
11	8.726	546.347	58.123	207.900	690.000	46.802	1039.015	79.050	98.982	7.140	0	999.999	96.104	639.653
12	7.550	455.518	152.565	138.300	159.000	331.250	2347.112	1445.107	529.904	10.980	100.000	999.999	481.728	67.452
13	87.863	59.042	47.598	1.200	0	13.400	64.000	14.166	101.875	6.180	250.000	999.999	1149.953	64.024
14	22.377	295.700	119.303	3.850	0	21.174	72.796	0	232.314	6.885	325.000	999.999	261.589	2607.149

999.999 indicates that the value for this cell is unknown

Table 4. Sample Imports of Specified Purchasing Sectors, Four-City Study, 1970.

Sector	Imports	Total Material Purchases	Local Material Purchases per $1 Total Material Purchases
	(thou. dollars)	(thou. dollars)	
1	687.408	1,637.045	.580
2	39,052.804	49,261.717	.156
3	2,526.124	3,004.202	.159
4	2,126.350	2,565.000	.171
5	2,539.000	4,300.000	.410
6	3,192.185	3,785.474	.157
7	40,593.941	52,034.294	.220
8	8,918.241	10,790.199	.173
9	176.250	1,420.597	.876
10	84.459	146.404	.423
11	6,725.000	7,750.000	.132
12	1/	1/	1/
13	708.298	3,012.969	.765
14	2,494.218	8,409.059	.703

1/ Not available.

Secondary Data In addition to the 35-sector Northeast Minnesota, Douglas County, Wisconsin transaction matrix discussed above,[6] two other sets of secondary data were used in this study. They include both private and public industry employment data.

Private Industry Employment. This data are compiled by using three sources: the U.S. Government's County Business Patterns,[7] the U.S. Census of Population of 1970,[8] and Duluth Area Employment Trends, County Business Patterns employment data has the advantage of giving employment by place of work. It can be used to obtain employment by industry for the counties in which the four cities are located. The census data had to be used to allocate totals between the urban and rural areas in estimating employment by industry in the four cities. Further adjustments are made for discrepancies between this data and the employment data reported in Duluth Area Employment Trends.[9]

Public Industry Employment. Federal government enterprises and state and local government enterprises, respectively, make up sectors 15 and 16 of the initial 16-sector transaction matrix and sectors 34 and 35 of the 35-sector transaction matrix. Federal government enterprise employment includes Post Office workers and employees of the Veterans Administration. The ratio of these employees to the total Minnesota population is obtained from the Statistical Abstract of the United States and multiplied by the four-city population to obtain the desired estimate.[10]

Employment in state and local government enterprises includes persons working in hospitals, local airports and water supply. The

Table 5. Sample Final Demands of Specified Gross Output, Four-City Study, 1970. 1/

Sector	Government			Household	Inventory Change
	Local	State	National		
	(thou. dollars)				
1	633.988	633.988	20.000	191.143	196.735
2	0	0	0	12,914.175	269.841
3	31.128	1,253	1.253	1,428.580	28.131
4	0	0	0	0	129.230
5	0	0	0	0	57.000
6	0	102.350	598.258	0	109.800
7	488.043	10,096.922	413.391	3,425.859	3,378.105
8	0	0	0	0	0
9	0	0	21.953	2,532.745	0
10	0	0	0	0	0
11	1,750.000	350.000	500.000	10,000.000	0
12	2/	2/	2/	2/	2/
13	2/	2/	2/	2/	2/
14	3,195.895	3,231.319	10,811.594	7,751.038	0

1/ Equal numbers indicate that the government demand in these sectors could not be disaggregated by government level. The government demand is then divided equally across the aggregated levels of the sector.

2/ Not available.

number of such employees (full-time-equivalent units) per ten thousand population is found in U.S. Department of Commerce's Census of Governments.[11/]

The annual rate of change of employees per ten thousand population for hospitals (1967-72) is used to update the 1967 value of that figure to a 1970 base. The 1970 estimate is multiplied by 14.62, the 1970 four-city population divided by 10,000. The same procedure is used to update the 1967 value for state and local employees working in local airports and in water supply facilities; except that for the former, the annual rate of increase is used for the category, "all other", and for the latter, "local utilities."

Developing a 35-Sector Transaction Matrix for the Four Cities

The basic approach used to develop the 35-sector, four-city transaction matrix was to first develop a 16-sector matrix using the available primary and secondary data, and then to expand this matrix into the desired 35-sector one. The organization of the preliminary 16-sector transaction matrix developed and its relationship to the 35-sector matrix is shown in Table 6.

The inter-industry transactions in the initial input-output table as shown in the first 16 rows and columns represent purchases and sales of the 16 private and public industrial sectors from one another. Row 17 and Column 17 give, respectively, the total local interindustry purchases and the total inter-industry sales. Columns 18 through 23 show the esti-

Table 6. Organization of the 16-Sector Transaction Table for Four-City Study, 1970.

Sector	Sector Title
Inter-industry Transactions:	
1	Construction (7)
2	Food and Kindred Products (8)
3	Publishing and Allied Industries (11)
4	Primary Metals (15)
5	Paper and Allied (10)
6	Machinery Manufacturing (17, 18)
7	Other Manufacturing (9, 12, 13, 14, 16, 19)
8	Railroad Transportation (20)
9	Other Transportation (21, 22)
10	Communications (23)
11	Utilities (24, 25, 26)
12	Commercial (27, 28)
13	Finance, Insurance and Real Estate (29)
14	Services (30, 31, 32, 33)
15	Federal Government Enterprise (34)
16	State and Local Government Enterprise (35)
17	Subtotals
Final Demands:	
18	Personal Consumption Expenditures
19	Fixed Capital Investment
20	Inventory Change
21	Exports
22	Federal Government Demand
23	State and Local Government Demand
24	Row Totals
Primary Inputs and Imports:	
18	Employee Compensation
19	Imports
20	Other Value Added
21	Column Totals

mated final demand purchases for the output of the 16 sectors, for primary inputs (labor, entrepreneurial ability), and for imports from without the regions. Finally, rows 18 through 20 represent the distribution of the primary inputs and imports to the industrial sectors and to the sources of final demand. Row 21 and column 24 are, respectively, the column and the row totals. Elements in row 21 represent the total value of outputs of the local industrial sectors (columns 1-16), the total value of demand from the various sources of final demand (columns 18 -22). Elements in column 24 represent the total value of outputs of the local industrial sectors (rows 1-16), and the total value of local primary inputs and imports used in the economy (rows 18-22). The element in the extreme lower right hand corner of the matrix represents the gross output of the four cities. Finally, this section provides an outline of the procedures used to construct this initial sixteen-sector transaction matrix and to expand it to a 35 sector transaction matrix of the four cities. A series of eight steps are identified:

1. Data set number 1 is used to estimate the ratios of total intermediate goods purchased to total output for each of the intermediate demand sectors to obtain the ratios of total local intermediate goods purchases to output (see column 4, Table 7).
2. Using matrices S and P (data sets numbers 1 and 2), two new matrices, S' and P' (in which each element, i.e., AS(K, L) and AP(K, L), is an estimate of purchases by sector L from

Table 7. Relation of Local Purchases to Total Purchases and Gross Output for Specified Sectors, Four-City Study, 1970.

Sector	Firms	Total Purchases	Gross Output	Local Purchases/ $1 of: Total Purchases	Local Purchases/ $1 of: Gross Output
		(thsn. dol.)	(thsn. dol.)	(dol.)	(dol.)
1	6	1,660.298	6,155.546	.580	.156
2	6	25,624.720	51,012.291	.156	.078
3	7	3,783.459	28,891.838	.159	.021
4	3	3,025.149	6,269.190	.171	.083
5	3	25,330.000	48,374.000	.410	.214
6	4	3,999.949	7,974.862	.157	.079
7	18	26,157.920	36,182.779	.220	.159
8	2	219,476.935	1,024,512.949	.173	.037
9	10	1,234.994	8,678.747	.876	.125
10	2	990.385	2,527.595	.423	.166
11	9999	7,750.000	28,601.098	.132	.036
12	9999	999.999	999.999	999.999[1/]	999.999[1/]
13	25	6,009.671	21,153.339	.765	.217
14	51	15,411.385	55,616.083	.703	.195

1/ Insufficient data for preparing survey-based estimate.

sector K as a fraction of the total local inter-industry purchases of sector L.) are computed.

3. Output estimates of each sector are obtained using data set number 1 to estimate the average productivity of labor in each sector and using the secondary employment data (Table 9).[12] this step are given in Table 8.
4. The outcomes of the above three steps are combined to produce a partial matrix of inter-industry transactions; to complete a matrix it is necessary to use additional secondary data.
5. Some components of final demand (government demand, household consumption, and changes in inventory) are obtained from the primary data (data set number 4), which is used to further build up the transactions table.
6. The 16-sector matrix is combined with the 35-sector Northeast Minnesota matrix (which had been collapsed to a 16-sector basis) to produce the Northeast Minnesota matrix.
7. The 35-sector input-output transactions matrix is balanced so that the value of gross output is the same whether measured as the sum of purchases of goods and services of the economy or the sum of sales of the same.
8. The 16-sector transaction matrix is expanded to a 35-sector transaction matrix using the Northeast Minnesota 35-sector transaction matrix.

Table 8. Estimated Employment Output Per Worker and Gross Output of Specified Sectors, Four-City Study, 1970.

Sector	Total Employment [1]	Output Per Worker	Gross Output
	(number)	(thsn. dol.)	(thsn. dol.)
1	1,951	32.728	63,852.328
2	3,172	37.399	118,629.628
3	961	56.319	54,122.559
4	3,094	36.662	113.432.228
5	1,675	31.391	52,579.925
6	859	31.357	26,935.663
7	2,604	37.714	98,207.256
8	3,077	21.641	66,589.357
9	1,849	28.014	51,797.886
10	972	22.512	21,881.664
11	784	35.704	27,991.936
12	18,177	6.755 [2]	122,776.891
13	3,396	44.722	151,875.912
14	11,563	11.463	132,546.669
15	1,124	4.471 [2]	5,025.404
16	783	9.012 [2]	7,056.396

[1] See footnotes 5, 6, and 7 for sources.

[2] Obtained from Northeast Minnesota Simlab data (see footnote 10).

Step 1: Estimating the Ratios of Local Intermediate Goods Purchases To Output.

Using data set number 1, total purchases and output are compiled for each of the 14 private sectors. Unfortunately, the data for total purchases of data set number 1 includes purchases of intermediate goods from outside the four cities. In the Northeast Minnesota transaction matrix, imports are treated as a residual.

To make the primary data compatible with the secondary data, it is necessary to estimate total sector intermediate purchases net of imports by using data sets numbers three and four (which give, respectively, total sample purchases for each sector and sample imports). These data are used to compute the ratio of local purchases to total purchases, RLMP(K), since this is equal to one minus the ratio of imports to total purchases.

The ratios of local intermediate goods purchases to output are computed by the formula,

$$RPQ(K) = (PUR(K)/X(K)\ RLMP(K),$$

where RPQ(K) is the ratio sought, (PUR(K) is total sample sector purchases, and X(K) is the estimate of total sample sector output; K represents each of the 14 private sectors.

Step 2: Obtaining Estimates of Interindustry Sales and Purchases.

The matrix S is compiled from data provided by individual firms in the sample concerning their sales to other firms in the four-city area. The columns of this matrix give some indication of the structure of intermediate

purchases for most of the 14 private sectors.

Because some of the elements of this matrix are missing (as indicated in Table 2 by the value 999.999), the column sums of matrix S are not obtained directly; they are estimated by (1) calculating the sample outputs of each sector (i.e., the summed outputs of the sample firms comprising each sector) and (2) multiplying these outputs by the ratios of local intermediate purchases to output calculated in step 1. Letting S_L represent the estimated sample sectors purchase totals, then

$$S_L = SALES(L) + F(L, 5) \times RPQ(L),$$

where SALES(L) gives the total inter-industry sales of sector L (obtained by summing elements of the Lth row of S), F(L, 5) is the total inventory change for firms in the Lth sector of the sample (obtained from data set number 5), and RPQ(L) is the ratio of local purchases to output for sector L computed in step 1.

The elements of S' are computed according to the formula,

$$AS(K, L) = S(K, L) / S_L .$$

Computation of the elements of P' is straightforward. With the exception of column 12, there are no missing elements in this matrix. Hence, column sums are obtained directly. Let these sums be represented by P_L. Then the elements of P' are obtained according to the formula,

$$AP(K, L) = P(K, L) / P_L .$$

Step 3: Obtaining Estimates of Sector Output.

The procedure for obtaining the sector output estimates is straightforward. Observations of data set number 1 are sorted into 16 sectors by their SIC codes. Then average output per worker, SAP16(K), is computed for each sector. These values are multiplied by total sector employment to obtain total output, i.e.,

$$XX(K) = SAPL16(K) \times LAB(K),$$

where XX(K) is the estimate of total sector output and LAB(K) is the estimated of total sector employment. XX(K), SAPL16(K), and LAB(K) are presented in Table 8.

Step 4: An Initial Matrix of Interindustry Transactions.

Let X represent a 14 by 14 matrix of inter-industry transactions and let X(K, L) represent the elements of this matrix. The products of steps 1, 2, and 3 are used to obtain a preliminary inter-industry matrix by adhering to the following rules: [13]

1. If AS(K, L) = 999.999 and AP(K, L)≥0, then an intermediate step variable ZX(K, L) = AP(K, L).
2. If AP(K, L) = 999.999 and AS(K, L)≥0, then ZX(K, L) = AS(K, L).
3. If AS(K, L) = 0, and AP(K, L)>0, then ZX(K, L) = AS(K, L).
4. If AP(K, L) = 0, and AS(K, L)>0, then ZX(K, L) = AS(K, L)
5. If AP(K, L) = 999.999 and AS(K, L) = 999.999, then ZX(K, L) = X(K, L) = 999.999.
6. If AP(K, L)>0, and AS(K, L)>0, then ZX(K, L) = [AS(K, L) + AS(K, L)]/2.

7. If AP(K, L) = 0, and AS(K, L) = 0, then ZX(K, L) = X(K, L) 0.

Where the conditions of rules 5 and 7 apply, the elements of X are found directly. Where the conditions of the other rules apply, further processing is necessary. Rules 1 and 2 indicate that when inter-industry purchases estimates are not available from one data set (data sets numbers 2 and 3), but available from the other, then the estimate is obtained using the latter data set.

Rules 4 and 5 are used since a zero in either data set could result from faulty questionnaire responses. Rule 6 indicates that when both data sets provide positive data, an arithmetic mean is used.

The intermediate variables ZX(K, L) comprise an intermediate matrix, ZX, which requires further processing. This consists of constraining the column sums of the matrix X so that the ratios of total local intermediate input purchases to output for each sector are equal to the RPQ(K) variables computed in step 1.

Let SZX_K be the Kth column sum of ZX. Then the estimates of the elements of the inter-industry transaction matrix for the private sectors are,

$$X(K, L) = [ZX(K, L) / SZX_K] \times RPQ(K) \times XX(K) .$$

Step 5: Estimating Final Demand.

Government demand columns of data set number 5 are combined into a single government demand column. This is necessary because not all respondents separated out the different governmental levels in

reporting government demand for their products.

Sample government demand in each sector is then divided by the sample output of that sector. This fraction is then multiplied by the estimate of total sector output to obtain the estimate of governmental demand for the output of that sector. This fraction is then multiplied by the estimate of total sector output to obtain the estimate of governmental demand for the output of that sector. The elements in each final demand column are restricted so that the ratio of the subtotal (the sum of the first 16 elements) to the total demand of that type was equal to the corresponding ratio in the Northeast model. The method used to calculate the total value of each type of demand is discussed below.

Estimates of personal consumption expenditures and inventory changes are obtained by the same method outlined above for combined government expenditures.

Gross Permanent Fixed Capital formation is not readily available from the primary data and have to be developed using the Northeast Minnesota transactions table. Export demand is treated as a residual. The estimation of these parts of final demand will be discussed below.

Step 6: Utilizing the Northeast Minnesota Transaction Matrix.

The 35-sector Northeast Minnesota transaction matrix is collapsed into a 16-sector matrix. Each element in this matrix is then divided by its corresponding column sum The resultant coefficients matrix is used to fill in gaps in the four-city transaction matrix. Some empty cells are

filled by multiplying the corresponding coefficients of the Northeast Minnestoa matrix by the four-city total outputs or estimated column sums (for column 18-24). Such estimates are entered in columns 12 and 19, representing, respectively, the commercial sector and gross private fixed investment; rows 15 and 16, which show the value of purchases from the two public enterprise sectors -- federal and state and local government; and rows 18 and 20, which show employee compensation (personal income) and other value added.

To effect the given approach for the final demand columns, the four-city column totals are estimated. This is done for all sources of final demand (except exports which are calculated as a residual) by multiplying the corresponding column total from the Northeast Minnesota model times the total four-city employment as a fraction of Northeast Minnesota and Douglas County Wisconsin employment.

Step 7: Balancing the Transaction Matrix.

The balancing of the transaction matrix forces consistency in the input-output tables. In particular, the balances assumes that (1) the gross output of the industrial sector is the same whether measured as the sum of payments by or the sum of payments to these sectors, and that (2) the final output of the economy is the same whether measured as payments to factors plus imports or as total final purchases (including exports) of the economy. To accomplish the first purpose, the first 16 elements in the last row, last column are set equal to the outputs of 16 industrial sectors.

The last element in column 17 is obtained by summing across all the subtotals in that column; the last element in row 17 is obtained by summing across all the subtotals in that row.

The process of estimating the column totals for the final demand columns is described in step 6.

It is necessary at this point to discuss the estimation of the row totals for the final payment rows, 18 and 20. Total employee compensation (row 18, column 24) is estimated by reducing the Northeast Minnesota figure by the proportion of the four-city employment to the Northeast employment figure. Total other-value-added (row 20, column 24) is assumed equal to the subtotal for this row (row 20, column 17); that is, the services of factors of production other than labor are assumed to be entirely indigenous to the area.

Since the row and column totals are determined in advance, it is necessary to have one element in each row and column act as a residual so that the elements in each row and column do in fact add up to the predetermined row and column sums. Thus, the elements in row 19 of each of the first sixteen columns are set equal to the difference between gross output and all the other elements in each of these columns (except the subtotals). These differences are taken to be the estimates of intermediate goods imported into the four cities by the industrial sectors.

Likewise, the difference between the value of the gross outputs and all the other elements in each of the first sixteen columns is taken to be

exports. These estimates appear in column 21.

The estimate in row 19 of the subtotal column is simply the sum of all the import estimates from the first sixteen columns; similarly, the estimate in column 21 of the subtotal row is simply the sum of all the export estimates from the first sixteen rows.

Residual elements in the final demand columns (investment and consumption, inventory change, and government demand) represent final purchases from outside the four cities. Residual elements in the final payment row (employee compensation) represent export of labor services.

At this stage, some of the elements in the export residual column are negative.[14/] Since negative exports may be interpreted as net imports, a procedure is employed which in effect, yields negative balances of trade in some sectors. Specifically, each negative element in the export column is set to zero, and the original value is allocated proportionally over all the other elements in its row. This decreases the estimates of purchases by industrial sectors and final users of the output of the row sector and increases the values of the row import residuals which are recalculated.

To obtain total exports, the elements of the export column are summed. The element of this column in the import row is held at zero (in as much as imports of exports does not make sense in an input-output framework). Total imports and total gross output are

calculated by summing across rows 19 and 21 respectively.

Step 8: Expanding the 16-Sector Transactions Matrix to a 35-Sector Matrix.

The procedure used for expanding the 16-sector transactions matrix into a 35-sector matrix is straightforward.

The first step consists of obtaining 35-sector column totals for the four cities. The final demand column totals are those used in the preliminary 16-sector transaction table.

To obtain output estimates for the 35 sectors of the four-city economy, the following procedures are employed:

1. Observations of data set number 1 are sorted according to SIC code into the 35 sectors. Then sample average labor productivities are computed, which are multiplied by the secondary employment statistics according to the formula,

$$X35(K) = SAPL35(K) \times LAB35(K),$$

where X35(K) is the output of the Kth sector, SAPL35(K) [15] is the average productivity of labor in that sector, and LAB35(K) is the four-city employment of that sector (Table 9).

2. The next step consists of dividing each column in the Northeast Minnesota transaction matrix by its column sum and multiplying the coefficients in each column by the four-city column sums.

3. Let ANE(I, J) be the I, J-th element of this new adjusted matrix. Suppose that sectors I = M1 through M2 and J = N1 through N2

Table 9. Estimated Employment, Output per Worker and Gross Output of Specified Sectors, Four-City Study, 1970.

Sector	Total Employment	Output per worker	Gross Output
	(number)	(thsn. dol.)	(thsn. dol.)
7 Construction	1,951	32.728	63,852.328
8 Food and kindred products	3,172	37.399	118,629.628
9 Lumber, furniture	472	28.156	13,289.632
10 Pulp and paper products	1,675	31.391	52,579.925
11 Printing and publishing	961	56.319	54,122.599
12 Chemicals and allied products	101	96.593	9,755.893
13 Petroleum refining	164	55.561	9,112.004
14 Stone, clay and glass	200	30.579	6,115.800
15 Primary metals	3,094	36.662	113,432.228
16 Fabricated metals	620	49.436	30,650.320
17 Machinery except electrical	622	32.282	20,079.404
18 Electrical manufactures	237	25.875	6,132.375
19 Other manufacturing	1,047	11.285	11,815.395
20 Railroad transportation	3,077	21.641	66,589.357
21 Trucking	602	22.374	13,469.148
22 Other transportation	1,247	30.302	37,786.594
23 Communications	972	22.512	21,881.664
24 Electric utilities	638	35.711	22,783.618
25 Gas utilities	104	35.345	3,675.880
26 Other utilities	42	22.000	924.000
27 Wholesale trade	4,621	10.256	47,392.976
28 Retail trade	13,556	5.378	72,904.168
29 F.I.R.E.	3,396	44.722	151,875.912
30 Hotels, personal services	2,452	9.594	23,524.488
31 Business services	1,263	20.762	26,222.406
32 Medical, educational	7,047	11.651	82,104.597
33 Other services	801	7,521	6,024.321
34 Federal government ent.	1,124	4.471	5,025.404
35 State and local government	783	9.012	7,056.396

of the 35-sector model correspond, respectively, to sectors K and L of the 16-sector model. Thus, the formula for apportioning each of the sixteen sector cells is,

$$A1(I, J) = X(K, L) * \left\{ \frac{ANE(I, J)}{\sum_{J=N_1}^{N_2} \sum_{I=M_1}^{M_2} ANE(I, J)} \right\},$$

where A1(I, J) is an initial estimate of the I, J-th element in the four-city 35-sector matrix, and X(K, L)is the element to be apportioned in the 16-sector matrix.

At this stage of development, there is no guarantee that the column sums would be such that the import residuals are non-negative. Thus, is is necessary to again restrict total intermediate purchases to a predetermined proportion of the column totals.

Computation of the ratios, RPQ35(K), local intermediate good purchases to output is accomplished in the same way in the 35-sector model as it was in the 16-sector model, with the following exceptions:

1. The data are sorted by SIC code into 35 rather than 16 sectors.
2. At the time of the computations, the ratio of local purchases to total purchases is not available on a 35-sector basis. Thus, RLMP(K) is assumed to apply to all sectors of the 35-sector model which corresponds to sector K of the 16-sector model.
3. For sectors 13, 26-28, and 34 and 35, the data do not permit the computation of the desired ratios. Hence, the ratios for these

sectors is taken directly from the Northeast Minnesota model.

The previous equation yields a matrix, A1, the elements of which are A1(K, L). Let $A1_J$ represent the column sums of this matrix. Then the elements of the first 35 rows of the 35-sector, four-city transaction matrix are computed according to the formula,

$$A(K, L) = A1(K, L)/A1_J \times RPQ35(K) \times X35(K),$$

where X35(K) represents the column sums.

Row 36 consists of the column subtotals, rows 39 and 37 are obtained by apportioning the elements of rows 20 and 18 of the 16-sector matrix by the elements of rows 39 and 37 of the adjusted (by the four-city column totals) Northeastern transactions matrix according to the above formula. As before, the export columns and the import rows are residuals obtained by subtracting the values of all other elements (except subtotals) in each column (row) from the column (row) sum.

Finally, this matrix is balanced using the procedure sketched above for the 16-sector transaction matrix. The 35-sector inter-industry transaction matrix is presented in the Appendix.

FOOTNOTES

1. Although a 35-sector model is continually referred to in this paper, in fact, we are dealing with a 29-sector model. Since we are dealing with an urban model, no activity is assumed to occur in the first six sectors which are agricultural and mining industries. A 35-sector format is followed to keep this work compatible with efforts for other regions in the State of Minnesota and with the models being developed for the state as a whole. Activities of these six sectors in the Northeast Minnesota region and Douglas County, Wisconsin are covered by the Simlab model for that region (see footnote number 10 below). The impact of these sectors on the urban model developed here will be felt through the final demand sectors.

2. For a discussion of economic conditions in the four-city area and of the methods of data collection and further discussion of the primary data collected see Richard Lichty and Wayne Jesswein, An Interim Report on the Economic Base of the Duluth-Superior Growth Center. Duluth, Minnesota: Arrowhead Regional Development Commission, 1973.

3. Wilbur Maki et. al., Interim Report on Economic Projections for Long-Range Energy Planning in Northeast Minnesota and Douglas County, Wisconsin, St. Paul: Minnesota Energy Agency, November, 1975.

4. Lichty and Jesswein, op. cit.

5. This table of sales transactions and the tables (of data sets numbers 3, 4, and 5) were compiled by the staff of Professor Lichty. For some sectors, the sales to the other thirteen sectors could not be broken down into sufficient detail. This problem was approached by the author of this paper by applying the row distributions of the purchase table to the sales table for those sectors where this difficulty existed. For a more detailed explanation of this procedure, the reader may contact the author.

6. Maki et. al., op. cit.

7. U.S. Bureau of the Census. County Business Patterns, 1970. Minnesota and Wisconsin. Washington, D. C., U.S. Government Printing Office, 1971.

8. U.S. Bureau of the Census. Census of Population: 1970, General Social and Economic Characteristics, Minnesota and Wisconsin. Washington, D. C., U.S. Government Printing Office, 1972. Table 123.

9. Duluth Area Employment Trends, Duluth, Minnesota: Minnesota Department of Manpower Services. All issues, 1970.

10. U.S. Bureau of the Census, Statistical Abstract of the United States: 1971. (92nd edition) Washington, D. C., 1971. Table 608, p. 392.

11. U.S. Bureau of the Census, Census of Governments, 1967. Vol. 3, No. 2, Compendium of Public Employment, Washington, D.C.: U.S. Government Printing Office, 1969. Table 14, pp. 30 and 31; and Census of Governments, 1972, Vol. 6, Topical Studies, No. 4: Historical Statistics on Governmental Finances and Employment, Washington, D.C.: U.S. Government Printing Office, 1974. Table 20, p. 127.

12. The employment data of Table 6 had to be collapsed to a 16-sector basis. Table 9 shows the correspondence between sectors on a 35-sector basis and those on a 16-sector basis.

13. Column 12 is excluded from these processes since at this stage it consists solely of "999.999's", due to deficiencies in the primary data.

14. By restricting the column subtotals of the producing sectors in the transaction matrix to be equal to pre-estimated ratios of total local interindustry purchases to output we insured that the import residuals for these sectors were non-negative. In the final demand columns, subtotals were restricted to the same fraction of the column totals as are those of the final demand columns of the Northeast Minnesota model. This insured that the import residual would be non-negative in those columns.

15. Data was insufficient to obtain an estimate of SAPL(26), average productivity in the "other utilities" sector. Thus, an "educated guess" was employed. Since this sector consists to a large extent of garbage hauling companies which use trucks, average productivity was guessed to be $22,000, a figure slightly lower than the $22,374 average productivity of the trucking sector (sector 21).

APPENDIX Table 1. Estimated Sales and Purchases of Specified Sectors, Four-City Study, 1970.

Sector	7 Construction	8 Food	9 Lumber
7. CONSTRUC	0	301.	152.
8. FOOD AND	0	6132.	7
9. LUMBER,	1673.	0	1304.
10. PULP AND	0	514.	81.
11. PRINT AN	2.	35.	6.
12. CHEMICAL	32.	0	6.
13. PETROL,	612.	0	27.
14. STONE, C	727.	0	15.
15. PRIMARY	0	0	2.
16. FABRIC,	1147.	0	59.
17. MACHINER	130.	103.	2.
18. ELECTRIC	99.	1.	0.
19. OTHER MA	61.	0	31.
20. RAILROAD	0	0	20.
21. TRUCKING	2.	37.	15.
22. OTHER TR	1.	77.	121.
23. COMMUNIC	29.	104.	2.
24. ELECTRIC	3.	257.	158.
25. GAS UTIL	0.	87.	22.
26. OTHER UT	0.	4.	1.
27. WHOLESAL	29.	325.	280.
28. RETAIL	33.	19.	19.
29. F.I.R.E.	55.	53.	6.
30. HOTELS,	0	41.	0.
31. BUSINESS	108.	152.	3.
32. MEDICAL	11.	11.	0.
33. OTHER SE	4.	4.	1.
34. FED. GOV	51.	50.	11.
35. STATE-LO	3176.	0	28.
36.SUBTOTAL	8015.	8557.	2553.
37.PERS.INC.	13280.	22548.	2955.
38.IMPORTS	43326.	80272.	2241.
39.OTHR.V.A.	-768.	7252.	541.
40.TOTAL	63852.	118630.	14290.

APPENDIX Table 2.

	10. PULP AND	11. PRINT AN	12. CHEMICAL	13. PETROL,	14. STONE, C	15. PRIMARY	16. FABRIC,	17. MACHINER	18. ELECTRIC
7. CONSTRUC	0	0	149.	333.	143.	0	252.	0	0
8. FOOD AND	0	0	0	0	0	0	0	0	0
9. LUMBER,	2732.	0	5.	0.	14.	20.	63.	0	0
10. PULP AND	0	0	148.	35.	127.	0	210.	0	0
11. PRINT AN	0	2488.	13.	2.	3.	0	218.	0	0
12. CHEMICAL	163.	0	73.	7.	4.	6.	11.	0	
13. PETROL,	299.	0	156.	239.	17.	20.	30.	0	
14. STONE, C	26.	0	8.	3.	98.	6.	21.	0	
15. PRIMARY	0	0	1.	0.	1.	2545.	64.	455.	65.
16. FABRIC,	213.	0	25.	5.	8.	40.	306.	0	
17. MACHINER	21.	2.	2.	1.	2.	0	27.	0	0
18. ELECTRIC	1.	0.	0.	0.	0.	0	3.	0	
19. OTHER MA	186.	0	9.	1.	8.	6.	42.	0	
20. RAILROAD	1037.	0	115.	22.	125.	0	216.	0	
21. TRUCKING	166.	35.	16.	13.	37.	90.	48.	31.	9.
22. OTHER TR	231.	10.	34.	464.	31.	363.	41.	41.	13.
23. COMMUNIC	25.	122.	1.	0.	1.	4.	5.	58.	22.
24. ELECTRIC	1987.	79.	191.	98.	172.	3631.	334.	93.	22.
25. GAS UTIL	511.	9.	60.	79.	92.	1080.	65.	9.	2.
26. OTHER UT	127.	2.	5.	6.	2.	63.	6.	1.	1.
27. WHOLESAL	580.	230.	135.	63.	96.	186.	392.	509.	144.
28. RETAIL	29.	55.	31.	3.	10.	6.	83.	150.	37.
29. F.I.R.E.	0	104.	7.	8.	4.	2.	14.	32.	8.
30. HOTELS,	0	14.	1.	0.	0.	0.	3.	8.	3.
31. BUSINESS	0	67.	9.	4.	2.	2.	8.	43.	18.
32. MEDICAL	0	16.	1.	0.	0.	0.	1.	9.	3.
33. OTHER SE	0	4.	0.	0.	0.	1.	1.	4.	1.
34. FED. GOV	13.	31.	13.	6.	6.	1.	23.	1.	0.
35. STATE-LO	19.	39.	43.	4.	71.	14.	36.	15.	7.
36.SUBTOTAL	8365.	3306.	1251.	1395.	1074.	8088.	2525.	1328.	356.
37.PERS.INC.	13806.	23700.	1587.	647.	1635.	26157.	7330.	5312.	1737.
38.IMPORTS	27982.	26227.	6064.	5987.	3081.	75964.	19759.	13072.	4004.
39.OTHR.V.A.	2426.	890.	854.	1083.	325.	3224.	1037.	367.	85.
40.TOTAL	52580.	54123.	9756.	9112.	6116.	113432.	30650.	20079.	6182.

APPENDIX Table 3.

	19. OTHER MA	20. RAILROAD	21. TRUCKING	22. OTHER TR	23. COMMUNIC	24. ELECTRIC	25. GAS UTIL	26. OTHER UT	27. WHOLESAL
7. CONSTRUC	75.	150.	0.	1.	184.	0	0	0	67.
8. FOOD AND	0	0	0	0	0	0	0	0	631.
9. LUMBER,	22.	0	0.	0.	0	0	0	0	174.
10. PULP AND	73.	0	0	0	0	0	0	0	414.
11. PRINT AN	13.	0	5.	4.	0	37.	2.	0.	161.
12. CHEMICAL	13.	0.	0.	0.	0	0	0	0	25.
13. PETROL,	8.	19.	11.	28.	0	0	0	0	517.
14. STONE, C	8.	0.	0.	0.	0	0	0	0	44.
15. PRIMARY	4.	0	0.	1.	0	0	0	0	58.
16. FABRIC,	54.	1.	0.	2.	0	0	0	0	75.
17. MACHINER	7.	38.	0.	4.	0	0	0	0	144.
18. ELECTRIC	3.	5.	0.	2.	0	0	0	0	64.
19. OTHER MA	182.	1.	1.	3.	0	0	0	0	121.
20. RAILROAD	56.	439.	0	0	0	0	0	0	24.
21. TRUCKING	15.	0	267.	56.	225.	0	0	0	134.
22. OTHER TR	20.	0	154.	2256.	170.	0	0	0	24.
23. COMMUNIC	2.	18.	16.	29.	538.	134.	18.	5.	669.
24. ELECTRIC	98.	58.	15.	329.	37.	0	0	0	176.
25. GAS UTIL	10.	4.	3.	31.	3.	0	0	0	10.
26. OTHER UT	3.	4.	1.	59.	2.	0	0	0	16.
27. WHOLESAL	173.	669.	107.	146.	36.	31.	0.	1.	546.
28. RETAIL	30.	61.	50.	32.	44.	7.	1.	0.	496.
29. F.I.R.E.	4.	8.	16.	45.	54.	105.	18.	9.	1425.
30. HOTELS,	1.	0	3.	17.	558.	31.	6.	5.	44.
31. BUSINESS	3.	0	8.	43.	390.	49.	7.	3.	585.
32. MEDICAL	1.	0	0.	11.	58.	11.	2.	0.	72.
33. OTHER SE	0.	0	4.	3.	88.	5.	1.	0.	59.
34. FED. GOV	15.	274.	128.	514.	44.	18.	0.	0.	106.
35. STATE-LO	20.	39.	1.	39.	131.	199.	6.	42.	17.
36.SUBTOTAL	913.	1790.	793.	3656.	2562.	628.	61.	65.	6490.
37.PERS.INC.	2599.	35680.	3049.	7487.	7780.	7511.	814.	124.	23777.
38.IMPORTS	7980.	24461.	8682.	24451.	5334.	7111.	1985.	610.	11516.
39.OTHR.V.A.	323.	4659.	905.	2193.	6206.	7534.	816.	124.	5210.
40.TOTAL	11815.	66589.	13469.	37787.	21982.	22784.	3676.	924.	47493.

APPENDIX Table 4.

	28. RETAIL	29. F.I.R.E.	30. HOTELS.	31. BUSINESS	32. MEDICAL	33. OTHER SE	34. FED. GOV	35. STATE-LO	36.SUBTOTAL
7. CONSTRUC	321.	0	107.	60.	628.	3.	39.	1251.	4218.
8. FOOD AND	21.	0	5.	0	1832.	294.	79.	0	3994.
9. LUMBER,	9.	0	5.	0	0	0.	0	0	6020.
10. PULP AND	748.	0	2.	1.	2.	0.	7.	8.	2372.
11. PRINT AN	113.	784.	3.	786.	209.	13.	19.	19.	4935.
12. CHEMICAL	5.	0	17.	3.	57.	0.	1.	7.	431.
13. PETROL,	494.	0	104.	24.	142.	4.	29.	70.	2851.
14. STONE, C	28.	0	10.	6.	4.	1.	0.	2.	1008.
15. PRIMARY	0	0	0	0	0	0	1.	6.	3100.
16. FABRIC,	52.	0	34.	37.	3.	8.	1.	3.	2271.
17. MACHINER	3.	0	157.	898.	16.	48.	1.	8.	1614.
18. ELECTRIC	4.	0	644.	84.	80.	29.	0.	3.	1024.
19. OTHER MA	48.	0	70.	31.	74.	7.	4.	5.	892.
20. RAILROAD	37.	0	0	0	0	0	262.	30.	2564.
21. TRUCKING	18.	259.	24.	30.	66.	4.	45.	9.	1761.
22. OTHER TR	35.	765.	93.	42.	142.	336.	246.	8.	5719.
23. COMMUNIC	517.	1724.	51.	520.	263.	2.	12.	29.	4919.
24. ELECTRIC	1078.	481.	472.	61.	2908.	10.	61.	629.	13409.
25. GAS UTIL	110.	56.	53.	30.	273.	1.	7.	65.	2654.
26. OTHER UT	35.	79.	29.	7.	200.	0.	2.	6.	666.
27. WHOLESAL	325.	1720.	1542.	677.	2857.	131.	26.	41.	11997.
28. RETAIL	193.	3041.	482.	697.	1519.	177.	5.	8.	7310.
29. F.I.R.E.	3271.	13441.	87.	48.	219.	3.	102.	141.	19291.
30. HOTELS,	291.	477.	1037.	249.	630.	105.	2.	4.	3531.
31. BUSINESS	607.	1254.	208.	478.	648.	8.	25.	48.	4779.
32. MEDICAL	95.	601.	121.	85.	2000.	7.	1.	4.	3128.
33. OTHER SE	27.	53.	33.	39.	94.	5.	1.	2.	433.
34. FED. GOV	799.	871.	81.	355.	734.	3.	1.	7.	4398.
35. STATE-LO	178.	1477.	65.	22.	445.	38.	0.	2.	5176.
36.SUBTOTAL	9464.	27083.	5534.	5270.	16046.	1239.	979.	2420.	133267.
37.PERS.INC.	41966.	30283.	7536.	6908.	37459.	289.	4831.	1436.	84 741.
38.IMPORTS	10513.	31236.	7395.	10895.	23431.	1037.	673.	1175.	49 455.
39.OTHR.V.A.	10961.	63274.	3059.	3150.	5169.	3460.	-1458.	1423.	135478.
40.TOTAL	72904.	151876.	23524.	26222.	82105.	6024.	5025.	7056.	402724.

APPENDIX Table 5.

	38.PERS.CON.	39.INVEST.	31.INV.CHNG.	32.EXPORTS	33.FEDERAL	34.ST.+LOC.	35.TOTAL
7. CONSTRUC	2678.	30339.	231.	20128.	2716.	3542.	63852.
8. FOOD AND	32328.	0	124.	77184.	0	0	118630.
9. LUMBER,	1848.	1007.	171.	3475.	548.	221.	13290.
10. PULP AND	0	0	-11.	50218.	0	0	52580.
11. PRINT AN	5584.	0	-3.	43546.	31.	28.	54123.
12. CHEMICAL	240.	0	34.	8629.	391.	31.	9756.
13. PETROL,	1990.	0	434.	684.	2956.	196.	9112.
14. STONE, C	82.	0	45.	4925.	41.	15.	6116.
15. PRIMARY	0	19.	516.	109797.	0	0	113432.
16. FABRIC,	179.	245.	130.	27419.	597.	10.	30650.
17. MACHINER	0	4518.	40.	13332.	541.	35.	20079.
18. ELECTRIC	0	758.	19.	3214.	1106.	11.	6132.
19. OTHER MA	1398.	1284.	334.	0	7777.	131.	11815.
20. RAILROAD	0	175.	0	63850.	0	0	66589.
21. TRUCKING	892.	49.	0	10817.	9.	0.	13469.
22. OTHER TR	20181.	34.	0	11774.	71.	7.	37787.
23. COMMUNIC	0	558.	0	16405.	0	0	21882.
24. ELECTRIC	8149.	0	0	0.	566.	660.	22784.
25. GAS UTIL	900.	0	0	0.	44.	79.	3676.
26. OTHER UT	233.	0	0	0.	21.	3.	924.
27. WHOLESAL	13944.	1229.	24.	18888.	1167.	146.	47393.
28. RETAIL	46556.	1057.	0	18073.	-19.	-72.	72904.
29. F.I.R.E.	65438.	675.	0	65645.	482.	344.	151876.
30. HOTELS,	10173.	0	0	7124.	2609.	87.	23524.
31. BUSINESS	582.	0	0	18311.	2200.	350.	26222.
32. MEDICAL	26927.	0	0	39759.	9402.	2884.	82105.
33. OTHER SE	143.	-44.	0	0.	3973.	1519.	6024.
34. FED. GOV	416.	0	0	-0.	177.	35.	5025.
35. STATE-LO	391.	0	0	267.	219.	3.	7056.
36.SUBTOTAL	241249.	41903.	2088.	633466.	37625.	10271.	1098808.
37.PERS.INC.	0	0	0	265236.	0	0	605997.
38.IMPORTS	364748.	19761.	1903.	0	128395.	29212.	1034484.
39.OTHR.V.A.	0	0	0	0	0	0	135376.
0.TOTAL	605997.	61664.	3991.	898702.	166020.	39482.	2874666.