

DOCUMENTOS
DE ECONOMÍA

From Old Wars to New Wars
and Global Terrorism

Neil Johnson
Michael Spagat
Juan Camilo Bohórquez
Nicolás Suárez
Elvira María Restrepo
Roberto Zarama
Jorge Alberto Restrepo



Facultad de Ciencias Económicas y Administrativas

Nº19

Septiembre 2005

Departamento de Economía

Colectivo: Documentos de Economía
ISSN 0014-1801-2

From Old Wars to New Wars and
Global Terrorism

ISSN 1798-1047-1



ISSN 1798-1047-1

NEIL JOHNSON

Universidad de Oxford

MICHAEL SPAGAT

Royal Holloway, Universidad de Londres

JUAN CAMILO BOHÓRQUEZ

Departamento de Ingeniería Industrial, Universidad de los Andes

NICOLÁS SUÁREZ

Departamento de Economía, Universidad Nacional de Colombia

ELVIRA MARÍA RESTREPO

CEIDE, Universidad de los Andes

ROBERTO ZARAMA

Departamento de Ingeniería Industrial, Universidad de los Andes

JORGE ALBERTO RESTREPO

PhD (candidato) en Economía, Royal Holloway, Universidad de Londres

MSc. en Economía, Royal Holloway, Universidad de Londres

Profesor de la Pontificia Universidad Javeriana.

Especialista en: Economía Política y Análisis Económico de Conflictos Internos.

Calle 40 No. 6- 23 piso 7

Teléfono: 571-3208320

Extensiones 5180, 5172

Fax: 571-2857289

jarestrepo@javeriana.edu.co



**PONTIFICIA UNIVERSIDAD JAVERIANA
FACULTAD DE CIENCIAS ECONÓMICAS Y ADMINISTRATIVAS
DEPARTAMENTO DE ECONOMÍA**

FROM OLD WARS TO NEW WARS AND GLOBAL TERRORISM

**Neil Johnson, Michael Spagat, Jorge A. Restrepo, Juan Camilo
Bohórquez, Nicolás Suárez, Elvira María Restrepo y Roberto Zarama**

Bogotá, Septiembre de 2005

From old wars to new wars and global terrorism

Neil Johnson^{1,7}, Michael Spagat^{2,7}, Jorge A. Restrepo^{3,7}, Juan Camilo Bohórquez⁴, Nicolás Suárez^{5,7}, Elvira María Restrepo^{6,7}, and Roberto Zarama⁴

¹ *Department of Physics, University of Oxford, Oxford, U.K.*

² *Department of Economics, Royal Holloway College, University of London, Egham, U.K.*

³ *Department of Economics, Universidad Javeriana, Bogotá, Colombia*

⁴ *Department of Industrial Engineering, Universidad de los Andes, Bogotá, Colombia*

⁵ *Department of Economics, Universidad Nacional de Colombia, Bogotá, Colombia*

⁶ *Department of Economics, Universidad de Los Andes, Bogotá, Colombia*

⁷ *CERAC, Conflict Analysis Resource Center, Bogotá, Colombia*

Abstract

Even before 9/11 there were claims that the nature of war had changed fundamentally [1]. The 9/11 attacks created an urgent need to understand contemporary wars and their relationship to older conventional and terrorist wars, both of which exhibit remarkable regularities [2]-[6]. The frequency-intensity distribution of fatalities in "old wars", 1816-1980, is a power-law with exponent $1.80(9)$ ¹ [2]. Global terrorist attacks, 1968-present, also follow a power-law with exponent $1.71(3)$ for G7 countries and $2.5(1)$ for non-G7 countries [5]. Here we analyze two ongoing, high-profile wars on opposite sides of the globe - Colombia and Iraq. Our analysis uses our own unique dataset for killings and injuries in Colombia, plus publicly available data for civilians killed in Iraq. We show strong evidence for power-law behavior within each war. Despite substantial differences in contexts and data coverage, the power-law coefficients for both wars are tending toward 2.5, which is a value characteristic of non-G7 terrorism as opposed to old wars. We propose a plausible yet analytically-solvable model of modern insurgent warfare, which can explain these observations.

Resumen

Desde antes del ataque del 11 de septiembre se decía que la naturaleza de la guerra había tenido un cambio fundamental [1]. Los ataques del 11 de septiembre crearon la necesidad urgente de entender las guerras contemporáneas y su relación con las guerras convencionales y el terrorismo, que exhiben regularidades notorias [2]-[6]. La distribución de la intensidad de las muertes en las "guerras antiguas", 1816-1980, sigue una ley de potencia con un exponente de $1,80(9)$ ¹ [2]. Los ataques terroristas en el mundo, de 1968 al presente, también siguen una ley de potencia con un exponente de $1,71(3)$ para los países del G7, y de $2,5(1)$ para los países que no pertenecen al G7 [5]. En este documento analizamos dos conflictos en marcha— Colombia e Iraq. Nuestro análisis utiliza una base de datos única para muertos y heridos en Colombia, además de datos disponibles públicamente para los civiles muertos en el conflicto de Iraq. Encontramos evidencia significativa de ley de potencia en cada caso. A pesar de las diferencias sustanciales en contexto y cubrimiento de los datos, los coeficientes de la ley de potencia en cada caso tienden hacia 2,5, el cual es un valor característico del terrorismo en países que no pertenecen al G7. También se presenta un modelo analítico de guerras insurgentes modernas, el cual puede explicar estos resultados.

Correspondence: M.Spagat@rhul.ac.uk (M. Spagat), n.johnson@physics.ox.ac.uk (N. Johnson), jarestrepo@javeriana.edu.co (J. A. Restrepo)

¹ Numbers in parentheses give the standard error on the trailing figure in each case.

I. Non-technical Description of Results

What is a power law?

A power-law is a particular type of statistical pattern. It is very different from the more familiar bell curve typically associated with random incidents. The main practical implication of a 'power-law war' is that big incidents, i.e. deadly ones, are always quite likely to happen. If the distribution of casualties were a Bell curve rather than a power law, then such big events would be virtually impossible.

A power-law, and its associated power-law index, represents a particularly potent indicator. The presence of a power-law has important practical implications concerning future risk: for example, the risk that future attacks will be of a particular intensity or larger. We are currently building up a toolbox of such indicators in order to fully characterize the spatio-temporal behavior of such incidents and hence fully characterize a war's evolution and the associated risks.

What exactly did we do in this particular project?

We analyzed incidents in Iraq and Colombia, asking what fraction of incidents has one casualty, two casualties, etc. We then graphed this and looked at the shape. All sorts of shapes are possible but the power-law gives a very close fit for Iraq, for Colombia -- and also for terrorist events in non-G7 countries.

An interesting feature of the mathematics of power-laws, is that it matters little whether we use the number killed or the number killed and wounded: as long as the number killed is a fixed fraction of the number killed and wounded, we will get the same power-law behavior. In addition, the same power-law behavior will arise even if the casualty figures are systematically over-or under-reported by some fixed fraction.

What is the significance of the number 2.5?

Power-laws are characterized by a number called the 'index'. As the power-law index decreases, the chance of big events increases. Conventional wars have an index of about 1.8. The Iraq war had exactly this index at the beginning when it was a conventional conflict. But over time the index has drifted upwards toward 2.5 as the war itself has become increasingly irregular. This reflects the fact that attacks are coming from small cells which cannot coordinate well with each other because of increasing coalition pressure. By contrast, the power-law index for Colombia has drifted downwards toward 2.5, implying that the guerillas are becoming increasingly centralized and are therefore more able to generate large incidents than before.

Global terrorist events in non-G7 countries also follow a power-law with index equal to 2.5. The implication is that the degree of centralization/coordination in global terrorism is now roughly equivalent to that characterizing insurgency in both Iraq and Colombia. Hence these

groups are all quite similar in terms of their structure and operations, irrespective of their original motivations and ideals or their particular field of combat.

What does 'steady state' mean?

This simply means a kind of equilibrium where the power-law index tends to neither increase nor decrease over time. If the Colombian armed forces increase their pressure on the guerrillas, one response will be for the guerrillas to regress back to smaller, more elusive units. This would impede the guerrillas' ability to generate big attacks and hence the power-law index will increase, i.e. it will climb back above 2.5.

What does this tell us about the progress of the war in Colombia?

The news is mixed from the government's perspective. The downward trend of the power-law index toward 2.5 suggests an enhanced ability of the guerrillas to coordinate, and hence generate, big attacks. This suggests that over the years the guerrillas have strengthened their central command lines. At the same time, our results suggest that the guerrillas are currently operating in the same way as other non-G7 terrorists -- either by their own design, or because the Colombian army is forcing them to fight in this way through its own military operations. The slight increase in the index over the last few years suggests that turning up the pressure on the guerrillas has caused some fragmentation of their command structures, thereby hindering their ability to coordinate big attacks.

Descripción de los resultados

¿Qué es una ley de potencia?

Una ley de potencia es un patrón estadístico particular. Es muy diferente de la familiar curva normal o gaussiana la cual se asocia con incidentes aleatorios. La consecuencia práctica más importante de un conflicto que sigue una 'ley de potencia' es que incidentes importantes, por ejemplo aquellos que son muy letales, tienen una probabilidad de suceder que no es nada despreciable. Si la distribución de las víctimas tomara la forma de una curva normal, en vez de una de ley de potencia, ese tipo de eventos serían virtualmente imposibles.

Una ley de potencia, y su índice de ley de potencia asociado, es un indicador particularmente potente. Su presencia tiene importantes implicaciones prácticas en lo que tiene que ver los riesgos futuros: por ejemplo el riesgo que ataques futuros tengan una intensidad dada. En la actualidad estamos construyendo un método de análisis sobre la base de tales indicadores, para caracterizar el comportamiento espacio-temporal de los incidentes en un conflicto y en consecuencia comprender su evolución y los riesgos asociados.

¿Qué hicimos exactamente en este proyecto?

Analizamos los incidentes del conflicto en Irak y en Colombia, preguntándonos qué fracción de estos tuvieron una muerte, dos muertes, etc. Después graficamos esto y miramos la forma que tomaba. Diversas formas pueden aparecer, sin embargo lo que obtuvimos fue un ajuste muy cercano al de la ley de potencia para Iraq, Colombia, el cual también se obtiene para los eventos de terrorismo en los países que no pertenecen al Grupo de los 7 o G-7.

Un rasgo interesante de la matemática de las leyes de poder es que importa poco si se usa el número de muertes o el número de muertos y heridos: siempre obtenemos el mismo comportamiento. También se podrá apreciar el mismo patrón si las cifras son sistemáticamente sobre-o sub-reportadas en una fracción constante.

¿Cuál es el significado del número 2,5?

Las leyes de poder están caracterizadas por un número llamado 'índice'. Cuando este índice disminuye, la posibilidad de que ocurran eventos con una mayor intensidad en términos de víctimas se incrementa. Las guerras convencionales tienen un índice de 1.8. La guerra en Irak tenía este mismo índice en sus inicios, cuando era un conflicto convencional. Pero con el tiempo este índice se ha movido hacia arriba hasta llegar a 2.5, ahora que la guerra se ha convertido en un conflicto irregular. Allí ataques son realizados por pequeñas células que no pueden coordinar adecuadamente unas con otras dada la presión cada vez más fuerte de la coalición liderada por los Estados Unidos. En contraste, el índice de la ley de potencia para Colombia se ha movido hacia abajo hasta llegar a 2.5, lo que implica que la guerrilla se ha ido centralizando y es capaz de generar más incidentes más letales que antes.

Los eventos de terrorismo internacional en países que no pertenecen al G-7 también siguen una ley de potencia con un índice igual a 2.5. La implicación aquí es que el grado de

centralización/coordinación del terrorismo global es aproximadamente igual al que caracteriza la insurgencia en Irak o en Colombia. En consecuencia, se puede afirmar que estos grupos son similares en términos de su estructura y operación, sin tener en cuenta sus motivaciones e ideales originales o su campo de batalla particular.

¿Qué quiere decir 'estado estacionario'?

Simplemente, significa que existe un tipo de equilibrio donde el índice de la ley de potencia no tiende ni a crecer ni a caer en el tiempo. Si las fuerzas armadas colombianas aumentan su presión sobre la guerrilla, una respuesta de la guerrilla es dispersarse en unidades más pequeñas y evasivas. Esto reduce la habilidad de la guerrilla de generar ataques más grandes por lo que el índice de la ley de potencia crecería; por ejemplo, se incrementaría más allá de 2.5.

¿Qué nos dice esto sobre el progreso del conflicto en Colombia?

Desde la perspectiva del gobierno las noticias parecen ser contradictorias. La tendencia de disminución del índice de la ley de potencia hacia 2.5 sugiere una mejor habilidad por parte de las guerrillas para coordinar, y generar así ataques de mayor intensidad. Esto sugiere que a lo largo de los últimos años la guerrilla ha fortalecido sus líneas de control y comando. Al mismo tiempo, nuestro resultado sugiere que la guerrilla está actualmente operando en el mismo sentido que el terrorismo en países no pertenecientes al G-7—por su propio diseño, o porque la fuerza pública colombiana la ha llevado a luchar de esta manera. El leve aumento del índice en los últimos años sugiere que al aumentar la presión sobre la guerrilla se ha causado alguna fragmentación en su estructura de comando, obstaculizando así su habilidad de coordinar ataques de gran intensidad.

II. Power Laws Analysis. Technical Results

In two celebrated papers [3],[4] Lewis Richardson showed that war casualties follow a power law distribution, i.e. the probability that a given war has x victims, $p(x)$, is equal to $Cx^{-\alpha}$ over a reasonably wide range of x , with C and α positive coefficients. This in turn implies that a graph of $\log[P(X \geq x)]$ vs. $\log(x)$ will be a straight line over this range of x , with negative slope $\alpha - 1$ ². These results were updated recently [6] to show that interstate wars, 1820-1997, obey a power law. Each data point is a casualty count for an entire war in these studies. Casualty numbers in global terrorist events, 1968 to present, also obey power laws where in this case each data point is a terrorist attack [5].

While many people believe that 9/11 fundamentally changed the nature of warfare, some analyst had discerned new wars emerging even before this disaster [1]. Thomas Hammes views “fourth generation wars”, as trenchantly exposted by Mao Tse-tung, as the prevalent form of contemporary warfare [7],[8]. These are conflicts in which incumbents with overwhelming military and economic superiority face extremely patient insurgents seeking to break their enemies’ political will through persistent and demoralizing attacks. The phenomenon covers numerous well-known cases including Viet Nam, Iraq, Afghanistan, Israel-Palestine and Al-Qaeda [7]. Thus, fourth generation warfare encompasses both global terrorism^[5] plus a variety of civil and/or international wars as commonly understood.

Here our contribution is threefold. First, we analyze detailed daily data for two specific ongoing wars in Colombia and Iraq and find that both obey power laws. Thus, we extend Richardson’s fundamental insight into the micro world of single conflicts. Second, we show that the power-law coefficients for both wars are drifting strikingly close to the global terrorism coefficient for non-G7 countries [5]. Thus, at least these two examples of modern warfare increasingly resemble both each other and global terrorism in non-G7 countries. This finding resonates strongly with the notion of the rise of fourth generation warfare [7]. Third we propose a micro conflict model that can explain our results.

Figure 1 shows log-log plots of the fraction of all recorded events for that particular war with x or more victims, $P(X \geq x)$, versus x . For Colombia we are able to work with the very broad measure of all conflict-related killings plus injuries. For the Iraq data we work with killings of civilians as provided by the Iraq Body Count Project. The straight lines over long ranges in Figure 1 suggest that both these wars follow power laws. The Colombia data displays an extraordinary fit for a social science application while the Iraq data also fits well except for a bulge in the 150 to 350 range. Since we have many more Colombia events than Iraq ones, the superiority of the Colombia curve is not surprising. Nevertheless, the rest of the Iraq curve fits well enough so as to suggest that we should expect more events in the 150-200 range in the future. (The inset to Figure 1 shows a shortage of events in the 150-200 range. The cumulative distribution therefore exhibits a bulge, which eventually disappears around 350).

² We will refer to $P(X \geq x)$ as the cumulative distribution obtained from $p(x)$.

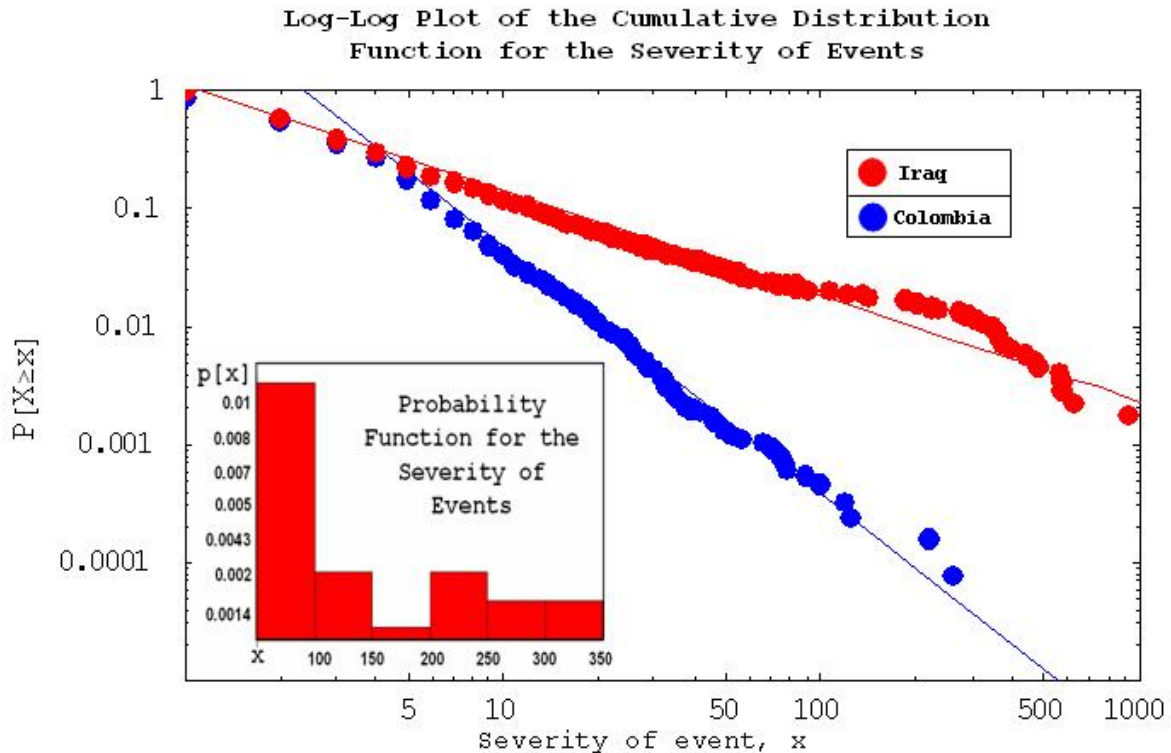


Figure 1

Figure 1. log-log plots of cumulative distributions $P(X \geq x)$ describing the total number of events with severity greater than x , for the ongoing wars in Iraq (red) and Colombia (blue). For Iraq, the severity is taken to be the lower estimate of civilian deaths from www.iraqbodycount.com. For Colombia, the severity is taken to be the total number of deaths plus injuries from the CERAC dataset [9]. Each line indicates the most likely power law that fits the data (see text). The inset shows a histogram of the Iraq data set and points to a shortage of attacks with severity in the range 150-200; this shortage creates the bulge in the Iraq line in the main figure.

Using well-established methods [2], as explained in the Methods and Supplementary Information sections, we have verified that each cumulative distribution in Figure 1 satisfies a power-law relationship over a very wide range. We also find robust power-law behavior for data collected over smaller time-windows, as discussed below, and have hence deduced the evolution of the power-law coefficient α over time by sliding this time-window through the data-series. Figure 2 shows these empirically-determined α values as a function of time for both conflicts. The α values in both cases are tending toward 2.5, which is the coefficient for global terrorism in non-G7 countries. The implication is that *both* these wars *and* global non-G7 terrorism are beginning to share a similar underlying structure. This finding is consistent with the idea of the increasing prevalence of fourth generation warfare^[7]. The Methods and Supplementary Information sections provide details of the tests we performed to verify the robustness of our results.

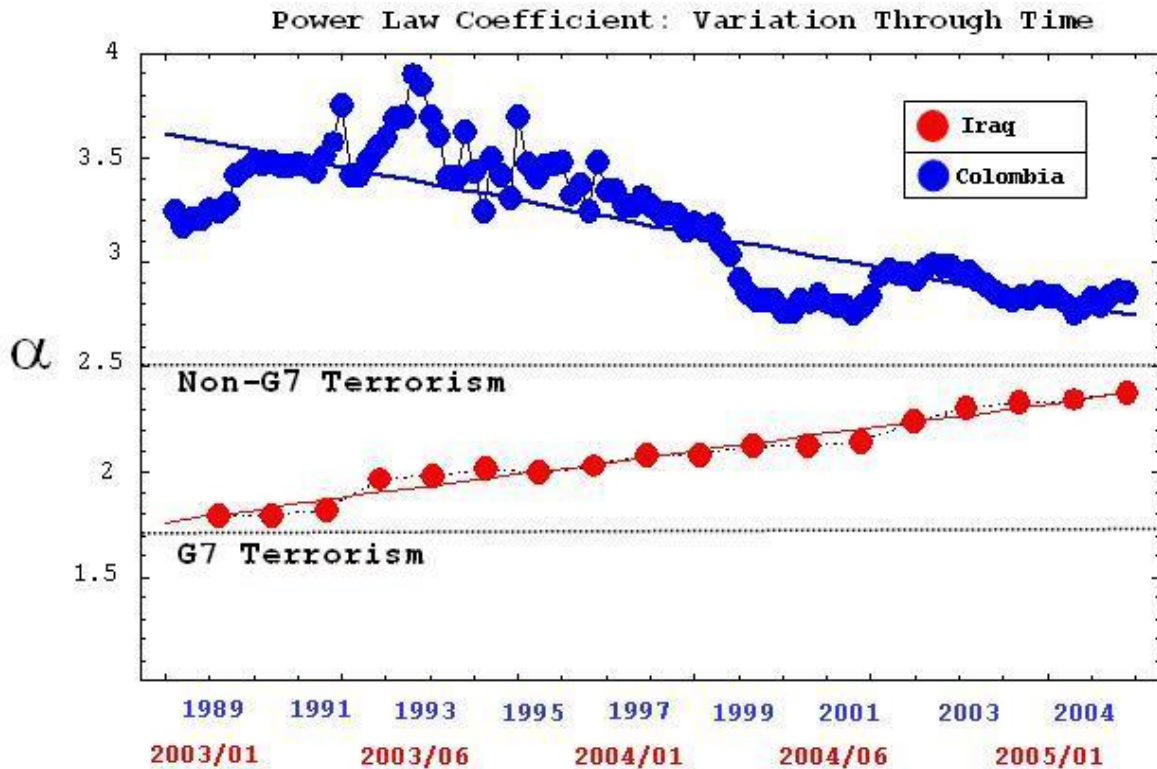


Figure 2

Figure 2. The variation through time of the power law coefficient α for Iraq (red) and Colombia (blue). The straight lines are fits through these points, and suggest a common value of approximately 2.5 for both wars in the near future. The values for G7 and non-G7 terrorism are also shown [5]. See text for details of how the variation through time of α is calculated.

There is a need for a model which can explain this common value of $\alpha \approx 2.5$. Standard physical mechanisms for generating power laws make little sense in the context of Colombia or Iraq [2]. One might instead guess that casualties would arise in rough proportion to the population sizes of the places where insurgent groups attack: given that city populations may follow a power law [2], it is conceivable that this would also produce power laws for the severity of attacks. However, we have tested this hypothesis against our Colombia data and it is resoundingly rejected.

Instead, we have developed a new model of modern insurgent warfare. As shown in Figure 3, and explained in detail in the Supplementary Information section, our model assumes that the insurgent force operates as a collection of fairly self-contained units, which we call 'attack units'. Each attack unit has a particular 'attack strength' characterizing the average number of casualties arising in an event involving this attack unit. As time evolves, these attack units either join forces with other attack units (i.e. coalescence) or break up (i.e. fragmentation). Eventually this on-going process of coalescence and fragmentation reaches a dynamical steady-state which is solvable analytically, yielding $\alpha = 2.5$. This value is in remarkable agreement with the α values to which both Colombia and Iraq appear to be tending (recall Figure 2). It also suggests that similar distributions of attack units might be emerging in both

Colombia and Iraq, with each attack unit in an ongoing state of coalescence and fragmentation. Our model also offers the following interpretation for the dynamical evolution of α observed in Figure 2. The Iraq war began as a conventional confrontation between large armies, but continuous pressure applied to the Iraqis by coalition forces has fragmented the insurgency into a structure in which smaller attack units, characteristic of non-G7 global terrorism, now predominate. In Colombia, on the other hand, the guerrillas in the early 1990's had even less ability than global terrorists to coalesce into high-impact units but have gradually been acquiring comparable capabilities.

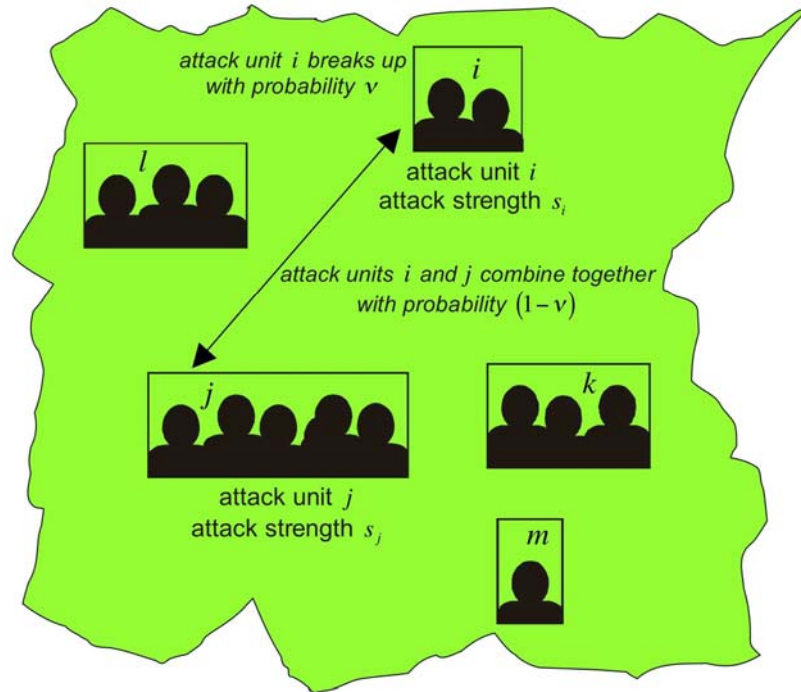


Figure 3

Figure 3. Our analytically-solvable model describing modern insurgent warfare. The insurgent force comprises *attack units*, each of which has a particular *attack strength*. The total attack strength of the insurgent force is being continually re-distributed through a process of coalescence and fragmentation. Mathematical details are provided in the Supplementary Information section.

More generally, our results – combined with those of Clauset and Young, and Richardson – suggest that there are power laws between wars, power laws for global terrorism and power laws within contemporary wars. That is, power laws have an extraordinary range of applicability to human conflict, both on the large and the small scale. In addition, our finding that the statistical patterns of the intra-war events in Colombia and Iraq appear to be trending toward the same value as global terrorist incidents in non-G7 countries (i.e. $\alpha = 2.5$) suggests that such global terrorist incidents can themselves be viewed as intra-war events within some larger, on-going yet ill-defined “global war”. This leaves open the possibility that the spatio-temporal correlations between events within a particular war, are related to those at play in global terrorism. We leave this intriguing discussion to a later publication.

III. Methods

Data Sources

We make extensive use of our own CERAC dataset for Colombia [9] plus publicly available data on Iraq (www.iraqbodycount.org). The CERAC data builds on primary source compilations of violent events by Colombian human rights NGO's and from local and national press reports. We distil from this foundation all the clear conflict events, i.e., those that have a military effect and reflect the actions of a group participating in the armed conflict. For each event we record the participating groups, the type of event (massacre, bombing, clash, etc.), the location, the methods used and the number of killings and injuries of people in various categories (guerrillas, civilians, etc.). This data set covers the years 1988-2004 and includes 20,251 events. The Iraq Body Count Project monitors the reporting of more than 30 respected online news sources, recording only events reported by at least two of them. For each event they log the date, time, location, target, weapon, estimates of the minimum and maximum number of civilian deaths and the sources of the information. The concept of civilian is broad, including, for example, policemen. The list of events, posted online, covers the full range of war activity, including suicide bombings, roadside bombings, US air strikes, car bombs, artillery strikes and individual assassinations. The data set covers the period from 2003 to the present and includes 1,746 events.

Power Law Calculation

First we used a Kolmogorov-Smirnov goodness-of-fit test to select x_{\min} , the smallest value for which the power law is thought to hold. The formula $\alpha = 1 + n \left[\sum_{i=1}^n \ln(x_i/x_{\min}) \right]^{-1}$ then estimates the power-law exponent while jackknife resampling estimates the error in α . To check these results, we then estimated α using least-square regression on the observations above x_{\min} . For both the Iraq and the Colombia data we obtained nearly identical point estimates of very high significance, with nearly null p -values using White-heteroskedasticity-corrected robust standard errors. We then performed robustness checks by excluding outliers and high-leverage observations from the regressions, finding only marginal changes in parameter estimates.

Variation through time of α

We apply the above procedures by varying x_{\min} for each estimate and also by using a fixed x_{\min} for all the estimates, and find no significant differences. The Colombian coefficients are calculated for two-year intervals displaced every 50 days. The Iraq coefficients are calculated for one-year intervals displaced every 30 days. The differences in calculation procedures were necessitated by the relatively shorter run of the Iraq data compared to the Colombia data.

IV. References

- [1] Kaldor, M. *New and Old Wars: Organized Violence in a Global Era* (Blackwell Publishers, Oxford, 1999).
- [2] Newman, M.E.J. Power laws, Pareto distributions and Zipf's law, *Contemp. Phys.* (2005) in press.
- [3] Richardson L.F. Variation of the Frequency of Fatal Quarrels with Magnitude, *Amer. Stat. Assoc.* **43**, 523-46 (1948).
- [4] Richardson, L.F. *Statistics of Deadly Quarrels*, eds. Q. Wright and C.C. Lienau (Boxwood Press, Pittsburgh, 1960).
- [5] Clauset A. and Young M., Scale invariance in Global Terrorism, e-print physics/0502014 at <http://xxx.lanl.gov>
- [6] Cederman L., Modeling the Size of Wars: From Billiard Balls to Sandpiles, *Amer. Pol. Sci. Rev.*, **97**, 135-50 (2003).
- [7] Hammes T., *The Sling and the Stone* (Zenith, St. Paul, MN, 2004).
- [8] Mao T., *On Protracted War* (People's Publishing House, Peking; 1954).
- [9] Restrepo J., Spagat, M. and Vargas, J.F. The Dynamics of the Colombian Civil Conflict, *Homo Oecon.* **21**, 396-428 (2004).

Acknowledgements The Department of Economics of Royal Holloway College provided funds to build the CERAC database. J. Restrepo acknowledges funding from Banco de la República, Colombia. J.C. Bohórquez acknowledges funding from the Department of Industrial Engineering of Universidad de los Andes.

V. Annexes: Supplementary Information

Annex 1: Detailed discussion of the model introduced in the paper

Here we provide details of the model of modern insurgent warfare, which we introduced in the main paper. Our goal is to provide a plausible model to explain (i) why power-law behaviour is observed in the Colombia and Iraq wars, and (ii) why the power-law coefficients for the Colombia and Iraq wars should both be heading toward a value of 2.5. In other words, *why should a modern war such as that currently underway in Colombia or Iraq, produce power-law behaviour and why should the value of 2.5 emerge as a power-law coefficient?*

Our model bears some similarity to a model of herding by Cont and Bouchaud³, and is a direct adaptation of the Eguiluz-Zimmerman model of herding in financial markets⁴. The analytical derivation which we present, is an adaptation of earlier formalism laid out by D’Hulst and Rodgers⁵, and also draws heavily on the material in the book *Financial Market Complexity* by Neil F. Johnson, Paul Jefferies and Pak Ming Hui (Oxford University Press, 2003). One of us (NFJ) is extremely grateful to Pak Ming Hui for detailed correspondence about the Eguiluz-Zimmerman model of financial markets, the associated formalism, and its extensions – and also for discussions involving the present model.

As suggested by Figure 3 in the paper, our model is based on the plausible notion that the total attack capability of an insurgent force in ‘fourth-generation’ warfare, is being continually re-distributed. Based on our intuition about such guerilla-like wars, we consider the insurgent force to be made up of *attack units* or *cells* which have certain *attack strength* (see below for a detailed discussion). One might expect that the total attack strength for the entire insurgent force would change slowly over time. At any particular instant, this total attack strength is distributed (i.e. partitioned) among the various attack units -- moreover the composition of these attack units, and hence their relative attack strengths, will evolve in time as a result of an on-going process of coalescence (i.e. combination of attack units) and fragmentation (i.e. breaking up of attack units). Such a process of coalescence and fragmentation is realistic for an insurgent force in a guerilla-like war, and will be driven by a combination of planned decisions and opportunistic actions by both the insurgent force and the incumbent force. For example, separate attack units might coalesce prior to an attack, or an individual attack unit might fragment in response to a crackdown by the incumbent force. Here we will model this process of coalescence and fragmentation as a stochastic process.

Each attack unit carries a specific label i, j, k, \dots and has an attack strength denoted by s_i, s_j, s_k, \dots respectively. We start by discussing what we mean by these definitions:

- *Attack unit* or *cell*: Here we have in mind a group of people, weapons, explosives, machines, or even information, which organizes itself to act as a single unit. In the case of

³ R. Cont and J.P. Bouchaud, *Macroeconomic Dynamics* **4**, 170 (2000)

⁴ V.M. Eguiluz and M.G. Zimmerman, *Phys. Rev. Lett.* **85**, 5659 (2000)

⁵ R. D’Hulst and G.J. Rodgers, *Eur. Phys. J. B* **20**, 619 (2001). See also Y. Xie, B.H. Wang, H. Quan, W. Yang and P.M. Hui, *Phys. Rev. E* **65**, 046130 (2002).

people, this means that they are probably connected by location (e.g. they are physically together) or connected by some form of communications systems. In the case of a piece of equipment, this means that it is readily available for use by members of a particular group. The simplest scenario is to just consider people, and in particular a group of insurgents which are in such frequent contact that they are able to act as a single group. However we emphasize that an attack unit may also consist of a combination of people and objects – for example, explosives plus a few people, such as the case of suicide bombers. Such an attack unit, while only containing a few people, could have a high attack strength. In addition, information could also be a valuable part of an attack unit. For example, a lone suicide bomber who knows when a certain place will be densely populated (e.g. a military canteen at lunchtimes) and who knows how to get into such a place unnoticed, will also represent an attack unit with a high attack strength.

- *Attack strength:* We define the attack strength s_i of a given attack unit i , as the average number of people who are typically injured or killed as the result of an event involving attack unit i . In other words, a typical event (e.g. attack or clash) involving group i will lead to the injury or death of s_i people. This definition covers both the case of one-sided attacks by attack unit i (since in this case, all casualties are due to the presence of attack unit i) and it also covers two-sided clashes (since presumably there would have been no clash, and hence no casualties, if unit i had not been present).

We take the sum of the attack strengths over all the attack units (i.e. the total attack strength of the insurgent force) to be equal to N . From the definition of attack strength, it follows that N represents the maximum number of people which would be injured or killed in an event, on average, if the entire insurgent force were to act together as a single attack unit.

Mathematically, $\sum_{i,j,k,\dots} s_i = N$. For any significant insurgent force, one would expect $N \gg 1$.

The power-law results that we will derive do not depend on any particular choice of N . In particular, the power-law result which is derived in this Supplementary Information section concerning the average number n_s of attack units having a given attack strength s , is invariant under a global magnification of scale (as are all power-laws).

The model therefore becomes, in mathematical terms, one in which this total attack strength N is dynamically distributed among attack groups as a result of an ongoing process of coalescence and fragmentation. As a further clarification of our terminology, we will now discuss the two limiting cases which we classify as the ‘coalescence’ and ‘fragmentation’ limits for convenience:

- ‘Coalescence’ limit: Suppose the conflict is such that all the attack units join together or *coalesce* into a single large attack unit. This is the limit of complete coalescence and would correspond to amassing all the available combatants and weaponry in a single place – very much like the armies of the past would amass their entire force on the field of battle. Hence there is one large attack unit, which we label as i and which has an attack strength N . All other attack units disappear. Hence $s_i \rightarrow N$. This ‘coalescence’ limit has the *minimum* possible number of attack units (i.e. one) but the *maximum* possible attack strength (i.e. N) in that attack unit.

- ‘Fragmentation’ limit: Suppose the conflict is such that all the attack units *fragment* into ever smaller attack units. Eventually we will have all attack units having attack strength equal to one. Hence $s_i \rightarrow 1$ for all $i=1,2,\dots,N$. This would correspond to all combatants operating essentially individually. This ‘fragmentation’ limit has the *maximum* possible number of attack units (i.e. N) but the *minimum* possible attack strength per attack unit (i.e. one).

In practice, of course, one would expect the situation to lie between these two limits. Indeed, it seems reasonable to expect that these attack units and their respective attack strengths, will evolve in time within a given war. Indeed, one can envisage that these attack units will occasionally either break up into smaller groups (i.e. smaller attack units) or join together to form larger ones. The reasons are plentiful why this should occur: for example, the opposing forces (e.g. the Colombian Army in Colombia, or Coalition Forces in Iraq) may be applying pressure in terms of searching for hidden insurgent groups. Hence these insurgent groups (i.e. attack units) might either decide, or be forced, to break up in order to move more quickly, or in order to lose themselves in the towns or countryside.

Hence attack units with different attack strengths will continually mutate via coalescence and fragmentation yielding a ‘soup’ of attack units with a range of attack strengths. At any one moment in time, this ‘soup’ corresponds mathematically to partitioning the total N units of attack strength which the insurgent army possesses. The analysis which we now present suggests that the current states of the guerilla/insurgency wars in Colombia and Iraq both correspond to the steady-state limit of such an on-going coalescence-fragmentation process. It also suggests that such a process might also underpin the acts of terrorism in non-G7 countries, and that such terrorism is characteristic of some longer-term ‘global war’.

Against the backdrop of on-going fragmentation and coalescence of attack units, we suppose that each attack unit has a given probability p of being involved in an event in a given time-interval, regardless of its attack strength. For example, p could represent the probability that an arbitrarily chosen attack unit comes across an undefended target – or vice versa, the probability that an arbitrarily chosen attack unit finds itself under attack. In these instances, p should be relatively insensitive to the actual attack strength of the attack unit involved: hence the results which we shall derive for the distribution of attack strengths, should also be applicable to the distribution of events having a given severity. When obtaining our analytic and numerical results, we assume that the war has been underway for a long time and hence some kind of steady-state has been reached. This latter assumption is again plausible for the wars in Colombia and Iraq.

Given the above considerations, it follows that if there are, on average, n_s attack units of a given attack strength s , then the average number of events involving an attack unit of attack strength s will be proportional to n_s . We assume, quite realistically, that only one insurgent attack group participates in a given event. For example, an attack in which 10 people were killed is necessarily due to an attack by a unit of attack strength 10. In particular, it could not be due to two separate but simultaneous attacks by a unit of strength 6 and a unit of strength 4 (i.e. $6+4=10$). Hence the number of events in which s people were killed and/or injured, is

just proportional to n_s . In other words, the histogram, and hence power-law, that we will derive for the dependence of n_s on s , will also describe the number of events with s casualties versus s . Indeed, if we consider that an event will typically have a duration of T , and that there will only be a few such events in a given interval T , then these results should also appear similar to the distribution describing the number of intervals of duration T in which there were s casualties, versus s . This is indeed what we have found in our analysis of the empirical data.

Given these considerations, our task of analyzing and deducing the average number of events with s casualties versus s over a given period of time, becomes equivalent to the task of analyzing and deducing the average number n_s of attack units of a given attack strength s in that same period of time. This is what we will now calculate. We will start by considering a mechanism for coalescence and fragmentation of attack groups, before then finally deducing analytically the corresponding power-law behaviour and hence deducing a power-law coefficient equal to 2.5.

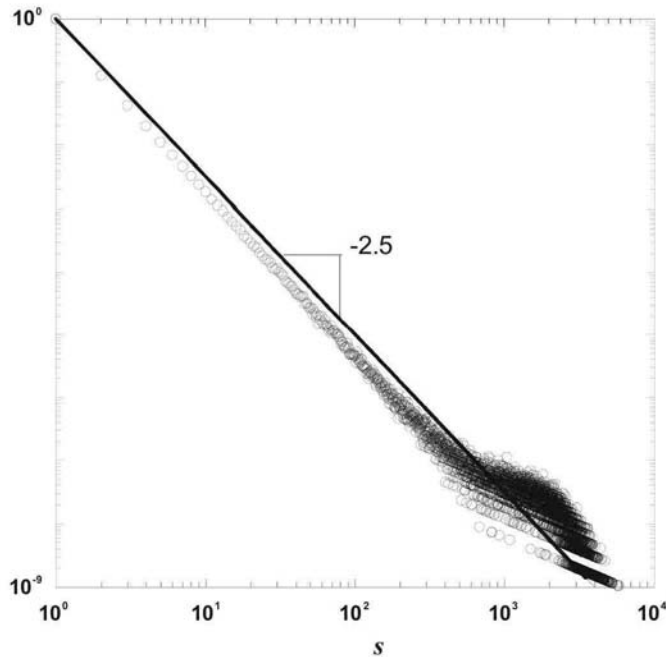
Consider an arbitrary attack unit i with attack strength s_i . At any one instant in time, labelled t , we assume that this attack unit may either:

- a) *Fragment* (i.e. break up) into s_i attack units of attack strength equal to 1. This feature aims to mimic an insurgent group which decides, either voluntarily or involuntarily, to split itself up (e.g. in order to reduce the chance of being captured and/or to mislead the enemy).
- b) *Coalesce* (i.e. combine) with another attack unit j of attack strength s_j , hence forming a single attack unit of attack strength $s_i + s_j$. This feature mimics two insurgent groups finding each other by chance (e.g. in the Colombian jungle) or deciding via radio communication to meet up and join forces.

To implement this fragmentation/coalescence process at a given timestep, we choose an attack unit i at random but with a probability which is proportional to its attack strength s_i . With a probability ν , this attack unit i with attack strength s_i *fragments* into s_i attack units with attack strength 1. A justification for choosing attack unit i with a probability which is proportional to its attack strength, is as follows: attack units with higher attack strength are likely to be bigger and hence will either run across the enemy more and/or be more actively sought by the enemy. By contrast, with a probability $(1 - \nu)$, the chosen attack unit i instead *coalesces* with another attack unit j which is chosen at random, but again with a probability which is proportional to its attack strength s_j . The two attack units of attack strengths s_i and s_j then combine to form a bigger attack unit of attack strength $s_i + s_j$. The justification for choosing attack unit j for coalescence with a probability which is proportional to its attack strength, is as follows: it is presumably risky to combine attack units, since it must involve at least one message passing between the two units in order to coordinate their actions. Hence it becomes increasingly less worthwhile to combine attack units as the attack units get smaller.

This model is thus characterized by a single parameter ν . The set up of the model is shown schematically in the figure at the front of this Supplementary Information section, and in Figure 3 of the paper. The connectivity among the attack units is driven by the dynamics of the model. For very small ν (i.e. much less than 1), the attack units steadily coalesce. This leads to the formation of large attack units. In the other limit of $\nu \rightarrow 1$, the system consists of many attack units with attack strength close to 1. A value of $\nu = 0.01$ corresponds to about one fragmentation in every 100 iterations. In what follows, we assume that ν is small since the process of fragmentation should not be very frequent for any insurgent force which is managing to sustain an ongoing war. Indeed if such fragmentation were very frequent, then this would imply that the insurgents were being so pressured by the incumbent force that they had to fragment at nearly every timestep. Hence that particular war would not last very long. It turns out that infrequent fragmentations are sufficient to yield a steady-state process, and will also yield the power-law behaviour which we observe for Colombia and Iraq.

A typical result obtained from numerical simulations, for the distribution of n_s versus attack strength s in the long-time limit (i.e. steady-state), is shown below in terms of n_s/n_1 :



Supplementary Figure 1: log-log plot of the number of attack units with attack strength s , versus attack strength s . Here $N = 10,000$ and $\nu = 0.01$. The results are obtained from a numerical simulation of the model. The initial conditions of this numerical simulation are such that all attack units have size 1. As time evolves, these attack units undergo coalescence and fragmentation as described in the text. In the long-time limit, the system reaches a steady state with a power-law dependence as shown in the figure, and with an associated power-law coefficient of 2,5 (i.e. $5/2$). The deviation from power-law behaviour at large s is simply due to the finite value of N : since there can be no attack unit with an attack strength greater than N , the finite size of N distorts the power-law as s approaches N .

We now provide an *analytic* derivation of the observed power-law behaviour, and specifically the power-law coefficient 2.5, in the steady-state (i.e. long-time) limit.

One could write a dynamical equation for the evolution of the model with different levels of approximation. For example, one could start with a microscopic description of the system by noting that at any moment in time, the entire insurgent army can be described by a partition $\{l_1, l_2, \dots, l_N\}$ of the total attack strength N into N attack units. Here l_s is the number of attack units of attack strength s . For example $\{0, 0, \dots, 1\}$ corresponds to the extreme coalescence case in which all the attack strength is concentrated in one big attack unit. By contrast, $\{N, 0, \dots, 0\}$ corresponds to the case of extreme fragmentation in which all the attack units have attack strength of 1 (i.e. there are N attack units of attack strength 1). Clearly, the total amount of attack strength is conserved $\sum_{i=1}^N l_i = N$. All that happens is that the way in which this total attack strength N is *partitioned* will change in time.

In principle, the dynamics could be described by the time-evolution of the probability function $p[l_1, l_2, \dots, l_N]$: in particular, taking the continuous-time limit would yield an equation for $dp[l_1, l_2, \dots, l_N]/dt$ in terms of transitions between partitions. For example, the fragmentation of an attack unit of attack strength s leads to a transition from the partition $\{l_1, \dots, l_s, \dots, l_N\}$ to the partition $\{l_1 + s, \dots, l_s - 1, \dots, l_N\}$. For our purposes, however, it is more convenient to work with the *average* number n_s of attack units of attack strength s , which can be written as $n_s = \sum_{\{l_1, \dots, l_N\}} p[l_1, \dots, l_s, \dots, l_N] \cdot l_s$. The sum is over all possible partitions. Since $p[l_1, \dots, l_N]$ evolves in time, so does $n_s[t]$. After the transients have died away, the system is expected to reach a steady-state in which $p[l_1, \dots, l_N]$ and $n_s[t]$ become time-independent. The time-evolution of $n_s[t]$ can be written down either by intuition, or by invoking a mean-field approximation to the equation for $dp[l_1, l_2, \dots, l_N]/dt$. Taking the intuitive route, one can immediately write down the following dynamical equations in the continuous-time limit:

$$\frac{\partial n_s}{\partial t} = -\frac{\nu s n_s}{N} + \frac{(1-\nu)}{N^2} \sum_{s'=1}^{s-1} s' n_{s'} (s-s') n_{s-s'} - \frac{2(1-\nu) s n_s}{N^2} \sum_{s'=1}^{\infty} s' n_{s'} \quad \text{for } s \geq 2 \quad (0.1)$$

$$\frac{\partial n_1}{\partial t} = \frac{\nu}{N} \sum_{s'=2}^{\infty} (s')^2 n_{s'} - \frac{2(1-\nu) n_1}{N^2} \sum_{s'=1}^{\infty} s' n_{s'} \quad (0.2)$$

The terms on the right-hand side of Equation (0.1) represent all the ways in which n_s can change. The first term represents a decrease in n_s due to the fragmentation of an attack unit of attack strength s : this happens only if an attack unit of attack strength s is chosen and if fragmentation then follows. The former occurs with probability $s n_s / N$ (see earlier discussion) and the latter with probability ν . The second term represents an increase in n_s as a result of the merging of an attack unit of attack strength s' with an attack unit of attack strength $(s-s')$. The third term describes the decrease in n_s due to the merging of an attack unit of attack strength s with any other attack unit. For the $s=1$ case described by Equation (0.2), the chosen attack unit remains isolated; thus Equation (0.2) does not have a contribution like the first term of Equation (0.1). The first term which appears in Equation (0.2) reflects the

increase in the number of attack units of attack strength equal to 1, due to fragmentation of an attack unit. Similarly to Equation (0.1), the last term of Equation (0.2) describes the merging of an attack unit with attack strength 1, with an attack unit of any other attack strength. Equations (0.1) and (0.2) are so-called ‘master equations’ describing the dynamics within the model. Note that for simplicity, we are only considering fragmentation into attack units of attack strength 1. However this could be generalized – indeed, we will look at more general fragmentations in future publications.

In the long-time steady state limit, Equations (0.1) and (0.2) yield:

$$s n_s = \frac{(1-\nu)}{(2-\nu)N} \sum_{s'=1}^{s-1} s' n_{s'} (s-s') n_{s-s'} \quad \text{for } s \geq 2 \quad (0.3)$$

$$n_1 = \frac{\nu}{(2-\nu)N} \sum_{s=2}^{\infty} (s')^2 n_s \quad (0.4)$$

Equations of this type are most conveniently treated using the general technique of ‘generating functions’. As the name suggests, these are functions which can be used to generate a range of useful quantities. Consider

$$G[y] = \sum_{s=0}^{\infty} s' n_{s'} y^{s'} \quad (0.5)$$

where $y = e^{-\omega}$ is a parameter. Note that $s n_s / N$ is the probability of finding an attack unit of attack strength s . If $G[y]$ is known, $s n_s$ is then formally given by

$$s n_s = \frac{1}{s!} G^{(s)}[0] \quad (0.6)$$

where $G^{(s)}[y]$ is the s -th derivative of $G[y]$ with respect to y . $G^{(s)}[y]$ can be decomposed as

$$G[y] = n_1 y + \sum_{s=2}^{\infty} s' n_{s'} y^{s'} \equiv n_1 y + g[y] \quad (0.7)$$

where the function $g[y]$ governs the attack-units’ attack-strength distribution n_s for $s \geq 2$. The next task is to obtain an equation for $g[y]$. This can be done in two ways. One could either write down the terms in $(g[y])^2$ explicitly and then make use of Equation (0.3), or one could construct $g[y]$ by multiplying Equation (0.3) by $e^{-\omega s}$ and then summing over s . The resulting equation is:

$$(g[y])^2 - \left(\frac{2-\nu}{1-\nu} N - 2n_1 y \right) g[y] + n_1^2 y^2 = 0 \quad (0.8)$$

First we solve for n_1 . From Equation (0.7), $g[1] = G[1] - n_1 = N - n_1$. Substituting $n_1 = N - g[1]$ into Equation (0.8) and setting $y = 1$, yields

$$g[1] = \frac{1-\nu}{2-\nu}N \quad (0.9)$$

Hence

$$n_1 = N - g[1] = \frac{1}{2-\nu}N \quad (0.10)$$

To obtain n_s with $s \geq 2$, we need to solve for $g[y]$. Substituting Equation (0.10) for n_1 , Equation (0.8) becomes

$$(g[y])^2 - \left(\frac{2-\nu}{1-\nu}N - \frac{2N}{2-\nu}y \right) g[y] + \frac{N^2}{(2-\nu)^2}y^2 = 0 \quad (0.11)$$

Equation (0.11) is a quadratic equation for $g[y]$ which can be solved to obtain

$$\begin{aligned} g[y] &= \frac{(2-\nu)N}{4(1-\nu)} \left(1 - \sqrt{1 - \frac{4(1-\nu)}{(2-\nu)^2}y} \right)^2 \\ &= \frac{(2-\nu)N}{4(1-\nu)} \left(2 - \frac{4(1-\nu)}{(2-\nu)^2}y - 2\sqrt{1 - \frac{4(1-\nu)}{(2-\nu)^2}y} \right). \end{aligned} \quad (0.12)$$

Using the expansion⁶

$$(1-x)^{1/2} = 1 - \frac{1}{2}x - \sum_{k=2}^{\infty} \frac{(2k-3)!!}{(2k)!!} x^k, \quad (0.13)$$

we have

$$g[y] = \frac{(2-\nu)N}{2(1-\nu)} \sum_{k=2}^{\infty} \frac{(2k-3)!!}{(2k)!!} \left(\frac{4(1-\nu)}{(2-\nu)^2}y \right)^k. \quad (0.14)$$

Comparing the coefficients in Equation (0.14) with the definition of $g[y]$ in Equation (0.7), the probability of finding an attack unit of attack strength s is given by:

$$\frac{sn_s}{N} = \frac{(2-\nu)}{2(1-\nu)} \frac{(2s-3)!!}{(2s)!!} \left(\frac{4(1-\nu)}{(2-\nu)^2} \right)^s. \quad (0.15)$$

⁶ The ‘double factorial’ operator !! denotes the product: $n!! = n(n-2)(n-4)\dots$

It hence follows that the average number of attack units of attack strength s is

$$\begin{aligned} n_s &= \frac{(2-\nu)}{2(1-\nu)} \frac{(2s-3)!}{s(2s)!!} \left(\frac{4(1-\nu)}{(2-\nu)^2} \right)^s N \\ &= \frac{(1-\nu)^{s-1} (2s-2)!}{(2-\nu)^{2s-1} (s!)^2} N \end{aligned} \quad (0.16)$$

The s -dependence of n_s is implicit in Equation (0.16), with the dominant dependence arising from the factorials. Recall Stirling's series for $\ln[s!]$:

$$\ln[s!] = \frac{1}{2} \ln[2\pi] + \left(s + \frac{1}{2} \right) \ln[s] - s + \frac{1}{12s} - \dots \quad (0.17)$$

Retaining the few terms shown in Equation (0.17) is in fact a very good approximation, giving an error of $< 0.05\%$ for $s \geq 2$. This motivates us to take the logarithm of both sides of Equation (0.16) and then apply Stirling's formula to each log-factorial term, as in Equation (0.17). We follow these mathematical steps (which were derived in the M.Phil. thesis of Larry Yip, Chinese University of Hong King, who was supervised by Prof. Pak Ming Hui). We hence obtain

$$\begin{aligned} \ln(n_s) &\approx \ln \left(\frac{(1-\nu)^{s-1}}{(2-\nu)^{2s-1}} N \right) + \left(2s - \frac{3}{2} \right) \ln(2s-2) + \ln(e^2) - \frac{1}{2} \ln(2\pi) - (2s+1) \ln(s) \\ &\approx \ln \left(\frac{e^2 4^s (1-\nu)^{s-1}}{2^{\frac{3}{2}} \sqrt{2\pi} (2-\nu)^{2s-1}} N \right) + \left(2s - \frac{3}{2} \right) \ln(s) - \left(3s - \frac{3}{2} \right) \frac{1}{s} - (2s+1) \ln(s) \end{aligned}$$

Combining the terms on the right-hand side into a single logarithm, it follows that

$$n_s \approx \left(\frac{(2-\nu)e^2}{2^{3/2} \sqrt{2\pi} (1-\nu)} \right) \left(\frac{4(1-\nu)}{(2-\nu)^2} \right)^s \cdot \frac{(s-1)^{2s-3/2}}{s^{2s+1}} N. \quad (0.18)$$

The s -dependence at large s can then be deduced from Equation (0.18):

$$n_s \propto N \left(\frac{4(1-\nu)}{(2-\nu)^2} \right)^s s^{-5/2}. \quad (0.19)$$

For small values of ν , the dominant dependence on s is therefore

$$n_s \sim s^{-5/2} \quad \text{hence} \quad n_s \sim s^{-2.5} \quad (0.20)$$

We have therefore shown analytically that the distribution of attack strengths will follow a power-law with a coefficient 2.5 (i.e. $5/2$). As discussed earlier, we assume that any particular

attack unit could be involved in an event in a given time interval, with a probability p which is independent of its attack size. Hence these power-law results which we have derived for the distribution of attack strengths, will also apply to the distribution of attacks of severity x . (Recall that the attack strength s is a measure of the number of casualties in a typical event, and that the severity x of an event is measured as the number of casualties). In other words, the same power-law exponent 2.5 derived in Eq. (0.20), will *also* apply to the distribution of attacks having severity x .

Hence our model predicts that any guerilla-like war which is characterized by an ongoing process of coalescence and fragmentation of attack units, and hence an ongoing re-distribution of the total attack strength, will have the following properties:

- (i) **The distribution of events with severity x will follow a power-law. This finding is consistent with the behaviour observed for the aggregated data in the Iraq and Colombia wars (see Figure 1 of the paper).**
- (ii) **The power-law distribution will, in the steady-state (i.e. long-time) limit, have a coefficient of 2.5. This is precisely the value to which the results for Colombia and Iraq currently seem to be heading (see Figure 2 of the paper).**

In the case of the Iraq war, we can go one step further by providing a simple generalization of the above model in order to offer an explanation for the evolution of the power-law coefficient throughout the war's entire history (recall Figure 2 of paper). The above model is characterized by the probability ν together with the mechanism for attack-unit coagulation and fragmentation. This value ν was chosen to be *independent* of the attack strength of the individual attack units involved. In this modification, we will keep the essential structure of the model, but we will add the modification that an attack unit will fragment with a probability which depends on its attack strength, and will coalesce with another attack unit with a probability depending on the attack strengths of the two attack units involved. With probability ν the randomly-chosen attack unit i (chosen with probability proportional to the attack strength) will fragment into attack units of attack strength 1, with a probability $f[s_i]$ which depends on s_i . With probability $(1-\nu)$ the attack unit of attack strength s_i coalesces with another randomly-chosen attack unit j having attack strength s_j , with probability $f[s_i]f[s_j]$. They remain separated otherwise. With the choice $f[s]=1$ the original model is recovered. Analytically, this particular formulation of the fragmentation and coagulation process can be readily treated by the generating function approach discussed earlier, as will be demonstrated below.

Before proceeding, we discuss why this probabilistic 'attack-unit-formation' process may indeed mimic certain aspects of guerilla warfare. One such aspect is the effect of the arrival of opposing troops in the area. Imagine that at a given timestep and with a given probability ν , the opposing army arrives in the vicinity of a given attack unit of attack strength s_i . If the overall conflict is such that the opposing army has the guerrillas/insurgents on the run, then this might suggest to the members of the insurgent attack unit that they should separate and move away from the area. However, if the state of the conflict is such that the

guerilla/insurgent force feels powerful, they are unlikely to just disband and run if they have a significant attack strength. Instead they will possibly stand their ground and fight. Hence their probability of fragmentation is likely to be a decreasing function of their attack strength. By contrast, with probability $(1-\nu)$, no opposing troops arrive in the vicinity of the attack unit. With probability $f[s_i](f[s_j])$ the attack unit i (j) decide to join forces. Thus, the two attack units will coalesce with probability $f[s_i]f[s_j]$. Again, this need for coalescing is likely to be less if the two attack units involved already feel powerful. Hence we would expect the probability of coalescence of the two attack units to be a decreasing function of their attack strengths. It is therefore quite plausible that -- depending on the state of the war from the insurgent force's perspective -- the probabilities of fragmentation and coalescence should depend on $f[s_i](f[s_j])$, i.e. they depend on the attack strengths of the attack units involved.

Analytically, the master equations for the specific example case in which $f[s] \sim s^{-\delta}$ can readily be written down:

$$\frac{\partial n_s}{\partial t} = -\frac{\nu s^{1-\delta} n_s}{N} + \frac{(1-\nu)}{N^2} \sum_{s'=1}^{s-1} (s')^{1-\delta} n_{s'} (s-s')^{1-\delta} n_{s-s'} - \frac{2(1-\nu) s^{1-\delta} n_s}{N^2} \sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'} \quad \text{for } s \geq 2 \quad (0.21)$$

$$\frac{\partial n_1}{\partial t} = \frac{\nu}{N} \sum_{s'=2}^{\infty} (s')^{2-\delta} n_{s'} - \frac{2(1-\nu) n_1}{N^2} \sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'} \quad (0.22)$$

with the physical meaning of each term being similar to that for Equations (0.1) and (0.2). The steady state equations become

$$s^{1-\delta} n_s = A \sum_{s'=1}^{s-1} (s')^{1-\delta} n_{s'} (s-s')^{1-\delta} n_{s-s'} \quad (0.23)$$

$$n_1 = B \sum_{s'=2}^{\infty} (s')^{2-\delta} n_{s'} \quad (0.24)$$

The constant coefficients A and B are given by

$$A = \frac{1-\nu}{N \nu + 2(1-\nu) \sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'}} \quad \text{and} \quad B = \frac{N \nu}{2(1-\nu) \sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'}}$$

Setting $\delta=0$ in Equations (0.23) and (0.24) recovers Equations (0.3) and (0.4) for the original model. A generating function

$$G[y] = \sum_{s'=0}^{\infty} (s')^{1-\delta} n_{s'} y^{s'} = n_1 y + g[y] \quad (0.25)$$

can be introduced where $g[y] = \sum_{s'=2}^{\infty} (s')^{1-\delta} n_{s'} y^{s'}$ and $y = e^{-\omega}$. The function $g[y]$ satisfies a quadratic equation of the form

$$(g[y])^2 - \left(\frac{1}{A} - 2n_1 y\right) g[y] + n_1^2 y^2 = 0 \quad (0.26)$$

which is a generalization of Equation (0.8). Using $n_1 + g[1] = \sum_{s=1}^{\infty} (s')^{1-\delta} n_{s'}$ and Equation (0.26), n_1 can be obtained as

$$n_1 = \frac{(1-\nu)^2 - \nu^2 A^2 N^2}{4(1-\nu)^2 A} \quad (0.27)$$

Solving Equation (0.26) for $g[y]$ gives

$$g[y] = \frac{1}{4A} \left(1 - \sqrt{1 - 4n_1 A y}\right)^2 \quad (0.28)$$

Following the steps leading to Equation (0.19), we obtain n_s in the modified model:

$$n_s \approx N \left[\frac{4(1-\nu) \left((1-\nu) + \frac{N\nu}{\sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'}} \right)}{\left(\frac{N\nu}{\sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'}} + 2(1-\nu) \right)^2} \right]^s s^{-(5/2-\delta)} \quad (0.29)$$

For $\delta=0$, $\sum_{s=1}^{\infty} (s')^{1-\delta} n_{s'} = N$ and hence Equation (0.29) reduces to the result in Equation (0.19) for the original model. For $\delta \neq 0$, it is difficult to solve explicitly for n_s . However the summation simply gives a constant, and thus for small ν the dominant dependence on the attack strength s is $n_s \sim s^{-(5/2-\delta)}$ and hence equivalently $n_s \sim s^{-(2.5-\delta)}$.

Most importantly, we can see that by decreasing δ from $0.7 \rightarrow 0$ (i.e. by increasing the relative fragmentation/coalescence rates of larger attack units) we span the entire spectrum of power-law exponents observed in the Iraq war from the initial value of 1.8, up to the current tendency towards 2.5. This effect of decreasing δ from $0.7 \rightarrow 0$ corresponds in our model to a relative increase in the tendency for larger attack units to either fragment or coalesce at each timestep. In other words, decreasing δ mimics the effect of decreasing the relative robustness or ‘lifetime’ of larger attack units.

Going further, we note that these theoretical results are consistent with, *and to some extent explain*, the various power-law exponents found for:

(1) Conventional wars. The corresponding power-law exponent 1.8 can now be interpreted through our generalized model with $\delta \approx 0.7$, as a tendency toward building larger, robust attack units with a fixed attack strength as in a conventional army -- as opposed to attack units

with rapidly fluctuating attack strengths as a result of frequent fragmentation and coalescence processes. There is also a tendency to form a distribution of attack units with a wide spectrum of attack strengths – this is again consistent with the composition of ‘conventional’ armies from the past.

(2) Terrorism in G7 countries. The corresponding power-law exponent 1.7 can be interpreted through our generalized model with $\delta \approx 0.8$, as an even stronger tendency for robust units (e.g. terrorist cells) to form. There is also an increased tendency to form larger units – or rather, to operate as part of a large organization.

(3) Terrorism in non-G7 countries. The corresponding power-law exponent 2.5 can be interpreted through our model with $\delta = 0$, as a tendency toward more transient attack units (e.g. terrorist cells) whose attack strengths are continually evolving dynamically as a result of an on-going fragmentation and coalescence process. Unlike a conventional army, there will be a tendency to form smaller attack units rather than larger ones.

Interestingly, we can now discuss the evolution of the wars in Colombia and Iraq in these terms:

War in Colombia. At the beginning of the 1990’s, the power-law exponent was very high (3.5). Then over the following 15 years, it gradually lowered to the present value and appears to be tending toward 2.5. Using our model, the interpretation is that the war at the beginning of the 1990’s was such that the guerrillas favoured having small attack units. This is possibly because they lacked communications infrastructure, and/or did not feel any safety in larger numbers. The decrease toward the value 2.5, suggests that this has changed – probably because of increased infrastructure and communications, enabling attack units with a wide range of attack strengths to build up.

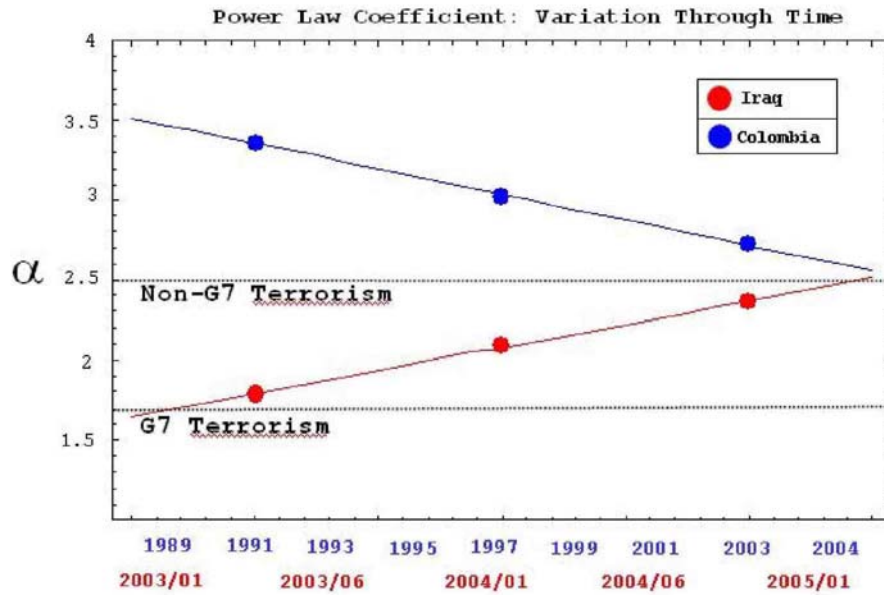
War in Iraq. At the beginning of the war in 2002, the power-law exponent was quite low (1.8) and was essentially the same value as conventional wars. This is consistent with the war being fought by a conventional Iraqi army against the Coalition forces. There is then a break in this value after a few months (i.e. the war ended) and following this, the power-law exponent gradually rises towards 2.5. This suggests that the insurgents have been increasingly favouring more temporary attack units, with an increasingly rapid fragmentation-coalescence process. This finding could be interpreted as being a result of increased success by the Coalition Forces in terms of forcing the insurgents to fragment. On the other hand, it also means that the Iraq War has now moved to a value, and hence character, which is consistent with generic non-G7 terrorism.

Annex 2: Supplementary tables and figures which help confirm the robustness of our results

Estimates of power-law coefficients for the entire time-series						
	α_1	Confidence Interval	Percentage of points inside interval	α_2	Confidence Interval	Adjusted R^2
K	3.1013	+/- 0.02	0.95%	2.8761	+/- 0.0151	0.9835
I	3.04	+/- 0.02	0.95%	2.9717	+/- 0.0211	0.9818
KI	2.93	+/- 0.015	0.95%	3.0061	+/- 0.0179	0.9796
CK _{min}	2.07	+/- 0.005	0.90%	2.1279	+/- 0.0057	0.9336
CK _{max}	2.02	+/- 0.003	0.90%	2.0966	+/- 0.0043	0.9496

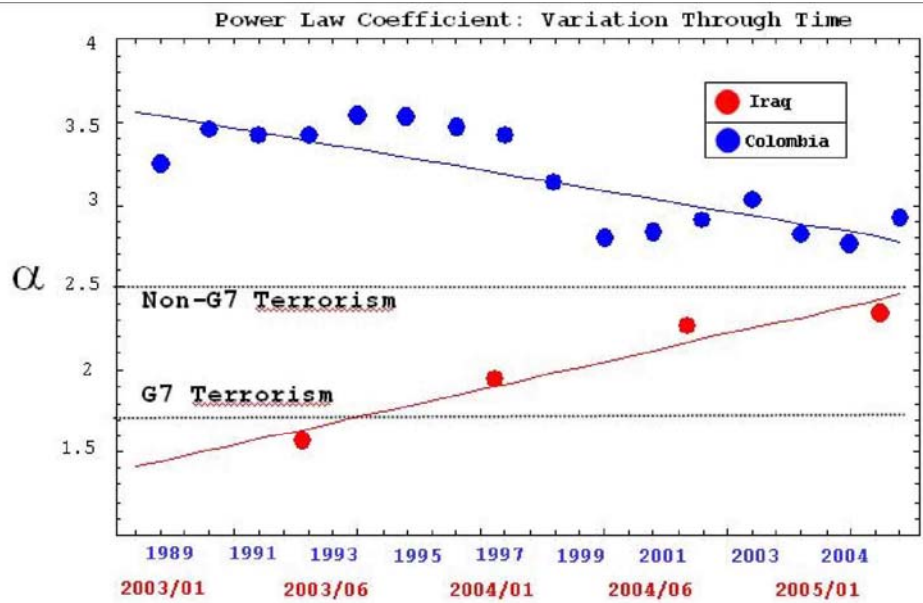
Supplementary Table 1 Shows two complementary estimates of the power-law coefficients for the variables K (reported deaths for Colombia), I (reported injuries for Colombia), KI (reported deaths plus injuries for Colombia), CK_{min} (minimum reported civilian deaths for Iraq) and CK_{max} (maximum reported civilian deaths for

Iraq). Our first estimate (α_1) uses $\alpha = 1 + n \left[\sum_{i=1}^n \ln \left(\frac{x_i}{x_{\min}} \right) \right]^{-1}$ while our second estimate (α_2) uses ordinary least-squares linear regression. The two results are always very similar and the results vary little as we vary the victimization measure for Colombia. For further discussion, see PART 3 of this Supplementary Information document.



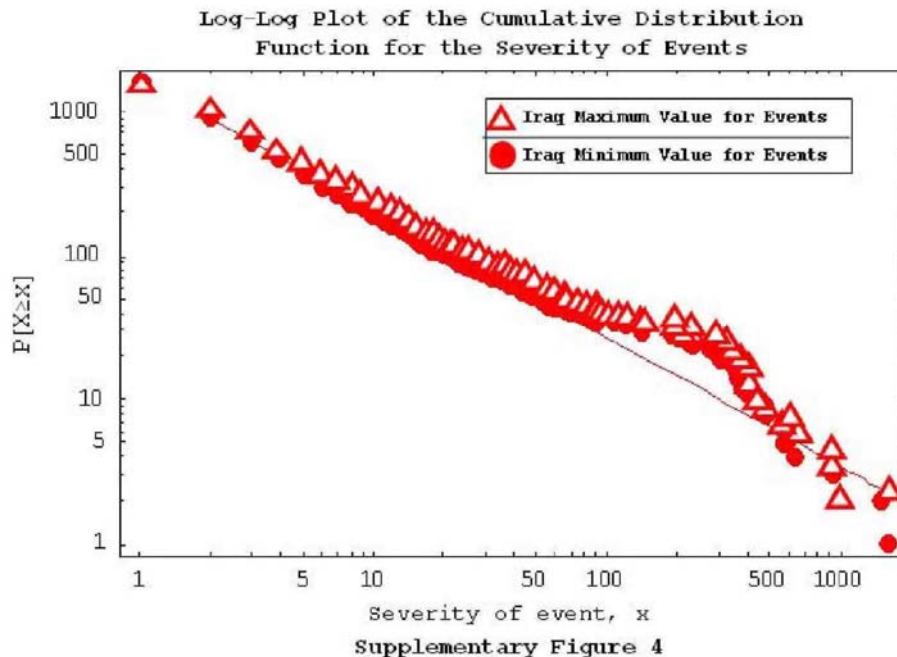
Supplementary Figure 2

Supplementary Figure 2 The variation through time of the power law coefficients for three 2,500 day intervals displaced by 1,855 days for the Colombian data and three 365 day intervals displaced every 258 days for the Iraq data. Despite this change in size of the windows and how they slide across time both curves seem to be tending toward 2.5, as in Figure 2 of the paper. For further discussion, see PART 3 of this Supplementary Information document.



Supplementary Figure 3

Supplementary Figure 3 The variation through time of the power law coefficients for two year intervals displaced every year for Colombia and 200 day intervals displaced every 200 days for Iraq. Again, they both seem to be tending toward 2.5, as in Figure 2 of the paper. For further discussion, see PART 3 of this Supplementary Information document.



Supplementary Figure 4

Supplementary Figure 4 log-log plots of cumulative distributions $P(X \geq x)$ describing events greater than x , for the minimum possible value and maximum possible value of each event in the Iraq dataset. The results are very much the same across the two measures. For further discussion, see PART 3 of this Supplementary Information document.

Annex 3: Supplementary Notes on Methods

If $p(x) = Cx^{-\alpha}$, then, $C = (\alpha - 1)x_{\min}^{(\alpha-1)}$ and $\alpha = 1 + n \left[\sum_{i=1}^n \ln \left(\frac{x_i}{x_{\min}} \right) \right]^{-1}$. We used two estimates for α (see Supplementary Information Table 1 in PART 2). We estimated α_1 using α with x_{\min} equal to the minimum value that satisfies the Kolmogorov statistic for the whole data set. We estimated α_2 using ordinary least-square regression analysis for all the values greater than x_{\min} .

We established x_{\min} as the minimum value of x where we could not reject the hypothesis that the data beyond x_{\min} followed a power law using a Kolmogorov-Smirnov goodness of fit test at 95% confidence.

In order to estimate the error of α , we used a minus-one jackknife resampling method. We created the datasets obtained by removing one value from the original dataset. For each of these surrogates, we estimated α and C . We then measured the maximum and minimum deviation from the mean of the parameters α obtained from the diverse jackknife datasets. We established an interval equal to the maximum distance from the mean of the sample times 5. Since this is not an analytical solution, we present the percentage of values that fall into that interval.

The Evolution of α : To test the robustness of our findings in Figure 2 of the paper, we have repeated the calculation of α for several different sizes of the time windows. We also tried varying the way in which these windows slide forward in time. All these changes barely affected our results.

As an example, in Part 2 of this document we have plotted the evolution of KI (deaths and injuries) for different time windows. For the Colombian data set we used a time window of 2,500 days moved every 1,855 days (see Supplementary Figure 2); then a moving time window of two years displaced every year (see Supplementary Figure 3), and finally a moving time window of two years, displaced every 50 days (Figure 1). For the Iraq data set we used a 365 day time window displaced every 258 days (Supplementary Figure 2), a 250 day interval displaced every 150 days (Supplementary Figure 3), and a 365 day time window displaced every 30 days (Figure 1). As can be seen, our results are essentially unchanged by these variations.

Further general comment on robustness testing: As a further test of the robustness of the results obtained in our paper, and in particular our main findings in Figures 1 and 2 of the paper, we ran the following variations of our calculations. For Colombia we used just killings and just injuries, rather than killings plus injuries as presented in the paper. For Iraq we used the maximum number of deaths rather than the minimum number of deaths as reported in the paper. These results are shown in Supplementary Figure 4. As can be seen, these variations do not affect our findings. This is reassuring, and is actually not too surprising since the

power-law coefficient α provides a statistical measure of the structure of the events' time-series, rather than the absolute number of killings and/or injuries. Hence the power-law coefficient α will be unaffected by any constant scale factors which are introduced as a result of a fixed ratio of injuries to killings.

An additional, but perhaps even more important, advantage of focusing on α , concerns possible over- or under-reporting of war casualties. In particular, α is insensitive to systematic over-reporting or under-reporting of casualties. This is because any systematic multiplication of the raw numbers by some constant factor, has no affect on the α value which emerges from the log-log plot.