AIDS and Economic Growth in South Africa

By

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Abstract

Morbidity and mortality effects are introduced into a three sector, Ramsey-type model of economic growth. The model is calibrated to South African national accounts data and used to examine the potential impact of HIV/AIDS on economic growth. Simulation results suggest a 10% decrease in the size of the effective labor force would lead to a 10% decrease in long run (steady state) GDP levels. Similarly, a 10% decrease in the number of laborers would lead to an 11% drop in long run GDP.

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1 Introduction

Of the 36,1 million people living with HIV/AIDS, 95 percent live in developing countries. Sub-Saharan Africa has 29.4 million people living with HIV/AIDS, of which ten million are young adults between the ages of 15 and 24, and almost 3 million children under 15 are living with HIV. Approximately 3.5 million new infections occurred there in 2002. HIV/AIDS claimed the lives of an estimated 2.4 million Africans in 2002 (FAO, 2002).

The *prevalence rate* of HIV/AIDS for a population is the percent of the population infected with the disease. In four southern African countries, the national HIV prevalence rate of adults between 15 and 49 exceeds 30% (see Table 1).

Table 1. Adult HIV Prevalence Rates (ages 15 - 49)

Country	Prevalence rate
Botswana	38.8%
Lesotho	31.0%
Swaziland	33.4%
Zimbabwe	33.7%

Source: UNAIDS, 2002.

The prevalence rates in other sub-Saharan countries include: Cameroon (11.8%), Central African Republic (12.9%), Zambia (20%), Somalia (1%), Uganda (5%). In South Africa estimates of adult HIV prevalence range from 15% to 25% (UNAIDS, 2002).

In many African countries HIV/AIDS began as an urban problem. Recently, however, it has become a greater threat in rural areas than in cities (UNAIDS 2002). Of the 25 most-affected African countries, more than two thirds of the population live in rural areas (Bell, Devarajan, Gersbach, 2003), with a disproportionate affect on economic sectors like agriculture, transportation and mining. As noted by Bell et al. (2003), and others, these industries are characterized by large numbers of mobile or migratory workers. Hence infections among workers in these industries have an opportunity to spill over into other regions as the workers move from one region to another. Since 1985 AIDS has killed over 7 million agricultural workers in the 25 hardest-hit countries in Africa, and it is estimated that 25% (16 million workers) will die from the disease by 2020. This is likely to impact food production, and possibly have adverse effects on aggregate gross domestic product (GDP), as currently over 1/3 of the GDP of the most-affected African countries comes from agriculture (FAO, 2002).

Several studies have examined the likely impact of HIV/AIDS on economic growth. For example, Arndt and Lewis (2000, 2001) predict relative to a no-AIDS scenario, annual aggregate GDP in South Africa would be 0.8% to 1% lower in the presence of AIDS. Over (1992) estimates AIDS could lead to a 0.56% to 1.08% drop in the level of annual aggregate GDP growth between 1990 and 2025. During the same period, Over estimates AIDS could lead to a 0.35% drop in per capita GDP. Sackey and Rarpala (2000) projected aggregate GDP growth in 2010 would drop from 4.0% without AIDS to 2.4% with the disease, and in 2015 drop from 4.0% to 1.3%. Relative to a no-AIDS case, Bonnel (2000) estimates that over a twenty year period, a prevalence rate of 20% would be accompanied by a 67% drop in aggregate GDP levels. In each of the above studies, comparisons are being made between GDP growth (or levels) without HIV/AIDS, and GDP growth (or levels) with the disease.

Like others, Bell, et al. (2003) argue AIDS destroys human capital, weakens mechanisms that generate human capital formation, and decreases incentive for adults to invest in their children's human capital formation. They then explicitly model the impact of AIDS on human capital. Using an overlapping generation model calibrated to South Africa data, they examine the impact of AIDS on average household income under various scenarios. In their baseline, no-AIDS, model the expected annual growth in household income between 1990 and 2050 (two generations) was 1.46% per year. Expected annual growth between 1990 and 2050 would be -1.2% given AIDS and little or no intervention, and would be 1.22% with AIDS and government intervention.

The models underlying the above predictions assume a subset of the following forces impact economic growth: (i) morbidity and mortality effects impact negatively, labor productivity; (ii) human and physical capital accumulation is impacted negatively by the disease; (iii) savings and investment decline as expenditures get reallocated towards medical care; (iv) labor productivity might improve as AIDS-induced mortalities reduce the pressure of population on existing land and capital.

The above studies take different approaches to ascertaining the impact of HIV/AIDS on economic growth. The studies by Bonnel, Over (1992), and Sackey and Raparla (2000) combine demographic modeling with cross country econometric analysis to ascertain the impact of the disease on aggregate GDP. These are single sector studies and are not designed to investigate the impact of the disease on sub-sectors of the economy. The studies by Arndt and Lewis (2000, 2001) are based on a multi-sector computable general equilibrium (CGE) model, and look at the short- and intermediate-run impact of the disease on GDP growth.

The Arndt and Lewis (2000, 2001) model is quite involved, and to simplify the analysis they assume the wage bill and the rate at which capital accumulates are exogenous. Although an engaging study, the Bell, et al. (2003) results are not easily related to the impact of AIDS on aggregate GDP growth (or levels) because the only productive resource in the model is labor augmented by human capital: there is no physical capital. Although these studies use different approaches to understanding the impact of HIV/AIDS on economic growth and development, they each lead to the same conclusion: HIV/AIDS is likely to affect, negatively, economic growth in sub-Saharan Africa.

In this paper, a simple, three sector Ramsy-type model of a small open economy is resurrected, and calibrated to South Africa national accounts data. The model is then used to calculate the long run impact of labor market shocks on aggregate GDP levels. Here, labor shocks are introduced simply by multiplying the country's labor endowment by a scalar whose value is between 0 and 1. The scalar has one of two interpretations. In one interpretation it indexes the proportion of time an average laborer is healthy enough to work. For example, letting γ represent the labor shock parameter, if $\gamma = 0.95$, on average a worker does not report to work 5% of time. This is the *effective labor* or *morbidity* interpretation. Under the morbidity interpretation HIV/AIDS is viewed as a disease that decreases the amount of time on the job: impacting the marginal product of a labor unit. In the other interpretation the parameter represents the proportion of individuals who die as a result of the disease. Again, letting γ represent the parameter, if $\gamma = 0.95$, then 5% of the workforce dies as a result of the disease. This is the *mortality* interpretation.

One basic result of the exercise is when the labor shock is viewed purely as a morbidity problem, a 1% drop in the size of the effective labor force leads to a 1% drop in the level of long run aggregate GDP. Likewise, when viewed purely as a mortality problem, a 1% drop in the size of the labor force also leads to a 1% drop in the level of long run aggregate GDP. When there is both a morbidity and a mortality problem, a 1% drop in the size of the labor force and a 1% drop in the effective labor force lead to a 2% drop in the level of long run aggregate GDP. This result, and the others reported later should be viewed as crude, "back-of-the-envelope" calculations of the disease's economic impacts. Note, given the manner in which the data is normalized, no attempt is made to determine per capita GDP levels.

The model's simplicity carries with it, several problems. For instance, potential health expenditures could possibly affect the rate at which capital accumulates, and hence, exacerbating the decrease in the long run level of GDP. A few more of the model's shortcomings are discussed in the conclusion, along with a brief overview of current attempts by the author and others to overcome some of these shortcomings.

The paper is organized as follows. The Ramsey-type model is presented in the next section. This model has the standard features one would expect in a general equilibrium model: three sectors; endogenous wages; endogenous rates of return to capital; and intertemporal optimizing behavior – savings. In addition, it adds a set of parameters whose objective is to represent labor force shocks. Section 3 presents the results of calibrating the model to South African data and simulating the impact of HIV/AIDS on economic growth. The last section concludes and outlines a few additional features to be included in future work. The interested reader can refer to the appendix for a formal definition of intertemporal and long run (steady state) equilibrium.

2 Basic model

We now describe a model economy endowed with land, labor, and capital that produces three types of output: agriculture, manufacturing, and services. The agricultural and manufacturing goods are traded goods, while services are not traded. The country is "small" in the sense that its production and export decisions have no appreciable effect on world prices.

2.1 Production

Denote the agricultural, manufacturing, and service goods by Y_a, Y_m, and Y_s respectively, where $(Y_a, Y_m, Y_s) = Y \in \mathbb{R}^3_+$. These goods are produced with three productive inputs: land, labor, and capital denoted respectively; $\bar{T}, L$, and K, where $\left(\bar{T}, L, K\right) = X \in \mathbb{R}^3_+$. Land is specific to agricultural production, while capital and labor are mobile across sectors. The output vector Y is produced with endowment X using a constant returns to scale (CRS) technology represented by the producible output set $\mathbf{Y}(X) = \{Y : X \text{ can produce } Y\}$.

Let L_a, L_m, and L_s denote the respective levels of labor allocated to agriculture, manufacturing, and services; and K_a, K_m, and K_s denote the respective levels of capital allocated to agriculture, manufacturing, and services. To introduce the labor shock we multiply L and/or L_j by the parameter $\gamma_1, \gamma_2 \in (0, 1]$. See below for more discussion of γ_i. Assuming production is nonjoint in inputs, with the labor shock parameter, technology can be represented by

$$\begin{aligned}\mathbf{Y}\left(L, \bar{T}, K\right) &= \{(Y_a, Y_m, Y_s) : Y_a \leq F^a(\gamma_1 L_a, K_a)\,\bar{T}, Y_m \leq F^m(\gamma_1 L_m, K_m), Y_s \leq F^s(\gamma_1 L_s, K_s);\\ &\qquad \gamma_2 L \geq L_a + L_m + L_s, K \geq K_a + K_m + K_s\},\end{aligned}$$

where F^a, F^m, and F^s are the production functions for agriculture, manufacturing, and services respectively. We assume each production function is increasing and strictly concave in each argument. Here, γ_1 is the morbidity parameter and γ_2 is the mortality parameter.

Table 2. Labor shock interpretations

	$\gamma_1 = 1$	$\gamma_1 \in (0,1)$
$\gamma_2 = 1$	No labor market shock	Pure morbidity interpretation
$\gamma_2 \in (0,1)$	Pure mortality interpretation	Morbidity and mortality

The per-unit minimum cost of the manufacturing and service output are

$$\begin{aligned} C^m(w,r;\gamma_1) &\equiv \min_{\{L_m,K_m\}} \{wL_m + rK_m : 1 \leq F^m(\gamma_1 L_m, K_m)\} \\ C^s(w,r;\gamma_1) &\equiv \min_{\{L_s,K_s\}} \{wL_s + rK_s : 1 \leq F^s(\gamma_1 L_s, K_s)\}, \end{aligned}$$

and the maximum net GDP (rents) in the agricultural sector is given by

$$G^a(p_a,w,r;\gamma_1)\,T \equiv \max_{\{L_a,K_a\}} \{p_a F^a(\gamma_1 L_a, K_a)\,T - wL_a - rK_a\}.$$

Here, w is the wage rate, r is the rental rate to capital, and p_a is the per unit price of the agricultural good (normalized with respect to p_m,the price of manufacturing). Under our assumptions, G^a is the per-unit rental payment to land. The corresponding cost and agricultural GDP functions under the mortality interpretation.

2.2 Households

Assume there are L, identical, infinitely lived households. A household earns income by selling labor at wage rate w, renting capital services at rate r, and renting land at rate τ. The household then allocates its income between savings and consumption.

Represent the household's utility at time t by $\mathbf{u}(t) \in \mathbb{R}_+$, and represent its intertemporal utility function by

$$U = \int_0^\infty Log(\mathbf{u}(t))\, e^{-\rho t} dt,$$

Suppressing the time argument, the minimum cost per household of achieving utility $\mathbf{u}$ is given by the expenditure function

$$E(p_a, p_s, \mathbf{u}) = \mu(p_a, p_s)\,\mathbf{u} \equiv \min_{(q)} \{(q_m + p_a q_a + p_s q_s) \mid \mathbf{u} \leq u(q_a, q_m, q_s)\}.$$

Here p_s is the price of the non-traded service good (normalized with respect to p_m, the price of manufacturing), and $u(q_a, q_m, q_s)$ is the instantaneous utility associated with consumption bundle $(q_a, q_m, q_s) \in \mathbb{R}^3_+$; q_a, q_m, and q_s are the per-capita levels of agricultural, manufacturing, and non-health service consumption.

The intertemporal budget constraint of the household is given by

$$\dot{K} = w + rK + \tau T - \mu(p_a, p_s)\,\mathbf{u}. \tag{1}$$

Suppressing the time argument, given preferences U and budget constraint (1), the household's present value Hamiltonian is

$$J = Log[\mathbf{u}] + \nu\{w + rK + \tau T - \mu(p_a, p_s)\,\mathbf{u}\},$$

where the costate variable ν is the shadow value of additional income. The household's problem has one state variable, K and one control variables, $\mathbf{u}$.

Let $E_{\mathbf{u}} = \partial E/\partial \mathbf{u}$. The utility maximizing choice for an interior solution satisfy: (i) the following necessary conditions:

$$\frac{\partial J}{\partial \mathbf{u}} = \frac{1}{\mathbf{u}} - \nu\mu(p_a, p_s) = 0, \tag{2}$$

$$\dot{\nu} = \rho\nu - \frac{\partial J}{\partial k} = (\rho - r)\,\nu, \tag{3}$$

(ii) the equations of motion (1), and (iii) the transversality condition

$$\lim_{t\to\infty}\left[v(t)\dot{K}(t)\right]=0. \tag{4}$$

Using expression (2), we solve for ν, take the time derivative of the resulting expression, and then make relatively straightforward substitutions into expression (3) to get the Euler condition:

$$\frac{\dot{\mathbf{u}}(t)}{\mathbf{u}(t)}+\frac{\mu_{p_s}}{\mu}\dot{p}_s=r(t)-\rho. \tag{5}$$

A definition of the competitive equilibrium for the above models is presented in the appendix, as is a characterization of the equilibrium.

2.3 Calibration Results

The production technologies in the calibrated economy are assumed approximated by the following functions:

$$\begin{aligned} y_a &= (\hat{\gamma}_1 l_a)^{\beta_1}(k_a)^{\beta_2}T^{1-\beta_1-\beta_2} \\ y_m &= (\hat{\gamma}_1 l_m)^{\alpha}(k_m)^{1-\alpha} \\ y_s &= (\hat{\gamma}_1 l_s)^{\eta}(k_s)^{1-\eta}, \end{aligned}$$

where the scalar coefficients $\beta_j\in(0,1)$, $j=1,2,3$, and $\alpha_j,\eta_j\in(0,1)$, $j=1,2$ are cost shares. For example, α is labor's share in the cost of producing the manufacturing good, while β_2 is capital's share in the cost of producing agriculture. Constant returns to scale requires $\beta_1+\beta_2+\beta_3=\alpha_1+\alpha_2=\eta_1+\eta_2=1$. Consumer preferences are represented by

$$u(q_a,q_m,q_s)=(q_a)^{\phi_1}(q_m)^{\phi_2}(q_s)^{\phi_3},$$

where the scalar coefficients $\phi_j \in (0,1)$, $j = 1,2,3$ are consumption share coefficients, where $\phi_1+\phi_2+\phi_3 = 1$. Here, ϕ_1 is the share of aggregate South African income spend on agricultural consumption.

Using South African national accounts data from the International Food Policy Research Institute (1997), the following cost (production) shares were derived:

Table 3. Production cost shares

	Labor	Capital	Land
Agriculture	$\beta_1 = 0.2950$	$\beta_2 = 0.3525$	$\beta_3 = 0.3525$
Manufacturing	$\alpha_1 = 0.6894$	$\alpha_2 = 0.3106$	–
Services	$\eta_1 = 0.4956$	$\eta_2 = 0.5044$	–

Consumption shares are also derived from South African national accounts data, and are given by

$$\phi_1 = 0.0546, \phi_2 = 0.3305, \phi_3 = 0.6149$$

With no labor market shocks, i.e., $\gamma_1 = \gamma_2 = 1$, long run GDP is projected to be equal to R 572,077 million. Also, in each scenario for both the mortality and morbidity models, the price of the non-traded good falls by about 20%. Given that we normalize over L, the following results can be viewed in level terms.

We first discuss the relationship between morbidity and long run economic performance. Then, results from the mortality model is presented, along with a brief comparison of the morbidity and mortality models. Table 4 relates effective labor rates to long run GDP.

Table 4. Morbidity and the value of manufacturing, services, capital and GDP

	Manufacturing $(163,518)^{\dagger,\ddagger}$		Services $(193,487)^{\dagger,\ddagger}$		Capital $(965,244)^{\dagger,\ddagger}$		GDP $(374,188)^{\dagger,\ddagger}$	
γ_1	Value‡	% change	Value‡	% change	Value‡	% change	Value‡	% change
1.0	213,908	–	336,318	–	4,298,770	–	572,077	–
0.9	191,458	−10.50	300,470	−10.66	3,881,200	−9.71	513,779	−10.19
0.8	169,008	−20.99	264,622	−21.32	3,463,630	−19.43	455,481	−20.38
0.7	146,558	−31.49	228,774	−31.98	3,064,050	−29.13	397,183	−30.57

† (Values in parentheses are calibrated base period values.)

‡ (Million South African Rand)

A quick inspection of Table 4 shows the elasticity of long run GDP with respect to γ_1 is about 1, i.e., a 10% drop in γ leads to (about) a 10.2% drop in the level of long run GDP (relative to the case where $\gamma_1 = \gamma_2 = 1$). Of course, since there is no attempt to determine the length of time required to reach the steady state, it is not possible to determine the impact of a 10% drop in γ_1 on short and intermediate run GDP growth rates. Not reported in Table 4 is, at $\gamma_1 = 0.66$ long run GDP is equal to $373,864$ – a level less than the initial period's GDP of 374,188. Hence, a large enough morbidity problem can trigger a scenario in which short or intermediate run GDP growth can be negative.

A major factor underlying the decrease in long run GDP is that the long run stock of capital falls as morbidity levels increase. As morbidity levels increase, the marginal product associated with a given level of labor falls. With the marginal product of labor falling, the value of an additional unit of capital falls, hence decreasing the incentive to invest. In such a case investment incentives fall, and less capital is accumulated in the short and intermediate run, hence leaving the economy with a smaller capital stock in the long run. Indeed, Table 4 shows that a 10% increase in morbidity leads to a 9.8% drop in the long run stock of capital.

Table 4 also shows the impact of an increase in morbidity is spread evenly across the manufacturing and service sectors: with a 10% increase in morbidity leading to about a 10.5% drop in both manufacturing and service sector output. Figure 1 plots agricultural land rents as a percent of aggregate GDP, and shows agriculture's share of long run GDP increases as morbidity levels increase.

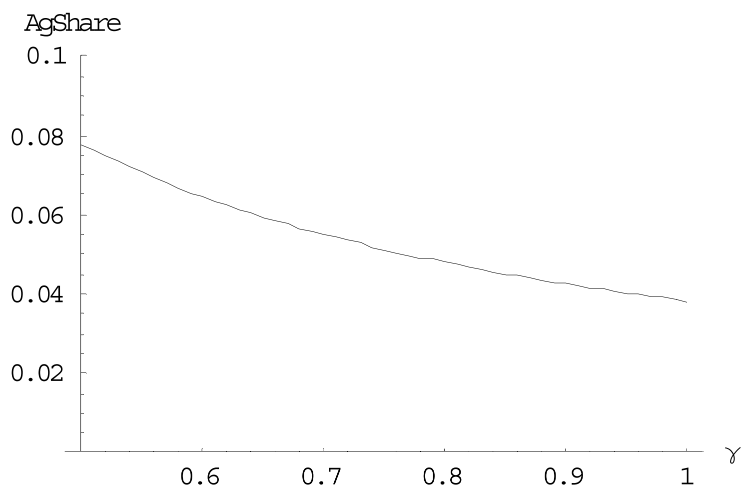

Figure 1. Agricultural rents as share of long run GDP

Figure 1 suggests with no morbidity problems, agriculture would comprise a slightly smaller share of aggregate income in the long run (3.8%). As morbidity problems increase, agriculture's share of aggregate income increases from 3.8% to 7.8%. Although not plotted here, similar, but opposite patterns emerge for manufacturing and the service sector: manufacturing's share drops from 37.4% with no morbidity to 35% when $\gamma_1 = 0.5$, and services' share drops from 58.8% with no morbidity to 56% when $\gamma_1 = 0.5$. In other words, morbidity effects have an impact on aggregate GDP levels, but a small impact on long run production and income distribution patterns. This result follows because morbidity essentially ends up increasing the per unit cost of labor: increasing wages from w to $w/\gamma_1 > w$. Or, viewed another way, morbidity acts like a tax that labor imposes on wages, where the tax is given by $\frac{1-\gamma_1}{\gamma_1}$.

The wage bill as a function of γ_1 is given by $w(\gamma) = 332437 * \gamma_1$, implying the total payment to labor is proportional to the base aggregate GDP. In the base year, labor's share of aggregate income was 57%. In the simulations, labor's share remained between 58% and 59%, increasing slightly from 58.1% to 59.2% as γ_1 falls. Given the tax interpretation, morbidity introduces a distortion that slightly favors labor. As an aside, the simulations examine the case where there is a single representative agent, hence not much can be said directly about the likely impact of HIV/AIDS on poverty. With aggregate income falling, however, and given the share of income received by labor remains relatively constant, the result does not augur well for the poor.

Table 5 suggests the impacts of mortality on long run GDP are similar to that of morbidity: a 10% drop in the labor endowment is accompanied by an 11% drop in the level of long run GDP (relative to the case where $\gamma_1 = \gamma_2 = 1$). As mortality levels increase, the marginal product of capital falls, decreasing the incentive to invest. Again, in such a case investment incentives fall, less capital is accumulated in the short and intermediate run, and the economy ends up with a smaller stock of capital in the long run. Table 5 shows that under mortality a 10% increase in morbidity leads to an 11.4% drop in the long run stock of capital.

Table 5. Mortality and the value of manufacturing, services, capital and GDP

	Manufacturing $(163,518)^{\dagger,\ddagger}$		Services $(193,487)^{\dagger,\ddagger}$		Capital $(965,244)^{\dagger,\ddagger}$		GDP $(374,188)^{\dagger,\ddagger}$	
γ_2	Value‡	% change	Value‡	% change	Value‡	% change	Value‡	% change
1.0	213,908	–	336,318	–	4,298,774	–	572,077	–
0.9	202,585	−5.29	284,993	−15.26	3,808,690	−11.40%	509,429	−10.95
0.8	191,261	−10.59	233,668	−30.52	3,318,620	−22.80%	446,780	−21.90
0.7	179,938	−15.88	182,344	−45.78	2,828,540	−34.20%	384,132	−32.85

†(Values in parentheses are initial values, e.g. time $t = 0$, values.)

‡ (Million South African Rand)

Although the morbidity effects tend to be distributed evenly across sectors, the effect of increased rates of mortality is more pronounced in the service sector than in agriculture and manufacturing. Table 5 shows the impact of mortality on the service sector is three times that on manufacturing, and Figure 2 shows the share of services in aggregate GDP drops significantly as AIDS mortality rates increase.

Given that morbidity impacts the service sector more than manufacturing and agriculture, HIV/AIDS policy debates will likely include discussions over whether the government should subsidize anti-retroviral drug prices. Manufacturing lobbyists, would then push for government subsidies on anti-retroviral drugs, while the service sector might stress investing public funds into HIV/AIDS awareness education.

Although not reported here, the model also examined scenarios with both morbidity and mortality. Under these scenarios, long run GDP falls about one percent for each one percent increase in morbidity and/or mortality. For example, if $\gamma_1 = \gamma_2 = 0.9$, long run GDP would be 20 percent lower than GDP with no AIDS.

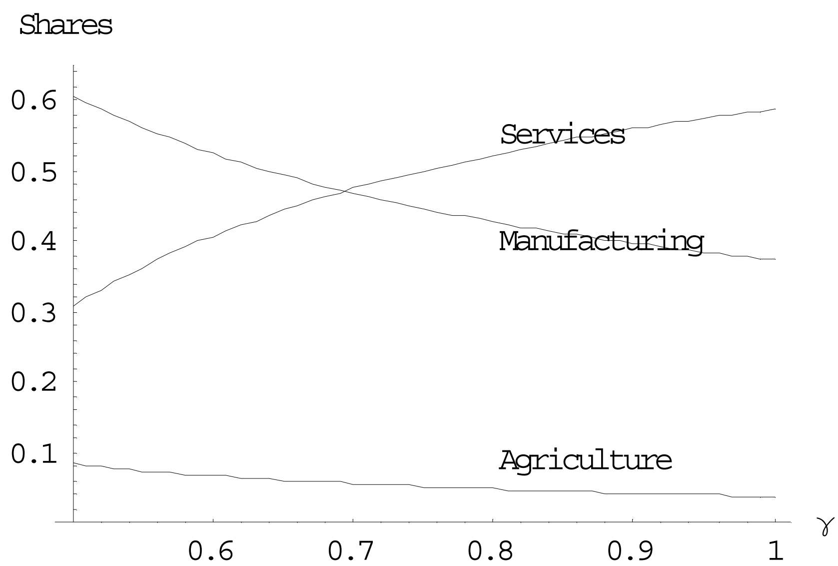

Figure 2. AIDS death rates and sectoral shares of aggregate GDP

3 Conclusion

The objective of this study was to investigate the potential morbidity and mortality impacts of HIV/AIDS on long run aggregate GDP, and on income distribution and production patterns. Morbidity and mortality parameters were introduced into a three sector, Ramsey-type model, and simulations conducted. The model was calibrated to South African national accounts data, and the following results were found: (i) long run GDP levels fell by about 1 percent for each one percent loss in time spent working; (ii) long run GDP levels fell by about 1 percent for each one percent loss in the labor force due to death; (iii) the long stock of capital falls about one percent for each one percent increase in morbidity and falls 1.1 percent for each one percent increase in mortality; and (iv) under mortality, service sector output falls three times as much as manufacturing. The analysis also suggests negative GDP growth scenarios are possible, but such outcomes are not likely unless morbidity and/or mortality levels are quite high.

As noted in the introduction, the results of this study should be viewed as preliminary, 'back-of-the-envelope' results, as the model comes with several shortcomings. First, analysis of the transition path needs to be conducted. After all, the model is set up to analyse the short, intermediate, and long-run impact of HIV/AIDS on economic growth. Second, regarding morbidity, the model only looks at the impact on long run GDP if, say, on average workers spend one percent less time working because of HIV/AIDS. The model does not, however, link HIV/AIDS prevalence rates to morbidity levels. The same can be said about death rates. Also, the model says nothing about the possible impact of health expenditures on long run GDP. Health expenditures would likely affect the rate at which capital accumulates, and hence, exacerbate the decrease in long run GDP levels. On the other hand, health expenditures would likely decrease morbidity rates and mortality. These two opposing forces suggest there might be an optimal level of investment in AIDS treatment. Hence, linking (endogenizing) mortality with health expenditures is a natural next step in which to take the above research. Another important direction in which to take future research is to introduce population and prevalence dynamics into the model. Linking prevalence and population dynamics is another way to endogenize mortality.

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Appendix A

A.1 Defining the competitive equilibrium

At each point in time both intertemporal and intra-temporal equilibrium conditions must be satisfied. Intra-temporal equilibrium requires that households maximize utility, firms earn zero profits, labor and capital markets clear, Walras' law is satisfied, and the Euler and transversality conditions hold.

Define $\hat{\mathbf{Y}}(L, T, K, \gamma_1, \gamma_2) \equiv \mathbf{Y}(L.T, K : \gamma_1, \gamma_2)$. A time t production plan $Y(t) = (Y_a(t), Y_m(t), Y_s(t))$ is *feasible* if $Y(t) \in \mathbf{Y}(L, T, K, \gamma, \hat{\gamma})$. Furthermore, let $\mathbf{u}(t) \in \mathbf{U}$ denote the household's (feasible) time t consumption plan. A sequence of consumption plans is feasible if for each $t \in [0, \infty)$, $\mathbf{c}(t) \in C$, and a sequence of production plans is feasible if for each $t \in [0, \infty)$, $Y(t) \in \hat{\mathbf{Y}}(L, T, K, \gamma_1, \gamma_2)$.

Definition 1 *Given* (γ_1, γ_2) *and initial condition* $k(0) = k_0$, *a competitive equilibrium is a sequence of prices* $\{p_a, p_s(t), w(t), r(t), \tau(t)\}_{t \in [0,\infty)}$, *feasible consumption plans* $\{\mathbf{u}(t)\}_{t \in [0,\infty)}$, *and feasible production plans* $\{Y_a(t), Y_m(t), Y_s(t)\}_{t \in [0,\infty)}$, *such that at each t: (i) given prices* $(p_a, p_s(t), w(t), r(t))$ *the manufacturing and service sectors earn zero profits*

$$1 = C^m(w(t), r(t), \gamma_1) \tag{6}$$

$$p_s(t) = C^s(w(t), r(t), \gamma_1), \tag{7}$$

the service good market clears

$$E_{p_s}(p_a, p_s(t), \mathbf{u}(t)) = Y_s(t) \tag{8}$$

and labor and capital markets clear

$$\gamma_2 \leq -G_w^a(p_a, w(t), r(t), \gamma_1) T + C_w^m(w(t), r(t), \gamma_1) Y_m(t) + C_w^s(w(t), r(t), \gamma_1) Y_s(t) \tag{9}$$

$$K(t) \leq -G_r^a(p_a, w(t), r(t), \gamma_1) T + C_r^m(w(t), r(t), \gamma_1) Y_m(t) + C_r^s(w(t), r(t), \gamma_1) Y_s(t), \tag{10}$$

(ii) the consumption plan $\mathbf{u}(t)$ *maximizes household utility given the budget constraint*

$$\dot{K} = w(t) + r(t) K(t) + \tau(t) T - E(p_a, \mathbf{u}(t)),$$

(iii) Walras' Law holds

$$[Y_m(t) - q_m(t) - \dot{K}(t)] + p_a [Y_a(t) - q_a(t)] = 0, \tag{11}$$

and (vi) Euler's condition (5) and the transversality condition (4) holds.

The above definition says, given the exogenous prices and factor endowments, the model must satisfy four sets of restrictions: (i) Constant returns to scale and no sector specific factors in manufacturing and services, requires that zero economic profits be earned in the two sectors. The supply of the non-traded service good must be equal to its demand, and the supply of labor and capital must be equal to their respective aggregate demands. (ii) Consumption decisions must maximize utility, given available income. (iii) The value of goods imported must equal the value of goods sold (net savings), and (iv) the value of an additional unit of income consumed must be equal to the value of an additional unit of consumption in the future (income saved).

A.2 Calibrating the steady state

In the steady state, $\dot{K} = \dot{E} = 0$. First, $\dot{E} = 0$ implies $r^{**} = \rho$. Suppressing the time argument, equations (6) and (**??**) are now:

$$\begin{aligned} 1 &= C^m(w, \rho, \gamma_1) \\ p_s &= C^s(w, \rho, \gamma_1), \end{aligned}$$

implying $w^{**} = w(\rho, \gamma_1)$ and $p_s^{**} = p_s(\rho, \gamma_1)$. Substituting into expressions (9) and (10):

$$\begin{aligned}\gamma_2 &= -G_w^a(p_a, w^{**}, \rho, \gamma_1) T + C_w^m(w^{**}, \rho, \gamma_1) Y_m + C_w^s(w^{**}, \rho, \gamma_1) Y_s \\ K &= -G_r^a(p_a, w^{**}, \rho, \gamma_1) T + C_r^m(w^{**}, \rho, \gamma_1) Y_m + C_r^s(w^{**}, \rho, \gamma_1) Y_s.\end{aligned}$$

which combined with

$$Y_m^* + G^a(p_a, w^{**}, \rho, \gamma_1) T = E_{p_a}(p_a, p_s^{**}, \mathbf{u}(t)) + E_{p_m}(p_a, p_s^{**}, \mathbf{u}(t))$$

yields $Y_m^* = Y_m(\rho, \gamma_1, \gamma_2)$, $Y_s^* = Y_s(\rho, \gamma_1, \gamma_2)$, and $K^* = K(\rho, \gamma_1, \gamma_2)$.

The values $\Omega = (r^{**}, w^{**}, p_s^{**}, \mathbf{u}^{**}, Y_m^{**}, Y_s^{**}, \gamma_1, \gamma_2)$ can be used to recover any remaining steady-state variables, e.g.,

$$Y_a^{**} = G_{p_a}^a(p_a, w^{**}, \rho, \gamma_1) T.$$

As noted in the introduction, a simple, albeit crude way to introduce HIV/AIDS into the Ramsey model is to assume the disease decreases the amount of time an individual spends working, i.e., impacts the effective labor supply of infected individuals. In such a case, the labor market clearing condition (9) can be written as

$$\gamma \leq -G_w^a(p_a, w^{**}, \rho, \gamma_1) T + C_w^m(w^{**}, \rho, \gamma_1) Y_m + C_w^s(w^{**}, \rho, \gamma_1) Y_s,$$

where γ_1 and γ_2 are the labor shock parameters.