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A Model for the Optimal Management of Sea Bass *Dicentrarchus Labrax* Aquaculture

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Abstract *A bioeconomic model for the simulation and the optimal management of a fish farm for sea bass Dicentrarchus labrax is presented. Growth and mortality, considered as a Markovian process, are described by a biological submodel, taking into account the effects of water temperature, feeding level, oxygen content, and water supply. Stochastic effects in growth and mortality, relating to the effects of genetic differences, can be also considered in the model. An economic submodel evaluates costs and revenues relating to plant management. The model exhibits good capabilities in predicting the effects of operating variables on fish growth and on economic outcomes and in determining the optimal strategies for plant management in different scenarios, considering the complex interactions of technical, biological, and economic aspects.*

Key words Aquaculture, mathematical model, optimal management.

Introduction

In recent years, many applications of mathematical models to aquaculture management have been described in the literature. General overviews presenting the different approaches are reported by Leung (1986) and Piedrahita (1988). Particular attention has been paid on optimal feeding schedules and harvesting time (Arnason 1992; Bjiorndal 1988; Heaps 1993). A model for simulation and optimal management of salmon aquaculture is described by Bjorndal (1990). A prawn *Palaemon Serratus* (Penn.) production management system has been modeled by Leung and Shang (1989) by a dynamic Markov decision approach. Similar methods have been adopted for the optimization of production planning in rainbow trout *Onchorbinchus muskis* (Walbaum) (Sparre 1977). Other models have been applied to social and economic aspects of aquaculture activity (IRPAI 1990).

An important contribution to the study of sea bass *Dicentrarchus labrax* aquaculture has been given by Querellou (1984), who has proposed simple relationships relating growth to water temperature. This model, though it does not consider all of the physiological and biological aspects involved, can nevertheless predict growth and mortality in typical plants with acceptable precision. It does not, however, represent a tool directly usable to define optimal management strategies, since it does not consider all biological, operating, and economic aspects involved in aquaculture.

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The other models described in literature, relating to different species, exhibit significant differences in structure and complexity. None of them are directly usable for the optimal management of aquaculture plants for species like *Dicentrarchus labrax* and *Sparus auratus*.

The present study therefore proposes a model for the simulation and optimization of sea bass aquaculture plants. It comprises a stochastic Markovian biological submodel and an economic submodel. The biological part is based on the model proposed by Querellou (1984), integrated by further relationships to consider feeding levels, oxygen consumption, and additional mortality due to harvesting and fish transfer between ponds. This base model has then been included in a stochastic model, in order to describe growth and mortality variability due to genetic and environmental conditions, through a Markovian transition matrix. An economic submodel expresses revenues due to harvesting and costs for stocking, food, water supply, auxiliary oxygenation, water heating, and fixed costs.

A Model for Sea Bass Aquaculture

The model is based on a biological submodel, derived from the relationships validated on typical aquaculture plants, which essentially describes the effects of water temperature on average growth and the physiological mortality (Querellou 1984). This model has been integrated by further relationships in order to consider feeding levels, oxygen and water consumption, and additional mortality relating to harvesting and pond transfer. Even when starting from a homogeneous population of fry, a significant spread in growth and in biomass distribution is usually observed. These stochastic effects on growth, due to genetic and environmental factors, have then been introduced in the model according to a Markovian process (Leung and Shang 1989; Sparre 1977).

The economic submodel describes revenues and the most relevant costs relating to aquaculture management (*i.e.*, stocked recruits, feeding, water and oxygen supply, water heating, fixed and financial costs). Particular options for energy saving, such as heat recovery, recirculation, and use of heat pumps, have not been considered, but they can be easily included in the model subsequently.

The model, which can be used for both simulation and optimization analysis, is formulated according to the following assumptions:

1. A population of sea basses, subdivided in weight classes and distributed in different groups of ponds, is considered. The entire fish farming process is described, from fry recruitment to harvesting. Multiple production cycles can be also considered.
2. Time horizon is divided in periods in which both external variables (temperatures, prices, costs) and decision variables (harvesting and stocking strategies) can be considered constant. The presented results refer to a time horizon of two years, divided in twenty-four periods of one month.
3. It is assumed that a predetermined fraction of the biomass which is contained in the upper weight classes can be independently harvested. An additional mortality, proportional to the harvested biomass, is considered for the fish left in the ponds.
4. The recruits exhibit homogeneous genetic behavior and weight distribution.
5. Each fish pond (or group of ponds) is defined by a minimum and a maximum weight class; biomass which exceeds maximum weight can be transferred to the next pond, with continuity, in each period. An additional mortality affects the transferred biomass.
6. Prices are considered to be independent of demand.

From a mathematical point of view, the fish farming process can be depicted as a multi-stage dynamic system (Brison and Ho 1975), where each stage represents a period of the process. For the kth stage, output variables y_k and state variables x_k (input for the next stage) are determined as a function of external variables v_k, decision variables u_k, and state variables x_{k-1}.

The Biological Submodel

The following relationship has been suggested for the prediction of the average weight W of a sea bass population (Querellou 1984):

$$W = k(S_{01})^x \tag{1}$$

where S_{01} represents the sum of a suitable daily temperature T' extended to the days from spawn hatch to the actual fish age:

$$S_{01} = T' \tag{2}$$

with t_0 = hatching time (days) and t_1 = fish age (days).

Two different formulations of T' have been adopted, since growth exhibits a slower dynamics below a critical temperature of about 10˚C:

$$T' = (T - 10) \qquad \text{if } T \geq 10 \tag{3a}$$

$$T' = (T - 10)/5 \qquad \text{if } T < 10. \tag{3b}$$

By observation of many sea bass populations in typical farms, it has been found that a satisfactory fitting for the average growth can be obtained assuming the following values for the constants in equation (1):

$$k = 0.9\ 10^{-8} \qquad x = 2.81 \tag{4}$$

In the original reference (Querellou 1984), limitations of the model were not explicitly reported, although the presented results refer to temperature range varying from 7˚C to 28˚C and to a maximum fish weight of 400 grams. Equation (1) can be easily rearranged in order to evaluate the weight at the time t_2, starting from the weight at t_1 and from the sum of the daily temperatures T' between t_1 and t_2. In order to take into account the feeding effects on growth, a feeding level, ϕ, has been defined as the ratio between the actual biomass of food B_f and the theoretical amount to be furnished ν:

$$\phi = B_f/\nu \tag{5}$$

The theoretical biomass ν is obtained from suitable tables provided by food producers, as a function of fish size and water temperature, *e.g.*, Feeding Chart for Sea Bass, 1990. Therefore, a further term $F(\phi)$, not present in the original relation proposed by Querellou, has been included in the growth equation:

$$W = k\ (S_{01})^x\ F(\phi) \tag{6}$$

In default of more detailed information about the influence of feeding on sea bass growth, this term has been assumed equal to the feeding level ϕ:

$$F(\phi) = \phi \tag{7}$$

with $0.5 < \phi < 1$. In order to avoid unrealistic results for the predicted growth, only feeding levels ranging from 0.5 to 1 have been considered.

The followed approach is consistent with the assumption of Markovian process; that is, the growth in a given period depends only on the environmental variables (temperatures, feeding) in that period and on the initial fish weight, irrespective of which combination of temperature and time each fish used to reach the initial weight. The Markovian assumption is realistic enough in typical and well-kept fish farms, and has been commonly adopted by other modelers (Sparre 1977; Leung and Shang 1989).

With regard to mortality, one should distinguish between a mortality due to particular pathologies and a physiological one. The former is not considered in the present model, since it substantially influences growth mechanism and requires *ad hoc* remedies. Physiological mortality, on a monthly basis, is estimated by the following relationship:

$$m_1 = \theta/a^{\gamma} \tag{8}$$

where a is the fish age in months, θ is the mortality rate for the first month and γ is an exponent relating to fish farming conditions. The following values have been suggested, for sea bass aquaculture (Querellou 1984):

$$\theta = 0.04 \quad \gamma = 1.17 \quad \text{(optimal conditions)} \tag{9a}$$

$$\theta = 0.07 \quad \gamma = 0.7 \quad \text{(poor conditions)} \tag{9b}$$

Due to the stochastic growth mechanism postulated, in a given weight class fish with different age can coexist. Since it would be difficult to compute the actual age distribution for each weight class considering the effects of stochastic growth, and since the relationship between age and mortality is described by a rather approximate model (8), a correlation to estimate age by average weight is considered adequate. For temperatures higher than 10˚C, fish age a in months can be related to its weight W by the relationship:

$$30a(T_{av} - 10) = S_{01} = (W/k)^{1/x} \tag{10}$$

where T_{av} represents the average temperature in the period $(t_0 - t_1)$. Physiological mortality coefficient m_1 can then be evaluated by equation (8). This approach is consistent with the Markovian approach adopted, which assumes that the growth is related only to the actual weight, regardless of previous history.

Two further coefficients m_2 and m_3 express mortality due to harvesting and to fish transfer between ponds; they are proportional respectively to the fractions of harvested and transferred biomass, in the previous period:

$$m_2 = B_h/B_{t,k}\,\chi_2 \quad m_3 = B_{tr}/B_{t,k}\,\chi_3 \tag{11}$$

In the following, values of order of 0.01 have been assumed for χ_2 and χ_3, on the basis of experience on typical plants (Massa and Bonfiglio, AGEI, pers. com.). The mortality probability in the given period is then given by:

$$p_m = m_T a + m_2 + m_3 \tag{12}$$

For the ith weight class, the biomass affected by mortality in the kth period can be expressed as a function of probability p_m and of total biomass $B_{t,k}$ or, equivalently, by number of fish $S_{t,k}$ and their average weight W_i:

$$B_{k,m} = p_m B_{t,k} = p_m S_{t,k} W_i \tag{13}$$

The Markovian Matrix

Mortality and transition from one weight class to another are described by a two dimension Markovian matrix $\mathbf{M}$, where $\mathbf{M}_{i,j}$ indicates the probability that fish belonging to ith weight class will grow to the jth class, during a given period, and $\mathbf{M}_{i,Ncl+1}$ represents probability of death due to physiological mortality or stresses caused by harvesting and pond transfer. It is assumed that, in each period, fish can either stay in the same class, grow to a larger class or die. Therefore, for each row, only the terms $\mathbf{M}_{i,j}$ ($i \le j \le N_{cl} + 1$) are greater than zero, and their sum is 1.

$$\textstyle\sum_j \mathbf{M}_{ij} = 1 \qquad i = 1,\ N_{cl} \tag{14}$$

Matrix $\mathbf{M}$ has been evaluated, for each time step and for a starting weight W_i, assuming that in the next period the weight of the fish from this weight class will be distributed normally around the average weight W predicted by equation (6). For a given standard deviation, the areas A_j subtended by probability curves which lie in each of the jth classes of arrival can be evaluated by numerical integration. Standard deviation (SD) is computed as:

$$\mathrm{SD} = c_1 \Delta W_i + c_2\,(W - W_i) \tag{15}$$

where ΔW_i represents width of the ith class. Standard deviation is then composed of two terms. The former is proportional to weight class width, and takes into account the effect of discretization due to the fact that the average weight W_i is associated to the entire biomass in the ith class. The latter term is proportional to the predicted average growth, and represents the scattering due to environmental and genetic differences. Probability p_j is assumed proportional to area A_j while it is zero for weight classes lower than the starting one. In order to avoid results that indicate unrealistically high growth for even a small part of the biomass (due to the asymptotic behavior of the Gaussian curve), the probability p_j, corresponding to areas less than a threshold value c_3, is considered zero. Therefore:

$$p_j = 0;\ \text{if } j < i \tag{16a}$$

$$p_j = A_j;\ \text{if } j \ge i \text{ and } A_j \ge c_3 \tag{16b}$$

$$p_j = 0;\ \text{if } j \ge i \text{ and } A_j < c_3 \tag{16c}$$

The truncations in equations (16a) and (16c) add reality to the model, but can produce asymmetric distributions, with a mean different from the average weight predicted by equation (6). The shifting between them can be both positive or negative, depending on the actual differences between the areas below the threshold value [equation (16c)] and the ones belonging to previous weight classes [equation (16a)], which can be influenced by the predicted average weight and by the adopted discretization. This effect of asymmetry can usually be neglected, except in cases of combination of very low growth with large weight classes and high standard deviations. The adoption of variable threshold values c_3 could in these cases avoid the occurrence of asymmetric distributions.

The elements of Markovian matrix for the given ith weight class are then computed by the following relationships:

$$\mathbf{M}_{ij} = 0 \qquad \text{for } j < i \tag{17a}$$

$$\mathbf{M}_{ij} = (1 - p_m)\, p_j / \sum p_k \qquad \text{for } j = i,\, N_{cl} \tag{17b}$$

$$\mathbf{M}_{i,Ncl+1} = p_m \tag{17c}$$

The terms $\mathbf{M}_{ij}$ in equation (17b) have been normalized, so that for the ith weight class the sum of transition probabilities p_i to classes from ith to N_{cl}th is equal to $(1 - p_m)$, and the sum of the entire ith row is 1.

For each period, transition matrix evaluation for the starting ith weight class is then composed by the following steps: (*i*) computation of the average growth [equation (6)], starting from weight W_i, considering temperature (2, 3, 4) and feeding (5, 7) effects; (*ii*) evaluation of stochastic weight distribution [equation (15)] around the average value [equation (6)], computation of subtended areas and transition probabilities [equations (16a, b, c)] for each jth class of arrival; (*iii*) estimation of physiological, additional and total mortality (812); and (*iv*) computation of Markovian matrix elements [equations (17a, b, c)].

Therefore, in each period, the transition matrix depends on water temperature, feeding level, fish age, harvested and transferred biomasses in the previous period and on model parameter values.

Model Data and Biomass Evaluation

The following data are considered known:[1] (*i*) the number of the weight classes N_{cl} and their upper limit W_i^+; (*ii*) the number of fish ponds N_p, their volume V_j, and the minimum and maximum weight class referring to each fish pond; (*iii*) the maximum allowable density per pond δ_{max}; (*iv*) the weight distribution $F_{st,i}$ for the recruits, among the first i_{st} classes: (*v*) the minimum class for harvesting i_h; (*vi*) the length of the period N_D and the number of periods N considered in the time horizon; (*vii*) the distribution of temperature T_k and fish prices P_k in the N periods; (*viii*) the distribution of the decision variables in the N periods (harvesting level α_k, stocking level β_k, feeding level, pond temperature increase;[2] in the case of optimization analysis, these values are assumed as initial values for the iterative process leading to the optimal values); (*ix*) biomass B_{t1} at the beginning of the process and the vector $\mathbf{x}_{i,1}$ describing the initial fish distribution among the N_{cl} weight classes; (*x*) the theoretical daily amount of food required, as a function of fish weight and water temperature; and (*xi*) unit costs for feed C_m and stocked recruits C_{st}.

Due to space constraints, only the formulas more related to economic aspects

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[1] The values adopted in this application for the most relevant variables are reported in table 1. More detailed information can be found in the previous papers (Rizzo and Spagnolo 1992) or obtained from the authors.

[2] In this application, only harvesting and stocking levels have been considered as decision variables.

are quoted in the following paragraphs. Mathematical details about the computation of biomass distribution in each period as a function of Markovian matrix and of harvesting and stocking levels can be found in previous papers (Rizzo and Spagnolo 1992).

Water and Oxygen Consumption

Oxygen consumption is computed from the biomass in the jth pond and the unit consumption per mass κ_{O2}:

$$\Psi_{O2} = \kappa_{O2}\, B_j\, 10^6 \qquad [\mathrm{mg/h}] \tag{17}$$

A constant value for oxygen requirement per unit weight and hour has been assumed, starting from literature data (Barnabé 1986):

$$\kappa_{O2} = 0.177 \qquad [\mathrm{mg/g/h}] \tag{18}$$

Typical values for oxygen concentration in the pond and in input water[3] have been assigned, while concentration in output water is assumed equal to the average concentration in the pond:

$$\begin{gathered} O_{2,p} = 4.5 \\ O_{2,\mathrm{in}} = 6 \\ O_{2,\mathrm{out}} = O_{2,p} \quad [\mathrm{mg/l}] \end{gathered} \tag{19}$$

The water flow rate, in m^3/h, theoretically needed to satisfy the oxygen requirement Ψ_{O2} can be then computed as a function of the oxygen concentration in the pond $O_{2,j}$, in the input water $O_{2,\mathrm{in}}$ and in output $O_{2,\mathrm{out}}$:

$$Q = \Psi_{O2}/(O_{2,\mathrm{in}} - O_{2,\mathrm{out}})10^{-3} \tag{20}$$

Actual mass flow rate in the pond Q_j is equal to Q, if $Q < Q_{\max}$, where this latter is the maximum mass flow rate that pumps can provide. Otherwise, it is assumed $Q_j = Q_{\max}$. In this case, further oxygen will be provided by auxiliary devices; their oxygen supply Ψ_{aux} is computed as:

$$\Psi_{\mathrm{aux}} = \Psi_{O2}\,(Q - Q_{\max})/Q \quad [\mathrm{mg/h}] \tag{21}$$

and the corresponding energy consumption can be evaluated, given a performance index I_{O2}:

$$E_{O2} = 24\, N_D\, \Psi_{\mathrm{aux}}\, 10^{-6}\, I_{O2} \quad [\mathrm{kWh}] \tag{22}$$

Water supply in terms of volumes per day can be finally evaluated:

$$\Gamma_j = 24\, Q_j/V_j \tag{23}$$

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[3] Possible dependence of oxygen concentration on input water temperature has not been considered, but it could be easily included in the model.

Pumping Energy Consumption

The power P_p for the pumps, in kW, is computed from total flow rate $Q_t = \Sigma_j Q_j$, from total head H_t and of pump efficiency η_p:

$$P_p = 9.81\ Q_t H/(3{,}600\ \eta_p) \tag{24}$$

The total head H_t is the sum of the geodetic term H_u, equal to the difference in height from the water reservoir and the fish ponds, and of head losses H_p (Colombo 1971) in pipes:

$$H_t = H_u + H_p \tag{25}$$

The energy consumption $E_{p,k}$ in the kth period is:

$$E_{p,k} = P_p\ 24\ N_D\ \ [\text{kWh}] \tag{26}$$

Feeding

For each weight class, the theoretical amount of food to supply ν_i is computed as a function of water temperature and fish length l_i from suitable data provided by food producers (Feeding Chart for Sea Bass with Trouvit Branzini). The actual amount $B_{f,k}$ is obtained considering the feeding level ϕ_k:

$$B_{f,k} = N_D \sum B_{ik} \nu_i \phi_k \tag{27}$$

At the end of the process, a global value of conversion ratio FCR can be computed as a ratio of the provided food and the harvested biomass:

$$FCR = \sum_k B_{f,k} \Big/ \sum_k B_{h,k} \tag{28}$$

Given water flow rate Q_j, it is possible to compute the thermal energy consumption E_R needed for heating water from external temperature T to the desired value T^*, in N_D days:

$$E_{eh} = 24\ N_D Q_j\ 10(T^* - T)\ \ [\text{kcal}] \tag{29}$$

It is assumed that temperature in ponds is equal to T^*, neglecting possible heat losses, due to the small temperature differences between pond temperature and the surroundings and to the relevant water flow rate. For the present application of the model, no heat recovery from output water has been considered.

Biological Constraints

In each period, density in each pond δ_{jk} must be lower than an upper limit $\delta_{\max}$. A suitable constraint is defined thus:

$$c_{jk} = \delta_{\max} - \delta_{jk} \tag{30}$$

The solutions provided by the optimization method correspond to densities not greater than the maximum value δ_{max}, which has been assumed equal to 25 kg/m^3. This result is obtained by selecting suitable strategies for the harvesting and stocking variables α_k and β_k.

Final Period: Residual Biomass

At the end of the selected time horizon, a residual biomass could remain in the ponds. Therefore, in order to homogeneously compare different management strategies, its value has to be considered. Given residual biomass distribution and price $P_{f,i}$ for each weight class, the residual value *RV* is computed thus:

$$RV = \sum RV_i = \sum B_{i,N+1} P_{f,i} \tag{31}$$

Economic Submodel: Costs and Revenues

For each period, it is assumed that all the harvested biomass can be sold to a price P_{ki}, variable in time but independent on the supplied biomass.[4] Therefore, revenue R_k can be obtained:[5]

$$R_k = \sum_i B_{h,ki} P_{ki} \tag{32}$$

The following costs are considered: (*i*) cost for pumping $C_{p,k}$ (*ii*) cost for auxiliary oxygen $C_{O2,k}$, proportional to the required energy and to the unit cost for electrical power K_p; (*iii*) cost for stocked fish $C_{st,k}$, proportional to the number of recruits supplied; (*iv*) cost for feeding $C_{f,k}$, proportional to the amount of food provided $B_{f,k}$; (*v*) cost for external water heating $C_{eh,k}$, proportional to required energy and to the unit cost of thermal energy; and (*vi*) in order to consider the incidence of other costs, such as fixed costs not depending on operative variables, a further cost term $C_{o,k}$ is introduced. Such cost is equally distributed over the periods.

Total cost per period $C_{t,k}$ is then given by:

$$C_{t,k} = C_{st,k} + C_{f,k} + C_{p,k} + C_{eh,k} + C_{O2,k} + C_{o,k} \tag{33}$$

Profit π_k is computed by:

$$\pi_k = R_k - C_{t,k} \tag{34}$$

In order to compare costs and revenues referring to different periods, all economic terms are multiplied for a discount coefficient *D*:

$$D = 1/(1 + i)^t \tag{35}$$

where *i* is the yearly discount rate, *t* is the time, in years, when the given term occurs.

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[4] Although prices are allowed to vary with weight classes, constant prices have been considered in the following.

[5] All economic terms are expressed in MLit (1,000,000 Lit, U.S.$667, 22 November 1996),

Simulation Analysis

In order to assess the predicting capability of the model, a simulation analysis has been performed, for the base plant considered. A maximum weight of 400 grams has been considered, divided in twenty classes with increasing width, in order to take into account the nonlinear nature of the growth process.

Values for k, x, θ, and γ in the biological model comply with indications provided by Querellou (1984). The parameters c_1, c_2, and c_3 have been assigned according to growth curve spread observed in typical cases of sea bass aquaculture by other researchers (Iandoli and Saroglia 1989).

The Base Case

The simulation analysis has been performed starting from a base plant described in table 1, also analyzing the influence of the most significant parameters on the results. The distribution of temperatures and variables describing harvesting, stocking, feeding level, and water heating are assumed constant over the periods. However, the model can also consider distributions varying with time. Harvesting strategy consists of extracting in each period 80% of the biomass present in the last five classes. The 20% remaining biomass will be affected in the subsequent period by a 1% mortality, due to harvesting stress. A further mortality relating to fish transfer between ponds has been assumed as 2% of the transferred biomass.

The stocking of recruits, with an average weight of 0.5 grams, is concentrated in the first month. The maximum water flow rate is computed to assure that the total pond volume can be changed twenty times a day. Auxiliary oxygenation occurs only when the required water flow is greater than this limit.

The results for the base case are described in tables 2-3 and in figures 7-8. For each period, table 2 reports: water temperature T (deg); harvesting level α (for the five classes in percent), stocking level β (percent of the initial biomass); feeding level ϕ (percent of the theoretical value); temperature increase due to auxiliary water heating (deg); average fish weight W_{av} (grams), total biomass B_t, evaluated at the middle of the period (tons); harvested biomass B_h, evaluated at the end of the period (tons); stocked biomass B_{st}, evaluated at the start of the period (tons); the amount of food supplied B_f (tons); the number of fish with respect to the initial value (percent); and the weight limits for the central 90% of biomass (5% has a weight less than W_{05}, while another 5% has a weight greater than W_{95}). Weight spread is computed according to the stochastic model.

The terms of the economic submodel, discounted and expressed in MLit, are reported in table 3. The last column displays the cumulative profit, which usually reaches its maximum value before the end of the period. The value of the residual biomass is also reported.

For the base case, sea bass harvesting is concentrated between the 8th and 19th period, with a maximum in the 13th month. The maximum amount of food and oxygen costs occur in the 11th month, where the maximum biomass is present in the plant. Auxiliary oxygen is supplied only from the 9th to 13th month, when water flow is inadequate to satisfy the whole oxygen requirement, and cost for oxygen is almost equal to pumping cost for the entire period. In this case, due to the particularly high unit cost assumed per auxiliary oxygenation, it should be more convenient to adopt higher water pumping rates.

In the 7th month, before the start of harvesting, about 95% of the initial fish survive, while the remaining 5% is affected by natural mortality. After the 16th month

Table 1. Description of the Base Case

Base Plant: Sector I			
Pond Volume	4,050 m^3	Min. and Max. Wt. [g]	0.5-81
Base Plant: Sector II			
Pond Volume	6,720 m^3	Min. and Max. Wt. [g]	81-400
Stocked Biomass	0.5 T	Mortality for Harvesting	1% of harvested biomass
Stocking Period	First month	Mortality for Pond Transfer	2% of transferred biomass
Average Wt. of Recruits	0.5 g	Max Water Supply	20 volumes/day
Water Temp.	22˚C	Cost of Recruits	1,000 Lit/fish
Harvesting Strategy	80% of biomass in Classes 16-20	Cost of Food	1.2 MLit/t
Feeding Level	100%	Cost of Electrical Energy	66.7 $\times 10^{-3}$ Lit/kJ (240 Lit/KWh)
Pond Heating	Not considered	Cost of Thermal Energy	11.7 $\times 10^{-3}$ Lit/kJ (0.049 Lit/Kcal)
Fish Price	22 MLit/t	Other Costs	400 MLit/year
Wt. Classes Harvested	16-20	Yearly Discount Rate	0.10

only 1% remains in the ponds, with an average weight of 280 grams. Maximum profit per month occurs in the 13th month, while maximum cumulated profit is reached in the 16th month. In the subsequent period revenues are lower than costs, due mainly to the incidence of fixed costs. At the end of the 24th month, however, a positive profit can be collected.

A total production of 241 tons of sea bass is achieved, with a conversion ratio of 3.04, maximum density of about 20 kg/m^3 and a maximum water flow rate corresponding to thirty-two volumes per day.

Biomass, harvest, and average weight are displayed in figure 7, while costs, revenues, and profit are shown in figure 8. The spread of the biomass around its average weight increases with time, and weight distribution becomes asymmetric when harvest occurs. Maximum pond density is reached in the second pond at about the 11th month. Water flow remains constant in the period from the 9th and 13th month, when auxiliary oxygen is provided.

Effects of the Operating Variables: Parametric Analysis

The model can be utilized to study the effects of the different variables. Relative variations of cost, production, revenues, and conversion ratio are plotted in figures 1-6. The most significant effects are briefly summarized:

1. Production and revenues increase with water temperature, while the opposite occurs for costs. Between 18˚C and 24˚C, profit increases of about 1,600 MLit, and conversion ratio changes from 3.93 to 2.31 (figure 1).
2. Stocking strategy can be varied acting on both the total amount of stocked biomass and on its time distribution. In the first series of results (figure 4), stocking is concentrated on the first period. When stocked biomass changes from 0.3 to 0.9 tons, an increase in production, revenues, density, and costs is observed. Profit is maximized when stocked biomass is 0.6 tons. For lower values, plant is not utilized enough, and high incidence of fixed costs results, while for values greater than 0.6 excessive costs for oxygen are required. In the second test (figure 2), a given biomass (0.5 tons) has been stocked in periods variable from one to sixteen months. Maximum profit is reached for a period of ten months,

Table 2. Simulation of the Base Case: Biomasses and Plant Variables

Period	Temp. (°C)	Harv. (%)	Stock. (%)	Feed (%)	Heating (°C)	Avg. Wt. (g)	Biomass (Tons) Total	Biomass (Tons) Harv.	Biomass (Tons) Stock.	Biomass (Tons) Feed	Fish (%)	(W_{05})	(W_{95})
1	22.0	80.0	100.0	100.0	0.0	1.5	1.45	0.00	0.50	2.12	100.00	0.1	2.4
2	22.0	80.0	0.0	100.0	0.0	4.3	4.17	0.00	0.00	5.69	98.21	0.8	6.3
3	22.0	80.0	0.0	100.0	0.0	9.0	8.68	0.00	0.00	10.76	97.20	2.9	12.5
4	22.0	80.0	0.0	100.0	0.0	16.0	15.30	0.00	0.00	17.35	96.48	6.8	27.0
5	22.0	80.0	0.0	100.0	0.0	25.7	24.40	0.00	0.00	26.07	95.93	12.5	41.5
6	22.0	80.0	0.0	100.0	0.0	38.5	36.33	0.00	0.00	36.83	95.48	19.0	63.5
7	22.0	80.0	0.0	100.0	0.0	54.8	51.47	0.00	0.00	49.18	95.09	28.2	88.0
8	22.0	80.0	0.0	100.0	0.0	75.0	70.16	0.21	0.00	63.14	94.69	40.4	120.4
9	22.0	80.0	0.0	100.0	0.0	99.8	92.84	2.91	0.00	78.90	94.09	56.1	160.8
10	22.0	80.0	0.0	100.0	0.0	129.7	118.36	13.74	0.00	95.46	92.35	73.7	221.9
11	22.0	80.0	0.0	100.0	0.0	161.5	137.79	35.59	0.00	106.57	86.32	93.7	239.2
12	22.0	80.0	0.0	100.0	0.0	190.3	134.85	59.43	0.00	101.19	71.70	115.0	266.2
13	22.0	80.0	0.0	100.0	0.0	213.1	101.29	61.26	0.00	74.43	48.09	136.1	291.5
14	22.0	80.0	0.0	100.0	0.0	231.9	56.09	40.16	0.00	40.57	24.47	156.5	324.2
15	22.0	80.0	0.0	100.0	0.0	250.1	23.58	18.40	0.00	16.82	9.54	167.1	352.6
16	22.0	80.0	0.0	100.0	0.0	269.6	7.98	6.49	0.00	5.62	2.99	182.4	367.8
17	22.0	80.0	0.0	100.0	0.0	290.5	2.30	1.90	0.00	1.60	0.80	188.6	373.5
18	22.0	80.0	0.0	100.0	0.0	311.4	0.59	0.48	0.00	0.40	0.19	197.5	375.9
19	22.0	80.0	0.0	100.0	0.0	330.7	0.14	0.11	0.00	0.09	0.04	211.8	377.1
20	22.0	80.0	0.0	100.0	0.0	346.8	0.03	0.02	0.00	0.02	0.01	230.9	377.8
21	22.0	80.0	0.0	100.0	0.0	359.1	0.01	0.01	0.00	0.00	0.00	253.9	378.1
22	22.0	80.0	0.0	100.0	0.0	367.7	0.00	0.00	0.00	0.00	0.00	283.1	378.3
23	22.0	80.0	0.0	100.0	0.0	373.3	0.00	0.00	0.00	0.00	0.00	318.3	378.5
24	22.0	80.0	0.0	100.0	0.0	376.7	0.00	0.00	0.00	0.00	0.00	342.9	378.5
Total								240.71	0.50	732.82			

FCR	Max. Density	Max. Volumes/Day
3.04	20.42	32.08

Table 3. Simulation of the Base Case: Economic Results (MLit)

			Costs								
Period	Price	Revenues	Feeding	Pumping	Stocking	Heating	Oxygen	Other	Total	Profit	Cumul.
1	22.0	0.0	2.5	2.0	1,000.0	0.0	0.0	33.3	1,037.9	–1,037.9	–1,037.9
2	22.0	0.0	6.7	5.7	0.0	0.0	0.0	33.3	45.8	–45.8	–1,083.7
3	22.0	0.0	12.7	11.8	0.0	0.0	0.0	33.3	57.8	–57.8	–1,141.5
4	22.0	0.0	20.3	20.7	0.0	0.0	0.0	33.3	74.3	–74.3	–1,215.8
5	22.0	0.0	30.2	32.7	0.0	0.0	0.0	33.3	96.2	–96.2	–1,312.0
6	22.0	0.0	42.3	48.4	0.0	0.0	0.0	33.3	124.1	–124.1	–1,436.1
7	22.0	0.0	56.0	68.2	0.0	0.0	0.0	33.3	157.9	–157.8	–1,593.9
8	22.0	4.3	71.4	92.6	0.0	0.0	0.0	33.3	198.0	–193.7	–1,787.6
9	22.0	59.6	88.5	99.5	0.0	0.0	71.4	33.3	293.2	–233.6	–2,021.3
10	22.0	279.2	106.2	98.7	0.0	0.0	177.6	33.3	416.0	–136.8	–2,158.1
11	22.0	717.4	117.6	97.9	0.0	0.0	256.8	33.3	505.7	211.7	–1,946.5
12	22.0	1,188.6	110.8	97.2	0.0	0.0	242.7	33.3	484.0	704.6	–1,241.8
13	22.0	1,215.5	80.9	96.4	0.0	0.0	103.6	33.3	314.2	901.2	–340.6
14	22.0	790.6	43.7	70.4	0.0	0.0	0.0	33.3	147.4	643.2	302.6
15	22.0	359.3	18.0	29.2	0.0	0.0	0.0	33.3	80.5	278.8	581.4
16	22.0	125.7	6.0	9.8	0.0	0.0	0.0	33.3	49.1	76.6	658.1
17	22.0	36.5	1.7	2.8	0.0	0.0	0.0	33.3	37.8	–1.4	656.7
18	22.0	9.2	0.4	0.7	0.0	0.0	0.0	33.3	34.5	–25.2	631.5
19	22.0	2.1	0.1	0.2	0.0	0.0	0.0	33.3	33.6	–31.5	600.0
20	22.0	0.5	0.0	0.0	0.0	0.0	0.0	33.3	33.4	–32.9	567.1
21	22.0	0.1	0.0	0.0	0.0	0.0	0.0	33.3	33.3	–33.3	533.8
22	22.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	33.3	–33.3	500.5
23	22.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	33.3	–33.3	467.2
24	22.0	0.0	0.0	0.0	0.0	0.0	0.0	33.3	33.3	–33.3	433.8
Res.Value		0.0									
Total		4,788.6	816.1	884.9	1,000.0	0.0	852.0	800.0	4,354.8	433.8	

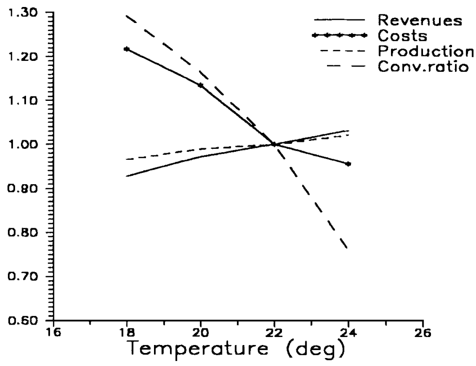

Figure 1. Effect of Water Temperature

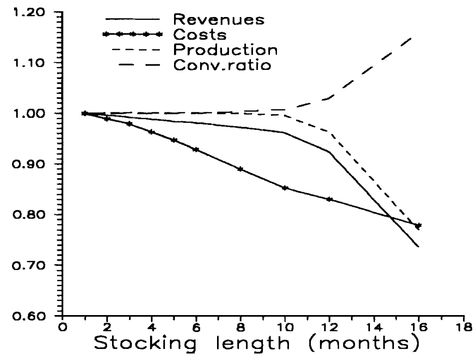

Figure 2. Effect of Stocking Period

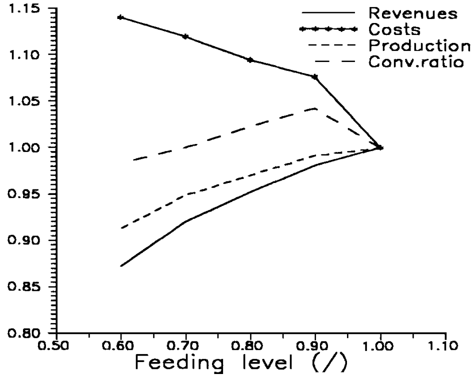

Figure 3. Effect of Feeding Level

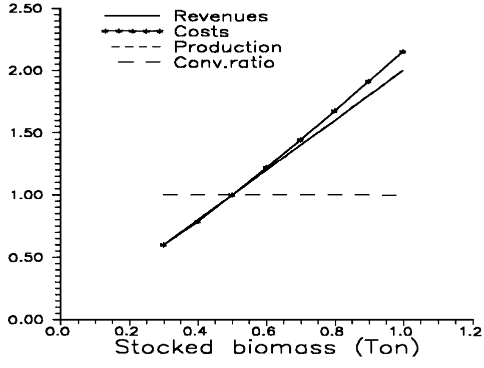

Figure 4. Effect of Stocked Biomass

which allows a reduction in maximum density (from 20 to 10 kg/m^3) and lower oxygen costs with respect to the base case. In case of more diluted distributions smaller fish are collected in the last period and a residual biomass is also present, thus resulting in lower revenues and profit.

3. The reduction of feeding levels from 100% to 60% of the theoretical value causes an increasing in process length (from sixteen to over twenty-four months) and a reduction of profit, production, and revenues. The lower costs for feeding do not offset the higher costs for pumping and oxygen, due to the longer process length and to the corresponding financial effects (figure 3).
4. Production, maximum density, revenues, costs, and process length increase with minimum harvesting weight (figure 5), while maximum profit is realized for the 16th class, corresponding to the base case.[6]
5. The increase of fish production and revenues achievable by heating the first pond do not compensate the higher energy cost (figure 6). However, in this case possible heat recovery from recirculating water has not been considered.

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[6] The model does not consider further weight increments after the 20th weight class (400 grams), and therefore the revenues obtained by harvesting only the last weight classes may be underestimated.

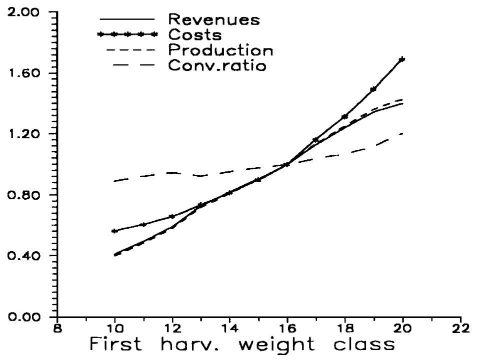

Figure 5. Effect of Harvesting Size

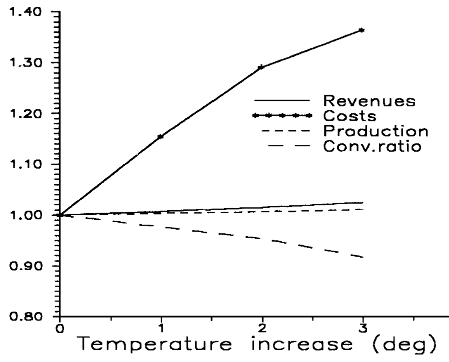

Figure 6. Effect of Water Heating

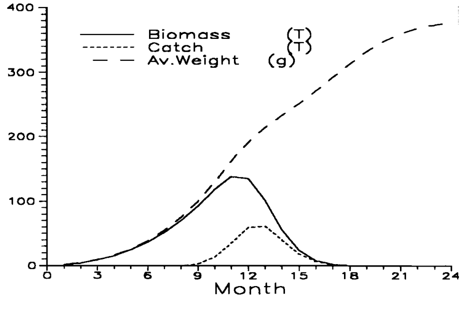

Figure 7. Base Case: Biomass

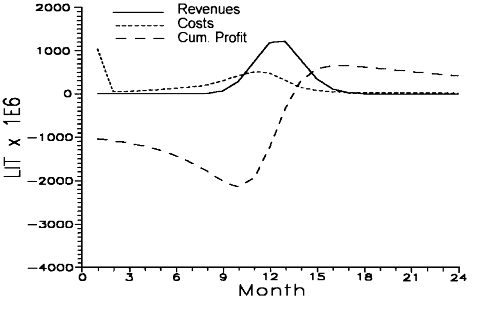

Figure 8. Base Case: Economic Results

This kind of analysis is particularly useful in order to clarify the influence of the single variables, but it may not provide exhaustive information on system behavior, since the effect of a variable may depend on the values assumed by other variables, due to the presence of interactions and nonlinearities in the model. In this case, optimization analysis represents the best tool to select the most suitable management strategies, considering the interactions between the variables.

Optimization Analysis

Management strategies can be optimized determining the distribution of the decision variables which maximizes profit in the given time horizon and respects the constraints on maximum density in ponds. A classical nonlinear constrained optimization problem (Gill *et al.* 1984) is then defined:

$$\min_u F(u) \tag{36}$$

$$c_i(u) \geq 0 \qquad i = 1, M \tag{37}$$

where the decision variables u represent stocking and harvesting strategies, and F is the sum of the profit π_k in the periods and of the possible residual value, at the end of the time horizon:

Table 4. Cases for Optimization Analysis

Case	Description	Initial	Optimal
A	Base	A1	A2
B	Base - Variable prices	B1	B2
C	As B - Height difference = 40 m	C1	C2
D	As C - Variable temperatures	D1	D2

$$F(u) = -RV - \sum \pi_k \tag{38}$$

The inequality constraints express the condition that pond density must be lower than a given maximum value:

$$c_i = \delta_{\max} - \delta \tag{39}$$

The nonlinear constrained optimization problem [equations (36-37)] has been solved using the Augmented Lagrangian approach (Gill *et al.* 1984).

Results of the Optimization Analysis

Some results obtained with optimization analysis are presented in the following.[7] The decision variables are: harvesting strategy (percent of the harvested biomass, for the weight classes from sixteen to twenty) and stocking strategy. Table 4 shows the four cases considered. The corresponding profile for prices are displayed in figure 11.

For each case, the optimal results have been compared with those obtained from the initial distribution of harvesting and stocking levels: 80% harvesting level for the weight classes upper to 15th, and biomass of 0.5 tons stocked in the first month. Global results are presented in table 5,[8] for all the cases described in table 4. Some detailed results are shown for the case A2 in figures 9-10, while the optimal time histories for the decision variables in the cases A2, B2, and D2 are plotted in figures 12-14.

In the case A, by the comparison of the initial and the optimal solutions it emerges that: (*i*) profit is increased of about 1,300 MLit due to a relevant increment in revenues, which largely offset the higher costs for food, pumping, oxygen, and stocking; (*ii*) stocked biomass and production grow, so resulting in better plant utilization and then a lower incidence of fixed costs (after the 24th months, a residual biomass is present in the plant); (*iii*) stocked biomass (figure 12) has been subdivided in different periods, spaced of about five months, in order to limit maximum pond density and to reduce auxiliary oxygenation; and (*iv*) harvesting level (figure 12) increases up to 100% in last periods (to reduce residual biomass) and when auxiliary oxygen should be needed, while decreases when density is lower (*e.g.*, months 21-23), in order to allow further growth and then greater revenues in the following periods.

The results for the optimal case are shown in figures 9-10. Average weight, harvested biomass and density exhibit more complex trends, with respect to the solu-

[7] More detailed information on mathematical procedures and results can be obtained from the authors.
[8] Two indexes synthetize biomass distribution. MP1 represent the number of months after which residual population is less than 1% (an asterisk indicates that more than 1% of the biomass remains after the end of the cycle). RP12 is the ratio, in percent, between the population after twelve months and the population in the first month.

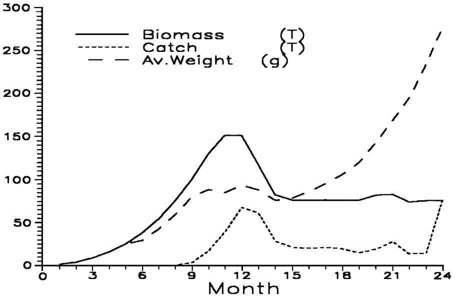

Figure 9. Optimal Results: Case A2—Biomass

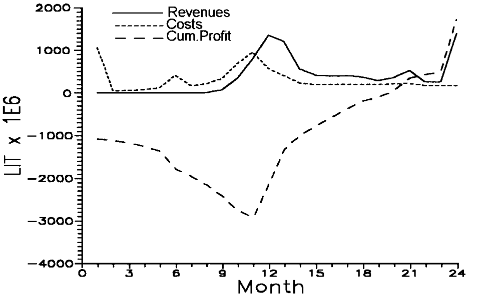

Figure 10. Optimal Results: Case A2—Economic Terms

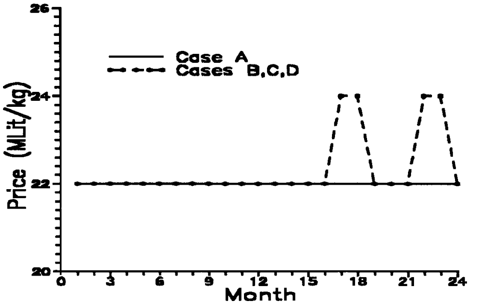

Figure 11. Fish Price for the Optimization Cases

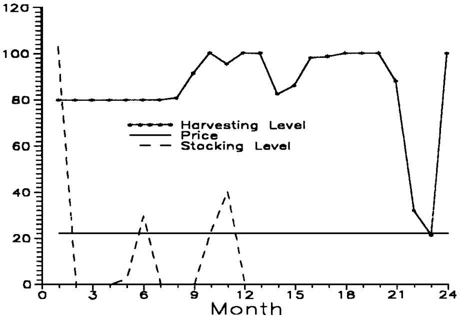

Figure 12. Price and Decision Variables: Case A2

tion obtained in the base case (figures 7-8) with standard management procedures. Density is usually higher, but always within the imposed upper limit of 25 kg/m^3, allowing a more complete productive utilization. This result has required suitable and not obvious management strategies (figure 12) in order to maximize profit by selecting the best compromise between density dependent and financial factors and to satisfy the constraint on maximum density.

The second case (*B*) is characterized by a nonuniform price distribution, with about 10% increase in months 17, 18, 22, and 23 (figure 11). The optimal solutions (figure 13) are similar to case A, but with higher harvesting levels in periods where price growths and lower levels in the previous periods. Profit increase (1,480 MLit) is greater than the previous case, getting advantage from the higher average price.

Case C is characterized by higher pumping height and then by increased energy consumption with respect to case B. It exhibits an unprofitable result (440 MLit) with standard solution, while optimal strategy gives rise to a positive result of about 300 MLit, obtained by increasing fish production from 240 to 350 tons; both revenues and costs increase with respect to the initial solution.

In the last case (D1-D2), finally, variable temperatures have been considered, with lower values in respect to previous cases. The initial solution gives rise to a loss of 1,370 MLit, which is reduced to about 800 MLit with optimized solution. This result is obtained by reducing fish production (from 0.5 to about 0.3 tons) and so eliminating costs for oxygen; stocking is concentrated in the first period, unlike

Table 5. Optimization Analysis: Global Results—Initial and Optimal Solutions

		Costs													
Case	Rev.	Feed	Pump.	Stock.	Heat.	Oxyg.	Others	Total	Profit	Prod.	FCR	MP1	RP12	Dens.	Vol./d
A1	4,788.6	816.1	884.9	1,000.0	0.0	852.0	800.0	4,354.8	433.8	240.71	3.04	16.9	71.70	20.4	32.1
A2	9,012.8	1,543.8	1,795.9	1,922.4	0.0	1,209.0	800.0	7,274.5	1,738.3	466.94	3.06	*	158.33	20.5	32.9
B1	4,792.8	816.1	884.9	1,000.0	0.0	852.0	800.0	4,354.8	438.0	240.71	3.04	16.9	71.70	20.4	32.1
B2	8,985.3	1,487.3	1,717.8	1,880.2	0.0	1,181.9	800.0	7,070.5	1,914.8	452.35	3.04	*	158.56	19.9	32.1
C1	4,792.8	816.1	1,763.7	1,000.0	0.0	852.0	800.0	5,233.6	–440.8	240.71	3.04	16.9	71.70	20.4	32.1
C2	7,075.8	1,153.6	2,977.9	1,445.2	0.0	399.5	800.0	6,778.7	297.2	354.30	3.03	*	234.40	13.3	32.1
D1	4,792.9	935.2	2,272.9	1,000.0	0.0	1,151.3	800.0	6,161.6	–1,369.0	240.29	3.55	20.6	93.10	19.5	32.1
D2	2,893.0	555.1	1,774.0	571.6	0.0	0.0	800.0	3,701.9	–808.9	143.03	3.55	20.6	93.09	11.2	31.6

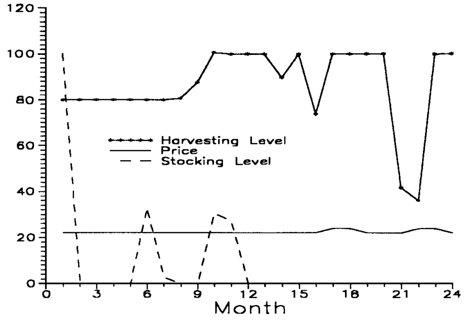

Figure 13. Price and Decision Variables: Case B2

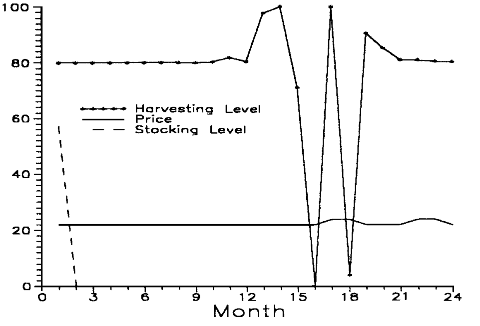

Figure 14. Price and Decision Variables: Case D2

the previous cases A, B, and C. Also in this case, harvesting level is maximized in the periods with higher prices, and reduced in the previous ones (figure 14).

It is also timely to remark that in all these results a fixed productive cycle of twenty-four months has been considered, and that some of the conclusions could be changed if longer cycles (*e.g.*, thirty-six months) are assumed. A complete analysis should therefore include a systematic investigation of the effects of productive cycle length.

These examples, even if extended only to harvesting and stocking strategies and not representative of all possible conditions occurring in aquaculture management, prove the considerable benefits obtainable with the use of an optimization model. This approach can account for the influence of structural, environmental, and economic variables and for their complex interactions on fish farming process and therefore can suggest articulate management strategies which realize the optimal solution for each operating condition.

Conclusions

A model for the simulation and the optimal management of sea bass aquaculture plants has been presented. The model accounts for the effects on growth and mortality of water temperature, harvesting and stocking strategies, feeding level, and oxygen consumption, and can evaluate costs and revenues. Cost-benefit analysis can be easily obtained examining the effects of each operating variable on the management outputs.

The results of the optimization analysis prove that, for each combination of environmental and economic conditions, articulate optimal harvesting and stocking strategies can be determined which tend to maximize plant utilization within the limit of maximum density and allow relevant benefits in profit, with respect to the more traditional management approach. In the examined cases, stocking is usually divided in different batches, over a time period of four to six months, and the harvesting level is reduced when price growth in the next period is expected. It is therefore evident that the management benefits depend on the accuracy in the predicted trends for the exogenous variables, especially prices. It is also clear that, due to the complex interactions of the many variables affecting the process, the optimal management strategies cannot be provided by simple rules.

Further work is in progress in order to include other management options, to improve the detail and the precision of the submodels, and to validate the model estimates and the optimal strategies by direct comparison on aquaculture plants.

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