The Effect of the Discount Rate on the Optimal Exploitation of Renewable Resources

Rögnvaldur Hannesson

Institute of Fisheries Economics
Norwegian School of Economics and Business Administration
Helleveien 30
N-5035 Bergen-Sandviken
Norway

1. Introduction

In a recent paper, Farzin (1984) has shown that the impact of the discount rate on the optimal rate of depletion of an exhaustible resource is ambiguous. This is so because not only is the discount rate an expression of time preference, it also reflects the opportunity cost of capital. As the extraction of an exhaustible resource typically requires investment in buildings, equipment, etc., a higher discount rate will raise the cost of extraction. In terms of the Hotelling rule, $dP/dt = r(P - c)$ where P is the price of the extracted product, c is the unit cost of extraction, and r is the discount rate, a higher r will be associated with a higher c, and so the implication for the rate of price increase is ambiguous. Furthermore, a higher discount rate is likely to affect the backstop price of the resource.

What, then, about renewable resources? Ever since Clark's famous paper on the discount rate and the extinction of animal species (Clark, 1973), it has been recognized that a higher dis-

Marine Resource Economics, Volume 3, Number 4
0738-1360/87/030319-00$03.00 + 0

count rate increases the optimal rate of exploitation of a renewable resource, and so increases the likelihood of extinction. This, however, ignores the capital cost implication of a higher discount rate, the effect discussed by Farzin.

In this paper we examine the effect of the discount rate on the optimal rate of exploitation and standing stock of a renewable resource such as fish. As is the case for non-renewable resources, this effect is ambiguous, and the ambiguity depends on the dual role of the discount rate. On the one hand, the discount rate expresses a required rate of return on a growing asset. For a renewable resource with a concave growth function, a higher rate of discount implies a smaller standing stock. On the other hand, the discount rate expresses the opportunity cost of capital to be invested in harvesting equipment. A higher discount rate thus means more costly harvesting, which in turn implies a less intensive optimal harvesting and a larger standing stock.

We shall distinguish between two cases. In the first case the unit harvesting cost is independent of the size of the stock being harvested (no "stock effect"). Here the effect of a change in the discount rate depends on whether or not the shadow price of the resource stock is positive. If it is, the optimal rate of exploitation is independent of the harvesting cost, except that revenues must exceed costs if any harvesting at all is to be optimal. The optimal rate of exploitation is then determined by the requirement that the marginal growth rate of the resource be equal to the discount rate. Hence a higher discount rate raises the optimal rate of exploitation and lowers the optimal standing stock, for a resource with a concave growth function. If, on the other hand, the shadow price of the resource stock is zero, the intertemporal aspect is irrelevant, and the optimum rate of exploitation is determined by an equality between the value of the current marginal harvest and current marginal harvesting costs. A higher discount rate then means higher harvesting costs, a lower optimal harvest rate, and a larger standing stock.

The second case we consider is the one of a stock-dependent unit cost of harvesting (a positive "stock effect"). Here the discount rate is active in both of its roles simultaneously, except in the special case of costless harvesting. In this case the effect of the discount rate on optimal harvesting and standing stock is truly ambiguous, its direction depending on the capital intensity of the harvesting process and the cost of capital.

2. The Model

Consider a renewable resource, the growth rate of which is given by the function $G(W)$, where W is the size of the resource stock. Let the rate of exploitation be given by the function $F(K, W)$, where K is the amount of capital invested in exploitation equipment.[1] To focus on the issue at hand, we shall assume that other factors of production are complementary to capital and ignore their costs. The net rate of growth of the resource stock then is (dots denote time derivatives):

$$\dot{W} = G(W) - F(K, W) \tag{1}$$

Optimum exploitation of the resource stock implies finding the time path of investment that maximizes the following integral:

$$\int_0^\infty [PF(K_t, W_t) - I_t]e^{-rt}\, dt \tag{2}$$

subject to (1) and

$$\dot{K}_t = I_t - aK_t \tag{3}$$

where I_t is investment at time t, a is the rate of depreciation of capital, and P is the price of the extracted product, assumed constant.[2] The time subscripts will henceforth be dropped whenever this causes no confusion.

The present value Hamiltonian of the problem defined by (1)–(3) is

$$H = [PF(K, W) - I + \gamma(I - aK) + \lambda(G(W) - F(K, W))]e^{-rt} \tag{4}$$

where γ and λ are the adjoint variables associated with the side constraints (1) and (3). From the maximum principle we get (subscripts to the equation symbols H and F denote partial derivatives):

$$H_I = (\gamma - 1)e^{-rt} \tag{5}$$

$$\dot{\gamma} - r\gamma = -[(P - \lambda)F_K - \gamma a] \tag{6}$$

$$\dot{\lambda} - r\lambda = -[(P - \lambda)F_W + \lambda G'] \tag{7}$$

Equation (5) determines the optimal investment policy:

$$\begin{aligned} I_t &= 0 && \text{if } \gamma_t < 1, \\ 0 \leqslant I_t &\leqslant I_{max} && \text{if } \gamma_t = 1, \\ I_t &= I_{max} && \text{if } \gamma_t > 1. \end{aligned}$$

where I_{max} is the upper limit on the flow of investment at any point in time.

If a stationary solution (W^*, K^*) exists, then $\gamma = 1$ and $I^* = aK^*$. From (6) we then have that $(P - \lambda)F_K(K^*, W^*) = r + a$, which says that the marginal product of capital, valued at the net price of the extracted resource, is equal to the discount rate plus the rate of depreciation. The net price is the market price of the extracted resource, minus the shadow price (λ) of the unextracted resource.

In a stationary solution, Equation (7) becomes

$$(P - \lambda)F_W = (r - G')\lambda \tag{7'}$$

3. Case I: Stock-Independent Harvesting Costs

If $F_W = 0$, the unit cost of extraction is independent of the size of the exploited resource. There are two possibilities for satisfying (7′) in this case:

$$\lambda > 0 \quad \text{and} \quad r = G'(W^*) \tag{7''a}$$

$$\lambda = 0 \quad \text{and} \quad r \neq G'(W^*) \tag{7''b}$$

If (7″a) holds, the optimal rate of exploitation is determined by

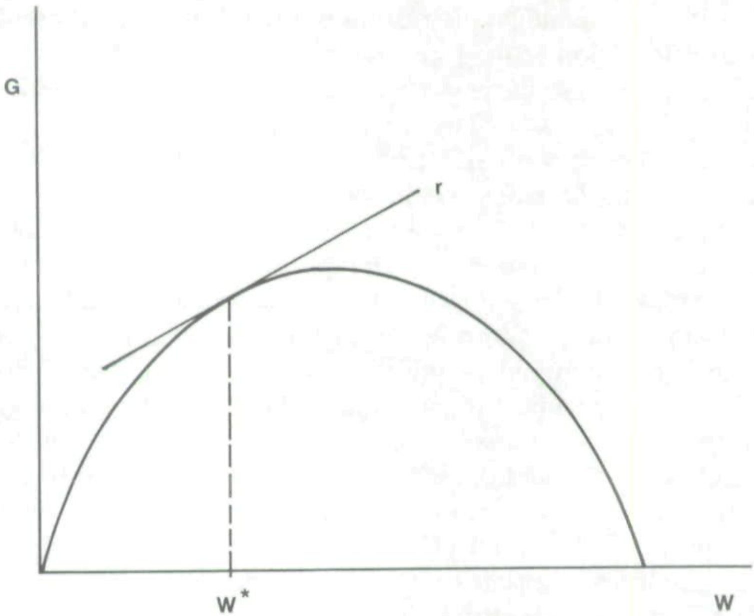

FIGURE 1. Optimal standing stock (W^*), as determined by the rate of discount when the marginal harvesting cost is independent of the stock and not very sensitive to the amount of harvesting effort.

the equality between the marginal rate of growth of the resource stock and the discount rate. The optimal investment is whatever is necessary to attain this rate. The intertemporal perspective is the only thing that matters, the price of the harvested product being high enough to cover the current harvesting cost. It is not surprising, therefore, to find that the effect of the discount rate on the optimal rate of exploitation is unambiguous in this case and points in the direction hitherto discussed in the literature, where the discount rate has been taken as expressing only time preference. Differentiating (7″a) gives $dW/dr = G'' < 0$ (assuming a monotonically decreasing marginal rate of growth); that is, a higher discount rate decreases the optimal standing stock (W^*) of the resource, as can be inferred from Figure 1. The amount of capital invested in the harvesting process would increase initially, in order to adjust the resource stock, but would thereafter fall to a level lower than before, because lesser effort will be needed to take the harvest which in the long run will decrease. Note again that $F_W = 0$ by assumption; that is, the size of the resource stock has no effect on the amount harvested (except, of course, that no stock would yield no harvest).

Consider now (7″*b*), the case in which the shadow price of the resource is zero. Here the intertemporal aspect is irrelevant; no future yields are forsaken by taking more in the current period. Kemp and Long (1980) referred to this case as one of a "non-scarce" or "potentially plentiful" resource. In Kemp and Long's model this was caused by marginal utility or revenue falling, or marginal harvesting costs rising, as the harvested quantity increases. In our model (which could easily be extended to cover quantity-dependent price) this occurs because of a falling marginal product of capital (F_K) as the harvesting capital is increased, which translates into rising marginal harvesting costs as the harvested quantity is increased.

This case is illustrated in Figure 2. To satisfy the intertemporal efficiency rule ($r = G'(W^*)$), the resource stock would have to be at W^*. This would require maintaining harvesting capital at K^*, such that $F(K^*) = G(W^*)$, cf. (1). But if $F_{KK} < 0$, it is possible that $PF_K(K^*) < r + a$, and (6) could not be satisfied in a stationary solution with $K = K^*$. Both (6) and (7) could, however, be satisfied in a stationary solution with $\lambda = 0$ and $K = K^{**} < K^*$, such that

$$PF_K(K^{**}) = a + r \tag{6'}$$

In this case the impact of the discount rate on the optimal rate of exploitation works through raising the harvesting cost. This decreases the optimal amount of capital invested and increases the optimal standing stock of the resource. Differentiating (6′) yields $dK/dr = (PF_{KK})^{-1} < 0$. This lowers the amount harvested, and the resource stock will grow until a new equilibrium is reached with a larger stock and a lesser rate of growth.

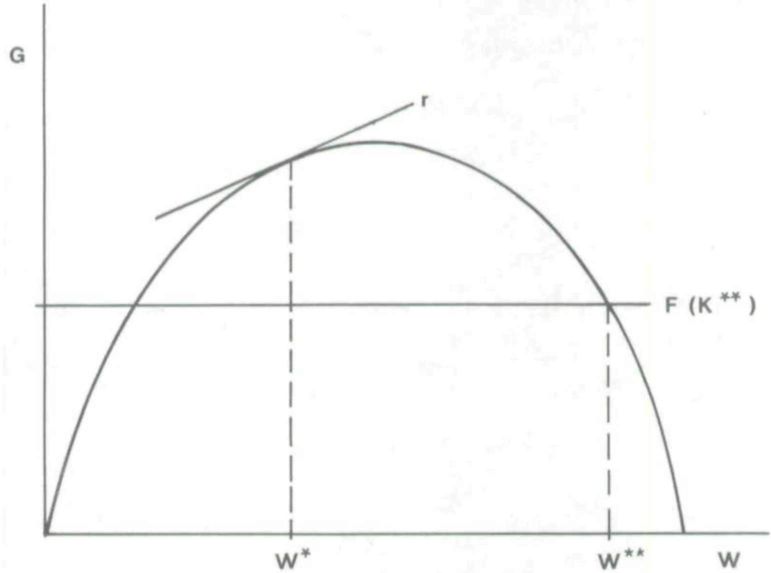

FIGURE 2. Optimal standing stock (W^{**}), as determined by $G(W^{**}) = F(K^{**})$ and $PF_K(K^{**}) = r + a$. Marginal harvesting costs are independent of stock but rise with harvesting effort (K), so that $PF_K(K^*) < r + a$, $G(W^*) = F(K^*)$.

4. Case II: Stock-Dependent Harvesting Costs

Solving (6) for the stationary value of λ and inserting in (7) gives

$$r - G'(W^*) = (r + a)F_W(K^*, W^*)/[PF_K(K^*, W^*) - (r + a)]. \quad (8)$$

The steady state relation $G(W) = F(K, W)$ implies a function $\psi(K, W) = 0$, which for certain forms of $G(\cdot)$ and $F(\cdot)$ can be solved to yield

$$K^* = \phi(W^*). \quad (9)$$

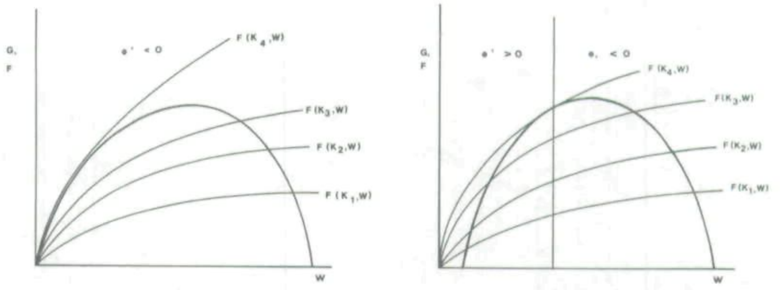

FIGURE 3. Cases where the function $W^* = \phi(K^*)$ is double valued (*b*) and monotonically decreasing (*a*).

Inserting this into (8) and differentiating gives

$$\frac{dW}{dr} = \frac{[PF_K - (r + a)]^2 - PF_K F_W}{G''[PF_K - (r + a)]^2 + [PF_K - (r + a)] \times (F_{WK}\phi' + F_{WW})(r + a) - PF_W(r + a)(F_{KK}\phi' + F_{KW})} \tag{10}$$

A monotonically decreasing relative rate of growth implies $G'' < 0$. Normally, we would expect $\phi' < 0$, but Figure 3b shows the possibility of $\phi' > 0$. Reasonable assumptions about the extraction function $F(K, W)$ are $F_K \geqslant 0$, $F_W \geqslant 0$, $F_{KW} = F_{WK} \geqslant 0$, $F_{KK} \leqslant 0$, $F_{WW} \leqslant 0$. Making these assumptions implies a negative denominator of (10). Since the sign of the numerator is ambiguous, the sign of dW/dr is also ambiguous. We see that the higher the cost of capital $(r + a)$ or the lower the value of the marginal product of capital, the more likely it is that a higher discount rate will imply a lower rate of exploitation. This ambiguity disappears if $F_W = 0$; i.e., if the size of the exploited stock has no influence on the yield per unit of capital (note that (10) was derived on the assumption that $\lambda > 0$). In that case, a higher discount rate implies a more intense exploitation ($dW/dr < 0$).

5. An Example

As an example, consider the familiar parabolic growth equation $G(W) = W(1 - W)$ and the "mass-encounter" production function $F(K, W) = KW$. Inserting this into (8) gives the following solution for the optimal stock size of the exploited resource:

$$W^* = (4P)^{-1}[\pm(8rP(r + a) + (P(r - 1) - (r + a))^2)^{1/2} - P(r - 1) + (r + a)] \qquad (11)$$

It may be noted that these functions satisfy the assumptions $G'' < 0$, $F_K > 0$, $F_W > 0$, $F_{KW} = F_{WK} > 0$, $F_{KK} = 0$, $F_{WW} = 0$. The function ϕ is given by setting $F(\cdot) = G(\cdot)$, which gives $K = \phi(W) = 1 - W$ and $\phi' < 0$.

Solving (11) for alternative values of P and r produces the values shown in Table 1. Consider, first, the case $a = 0$, which implies that capital equipment does not depreciate. The figures

Table 1
Solutions of Equation (11) for Alternative Values of P and r

r		$a = 0$	
P	0	0.1	0.5
0.5	0.5	0.57	1.00
1.0	0.5	0.51	0.68
2.0	0.5	0.48	0.50
4.0	0.5	0.47	0.39

r		$a = 1$	
P	0	0.1	0.5
1	1.00	1.00	1.00
2	0.75	0.76	0.85
4	0.63	0.61	0.60

Note: The solution of (11) implies $W > 1$ when $a = 1$, $P = 1$, and $r > 0$. Since $W = 1$ is the natural equilibrium, we set $W = 1$ even in these cases.

show that for a "low" price of the extracted product ($P = 0.5$ and $P = 1$), a higher discount rate implies a greater optimal stock of the resource; that is, a less intense exploitation. Higher prices of the extracted product, such as $P = 2$ and $P = 4$, show the "traditional" relationship between the discount rate and the degree of exploitation. In this case the capital cost is less relatively speaking, and the cost-increase effect of raising the discount rate counts for less than the lessened willingness to invest in the resource by leaving a part of it to grow.

The case $a = 1$ implies that capital equipment fully depreciates in one time period. This raises the opportunity cost of capital, and is reflected in a higher optimal standing stock (lower optimal rate of exploitation) of the resource for any given configuration of P and r. Otherwise the same pattern emerges; if P is low enough, then a higher discount rate implies less intense exploitation.

A final point to note about the two cases is that the no depreciation—no discounting case represents the zero cost—no discounting case, in which the resource should be exploited to give maximum sustainable yield, which occurs at $W = 0.5$ ($W = 1$ is the no exploitation equilibrium stock). When capital depreciates, the zero discounting case no longer represents the zero cost case. With $P = 1$ the exploitation is unprofitable, as the lowest possible exploitation cost per unit is $K/KW_{max} = 1$, $W_{max} = 1$ being the size of the resource stock when left unexploited.

Notes

1. We express capital in money terms, implicitly setting the price of real capital equal to one. Changing the price of real capital has the same effect as changing the marginal productivity of capital in the opposite direction.

2. This formulation of the optimal harvesting problem is similar to Clark, Clarke and Munro's (1979) case of quasi-malleable capital; i.e., disinvestment is limited to the depreciation of capital. They demonstrated various approach paths to an optimal stationary solution and how these depend on the starting point. In particular, if one starts with a heavily overcapitalized fishery, it is desirable not to use all the harvesting capital available and to rebuild the stock more rapidly. To take

this into account, it would be necessary to distinguish between available and utilized capital in the above problem. As we have nothing to add to Clark, Clarke and Munro's analysis on this point, we shall stick to the simpler formulation, even if it is, strictly speaking, appropriate only for a fishery that is not too heavily overcapitalized.

References

Clark, C. W. 1973. Profit maximization and the extinction of animal species. *Journal of Political Economy* 81: 950–961.

Clark, C. W., F. H. Clarke and G. R. Munro. 1979. The optimal exploitation of renewable resource stocks: problems of irreversible investment. *Econometrica* 47: 25–47.

Farzin, Y. H. 1984. The effect of the discount rate on depletion of exhaustible resources. *Journal of Political Economy* 92: 841–851.

Hannesson, R. 1983. A note on socially optimal versus monopolistic exploitation of a renewable resource. *Zeitschrift für Nationalökonomie* 43: 63–70.

Kemp, M. C., and N. V. Long. 1980. The management of a renewable resource: monopoly versus competition. In M. C. Kemp and N. V. Long, eds. *Exhaustible Resources, Optimality, and Trade*. North-Holland.