Marine Resource Economics, Volume 8, pp. 145–154
Printed in the USA.
0738-1360/93 $3.00 + .00

Dynamic Regulation of Fisheries: the Case of the Bowhead Whale

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Abstract *The regulation of fisheries often requires finding numerical solutions to dynamic optimization problems. This paper presents a version of the "multiple shooting" algorithm and uses it to approximate the dynamic solution to a fisheries problem examined by Conrad (1989): the hunting of the Bowhead whale in the Western Arctic. It is found that the inclusion of dynamic considerations can significantly alter the nature of the policy if the regulated population is not near its steady state.*

Keywords Bowhead whale, multiple shooting, numerical methods, regulation of fisheries.

Introduction

Regulating marine populations often requires the explicit solving of a dynamic optimization problem. While it may not be difficult to write out the Euler equations describing the transition path to the steady state, in practice, solving for numerical solutions may be problematic. This paper shows how to apply a simple technique (multiple shooting) for solving for the dynamics of a specific class of problems and uses this technique to approximate the solution to an actual fisheries problem presented in Conrad (1989).

The fishery that the technique is applied to is the Bowhead whale population of the Western Arctic. For the Alaskan Eskimo, whaling is an essential part of the culture, however, the whale population is also valued in and of itself, which has led to government imposed limits on the extent of whaling. This desire for a limit on whaling has conflicted with local interests and so the question of the appropriate limit has arisen. Conrad (1989) has constructed a model of Bowhead whaling which will serve as a good problem to use to illustrate the multiple shooting algorithm.[1] The model is expressed as a dynamic optimization problem over an infinite horizon, where both the stock of whales and the whale harvest are valued. The steady state policy is determined and is offered as an appropriate level of whaling. But because past harvesting has depleted the whale population, it is likely that the current whale population is below the desired steady state. In order to determine the optimal amount of whaling today, Conrad's work must be extended by determining the best way to move to the steady state. The paper solves

I would like to thank Lars Olson and the anonymous referees for helpful comments.
[1] Conrad also provides a short history of whaling in this region and a discussion of why whaling has been constrained by the regulatory limits in recent years.

for an approximate dynamic solution and finds that the short run optimal policy may be quite different from the long run policy, depending on the relationship between current and steady state stocks.

While this specific fishery provides a good application of the numerical techniques used in this paper, they can also be applied to other problems. Given that many regulated fisheries have populations below their steady state level (which is probably why they are regulated) the need for dynamic analysis is clear. It is hoped that these techniques may prove to be useful for determining the optimal regulation of other fisheries which are characterized by dynamic optimization and multi-argument welfare functions.

The paper proceeds as follows: Section II presents the model developed by Conrad and shows how the optimal steady state is determined. Section III presents an alternative structure for the welfare function that allows a non-linear relationship between stocks, harvesting and utility. Because of the form of the recruitment function, analytical solutions to the dynamic problem are unavailable, and we will be forced to use numerical methods to solve an approximate problem to determine the optimal dynamics. Section IV presents the solution algorithm. Section V applies the solution method to the problem of the Bowhead whales. The sensitivity of the solution to the various parameters, and the accuracy of the numerical procedure are discussed. The final section examines how these results might be implemented.

The Model and the Determination of the Steady State

The model developed in Conrad (1989) posits a social planner that maximizes a stream of discounted utility over an infinite horizon. Utility is determined by the size of the whale population (X_t) and the number of whales harvested (Y_t). Next year's stock of whales is equal to this year's stock, reduced by hunting and mortality, plus recruits to the population by birth. The recruitment term enters with a lag to capture the fact that it takes several years for whales to reach adulthood. Formally, the problem is,

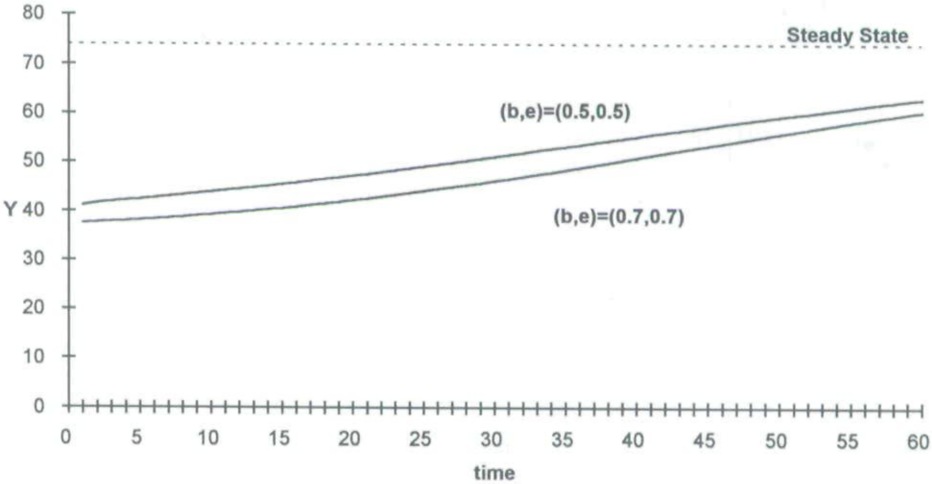

Figure 1. "Optimal path of harvesting."

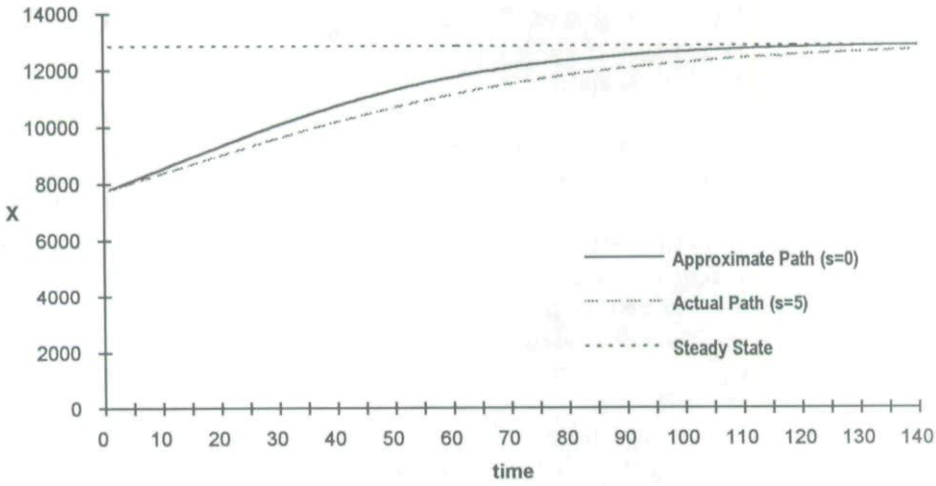

Figure 2. "Actual and approximate stocks."

$$\text{Max} \sum_{t=0}^{\infty} \rho^t W(X_t, Y_t)$$

subject to:

$$X_{t+1} = (1 - m)(X_t - Y_t) + F(X_{t-s} - Y_{t-s})$$

$$X_0, X_{-1}, \ldots, X_{-s}; \text{ given}$$

X_t: stock of whales at time t
Y_t: whale harvest at time t
W(•): welfare function
ρ: discount factor
m: mortality rate
s: lag between birth and recruitment
F(•): recruitment function[2]

In order to determine the steady state, Conrad approximates the value function around the steady state with the linear funtion, $W = \gamma X + \phi Y$. Given the parameter values of the recruitment function, the steady state solution depends on the values of the preference parameters: γ, ϕ, and ρ. ϕ was set equal to 1 and so γ was used to set the relative value between X and Y. Conrad determines the steady state for values of δ ($\delta = 1/\rho - 1$) and γ ranging from 0.00 to 0.05. The optimal steady state populations range from 0 to 14,785. The zero solution arises because the linear approximation does not work well when the population is away

[2] Following Conrad, the recruitment function takes the form $F(Z) = rZ(1 - (Z/k)^{\alpha})$; $Z = X - Y$. The parameter values are: $\alpha = 2.39$, $k = 25{,}000$, $s = 5$, and $r = 0.07$. The mortality rate (m) is 0.05.

from the steady state and near zero. It would be agreed by all the participants that this solution is not desirable.

In discussing the appropriate regulation Conrad provides two typical steady states: $X_{ss} = 7{,}298$ ($\delta = 0.03$, $\gamma = 0.03$) and $X_{ss} = 13{,}005$ ($\delta = 0.00$, $\gamma = 0.03$).[3] If the solution was the former the regulatory authority would be in a fortunate position. The current Bowhead whale population has been estimated to be about 7,800 (±2000), so that harvesting at the steady state level would be appropriate. However, if we adopt the second parameterization, the population is significantly below the desired level, and it is no longer sufficient to simply determine the steady state. In order to demonstrate the value of having a dynamic solution, the optimal dynamic policy will be determined for this steady state.

If the initial whale population is relatively low, the slow growth of the population ensures that for the near future we will be concerned with behavior away from the steady state. Using the growth function along with the estimated stock of whales, the time path of the whale stock can be calculated for a given path of whaling. If there were no whaling, it would take over 60 years for the population to reach the second steady state level. With whaling at the current regulated maximum of 35, the time would increase to over 85 years. At the steady state level of whaling of 71 the stock of whales asymptotically approaches the steady state, taking over 125 years to reach 95% of the steady state. With such slow growth of the population, time spent away from the steady state will be significant, so it will be necessary to solve for the optimal dynamics of the problem in order to determine the correct regulatory policy.

Welfare Functions

Linear welfare functions work well in determining behavior at or near the steady state, but away from the steady state they may produce inaccuracies or lead to corner solutions, implying optimal extinction. The recent recognition of the value of preserving species, particular marine mammals, has led to the hunting of these species being limited to cultural or research purposes. This will result in whales being valued in a non-linear manner. If whales are near extinction the value of an additional whale is much higher than if there is a thriving population. What is valued here is not so much a given whale, but the whale population as a whole. The value of an additional whale comes from increasing the survivability of the whale population. This justification of the welfare function contrasts with the common approach to valuing stocks in resource use. Commonly larger stocks are valued because they reduce fishing costs, so stocks are only valued to the extent that they support harvesting. The presumption of this paper is that these cost savings are less important than the non-use valuation of the whale stock and that the non-use value implies that the function should be non-linear. When the whale population is low the value function should be relatively steep, while being much flatter near the steady state. Overall, the welfare function will reflect both the

[3] Page 985. Conrad presents a number of steady states, the level of X and Y depending on the values of ρ and γ. The two steady states mentioned are typical in that they represent a range of solutions, and they are not pathological (e.g. they do not imply extinction). From a dynamics point of view, they also represent two extremes. The smaller steady state is close to the current population, while the larger state is quite distant.

value of whale harvesting to the hunters and the general value of a larger whale stock. To capture the non-linearities, a function of the following form will be used:

$$W(X,Y) = (a/b)(X - c)^b + (d/e)(Y - f)^e$$

This function allows variable marginal values of whaling and the whale stock, and also minimum values of each. The solution method allows for more complicated functions, so long as welfare this period only depends on the value of X and Y this period. Since there is no detailed information on the cost and benefits of whaling this general functional form will be used, treating it as a conglomerate function that includes both costs and benefits.

c sets the minimum value of X, so that the marginal value of the stock approaches infinity as the stock approaches the minimum level. b is used to determine the curvature of the function. a is used to set the relative value of the whale stock. d, e, and f serve the same roles for Y as a, b, and c do for X. The linear function can be reproduced by setting $b = e = 1$ and $c = f = 0$.

To illustrate the use of the multiple shooting algorithm, a parameterized version of the model will be solved. The minimum levels (c,f) will be specified conservatively. Since the current push is for increasing the rate of harvesting, the minimum level of whaling will be set at the current regulated level of 35. As reported in Conrad (1989), estimates of the whale stock below 2000 animals triggered concerns about extinction and a desire to ban whaling in the region. Again, to be conservative, the minimum whale stock will be set at 3,000. The minimum whale stock will not be terribly important since the current stock is significantly above this level and rising. The minimum level of whaling will be more important since some parameterizations imply whaling near this limit. The steady state consumption levels are determined by the marginal rate of substitution between stock and harvesting. To facilitate comparison of the results of this paper with Conrad, the values of a and d were chosen so that marginal utilities of the welfare function will equal the marginal utilities from Conrad at the specific steady state to be used in the simulation. Since the welfare function is separable, a can be determined from b, c and the specific steady state level of X, and d from e, f and the level of Y.[4] In this way, the tradeoff between stock and harvesting, at the steady state, will be the same for both papers. b and e determine the curvature of the welfare function and must be less than one for the marginal value to be decreasing in quantity. As b and e become smaller the marginal utility becomes higher (for stocks or harvesting below the steady state) but marginal utility diminishes faster (since marginal utility is the same at the steady state). As will be shown, changes in curvature will affect the solution by changing the relative value of the two goods when away from the steady state.

Solving a Dynamic Optimization Problem

The solution to the model is given by the Euler equations from the optimization problem:

[4] Since the values of a and d depend on b and e and the targeted steady state they will differ across simulations. The values used in section V are $(a,d) = (0.63,3.02)$ for $(b,e) = (0.7,0.7)$ and $(a,d) = (3.97,6.33)$ for $(b,e) = (0.5,0.5)$.

$$W_y(X_t,Y_t) = (1 - m)\rho\lambda_{t+1} + \rho^{s+1}\lambda_{t+s+1}F'(X_t - Y_t) \quad (1)$$

$$\lambda_t = W_x(X_t,Y_t) + (1 - m)\rho\lambda_{t+1} + \rho^{s+1}\lambda_{t+s+1}F'(X_t - Y_t) \quad (2)$$

$$X_{t+1} = (1 - m)(X_t - Y_t) + F(X_{t-s} - Y_{t-s}) \quad (3)$$

where λ is the Lagrangian multiplier associated with the law of motion of the whale stock (3). Using the first two equations λ can be written as a function of X and Y,

$$\lambda_t = W_y(X_t,Y_t) + W_x(X_t,Y_t)$$

The steady state can be determined by setting $\lambda_t = \lambda$, $X_t = X$ and $Y_t = Y$ for all t. Expressing both X and Y in terms of Z ($Z = X - Y$), (1) and (2) reduce to one equation which can be solved for Z. From Z, both X and Y can be found using (3).

For non-linear welfare functions, the dynamic solution to the equations cannot be solved for analytically, and will prove difficult to solve numerically.[5] A variety of numerical methods which might be used are ruled out by the large computational demands implied by the function. One could approximate the infinite horizon problem with a finite horizon problem that can be solved numerically. The solution to the system will be a finite number of Euler equations, with an equal number of variables, which can be solved by standard means. The length of the horizon will be the critical value in determining the accuracy of the approximation. Values that are too low will result in whaling levels that are too high. As T becomes large the quality of the approximation improves, but the solution costs increase. It will be seen that values of T around 100 to 150 would be appropriate. This implies a problem with 300 non-linear equations combined with 100 inequality constraints, and the terminal conditions. It is possible to solve these problems, but at a high computational cost.

Another approach is to use the optimality conditions at the steady state as an approximate solution when away from the steady state. Kolberg (1992) explores the accuracy of this kind of approximation. The first approximation is made using the method of "backward induction," which is very similar to the method of this paper. It uses a starting point (the steady state) and works backwards to find a path that meets the initial state. The advantage of multiple shooting (forward) is that the potential paths can be indexed by first period harvesting and can bound

[5] One can solve for the dynamics of the linear welfare function fairly easily. Replace λ with $(\gamma + \phi)$ in (1). This produces an equation in X and Y. Given an initial value of X one can solve for Y and then determine X one period ahead using (3). One can continue in this way and determine the entire time path of X and Y. To introduce the constraint that $Y \geq Ymin$ it is simplest to get the solution by solving the unconstrained problem each period, and if the value of Y is less than Ymin, set Y equal to the minimum and then update X. Given the linearity of the objective function, the solution will be a "most rapid approach path," where whaling will equal the minimal value until the stock of whales is at the steady state value. Using this objective function, with a minimum of 35 whales, the solution is to fish at the minimum level for about 85 years, and then fish at the steady state level from then on. This solution provides a limit to the problem as *b* and *e* approach 1.

the true path both above and below, and from this, the first period harvesting can also be bounded above and below. The backward induction path is taken by Kolberg as the best solution to the model and the other approximation methods are evaluated by how well they follow that path. His "first order approximation" will not work well in situations where current harvesting is far from steady state harvesting, as in the example of this paper. He finds that a "second order approximation" works quite well, although this method is restricted to cases where next period's stock can be written as a function of resource escapement alone. While this type of function is common (and is used in this paper), the multiple shooting method is more general and can be used when resource growth is a function of the stock and harvesting level independently.

The technique of "multiple shooting" is a trial and error method of solving difference or differential equations.[6] However, it is infeasible to use this method with large values of s (the period to adulthood) since search costs increase exponentially with $s + 1$. This section will show how to solve the problem with $s = 0$ and to use this method to approximate the actual solution. Theoretically, reducing the value of s implies that the population will respond faster to harvesting when below the steady state level. Section V provides numerical evidence on the size of this error. It will be found that the procedure works quite well, producing only a small overestimation of the optimal harvesting.[7] For the approximation, the law of motion of the whale stock will take the form,

$$X_{t+1} = (1 - m)(X_t - Y_t) + F(X_t - Y_t) \qquad (4)$$

A candidate time path for whaling can be constructed from the Euler equations of the optimization problem along with a guess of the first period level of whaling. The multiple shooting technique involves constructing many candidate solutions, rejecting solutions that prove to be unstable, which will place bounds on the true path. The objective function can be described in terms of the Y's and X's, or the X's alone by substituting the law of motion of the whale stock into the objective function. It will be more convenient to express the problem in terms of the X's, so the law of motion (4) is rewritten as

$$Y_t = h(X_t, X_{t+1})$$

This is then substituted into the objective function. Taking the part of the objective function than includes X_{t+1}, we get,

$$\ldots + \rho^t W(X_t, h(X_t, X_{t+1})) + \rho^{t+1} W(X_{t+1}, h(X_{t+1}, X_{t+2})) + \ldots$$

The Euler equation for X_{t+1} takes the form,

[6] This method is commonly used with differential equations [see, for example, Hildebrand (1987)]. Dow (1987) uses a discrete time analogue for solving a system of difference equations derived from an optimization problem.

[7] One could also reinterpret the time period to be a longer period of time, say five years, so that s equal to zero is a closer approximation. The reparameterized model could then be solved with Y represented the average level of harvesting over those years.

$$W_2(X_t,h(X_t,X_{t+1}))h_2(X_t,X_{t+1}) + \rho W_1(X_{t+1},h(X_{t+1},X_{t+2})) + \rho W_2(X_{t+1},h(X_{t+1},X_{t+2}))h_1(X_{t+1},X_{t+2}) = 0$$

or

$$g(X_t,X_{t+1},X_{t+2}) = 0$$

The solution to the problem is then a system of equations

$$g(X_0,X_1,X_2) = 0$$
$$g(X_1,X_2,X_3) = 0$$
$$g(X_2,X_3,X_4) = 0$$
$$\vdots$$
$$, X_0 \text{ given.}$$

Given an initial guess of the value of X_1 (equivalent to choosing Y_0) one can calculate X_2. Given X_1 and X_2, one can calculate X_3, and so on. In this manner the entire time path can be constructed from the initial guess of X_1. Each of these time paths associated with a guess of X_1 is a candidate solution. Candidate solutions can be rejected if they result in values of X that explode past the steady state (which means that the initial choice of X_1 was too high), or if X peaks and then declines towards zero (the initial choice of X_1 was too low). By continually rejecting solutions and choosing new values of X_1 one can place arbitrarily close upper and lower bounds on the true value. A value of X_1 between the bounds is chosen as the approximate value and the start of the approximate path. The values of Y can be recovered from the time path of X and the law of motion of X.

It is likely that analytical solutions to the $h(\bullet)$ and $g(\bullet)$ equations will not be available, as is true in this case, and so the values of these functions and their derivatives must be solved for numerically.

The Optimal Dynamic Policy

The multiple shooting technique will now be applied to the problem of section II. A slightly different steady state than the one discussed in section II will be used since ρ needs to be less than one to ensure that the welfare function is bounded. The parameter values ($\delta = 0.01$, $\gamma = 0.04$) will be substituted for ($\delta = 0.0$, $\gamma = 0.05$) which implies a steady state of X = 12,858 and Y = 75. These values are similar to the previous values of X = 13,005 and Y = 71.

Figure 1 presents the optimal paths for two different degrees of curvature of the utility function (the linear solution $(b,e) = (1.0,1.0)$ already having been discussed). The two curves are for (b,e) equal to (0.5,0.5) and (0.7,0.7). The horizontal line represents the steady state level of whaling. The pattern is for the level of whaling to initially start at a low level and to increase as the stock increases. Because of the curvature of the welfare function, the marginal values of the stock and harvest are higher the closer they are to their minimum levels. As b and e

become smaller this effect becomes more pronounced, so that an equal decrease in b and e increases the initial relative value of the good closest to its minimum. In this case, harvesting is closer to the minimum than the stock is, so that a decrease in b and e increases the relative importance of harvesting at levels below the steady state. Because of this, at (0.5,0.5) we see a higher level of harvesting and a slower movement of the stock to the steady state.

To evaluate the accuracy of this solution, we need to consider the quality of the approximation. The accuracy of the numerical procedure for solving a problem with s equal to zero is quite high. The initial level of whaling for this problem is within 0.1% of the true value. Formally, the difference between initial Y's which begin paths that diverge from the steady state in opposite directions is less than 0.1%. Since the chosen path is in-between these bounds, the actual error will be less. There will also be calculation errors in numerically solving the equations. To determine the effect of this, the fineness of the calculations was reduced significantly and it was found that this changed the answers by less than 1%.

Another potential source of error is the approximation of a problem with s equal to 5 by a problem with s equal to 0. The basic effect of this error is to underestimate the time it will take for the whale stock to grow early on, so that the procedure will overestimate the stock of whales. The size of the error will depend on the desired rate of growth of the whale stock. To determine the size of this error, the optimal whaling policy determined by the numerical procedure has been used to recalculate the path of the whale stock using the true recruitment function (3). This path and the approximate path are shown on figure 2. As we can see, the quality of the approximation is best at the beginning and the end, with the approximate path overestimating in the middle. The largest error is 5.8%, with the error near the steady state being much smaller, on the order of 1%. Thus, the predicted stock of whales will match the true stock of whales at the terminal date, but will overestimate the stock of whales during the intermediate years. The true solution to this problem would thus involve slightly less whaling initially to create a slightly larger stock. Again, this effect is small in percentage terms. The steady states between the two problems ($s = 0$ and $s = 5$) will also differ slightly, the steady state harvest for s equaling 5 being greater by about one whale. An ad-hoc correction for the overall approximation error would be to reduce the optimal level of whaling determined from the procedure by perhaps two whales.

Implementing the Optimal Policy

The benefit of having the optimal dynamic solution to the problem is to use it as guide for implementing policy. We have seen that the inclusion of dynamic considerations can make a significant difference in the short run policy. There are two major issues in the practical implementation of this algorithm. The first is translating the solution to the optimization problem into a form usable for policy. Even though numerical approximation methods were used to find an optimal solution, in some ways we have generated much more accuracy than we can use. For the steady state used in section IV (with $b = 0.5$), the procedure can reject paths with an initial level of whaling equal to 41.5 as being too high and those with an initial level of 41.4 as being too low. However, whaling strategies expressed in fractional whales represents spurious accuracy.

The second difficulty is that there is a fair amount of uncertainty in this problem, both about the initial stock of whales, and about the parameters of the recruitment and welfare functions. Using a deterministic optimization algorithm is implicitly approximating the problem by one where we have separated estimation and control [see Bertsekas (1976) for a discussion of sub-optimal control issues]. This separation implies a sequential strategy, where the problem is initially solved treating the estimates of the parameters as being certain. Each period the functions are re-estimated using any new information available, and the certainty problem is again solved. This approach implies that only the first periods of the calculated path will actually be used, as a new path will be determined in the future. Two types of errors are likely to result from this simplification. The first is that the fishing strategy is not chosen to optimize the collection of information about the parameters. By varying the harvest, we can learn about the nature of the recruitment function. However, the whaling population is unlikely to agree to large fluctuations in whaling harvest, so not much information would be obtained this way. The second is that the risk aversion of the participants in the fishery is ignored. Ad-hoc adjustments can be made for ignoring risk aversion if one is willing to specify the qualitative effects. The known results about uncertainty in the recruitment function can best be described as special cases [see Anderson and Sutinen (1984) for a review], but often seem to imply lower levels of fishing, for example, Reed (1979), Blanchard and Fischer (1989) [the implication of precautionary saving] and Nyarko and Olson (1991).

Given the various sorts of uncertainties in this problem any policy must be treated as a rough guide rather than an exact determination of the optimal harvest. The point of this paper is that since significant improvements can often be made in the quality of an approximate solution by moving from a steady state policy to a dynamic policy, tractable numerical methods will prove to be important. The paper has present one such method, multiple shooting, which is a convenient way of solving dynamic problems with non-linear welfare and recruitment functions.

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