Regulatory Mechanisms and Information Processing in Uncertain Fisheries

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Abstract We study the effects on fisherman decision processes of periodic (e.g., weekly) individual quotas. In the model, the fisherman must choose at the start of each week which of two grounds to fish on. The catch per week on each ground is a random variable and the fisherman does not know with certainty the parameters of the distribution of that variable. He does have estimates on each parameter and can improve these estimates by Bayesian updating. The choice of a fishing ground takes into account the expected catch on that ground and the expected improvement in information from fishing on that ground. Our study is concerned with the effect of weekly quotas on the joint production of information and fish. Various policy implications are discussed, and the results are compared with the policy analysis of Clark (1980) in the deterministic case. We show that the quota affects the value of information and that if quotas are transferable, then the quota may limit its own value.

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Introduction

Most fisheries are characterized by high levels of uncertainty in stock size and location of fish in a particular year. These uncertainties, coupled with environmental fluctuations, lead to catch rates for individual fishermen that are highly erratic. They also pose difficult management problems that somehow must be overcome in trying to deal with a particular fishery. Although management may aid in reducing uncertainty through surveys, satellite projections, and weather forecasting, a major reduction in uncertainty comes from the fishermen themselves. In particular, the process of fishing produces information about the location, size, and quality of the stock, as well as producing fish. This paper is concerned with the effect of quotas on the joint production of information and fish.

Mangel and Clark (1983) modeled uncertainty in fisheries by considering the search component of the fishing operation to be the most important stochastic consideration. This choice rests on a number of factors. First, the individual fisherman can do little about the uncertainties in weather or stock size and quality, whereas he or she can accomplish much in the way of locating fish. Second, in most cases, the fisherman is still fundamentally a hunter (e.g., tuna vessels may spend up to 80% of their time at sea looking for tuna). Mangel and Clark used decision analysis to determine the optimal allocation of fishing effort over time and space, assuming that the individual fishermen were profit maximizers and that the fishery was an open access one.

In the present paper, we study the question of uncertainty and fishermen's behavior in a fishery with catch quotas for individual fishermen. There are many reasons for studying the seasonal quota and its effect on the fishery. First, in a deterministic setting, the seasonal quota can promote optimal utilization of the resource (Clark, 1980), so that as a regulation tool quotas appear to be useful. Second, there is empirical evidence (Swierzbinski et al., 1981; McKay, 1980) that fishermen may set quotas for themselves. In some cases (Swierzbinski et al., 1981) the quota is apparently a seasonal income target level. The informational problems associated with a seasonal quota are similar to the ones studied by Mangel and Clark (1983). Fishing stops, in this case, either when the quota is reached or the season ends.

There is another kind of quota, however, that is also of interest. This is the periodic (e.g., weekly) quota. For example, in some fisheries (McKay, 1980) the quota is the result of a cooperative of fishermen trying to ensure equitable weekly compensations. Quotas may also be set by processors (Fletcher, 1982), who limit the amount of fish that they will buy from an individual fisherman.

In this paper, we will study the effects of periodic quotas on the joint production of information and fish. The effects of periodic quotas will be compared with the effects of seasonal quotas. We are most interested in the effect of the different kinds of quotas on information processing early in the season. As data are accumulated through the season, it becomes progressively easier to make good decisions. The most difficult decisions are the ones made early in the season, when there is a paucity of information.

There is an interesting analytical duality between profit maximization and quota regulation. In the profit maximizing (*PM*) case, the seasonal return (R_{PM}) to the fisherman takes the form

$$R_{PM} = p\tilde{B} - C(T) \tag{1}$$

where $\tilde{B}$ is the (uncertain) biomass of the harvest, p is the price per unit biomass, and $C(T)$ is the cost of operating for a season of length T. In the case of quotas, the return takes the form

$$R_Q = pB_Q - C(\tilde{T}_Q) \tag{2}$$

Here B_Q is the biomass of the quota and $\tilde{T}_Q$ is the (uncertain) time to achieve that quota. In Equation (1) the uncertainty affects the revenue term and the cost, since the operating time is a control variable chosen by the fisherman, but in Equation (2) the uncertainty affects mainly the cost term (if the quota is reached). Although the net revenue functions in Equations (1) and (2) have considerable symmetry, it turns out that the presence of the quota leads to considerable differences in results. The case of the periodic quota is completely covered here; some aspects of the seasonal quota are discussed. Clark (1985) also discusses how a seasonal quota may reduce the expected catch of a fisherman. For simplicity of exposition, we consider the case in which the fisherman makes repeated trips of a fixed duration, say one week, to one of two fishing grounds. Our analysis is primarily concerned with determining the effects of an imposed weekly quota on the information used by the fisherman to decide which ground to visit. In the next section we give a formulation of our model and discuss some of its aspects. In the third section we describe the results of simulation studies using the model. The fourth section contains a discussion of some of the policy implications of our results.

Theoretical Formulations

This section contains two subsections. The first subsection contains a review of the model of Mangel and Clark (1983) as well as the essential theory of this paper. The second subsection contains a discussion of how the theory developed here can be extended to many cooperating fishermen. Mangel and Clark (1983) provide further discussion of the situation in which many fishermen cooperate.

Weekly Quotas, Seasonal Quotas, and a Single Fisherman

This section contains the necessary theory to compare regulation by weekly quotas with fishing under no imposed limitation or seasonal quotas. We assume that the fisherman is risk-neutral and maximizes expected profits. Other utility choices, such as risk aversion, may be treated in a straightforward way. The model for fishing under no limits is the simpler of the two cases; it was formulated by Mangel and Clark (1983). For the study of the joint production of information and fish, the difference between the profit maximizing case and the seasonal quota is simply the length of the season. Clearly, if no kind of limit is imposed and stock depletion is not important, the fisherman can always do better when there is no quota than if a limit is imposed. For a single fisherman, it is possible to ignore stock depletion. This allows us to concentrate completely on the effect that the quota has on information processing. Some aspects of stock depletion are discussed in the next section.

Assume that the schools of fish are encountered in a way that may be modeled as a Poisson process with parameter λ. Therefore the probability ($\Pr\{n, \lambda\}$) of encountering n schools in a period of length Δ is given by

$$\Pr\{n, \lambda\} = \frac{(\lambda\Delta)^n}{n!} e^{-\lambda\Delta} \tag{3}$$

Using the Poisson model ignores stock depletion effects in subsequent periods. Mangel and Clark (1983) show how to include depletion; the calculations are possible but more complicated (also see the following section).

In this model, the expected catch during a period Δ is equal to $\lambda\Delta$. The parameter λ is assumed to be a random variable whose value depends on which of the two grounds the fisherman is on. We assume that λ has a gamma distribution so that it forms a conjugate family with the Poisson fishing process (De Groot, 1970). That is, we assume

$$\Pr\{\lambda \epsilon (\hat{\lambda}, \hat{\lambda} + d\lambda)\} = \frac{\alpha^\nu \hat{\lambda}^{\nu-1}}{\Gamma(\nu)} e^{-\alpha\hat{\lambda}} \, d\lambda \tag{4}$$

where the values of α and ν depend on the fishing ground. Given parameters ν and α, the expected value of λ is ν/α. Using Bayesian estimation (DeGroot, 1970), if the prior distribution on λ has parameters ν and α and n fish are caught in a time interval Δ, then the posterior distribution has parameters $\nu + n$ and $\alpha + \Delta$.

Take $\Delta = 1$ week as the basic time period. If the fisherman only plans to fish for one more week during the season, he would simply choose the fishing ground that maximizes his expected profit. The expected gross income is the expected catch times the unit price of the fish, which is assumed to be fixed. We neglect fixed costs in computing costs (while there certainly are fixed costs, it is reasonable to assume that they do not significantly depend on the choice of fishing ground and therefore do not influence that decision). We assume that the variable costs are proportional to the amount of time spent fishing. Let $c\Delta$ denote the cost for one week's fishing. The expected profit is given by $p(\nu/\alpha) - c\Delta$ where p is the unit price of the catch, so if J_1 is the maximum expected profit with one week remaining, the fisherman will choose the fishing ground according to the rule

$$J_1 = \max_{i=1,2} \{p(\nu_i/\alpha_i) - c\Delta\} \tag{5}$$

Here ν_i and α_i are the current estimates of ν and α on ground i. At the start of the season these values are presumed to be known from historical data. They are updated as information is obtained.

If more than one week remains, the fisherman may decide to visit a ground other than the one specified by the rule in Equation (5). He will do this if the expected gain of information is more valuable in the long run than the opportunity cost of not visiting the most profitable ground right away.

As in Mangel and Clark (1983), we assume the fisherman follows a "myopic Bayes" strategy in which he only anticipates events one week into the future. The myopic Bayes solution for the dynamic programming problem is chosen for two reasons. First, it simplifies the computational considerations and avoids problems associated with the "curse of dimensionality." Second, and more important, it allows one to see clearly the joint production of catch and information and how they are related. The choice of fishing ground is therefore determined by maximizing expected profit over a two-week interval, taking into account the information gained in the first week of the interval. The use of a myopic Bayes procedure may underestimate the value of information, since there is only one week in which to exploit it. (An interesting related question concerns how much the myopic Bayes procedure underestimates the value of information. That is, how suboptimal is the myopic Bayes procedure? In particular, one would like to know how much the value of information is underestimated in the case of weekly versus seasonal quotas. In order to answer this question, one would have to solve the full dynamic programming equations and compare them with the solutions of the myopic Bayes equations. Based on heuristic arguments, one would expect that the value of information is underestimated more in the case of a weekly quota than in the case of a seasonal quota. In the case of a weekly quota, the catch is bounded in the week in which the information has value.)

The value of the maximum expected profit and the corresponding choice of fishing ground may be obtained by solving the appropriate two-period dynamic programming problem. The Bellman equation for this problem is

$$J = \max_{i=1,2} \{p(\nu_i/\alpha_i) - c\Delta + E\{J_2 \mid i\}\} \tag{6}$$

where $E\{J_2 \mid i\}$ is the expected second period profit given that the fisherman's ground i was visited in the first period. The value of $E\{J_2 \mid i\}$ is calculated from the formula

$$E\{J_2 \mid i\} = \sum_{n_i=0}^{\infty} P_{n_i} \Pr\{n_i\} \tag{7}$$

where P_{n_i} is the expected profit conditioned on finding n schools in ground i, given by

$$P_{n_i} = \max_{i,j} \left\{ p \frac{\nu_i + n_i}{\alpha_i + \Delta} - c\Delta, \quad p \frac{\nu_j}{\alpha_j} - c\Delta \right\} \tag{8}$$

and $\Pr\{n_i\}$ is the probability of finding n schools on ground i, given by Mangel and Clark (1983) as:

$$\Pr\{n_i\} = \frac{\Gamma(\nu_i + n_i)}{n_i!\Gamma(\nu_i)} \left(\frac{\alpha_i}{\alpha_i + \Delta}\right)^{\nu_i} \left(\frac{1}{\alpha_i + \Delta}\right)^{n_i} \tag{9}$$

It is worth noting that the dynamic programming formulation does not determine which ground will be visited in the second period; this depends on the outcome of the events of the first period. Further details for the profit maximizing case can be found in the paper of Mangel and Clark (1983), which also discusses some aspects of the myopic Bayes approach.

In order to derive analogous equations for the case of a weekly quota of Q schools, it is easiest to separate the decision action—that is, which ground to visit first—and the value of that action. Let V_i be the value of visiting ground i in the first period. This term involves the expected net return from the first period and the updated expected net return from the second period.

When each period has a quota Q, the possible events are finding 0 to Q schools; so that the distribution in Equation (3) must be renormalized to sum to 1 as n goes between 0 and Q. Let

$$C_N(\lambda, Q) = \frac{1}{\Sigma_{m=0}^{Q} \Pr\{m, \lambda\}} \tag{10}$$

be this normalization constant.

Assume that ground i is visited in the first period. From Equations (3) and (10), the expected net income r_1 from the first period's visit is

$$r_1(\lambda_i) = C_N(\lambda_i, Q) \sum_{n=0}^{Q} R_1(n, \lambda_i)\Pr\{n, \lambda_i\} \tag{11}$$

where $R_1(n, \lambda_i)$ is the expected net revenue in the first week if n schools are caught. If $n < Q$ then we have simply

$$R_1(n, \lambda_i) = pn - c\Delta \tag{12}$$

That is, under the assumptions of this model, if less than Q schools are caught then the entire week is spent on the ground.

The situation is different, however, if $n = Q$—that is, if the quota can be filled at any time in the week. Thus one must consider the density for the time t at which the quota is filled. The density for the time at which the Qth event of a Poisson process occurs is a gamma density with parameters Q and λ (Ross, 1980). Since T is constrained to be in $[0, \Delta]$, the density for t is, for fixed λ,

$$h(t; Q, \lambda_i) = \begin{cases} \dfrac{t^{Q-1}e^{-\lambda_i t}}{\displaystyle\int_0^{\Delta} t^{Q-1}e^{-\lambda_i t}\,dt} & 0 \le t \le \Delta \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

Therefore

$$R_1(Q, \lambda_i) = \int_0^{\Delta} (pQ - ct)h(t; Q, \lambda_i)\,dt \tag{14}$$

To compute the expected profit in the second period, assume for definiteness that ground 1 is visited in the first period. If $n < Q$ schools are found in the first period, then the posterior distribution of λ_1 has parameters $\nu_1 + n$ and $\alpha_1 + \Delta$. If $n = Q$ schools are found in the first period, then the posterior distribution of λ_1 has parameters $\nu_1 + Q$ and $\alpha_1 + t_1$, with probability density $h(t_1; Q, \lambda_1)$.

The expected revenue in the second week is

$$r_2(\lambda_1) = C_N(\lambda_1, Q)\sum_{n=0}^{Q} R_2(n, \lambda_1)\Pr\{n, \lambda_1\} \tag{15}$$

where $R_2(n, \lambda_1)$ is the expected revenue in the second week given that n fish were caught on ground 1 in week 1. Both R_1 and R_2 depend on the values of ν_1, ν_2, α_1, and α_2, but for clarity this dependence is suppressed. As in the case of fishing under no limit, the updated expectation of return on ground 1 is compared to the expected return from ground 2. Let $E_{\lambda}^{(\nu,\alpha)}(\cdot)$ denote the expectation over λ using a gamma density with parameters ν and α. For $n < Q$ schools caught on ground 1 in week 1 the updated parameters on ground 1 have values $\nu_1 + n$, $\alpha_1 + \Delta$. Therefore

$$R_2(n, \lambda_1) = \max\{E_{\lambda_1}^{(\nu_1+n,\alpha_1+\Delta)}(r_1(\lambda_1)); E_{\lambda_2}^{(\nu_2,\alpha_2)}(r_1(\lambda_2))\} \tag{16}$$

For $n = Q$ we must take the expectation over t for $0 \leq t \leq \Delta$, as in the first period. This yields

$$R_2(Q, \lambda_1) = \int_0^{\Delta} h(t; Q, \lambda_1) \max\{E_{\lambda_1}^{(\nu_1+Q,\alpha_1+t)}\, r_1(\lambda_1), \\ E_{\lambda_2}^{(\nu_2,\alpha_2)}\, r_1(\lambda_2)\}\, dt \qquad (17)$$

In this way we obtain $r_1(\lambda_1)$ and $r_2(\lambda_1)$. Both of these quantities are conditional on ν_1 and α_1 having particular values. Therefore, to obtain V_1, the expected value of return from fishing on ground 1 during the first period, we must take a final expectation over λ_1. Thus

$$V_1 = E_{\lambda_1}^{(\nu_1,\alpha_1)}\,(r_1(\lambda_1) + r_2(\lambda_1)) \qquad (18)$$

The value V_2 is defined analogously. If $V_i > V_j$, then ground i is visited in the first period. The extension of this kind of analysis from the myopic Bayes strategy to the full dynamic programming problem is straightforward, although the computations become burdensome very quickly.

In Appendix 1, we discuss certain other aspects of the formulation and interpretation of the dynamic programming problems. In particular, observe that while the value function in Equation (18) involves the true value of λ, the dynamic programming Equation (6) does not. In Appendix 1, we discuss this difference in more detail.

Behavior of Many Fishermen with Seasonal Quotas

Mangel and Clark (1983) showed that the advantage to fishermen of cooperating and sharing information may be considerable for the case of profit maximizing behavior. In this section, we consider the analogous question for the case of seasonal quotas.

Imagine N fishermen, each with a seasonal quota of Q schools. Rather than fishing alone, these N fishermen may cooperate by pooling their quotas together to form an aggregate quota NQ. They will then fish until the aggregate quota is filled.

There may be two reasons for cooperation. First, the time to achieve the total quota NQ may be less than the time needed to fill a single quota. Second, cooperation may reduce uncertainty about the time to fill the quota. Assume that $\lambda(N)$ is the rate at which N individuals find schools of fish and let T_{NQ} be the time that it takes N individuals to achieve the quota Q per individual.

First, let us ignore depletion of the stock. In Appendix 2, we show that the mean $E\{T_{NQ}\}$ and variance $\mathrm{Var}\{T_{NQ}\}$ of T_{NQ} are given by

$$\begin{aligned} E\{T_{NQ}\} &= \frac{NQ}{\lambda(N)} \\ \mathrm{Var}\{T_{NQ}\} &= \frac{NQ}{\lambda(N)^2} \end{aligned} \tag{19}$$

Equation (19) is derived by using the assumption that encounters with schools of fish are a Poisson process with parameter $\lambda(N)$. If the fishermen search independently, then $\lambda(N) = N\lambda_1$, where λ_1 is the search rate for an individual fisherman. In this case, Equation (19) becomes

$$\begin{aligned} E\{T_{NQ}\} &= \frac{Q}{\lambda_1} \\ \mathrm{Var}\{T_{NQ}\} &= \frac{Q}{N\lambda_1^2} \end{aligned} \tag{20}$$

Equation (20) shows that on the average, there is no advantage to cooperation, since the mean time for N fishermen to achieve the quota NQ is the same as the mean time for an individual to achieve the seasonal quota of Q schools. The advantage of co-operation, on the other hand, comes from a reduction in the uncertainty about the time to fill the quota. This reduction of uncertainty was also noted by Clark and Mangel (1983) in a different context.

When many fishermen are present, it seems reasonable to assume that depletion of the stock should be considered when one is modeling the discovery and search rates. Following Mangel

Table 1
Value of Information Sharing in an Uncertain Fishery Regulated by Quota

Q	δ	N	$E\{T\}$	$\mathrm{Var}\{T\}$
20	.04	1	38.3	88.8
		2	39.3	47.1
		3	39.6	32.0
		4	39.7	24.3
		5	39.8	19.5
20	.01	1	22.23	24.72
		2	22.25	12.43
		3	22.27	8.30
		4	22.28	6.23
		5	22.29	4.99
10	.06	1	14.5	22.5
		2	14.9	11.9
		3	15.0	8.0
		4	15.1	6.1
		5	15.1	4.9

and Clark (1983), this can be done by assuming that each encounter reduces $\lambda(N)$ by a fixed amount δ. Thus after k schools have been caught, the encounter rate is $\lambda(N) - k\delta$. In Appendix 2, we show that for this case

$$
\begin{aligned}
E\{T_{NQ}\} &= \sum_{k=0}^{NQ-1} \frac{1}{\lambda(N) - k\delta} \\
\mathrm{Var}\{T_{NQ}\} &= \sum_{k=0}^{NQ-1} \frac{1}{(\lambda(N) - k\delta)^2}
\end{aligned}
\tag{21}
$$

Table 1 shows results of calculations using Equation (21) with $\lambda(N) = N\lambda_1$. Once again, the reduction in uncertainty over the time to achieve the quota is apparent.

Mangel and Clark (1983) also considered the multiperiod problem for a number of profit maximizing fishermen. The analysis, which must include depletion, becomes exceedingly complex.

Consequently, the case of many periods for fishermen with quotas is not considered here.

Numerical Comparison of Seasonal and Weekly Quotas for Fishermen

There are essentially three cases of potential interest. They are (1) the fisherman who maximizes profit, (2) the fisherman with a seasonal quota, and (3) the fisherman with a periodic—say, weekly—quota. To compare these we consider the simplest case of a fisherman who each week must choose between one of two fishing grounds for a season of M weeks. In light of the theoretical development of the previous section, we will actually consider separate two-week intervals in order to assess the value and impact of information on the fisherman's behavior.

In the Monte Carlo simulation, a single fisherman fishes for a single season during each simulation run. The season consists of 30 weeks, and in both cases (seasonal quota SQ or weekly quota WQ) there is a maximum allowed catch of 10 schools, so that fishing ends when 10 schools are caught. In the WQ case there also is a weekly quota of two schools.

The input data for the simulation are the parameters Q, p, c, ν_1, α_1, ν_2, and α_2 of Equation (4). At the start of each simulated season, true values λ_1^T, $i = 1, 2$, are selected at random from a gamma distribution with parameters ν_i and α_i. These values λ_i^T define the true values of each ground i during the entire season. On each ground the times between encounters with a school of fish are exponentially distributed, with parameter λ_i^T.

Consider first the case of a seasonal quota. At the start of the season the fisherman solves Equation (6) to determine the ground to fish initially. He fishes on ground i for the entire week. Suppose the catch is m schools of fish during that week. At the end of the period the parameters ν_i and α_i are updated to $\nu_i + m$ and $\alpha_i + 1$, leaving the values of ν_j and α_j unchanged. This new set of parameters is then used in Equation (6) to determine which ground to fish on in the next week. This process updating after each week continues until 10 schools are caught or 30 weeks elapse.

In the case of weekly quotas a similar procedure is used. Equation (18) replaces Equation (6). In each week the fisherman stays on the ground until he or she catches Q schools or the week ends. The updated parameters are either $\nu_i + Q$ and $\alpha_i + t$ (if the quota is caught) or $\nu_i + m$ and $\alpha_i + 1$ (if $m < Q$ schools are caught).

Before describing the results of the simulation, let us discuss the properties of the solutions of Equations (6) and (18). For purposes of comparison, all the parameter values except α_2 are fixed. The fixed values are $\nu_1 = 2$, $\alpha_1 = 1$, $\nu_2 = 4$, $Q = 2$, $p = 1$, and $c = 0.5$. Figure 1 shows the expected return (E_i) for the two-week period as a function of the expected catch rate on ground 2, $R_2 = \nu_2/\alpha_2$. The lines marked 1 or 2 refer to visiting ground 1 or 2 in the first week. The ground visited in the second week is determined in the solution of the dynamic programming equation.

The value of information as a function of R_2 can be computed in the following way. The E_i values shown in Figure 1 are the expected return if ground i is visited in the first week; the information obtained is then used to determine which ground is visited in the second week by Bayesian updating. For a fixed value of R_2, the fisherman chooses the ground for which E_i is larger.

If the fisherman does not update at all, then he or she will visit in both weeks the ground that has the higher prior expected return. The prior expected return on ground i is ν_i/α_i (the mean of the gamma distribution). Thus the value of information (VOI) is computed by

$$\text{VOI} = \max_i \, (E_i) - \max_i \, 2[\nu_i/\alpha_i - c\Delta] \tag{22}$$

Figure 1 can thus be used to calculate the value of information in either case of seasonal quotas or weekly quotas.

Consider, for example, the SQ case in which $R_2 = 1$. In such a case, ground 1 is a priori better, and the prior expected return for the two-week period, assuming that ground 1 is visited in both periods, is $J = 2 \times 2 - 2(0.5) = 3$. From the curve marked

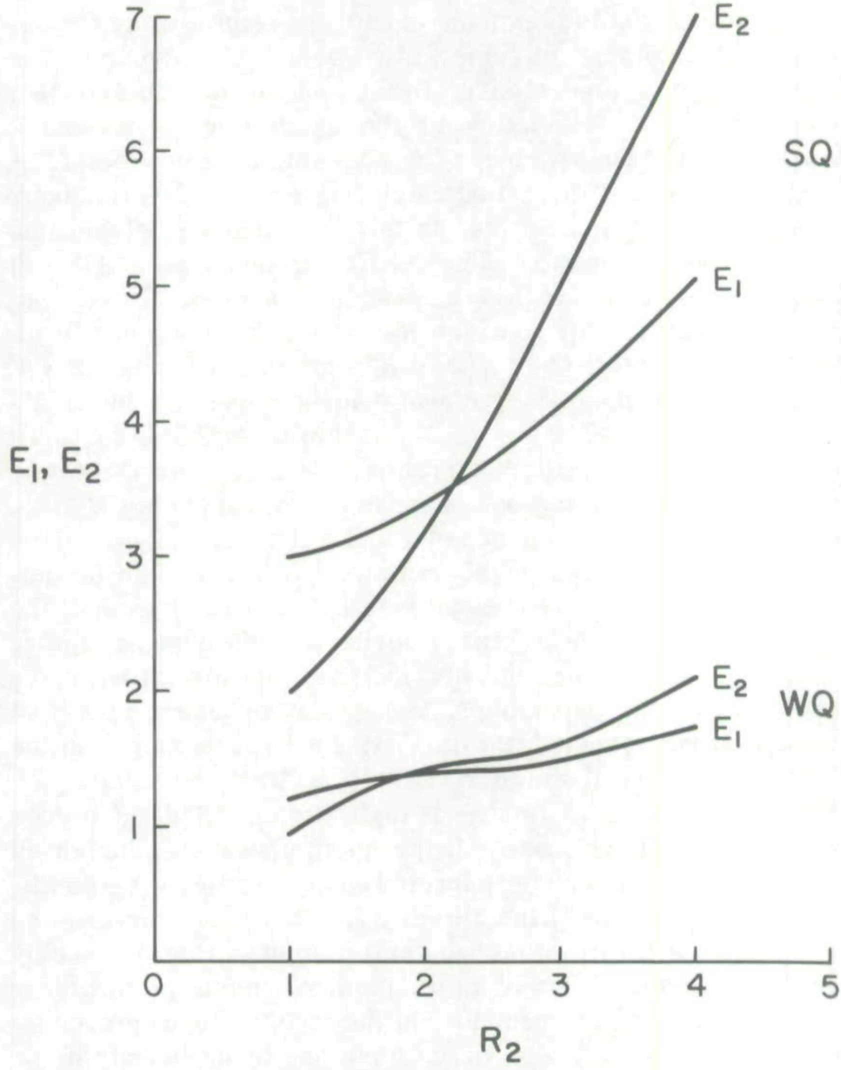

FIGURE 1. Comparison of the expected returns for the two-week fishing period as a function of the catch rate on the second ground and the choice of ground visited in the first period. The catch rate on the first ground is two schools per week. The value of each school is $p = 1$ and the operating cost is $c = 0.5$ per unit time.

E_1, it is seen that the optimal value of the return, using the dynamic programming solution, is also about 3. Consequently, the value of information is small. To understand this, observe that the coefficient of variation of the catch rate on ground 2 is $1/\sqrt{\nu_2} = 0.5$. Thus when R_2, the mean catch rate on ground 2, is 1, the chance that the actual catch rate exceeds 2 is relatively small (in fact, it is about .04). In this case, there is only a small chance that information obtained from probing ground 1 will improve the catch over the deterministic solution. Next, consider the case for SQ in which $R_2 = 2$. There the value of information is larger. The value of E_1 when $R_2 = 2$ is about 3.4, so that VOI $= 0.4$. Observe also that the curves E_1 and E_2 do not cross at $R_2 = 2$, but at $R_2 > 2$ (about $R_2 = 2.3$, in fact). To understand this, recall that ground 1 is more uncertain than ground 2, so that the grounds are not identical even if $R_2 = 2$ (i.e., there is a larger chance of a high catch rate on ground 1).

Similar calculations of the value of information can be performed for the WQ case, using Figure 1. For each ground, the expected return in the SQ case is higher than the expected return in the WQ case. As the value of Q increases, the lower two curves rise and join the upper ones. The quota appears to have two effects on the expected return. First, the expected catch in the WQ case is lower than the SQ case; it is clearly bounded by Q. Thus when the catch rate R_2 is high, the expected catch does not rise accordingly. Second, the quota causes a reduction in the difference between the expected catch from the two grounds. Since both the size of the curves E_1, E_2 and the difference $E_1 - E_2$ are a measure of information, it appears that the weekly quota affects the quality of information. When interpreting these results, one should remember that the myopic Bayes procedure underestimates the value of information. In particular, if the value of information is underestimated more in the case of weekly quotas than in the case of seasonal quotas, then the results presented here can be viewed as an upper bound for the difference between the two.

It is important to remember that Figure 1 pertains to the value of information at the start of the first period of fishing. The value of information at the end of the first period may be quite different. For example, if the entire seasonal quota is caught in the first period, then any information from that period is valueless. For a multiweek season, one would expect that higher catches in the early part of the season (under a seasonal quota) will mean that information obtained later in the season has less value.

In addition to the standard value of information, it is helpful to consider two other measures in studying how the quota affects the processing of information. The first of these is the function φ_i, defined by

$$\varphi_i = \max\left[\begin{array}{c}\text{expected return on ground } i \text{ in the second}\\ \text{period using updated parameters}\\ -\text{ expected return on ground } i \text{ on the first}\\ \text{period using prior parameters, } 0\end{array}\right] \tag{23}$$

This function φ_i has the property that it is nonzero only when the posterior expected catch on ground i (i.e., after one week of fishing) is larger than the prior expected catch on ground i. That is, φ_i will be positive only if ground i looks better after one week of fishing than it did before fishing. Figure 2 shows φ_1 and φ_2 for both cases. The solid line indicates φ_i when the ground is the superior of the two grounds (as determined from the solution of the dynamic programming equation). The dashed line indicates φ_i when the ground is the inferior of the two grounds. Observe that in the *WQ* case the value of φ_i is positive only when the ground is suboptimal. This is somewhat surprising, since it shows that the posterior expected catch seems to be reduced by the presence of the quota (cf. Mangel and Clark, 1983). Clark (1985) also observed this phenomenon.

In order to further understand what is happening here, let us consider the posterior expectations explicitly. For the *WQ* case, if the quota were met in time t, then the posterior expectation is

$$E_{WQ} = \frac{\nu + Q}{\alpha + t} \tag{24}$$

For the *SQ* case, if the seasonal quota is not met in the first

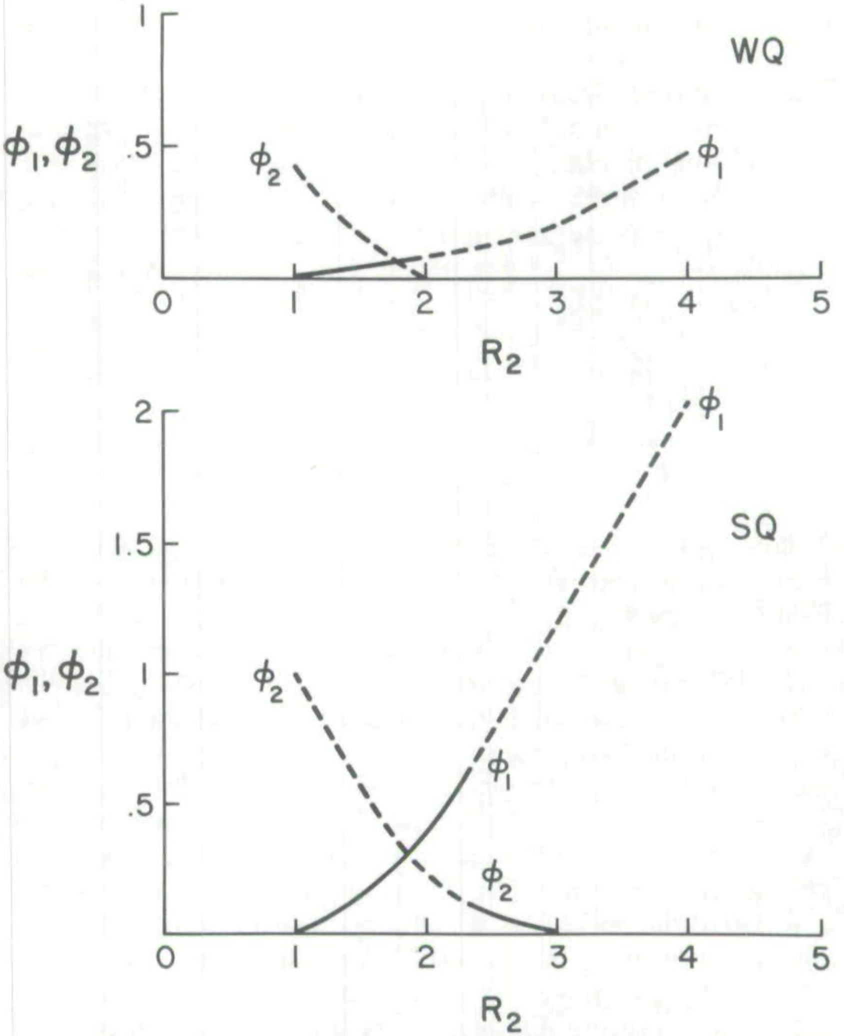

FIGURE 2. The function φ_i, characterizing the expected posterior and prior catch rates on each ground, as a function of R_2.

week, then the expected catch in the second week, if n schools are caught in the first week, is

$$E_{SQ} = \min\left[Q_s - n, \frac{\nu + n}{\alpha + 1}\right] \tag{25}$$

Here Q_s is the seasonal quota.

When considering these equations, observe the following. For the WQ case, if the fish stock is very large (i.e., if the catch rate is very high) the quota will be met quickly. From Equation (24),

$$\lim_{t \to 0} E_{WQ} = \frac{\nu + Q}{\alpha} \tag{26}$$

Even if the fisherman found the quota "instantaneously," the posterior expectation is bounded by $(\nu + Q)/\alpha$. On the other hand, $(\nu + n)/(\alpha + 1)$ is unbounded as $n \to \infty$. Consequently, there is no bound (other than the seasonal quota) to the posterior expectation. In general, the posterior expected catch, given that n schools were found in time t, is

$$E = \frac{\nu + n}{\alpha + t} \tag{27}$$

Our results show that n and t in Equation (27) have different and nonexchangeable roles; there is an asymmetry in the dependence upon n and t. One question that cannot be answered is whether this difference is a quirk of the model or a general phenomenon. Clark (1985), using a somewhat different model, observed a similar phenomenon.

To highlight the different roles played by n and t in Equation (27), consider the SQ case in which n schools are caught in the first week. Then Equation (27) is the same as (25). We now ask, how quickly would the fisherman have to catch the weekly quota to have the same posterior expected catch? Although the economic return on the second week is not solely determined by the expected catch, the information obtained from the first week's fishing is succinctly summarized in terms of this expectation. Let $t_Q(n)$ be the time in which the quota must be met to have the same posterior expectation as when n schools were caught in one week. In Appendix 3, we show that $t_Q(n)$ is given by

$$t_Q(n) = \left(\frac{\nu + Q}{\nu + n}\right)(\alpha + 1) - \alpha \tag{28}$$

Observe that if n is large enough, $t_Q(n)$ is *negative*; this simply shows that it is impossible for the fisherman to meet the weekly quota quickly enough to match Equation (25). The second additional measure that we introduce is an elasticity in the posterior expectation. That is, suppose first that $t_Q(n) > 0$. If n is changed by an amount Δn, then $t_Q(n)$ is decreased. In Appendix 3, we show that the change in $t_Q(n)$ that keeps the posterior expectations the same is

$$\Delta t_Q(n) = -\frac{(\nu + Q)}{(\nu + n)^2}(\alpha + 1)\Delta n \tag{29}$$

The changes Δn and $\Delta t_Q(n)$ cause corresponding changes in the posterior returns E_{WQ} and E_{SQ}. We shall denote these changes by ΔE_{WQ} and ΔE_{SQ}. We now define elasticities ϵ_{SQ} and ϵ_{WQ} by

$$\begin{aligned} \epsilon_{SQ} &= \frac{\Delta E_{SQ}}{E_{SQ}} \\ \epsilon_{WQ} &= \frac{\Delta E_{WQ}}{E_{WQ}} \end{aligned} \tag{30}$$

In Appendix 3, we derive explicit formulas for these elasticities.

Table 2 shows these elasticities for a range of parameter values. Observe that ϵ_{WQ} is always less than ϵ_{SQ}; the quota limits the proportional change in the expected return. This limit can be considerable.

Having discussed the Bellman equations, we now turn to the simulation results. Each simulation run proceeds as follows. The

Table 2
Elasticities in Posterior Expected Catch*

ν	α	n	$t_Q(n)$	ϵ_{WQ}	ϵ_{SQ}
2	1	3	.6	.023	.038
		4	.33	.019	.025
		5	.14	.011	.013
4	1	2	1	.025	.050
		3	.71	.024	.042
		4	.5	.022	.033
		5	.33	.019	.025
		6	.2	.014	.017
4	2	2	1	.011	.033
		3	.57	.009	.022
		4	.25	.005	.011

* Parameters: $Q = 2$.

season lasts for 30 weeks, with a seasonal quota of 10 schools. At the start of each two-week period, in the *SQ* case the fisherman solves the dynamic programming Equation (6). The solution indicates which of the two grounds should be fished in the current week. An alternate formulation of Equation (6), which is less myopic, proceeds as follows. If there are $\hat{m}$ weeks left in the season, the first period in Equation (6) has a length of one week, and the second period has a length of $\hat{m} - 1$ weeks. Such a formulation emphasizes the value of information more than one in which each period has the same length. Once the seasonal quota is met (or the season ends, if that happens first), the simulation run ends.

In the *WQ* case, each simulation run is essentially the same. The differences are (1) that Equation (18) replaces Equation (6), and (2) if the weekly quota is met in any week, then fishing stops.

In all runs, the average catch rate on ground 1 was set equal to two schools per week with a coefficient of variation of $1/\sqrt{2}$ (71%). The catch rate of the second ground was more certain, with a coefficient of variation of 50%; three different values for the average catch rate (four, two, and one schools per week) on the second ground were used. Price per school p and variable

Table 3
Results of the Simulation

Case	(ν_2, α_2)	$\overline{P}$	CV_p	f_2	f_s
SQ	(4, 1)	8.15	.13	.91	0
WQ	(4, 1)	7.95	.16	.80	0
SQ	(4, 2)	7.67	.13	.58	0
WQ	(4, 2)	7.29	.16	.65	0
SQ	(4, 4)	4.60	1.06	.33	.068
WQ	(4, 4)	4.33	1.05	.60	.06

operating cost c were held constant at $p = 1$ and $c = 0.5$, respectively.

The following values are computed: the average profit $\overline{P}$, coefficient of variation of profit CV_p, the fraction of time spent on ground 2 f_2, and the fraction of simulations where the seasonal quota was not met f_s. The results are shown in Table 3.

Note that f_2, the fraction of time spent on ground 2, follows the results of the dynamic programming calculations (Figures 1 and 2). The decrease in f_2 as R_2 changes is much more dramatic in the *SQ* case than in the *WQ* case, which reflects the fact that the difference between E_1 and E_2 is greater (i.e., the value of ϕ_i is greater) in the *SQ* case.

The results of the simulation indicate that if only variable costs are considered, the difference between seasonal and weekly quotas is not large. If there is a weekly fixed cost, then the difference between the seasonal and weekly quota will rise accordingly, since the fishing season with a seasonal quota will end sooner. For example, if a weekly fixed cost of 0.5 units is included for the case of $R_2 = 2$, the expected profit is 2.37 in the *SQ* case and 1.35 in the *WQ* case. Part of the reason for the closeness may be the following. Suppose that the average weekly catch is the quota Q. The seasonal quota allows the fisherman to "catch up" in later weeks if his catch falls below Q.

Policy Implications

Clark (1980) considered four kinds of regulations in a deterministic model of the fishery. They are:

1. Total catch quotas.
2. Vessel licenses.
3. Taxes on catch/effort.
4. Allocated catch/effort quotas.

Clark found that total catch quotas were not effective instruments for economic control and that vessel licenses could not serve the purpose of economic optimization. For the deterministic case, he found that taxes and allocated quotas were theoretically equivalent and that both could serve the role of optimizing exploitation of a common-property fishery. The models he used were deterministic and without uncertain parameters.

Clark (1980) shows that catch taxes and allocated transferable vessel quotas are mathematically equivalent in their effect on effort use. He explains this equivalence by noting that quotas acquire a scarcity value and that a market for quotas develops. A fisherman not selling the quota faces an opportunity cost, which in the deterministic setting is the analytical equivalent of a tax on his catch. Although Clark's work pertains to seasonal quotas, it is worthwhile to consider the case of periodic quotas, transferrable among fishermen within weeks but not across weeks. In such a case, the results presented in this paper show that the quota may limit its own value. Namely, the transferable quota becomes valuable when the stock size or catch rate is high. If the only information about stock size is provided by fishing, then the quota limits the catch and thus the posterior estimation of catch rate, as in Equation (24). Consequently, the stock size and catch rate may be underestimated with the weekly quota, and the quota may then lead to less weekly effort than the case in which there is only a seasonal quota. If there is a market for quotas, then weekly quotas may tend to be undervalued. In this case, the weekly quota will ultimately cause a lower net return in the fishery than a seasonal quota.

The ultimate optimality of seasonal rather than weekly quotas when uncertainty is present cannot be fully assessed by using the approach of this paper, since the effects of depletion were not taken into account in the dynamic programming equations.

Some preliminary observations about the relationship between quotas and taxes follow. For the situation analyzed in this paper, with uncertainty in the fishery only, the effect of a tax on the catch is simply to reduce p in the dynamic programming equation (16). A reduction in the price per school affects each ground equally and does not affect the information updating at all. In particular, the tax directly affects the overall economic return, but has no effect on the fisherman's posterior estimate of catch rate or stock size, once he or she decides to go fishing.

The quota affects the overall economic return only by bounding it, but if the quota for the individual fisherman is small enough a main effect of the weekly quota is in the posterior estimate of catch rate, once the fisherman decides to go fishing. In this sense, quotas and taxes should not be interchangeable. That is, even if taxes and quotas lead to the same level of effort over a season, the posterior estimates of stock may differ, in the same way that Equations (24) and (25) differ in the limits $t \to 0$ and $n \to \infty$ respectively. Consequently, it is not clear that taxes and quotas will have the same effects on effort. Even if a tax were adopted to produce the same effort pattern as a weekly quota, the information processing aspects need not be the same. That is, identical levels of effort may lead to different information updating whenever the weekly quota is reached, since in one case fishing stops and in the other case it does not.

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Appendix 1. *The True Value of λ Is Not Needed in the Dynamic Programming Equation for Profit Maximization*

Although the true value of the parameter λ_i, reflecting the quality of ground i, enters into the calculation of the second period profit in the case of weekly quotas in Equations (16) and (17), it does not enter into the corresponding Equation (6) for the profit maximization case. This may seem paradoxical. In this appendix we resolve this apparent paradox by rederiving the results represented by Equation (6). Since the value of c does not affect the decision rule, let $c = 0$. Equation (6) can be restated as follows. Let V_1, V_2 be the values of visiting grounds 1 and 2 respectively on the first week. The ground on which V_i is largest is visited first. Explicitly,

$$\begin{aligned} V_1 &= E_{\lambda_1}^{(\nu_1,\alpha_1)} \left\{ \lambda_1 + \sum_{n=0}^{\infty} \Pr\{n, \lambda_1\} \max\left(\frac{\nu_1 + n}{\alpha_1 + \Delta}, \frac{\nu_2}{\alpha_2} \right) \right\} \\ V_2 &= E_{\lambda_2}^{(\nu_2,\alpha_2)} \left\{ \lambda_2 + \sum_{n=0}^{\infty} \Pr\{n, \lambda_2\} \max\left(\frac{\nu_1}{\alpha_1}, \frac{\nu_2 + n}{\alpha_2 + \Delta} \right) \right\} \end{aligned} \tag{31}$$

The true value of the parameter λ_i can be included in the value function in the following way.

Let $n_1{}^*$ and $n_2{}^*$ satisfy

$$\frac{\nu_1 + n_1{}^*}{\alpha_1 + \Delta} = \frac{\nu_2}{\alpha_2} \qquad \frac{\nu_2 + n_2{}^*}{\alpha_2 + \Delta} = \frac{\nu_1}{\alpha_1} \tag{32}$$

so that if ground 1 is visited first and fewer than $n_1{}^*$ schools are found, ground 2 appears better in the second week. The values $\hat{V}_i$ associated with visiting the ith ground are then

$$\begin{aligned} \hat{V}_1 &= E_{\lambda_1\lambda_2}^{(\nu_1,\alpha_1),(\nu_2,\alpha_2)} \left\{ \lambda_1 + \sum_{n=0}^{n_1} \Pr\{n, \lambda_1\}\lambda_2 + \sum_{n=n_1{}^*+1}^{\infty} \Pr\{n, \lambda_1\}\lambda_1 \right\} \\ \hat{V}_2 &= E_{\lambda_1\lambda_2}^{(\nu_1,\alpha_1),(\nu_2,\alpha_2)} \left\{ \lambda_2 + \sum_{n=0}^{n_2{}^*} \Pr\{n, \lambda_2\}\lambda_1 + \sum_{n=n_2{}^*+1}^{\infty} \Pr\{n, \lambda_2\}\lambda_2 \right\} \end{aligned} \tag{33}$$

Therefore, in this derivation, expectations are taken over both λ_1 and λ_2, as in the case of a weekly quota. Equation (33) is similar in form to Equation (18). On the surface, it appears that Equations (33) and (31) represent different functions.

To see that $\hat{V}_i = V_i$, begin by writing V_1 as

$$V_1 = E_{\lambda_1}^{(\nu_1,\alpha_1)} \left\{ \lambda_1 + \sum_{n=0}^{n_1^*} \Pr\{n, \lambda_1\} \frac{\nu_2}{\alpha_2} + \sum_{n=n_1^*+1}^{\infty} \Pr\{n, \lambda_1\} \frac{\nu_1 + n}{\alpha_1 + \Delta} \right\} \quad (34)$$

A comparison of Equation (34) and the first Equation (33) shows that $V_1 = \hat{V}_1$ if

$$E_{\lambda_1}^{(\nu_1,\alpha_1)} \left\{ \sum_{n=n_1^*+1}^{\infty} \Pr\{n, \lambda_1\}\lambda_1 \right\} = E_{\lambda_1}^{(\nu_1+\alpha_1)} \left\{ \sum_{n=n_1^*+1}^{\infty} \Pr\{n, \lambda_1\} \frac{\nu_1 + n}{\alpha_1 + \Delta} \right\} \quad (35)$$

This equality must hold termwise, which means that

$$E_{\lambda_1}^{(\nu_1,\alpha_1)} \{\Pr\{n, \lambda_1\}\lambda_1\} = E_{\lambda_1}^{(\nu_1,\alpha_1)} \{\Pr\{n, \lambda_1\}\} \frac{\nu_1 + n}{\alpha_1 + \Delta} \quad (36)$$

To demonstrate the validity of Equation (36), note that $(\nu_1 + n)/(\alpha_1 + \Delta)$ is the posterior expectation of λ_1, given that n schools were found. If $f(\lambda_1)$ is the prior density on λ_1, then this posterior expectation is

$$E\{\lambda_1 \mid n \text{ found}\} = \frac{\int \lambda_1 f(\lambda_1) \Pr\{n, \lambda_1\}\, d\lambda_1}{\int f(\lambda_1) \Pr\{n, \lambda_1\}\, d\lambda_1} \quad (37)$$

Writing the right hand side of Equation (37) in terms of expectations gives

$$E_{\lambda_1} (\Pr\{n, \lambda_1\}\lambda_1) = E_{\lambda_1} (\Pr\{n, \lambda_1\}) E(\lambda_1 \mid n \text{ found}) \quad (38)$$

Equation (36) is a special case of (38), therefore $V_i = \hat{V}_i$. This confirms the statement that the values of λ_i may be eliminated from the equations for the return in the profit maximization case.

Appendix 2. *Results for More Than One Fisherman*

In this appendix, we show how to derive the results in the section on seasonal quotas for many fishermen for the times for N fishermen to achieve a quota of Q schools. The time T_{NQ} is a random variable given by

$$T_{NQ} = \sum_{i=1}^{Q} T_i \tag{39}$$

Here T_i is the random variable that is the time it takes to find the ith school. The generating function for T_{NQ} is

$$\varphi(\omega) = \prod_{k=0}^{NQ-1} E\{e^{\omega T_k}\} \tag{40}$$

If the T_k are identically distributed exponentially with parameter $\lambda(N)$, where N is the number of fishermen, Equation (40) becomes

$$\varphi(\omega) = \left(\frac{\lambda(N)}{\lambda(N) - \omega}\right)^{NQ} \tag{41}$$

The moments of T are found by differentiating $\varphi(\omega)$ and setting $\omega = 0$. This gives

$$\begin{aligned} E\{T\} &= \frac{NQ}{\lambda(N)} \\ \mathrm{Var}\{T\} &= \frac{NQ}{\lambda(N)^2} \end{aligned} \tag{42}$$

If the fishermen act independently, then $\lambda(N) = N\lambda_1$, where λ_1 is the mean search rate for a single fisherman. Thus

$$E\{T\} = \frac{Q}{\lambda_1} \qquad \mathrm{Var}\{T\} = \frac{Q}{N\lambda_1} \tag{43}$$

Equations (43) show that on the average, it takes an individual fisherman in a group of N just as long to fill the quota as it would take alone, but that the variance of this time decreases with $1/N$. By acting cooperatively, the fishermen thus reduce their risk.

Depletion can be included by assuming that the time to the kth detection is exponentially distributed with parameter $\lambda(N) - (k - 1)\delta$ (Mangel and Clark, 1983; Clark and Mangel, 1984). Here δ measures the decrease in the search rate caused by the removal of a school. The generating function becomes

$$\varphi(\omega) = \prod_{k=0}^{NQ-1} \frac{\lambda(N) - k\delta}{\lambda(N) - k\delta - \omega} \tag{44}$$

so that

$$\log \varphi(\omega) = \sum_{k=0}^{NQ-1} \log (\lambda - k\delta) - \sum_{k=0}^{NQ-1} \log (\lambda(N) - k\delta - \omega) \tag{45}$$

The first moment of T is

$$E\{T\} = \sum_{k=0}^{NQ-1} \frac{1}{\lambda(N) - k\delta} \tag{46}$$

and the variance is

$$\mathrm{Var}\{T\} = \sum_{k=0}^{NQ-1} \frac{1}{[\lambda(N) - k\delta]^2} \tag{47}$$

Setting $\lambda(N) = N\lambda_1$ gives

$$
\begin{aligned}
E\{T\} &= \sum_{k=0}^{NQ-1} \frac{1}{N\lambda_1 - k\delta} \\
\mathrm{Var}\{T\} &= \sum_{k=0}^{NQ-1} \frac{1}{(N\lambda_1 - k\delta)^2}
\end{aligned}
\tag{48}
$$

Appendix 3. *Posterior Expectations with Weekly and Seasonal Quotas*

In this appendix, we begin by finding $t_Q(n)$, the time in which the quota must be caught to match the posterior expectation obtained by catching of schools in the first week. Namely, $t_Q(n)$ satisfies

$$\frac{\nu + Q}{\alpha + t_Q(n)} = \frac{\nu + n}{\alpha + 1} \tag{49}$$

Solving Equation (49) for $t_Q(n)$ gives

$$t_Q(n) = \left(\frac{\nu + Q}{\nu + n}\right)(\alpha + 1) - \alpha \tag{50}$$

Differentiating Equation (50) gives

$$dt_Q(n) = -\frac{(\nu + Q)}{(\nu + n)^2}(\alpha + 1)\, dn \tag{51}$$

which is the differential form of Equation (29).

Consider a situation in which n changes by an amount dn, given exogenously, and $t_Q(n)$ then changes by an amount $dt_Q(n)$. Assume that $(\nu + 1)/(\alpha + 1) < Q_s - n$. The changes in the posterior expectations are then

$$
\begin{aligned}
E_{WQ}[t + dt_Q(n)] &= E_{WQ}(t) + dE_{WQ} \\
E_{SQ}(n + dn) &= E_{SQ}(n) + dE_{SQ}
\end{aligned}
\tag{52}
$$

Here $E_{WQ}(t) = (\nu + Q)/(\alpha + t)$ and $E_{SQ}(n) = (\nu + n)/(\alpha + 1)$. A Taylor expansion of these functions gives

$$\begin{aligned} \frac{dE_{WQ}}{E_{WQ}} &= \frac{-dt_Q(n)}{[\alpha + t_Q(n)]^2} \\ \frac{dE_{SQ}}{E_{SQ}} &= \frac{\nu dn}{\nu + n} \end{aligned} \tag{53}$$

Equations (53) are the differential forms of the elasticities defined by Equation (30).

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