Chaos For the Halibut?

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Abstract *A generalized method for analyzing stability potential in discrete-time renewable resource models subject to market-driven harvest is discussed. Two means by which harvest activity can influence dynamical properties of renewable resource models are identified: the "growth factor" and the "market response effect". The growth factor is a systematic influence on stability tied to changes in the position of the bioeconomic equilibrium point along a given open access supply locus. The market response effect involves variation in harvest in response to stock level changes.*

The analysis is applied to a model of the Pacific Halibut Fishery; a modified discrete-time version of the traditional Schaefer model. In order to investigate potential instability, we vary certain parameters of the model and study the resulting effects on stability.

We find that enhancing harvest response by changing the slope of the demand schedule can thrust the model into instability, chaos, and extinction, without changing the bioeconomic equilibrium point for the Pacific Halibut Fishery Model. We also show that enhancing harvest response via slope-preserving increases in market demand can push the model into instability, chaos, and even extinction. Finally, we show that similar adjustments in market demand may be capable of eliminating instability and chaos rooted in powerful intrinsic growth of the stock.

Keywords Stability Analysis, Chaos, Open-Access Fisheries, Renewable Resource Models.

Introduction

Nearly all fisheries world-wide display large fluctuations in stock levels and yields year to year. In the mid-1970's this phenomena became commonly considered in bioeconomic fishery models (Andersen and Sutinen 1984). Most practitioners attempting to account for sources of stock fluctuations have focused on the population dynamics rather than harvest activity. Stock assessment surveys consistently reveal fluctuating stock levels regardless of whether or not they are harvested. Indeed, it is well known that fluctuations in food availability, habitat, water temperature, and in stocks of competing species all provide a rich source of impacts which might explain the fluctuations and related instability in any one fish stock. Stock fluctuations in bioeconomic fisheries models therefore are usually generated via additive or multiplicative exogenous random shocks applied to the stock growth and/or recruitment relation. While it is obvious that stock fluctua-

tions should in turn be associated with fluctuating harvests, comparatively little attention has been given to any role the harvest sector might play in actually generating stock fluctuations. It is not surprising then that initial interest in applying May's work (May 1974) on deterministic chaos in biological growth processes would be focused on the growth sector of bioeconomic models.[1] Hassel, Lawton, and May (1976), however, found that the intrinsic growth "energy" required for a chaotic stock growth process is significantly greater than actual intrinsic growth of fish stocks world-wide. Thus, it appears that fish stock growth relations themselves cannot generate deterministic chaotic fluctuations. These results suggest ecological deterministic chaos will not be an endogenous component of a properly specified bioeconomic fisheries model unless the model is extended to include a more comprehensive "chunk" of the ecological system to be studied.

The research reported here should be of interest because we search for roots of deterministic chaotic processes in both the population dynamics sector and the harvest sector of a simple, single species discrete-time open-access bioeconomic model in which harvest is completely determined by current period market conditions. Others have noted that harvest activity can generate instability in simple discrete-time bioeconomic models (e.g. Hilborne and Walters (1992), Opsomer and Conrad (1994)). In these models, harvest adjustments depend on previous period profit with a multiplicative adjustment sensitivity parameter that is exogenously determined. In the model we study, current period harvest is instead completely determined by current period ex-vessel landings supply and demand. Does market-driven harvest activity exert a stabilizing or destabilizing impact on the model? Under what conditions would such impacts be expected to emerge? These issues are the focus of the research summarized in this paper.

We find that open access market-based harvest activity may be just as influential as stock growth characteristics in determining the stability of discrete-time renewable resource models. Actual stability is shown to depend on the interplay of population dynamics and market conditions. Under certain market and growth conditions, harvest can be dramatically destabilizing. Notably, even when intrinsic growth is well within the ranges commonly observed in existing fisheries around the world, chaotic fluctuations in stock size can occur in a wide class of discrete-time renewable resource models based on market-oriented harvest. These predictions suggest that potential chaos may be lurking in unexpected places in harvested renewable resource models. The analysis also shows that under other circumstances, market-oriented harvest may also have a profoundly stabilizing influence, even when applied to stocks characterized by extreme intrinsic growth.

A generalized analysis of stability potential for discrete-time renewable resource models is discussed in the second section of the paper. The methods employed in this analysis do not require complicated mathematics and can be applied to study a wide range of models. Two means by which harvest activity can influence dynamical properties of renewable resource models are identified: the "growth factor" and the "market response effect". The growth factor is a systematic influence on stability tied to changes in the position of the bioeconomic

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[1] For a brief general review of economic applications of chaos theory, see Baumol and Benhabib (1989). For a variety of biological applications see Murray (1989).

equilibrium point along a given open access supply locus. The market response effect involves variation in harvest in response to stock level changes.

In the third through sixth sections this analysis is applied to a model of the Pacific Halibut Fishery specified by Cook and Copes (1987), a modified version of the traditional Schaefer model. Single species deterministic population dynamics are modeled with a discrete-time logistic growth equation, harvest is specified as proportional to fishing effort, while demand is linear and downward-sloping. In order to investigate potential instability, we vary certain parameters of the model and study the resulting effects on stability.

We find that enhancing harvest response by changing the slope of the demand schedule can thrust the model into instability, chaos, and extinction, without changing the bioeconomic equilibrium point reported by Cook and Copes for the Pacific Halibut Fishery Model. We also show that enhancing harvest response via slope-preserving increases in market demand can push the model into instability, chaos, and even extinction. Finally, we confirm predictions that similar adjustments in market demand may be capable of eliminating instability and chaos rooted in powerful intrinsic growth of the stock. Conclusions and policy implications are reviewed in the seventh section.

Stability Potential in Discrete-Time Bioeconomic Models of Renewable Resource Stocks

The models we will be considering in this paper are discrete first order dynamical systems of the general form: $X_{t+1} = T(X_t)$ for some transition function T. The system will have an equilibrium (fixed point) at a value x_e whenever $T(x_e) = x_e$. We will call an equilibrium x_e *stable* if there is some open interval U containing x_e so that if X_t ever attains a value in U, the system will be "attracted" to x_e: i.e., $\lim_{t \to \infty} X_t = x_e$. The equilibrium x_e will be called *unstable* if there is some open interval U containing x_e such that the system starting in any point of U other than x_e will be "repelled" from x_e: $|X_t - x_e| > |X_0 - x_e|$ for some, but not necessarily all, values of $t > 0$. Intuitively, an equilibrium x_e is unstable if eventually X_t moves farther away from x_e. This includes the possibilities that X_t moves away and stays away from x_e (perhaps going to some other equilibrium) or that X_t repeatedly gets close to x_e only to repeatedly move away. It is well known that if the transition function is continuously differentiable in a neighborhood of x_e, then x_e will be stable if $|T'(x_e)| < 1$ and unstable if $|T'(x_e)| > 1$ (e.g., Clark 1990). The sign of $T'(x_e)$ also has significance. When X_0 is sufficiently close to x_e, the system will overshoot the equilibrium value when $T'(x_e)$ is negative and will stay on one side of the equilibrium when $T'(x_e)$ is positive. We will call the equilibrium *oscillatory* when $T'(x_e)$ is negative and *non-oscillatory* when $T'(x_e)$ is positive.

We will refer to the value of $T'(x_e)$ as the *stability index* of the equilibrium. Table 1 gives a classification of equilibria based on the value of their stability index. For classification purposes we will denote equilibria that are unstable and non-oscillatory as Class I equilibria. In this class, the value of the system near the equilibrium will be simply pushed farther away from the equilibrium, eventually (typically) to the domain of a different equilibrium. Equilibria that are both stable and non-oscillatory are approached monotonically, and are grouped as Class II. Equilibria which are stable and oscillatory are grouped in Class IV, and are approached with a damped oscillation. Class III equilibria occur on the border

between the oscillatory and non-oscillatory stable equilibria. Class III equilibria (when the stability index is 0) represent maximal stability; in general, the system will return to equilibrium most rapidly if the equilibrium is nearly Class III.[2] Equilibria that are unstable and oscillatory are grouped as Class V. The dynamics associated with a Class V equilibria can become extremely complex. In this case the system may fall into a periodic or chaotic pattern. Note that the classification of equilibrium stability does not always completely determine the dynamics of the model. Equilibrium classification will predict the behavior of the model when the model is near equilibrium, but bifurcation diagrams may be needed to supplement this classification to investigate the range of the dynamics of a system with several possible competing equilibria.

For the renewable resource models we consider, we will assume the following intra-period sequence of events (see Appendix I for a discussion of other possible sequencing specifications):

a) Stock (X_t) becomes available for the harvest season;
b) Harvest (H_t) occurs;
c) Escapement $S_t = (X_t - H_t)$ remains after harvest; and
d) Interseasonal growth occurs $X_{t+1} = S_t + G(S_t)$.

In this model, the stock level at the beginning of period (t) determines the harvest supply schedule for the period. Open access harvest is then determined by supply and demand in period (t).

We will assume throughout that interseasonal growth is governed by the logistic growth equation[3], but the method of analysis applies to other growth specifications. The transition functions in these models take the form:

$$X_{t+1} = T(S_t) = S_t + G(S_t), \text{ where} \tag{1}$$

$$G(S_t) = g\, S_t\, (1 - S_t/K) \tag{1a}$$

In (1a), g is the intrinsic growth parameter and K is the environmental carrying capacity for the species population. Using the chain rule,

$$\frac{dT}{dX_t} = \frac{dT}{dS_t} \cdot \frac{dS_t}{dX_t} = \left(1 + g - 2g\,\frac{S_t}{K}\right)(1 - H'(X_t)) \tag{2}$$

we find that there are two multiplicative factors which impact on equilibrium stability and stock dynamics when (2) is evaluated at an equilibrium. For reasons

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[2] More precisely, in the neighborhood of the equilibrium the rate of convergence will be quadratic for Class III equilibria and only linear for Class II and Class IV equilibria. See, for example, Burden and Faires (1993) for further details.

[3] Note that it is possible for the logistic growth model to fail under certain circumstances (e.g., when X_t is very large or when $g > 3$ in the absence of harvest) since the model has the potential to predict negative populations. A similar model based on an exponential version of the logistic growth model:

$$T(X_t) = X_t\, \exp[g\, (1 - X_t/K)]$$

avoids the problem of potentially negative populations. In this paper we will use the classical logistic model, but note that the exponential version will qualitatively exhibit the same properties.

we discuss below, we will refer to $dT/dS_t = 1 + g - 2gS_e/K$ as the *growth factor* and to $dS_t/dX_t = 1 - H'(x_e)$ as the *escapement response factor*.

The growth factor, dT/dS_t, is determined by escapement-based growth conditions at the bioeconomic equilibrium point. Biological equilibrium occurs where:

$$H_t = g \, S_t(1 - (S_t/K)) \qquad (3)$$

Solving Equation (3) for S in terms of H shows that there is a maximal equilibrium harvest (the MSY) when $H_e = H_{MSY} = gK/4$, and for any equilibrium harvest less than H_{MSY} there will be two possible equilibrium escapements:

$$S_e = (K)/(2) \sqrt{(1 \pm 1 - (4H_e)/(gK))}.$$

Then from (2) we see that the growth factor can take the values:

$$\frac{dT}{dS_t} = 1 \pm g \sqrt{1 - \frac{4H_e}{gK}} \qquad (4)$$

Note that the growth factor has a value of 1 at the H_{MSY} equilibrium and approaches its extreme values of $1 \pm g$ as equilibrium harvest decreases to zero. For a species with a given value of g, market forces will influence the growth factor by determining the size of equilibrium harvest and escapement, thereby determining just where the value of the growth factor actually lies within the established bounds for that particular stock.

The escapement response factor, $(1 - H'(X))$, measures the responsiveness or sensitivity of the escapement size to changes in the stock size via adjustments in harvest. In the following sections we will focus analysis on $H'(X)$ and denote it as the *harvest response effect* on the stability index. For stability analysis it is useful to classify the harvest response effect into four broad categories depending on the value of the harvest response term at bioeconomic equilibrium, $H'(x_e)$. When harvest is invariant and unresponsive to stock level changes, $H'(x_e) = 0$ so that the escapement response factor has a value of 1. Here, all of the variation in X_t is transmitted into variation in S_t, which in turn will impact on X_{t+1}. This could occur when the stock is unharvested or with perfectly inelastic demand (or supply) in the ex-vessel market for landings. In this category, the stability index is completely determined by the growth factor. When harvest is "moderately responsive" to the stock size $(0 < H'(x_e) < 1)$, the market driven harvest will tend to reduce the absolute value of the stability index from what it would have been with the growth factor alone. Under these conditions, current period market equilibrium harvest variation would absorb some variation in the current period stock and reduce variation in current period escapement, enhancing stability. "Maximum stability" is attained when $H'(x_e) = 1$, so the escapement response factor and stability index are zero. Here exactly all the variation in X_t is absorbed by adjustments in $H(X_t)$ so that $S(X_t)$ remains invariant. When the harvest is still more sensitive $(H'(x_e) > 1)$, the escapement response factor becomes negative and the sign of the stability index is reversed from what it would have been under the growth factor alone. As $H'(x_e)$ becomes greater than one, changes in the harvest begin to "overshoot" the changes in the stock which initiated the harvest response. We will refer to this range of values for the harvest response effect (when it is greater than 1) as "harvest overshoot".

The stability index tied to the values of the growth factor and the harvest

response effect are summarized in Figure 1. In Figure 1, equilibrium values for dT/dS_t, the growth factor, are plotted on the vertical axis and values for the harvest response effect $H'(x_e)$, are plotted on the horizontal. Contour lines representing the stability index, $dT/dX_t = -1, 0$, and 1 are provided. Figure 1 may be used to easily classify the stability of discrete-time renewable resource model equilibria by using Table I and by plotting the appropriate values of dT/dS_t and $H'(x_e)$ for any bioeconomic equilibrium. We can conclude from Figure 1 that both the growth factor and the market response effect may strongly influence model stability. For most fisheries world-wide, however, growth characteristics reflect values for intrinsic growth which range between 0 and 1. Using Equation (4), this implies we can expect growth factors to range from 0 to 2. In Figure 1, the range of possibilities we would expect to observe is therefore restricted to the shaded area. This suggests that any instability with roots in the growth factor must be restricted to Class I (when harvest response is relatively low) and will preclude the possibility of chaos. As market equilibrium harvest response is increased from zero, however, dramatic impacts on stability may occur. For an equilibrium with a given growth factor within the shaded area of Figure 1, increases in the harvest sensitivity (moving from left to right on Figure 1), initially tend to stabilize the equilibrium towards maximum stability, but further increases then destabilize the equilibrium, eventually to the extremes of Class V instability. In what follows, we investigate whether such a result is reasonable by focusing on the determinants of harvest response in the context of the Pacific Halibut Fishery model of Cook and Copes. Before turning to this issue, however, we first focus on an analysis of the
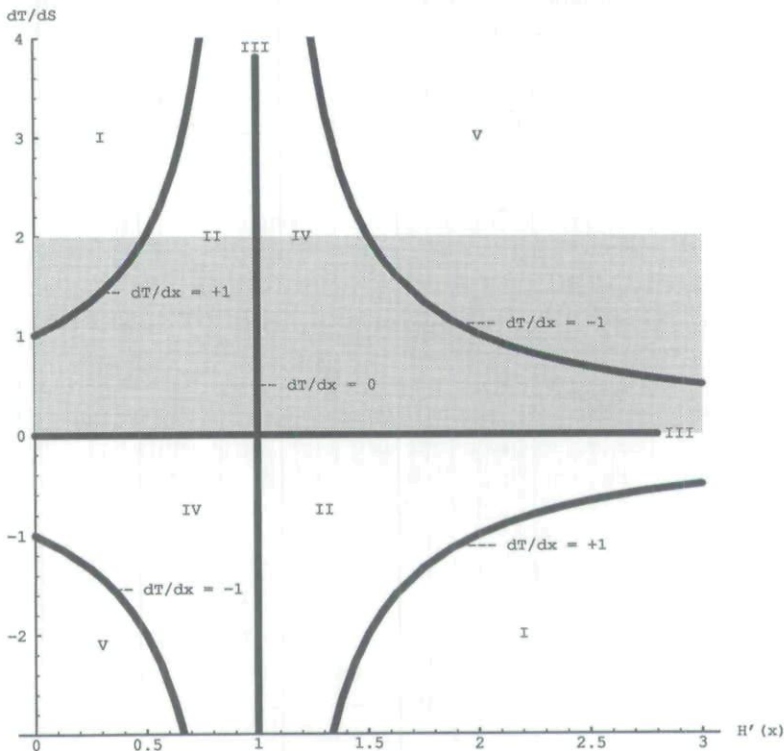

Figure 1. Contour Map of the Stability Index, dT/dX.

Table 1
Stability in Renewable Resource Bioeconomic Models

Nature of Bioeconomic Equilibrium	T'(X) at Bioeconomic Equilibrium	Typical Behavior Near Bioeconomic Equilibrium
Class I. Unstable, Non-Oscillatory	$T'(x_e) > 1$	Stock moves away from X_e
Class II. Stable, Non-Oscillatory	$0 < T'(x_e) < 1$	Stock approaches x_e monotonically
Class III. Maximum Stability	$T'(x_e) = 0$	Stock approaches x_e monotonically and most rapidly
Class IV. Stable, Oscillatory	$-1 < T'(x_e) < 0$	Stock approaches x_e with damped oscillations
Class V. Unstable, Oscillatory	$T'(x_e) < -1$	Cycles of various degree, Chaos, or Stock moves to different attractor

growth factor in the context of the PHF model *without restricting the analysis to reasonable bounds on g*. This is done to provide a numerical test of the dynamical possibilities suggested by Figure 1 vis a vis the growth factor. Then we return to an analysis of the harvest response factor and its potential impact on model stability in the PHF model.

The Ex-vessel Market for Landings From The Pacific Halibut Fishery

The landings market model we will be working with is based on the model developed in Cook and Copes (1987) to study the Pacific Halibut Fishery (PHF). The inverse demand schedule for ex-vessel harvested halibut is assumed linear:

$$P_t = d_1 - d_2 H_t, \tag{5}$$

with positive constants d_1 and d_2; Cook and Copes estimate the values $d_1 = 0.60$ and $d_2 = 0.001$ for H_t in units of 10^5 pounds and P_t in units of 1961 Canadian dollars.

Assuming costless entry/exit of fishing inputs, additional inputs will be attracted to (removed from) this industry within each period as long as returns are better than (less than) normal, so industry harvest will adjust in each period to the point where average pecuniary cost per pound (APC) just equals ex-vessel price per pound:

$$P_t = APC(H_t, X_t) = s_1(X_t) + s_2(X_t) H_t \tag{6}$$

In this model the APC schedule is derived from a fishing effort-based Schaefer production model of the PHF (see Appendix II), resulting in $s_1(X_t) = C_1/qX_t$ and $s_2(X_t) = C_2/q^2 X_t^2$; estimated values in the Cook and Copes model are $C_1 = 38.65$, $C_2 = 9*10^{-6}$, and $q = 1.15*10^{-6}$. Equation (6) gives the long run inverse normal profit industry supply locus, given X_t. For a given initial stock level, this inverse market supply schedule is linear and upward sloping, reflecting increasing factor prices required to attract inputs to effort in this fishery from their best alternative allocation. This inverse supply schedule is illustrated in Figure 2 for three differ-
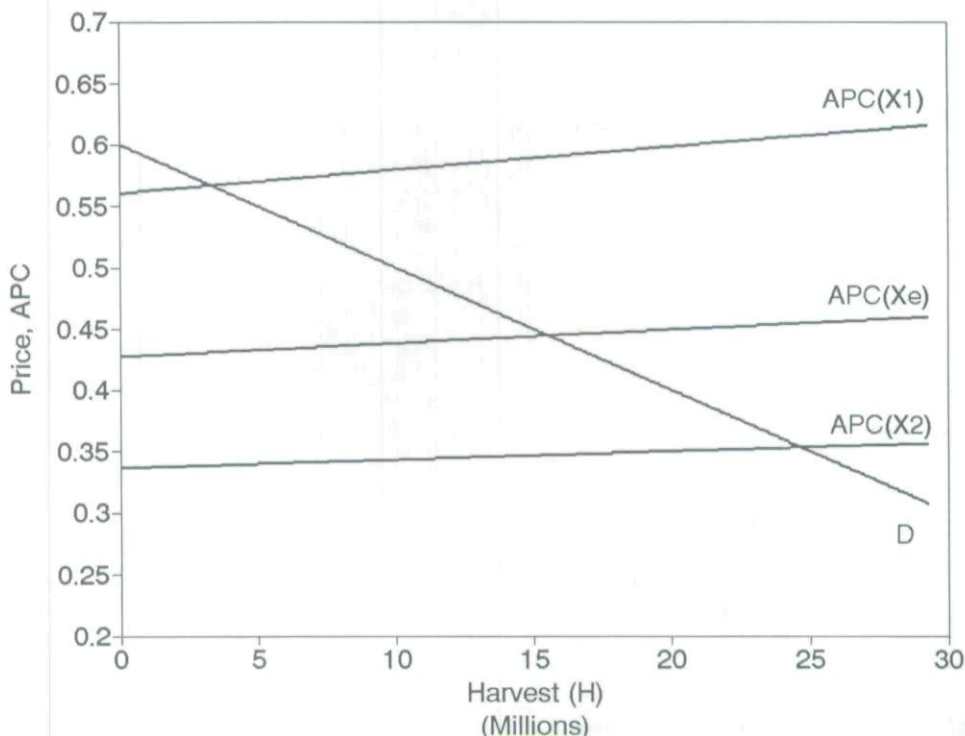
Figure 2. Ex-Vessel Landings Market Model for the Pacific Halibut Fishery Model. X_e = 78,667,219; X_1 = 60,000,000; X_2 = 100,000,000.

ent initial stock levels. Note that changes in the stock size (X_t) will shift and rotate the APC schedule.

Using (5) to substitute for P_t, (6) can be solved for the long run normal profit market equilibrium harvest in terms of X_t:

$$H(X_t) = (d_1 - s_1(X_t))/(d_2 + s_2(X_t)). \qquad (7)$$

Assuming market equilibrium is fully attained within each period[4], Equation (7) provides an open access default market-driven current period harvest decision rule[5]. Given stable demand for fish, (7) provides a means for determining appropriate market equilibrium harvest response in any period (t) when $0 < H(X_t) < X_t$. (If (7) yields $H(X_t) < 0$ then no harvesting will occur, while $H(X_t) > X_t$ implies that the entire stock would be harvested.)

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[4] Note that the assumption of full attainment of market equilibrium with these cost parameters and assumptions implies a high degree of flexibility in harvest response from period to period under changing market conditions (*i.e.* market supply is highly responsive to changes in ex-vessel price). This is due in part to the linear production function in effort, and in part to the assumption that entry/exit of inputs is costless with little inertia involved.
[5] Notice that Equation (7) represents a "myopic extreme" in that neither information on past stock conditions or concern for future stock levels is considered. An alternative which considers past information while ignoring the future might be used to specify current period harvest as a function of the previous period's profit. See, for example, Opsomer and Conrad (1994) for such an approach with fishing effort as the decision variable.

Sustainable harvest supply is defined as the harvest levels at which the indus-try earns a normal pecuniary profit and the stock is in biological equilibrium. Equations (3) and (6) can be used to plot the classic open access backward-bending sustainable harvest supply locus, plotting price and harvest level. This is done for the base case PHF parameters given in Table II, and is shown in Figure 3. Bioeconomic equilibrium occurs where market demand intersects the sustain-able harvest locus (point A_{PHF} in Figure 3 for the PHF).

The Growth Factor and Stability in the Pacific Halibut Fishery Model

In order to appreciate the influences that market-based open access harvest ac-tivity may exert on the dynamics of the PHF model, we first focus on the growth factor and relate its value to the position of the bioeconomic equilibrium on the sustainable supply locus of Figure 3. For the PHF model with $g = .58$ and $K = 1.098 \times 10^8$, the value for the growth factor, dT/dS, will increase from 1 at H_{MSY} (point A_{MSY} in Figure 3) towards a limiting value of $(1 + g = 1.58)$ where the backward bending portion of the sustainable supply locus approaches the vertical axis at point A_3 (not shown). As the equilibrium moves away from point A_{MSY} down the lower portion of the sustainable supply locus, the value of the growth factor diminishes from 1 to a minimum value of $(1 - g = 0.42)$ at the equilibrium associated with an unharvested stock (point A_1 in Figure 3). The market's influ-ence on the growth factor is its contribution to determining the bioeconomic equilibrium escapement level via the determination of bioeconomic equilibrium price, fishing effort level, and the resulting equilibrium sustainable harvest.

Changes in the value of g can dramatically change the range of possible values for the growth factor. Equation (4) indicates that increasing g increases the growth factor along the backward-bending portion of the locus, while reducing it along the
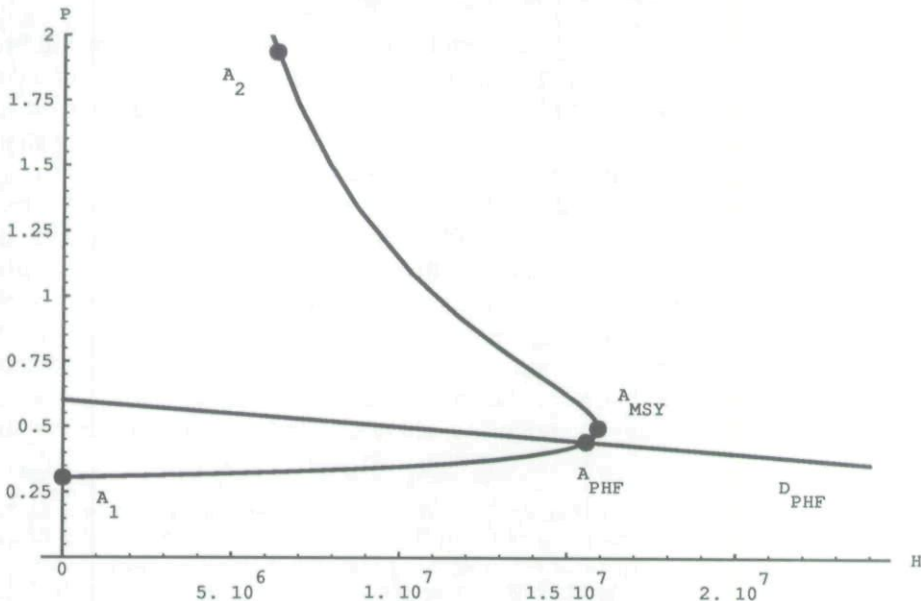

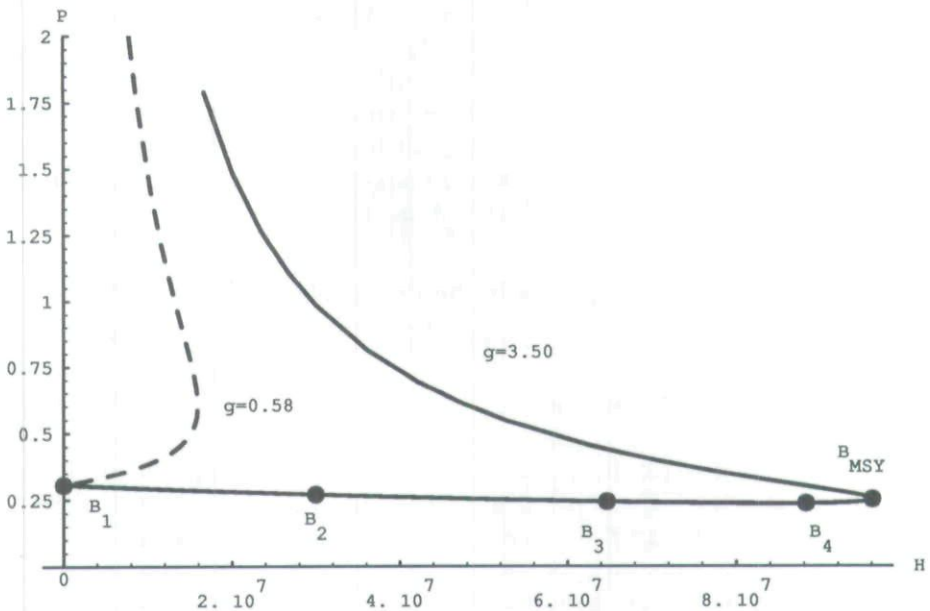**Figure 3.** Sustainable Supply Locus for $g = .58$, and Market Demand: Pacific Halibut Fishery Model.

Figure 4. Sustainable Supply Locus for the Pacific Halibut Fishery Model with g = 3.5.

lower portion. In order to illustrate this in the context of the PHF, g is increased from 0.58 to 3.5, keeping other parameters the same. The resulting sustainable supply locus is plotted as the solid line in Figure 4 (for comparison the locus for g = 0.58 is included as the dashed curve).[6] The range for the growth factor in this case has expanded to a minimum of −2.5 at the unharvested equilibrium (B_1) to a limit of 4.5 as the backward-bending portion of the locus approaches the vertical axis (B_5).

To isolate the influences of the growth factor on stock dynamics, we will first hold the harvest response effect constant at a value of 0, so that the stability index is completely determined by the growth factor. This could occur if demand (or supply) for landings is perfectly inelastic (this includes the case in which the stock is not harvested, as considered by May (1974)). Under these conditions, the stability index can be located along the left-side vertical axis in Figure 5 which summarizes the stability potential of the PHF model based on analysis of equation (2). When g = 0.58 in this case, the stability and dynamics are straightforward. Along the lower portions of the sustainable supply locus, the equilibria are stable Class II as harvest is increased from 0 at A_1 towards MSY. From MSY to A_3 (along the backward bending portion of the sustainable supply locus) the equilibria are Class I—unstable and non-oscillatory. Stock levels near these equilibria will either be pushed to the stable equilibrium along the lower portion of the sustain-

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[6] The reader should note that, unlike the case for g = 0.58, a portion of the lower part of the sustainable supply locus slopes downward (B_1 to B_3) when g = 3.5. This "forward-falling" character may be explained with the aid of Figure 9 in the text, which is a graph of Equation (1) with g = 3.5. Note that over the negatively sloped portion of $T(S_t)$, increased harvests (reductions in escapement) lead to larger stocks (X_{t+1}), resulting in reductions in APC(X_t) and the forward-falling character of the sustainable supply locus for large stock levels.
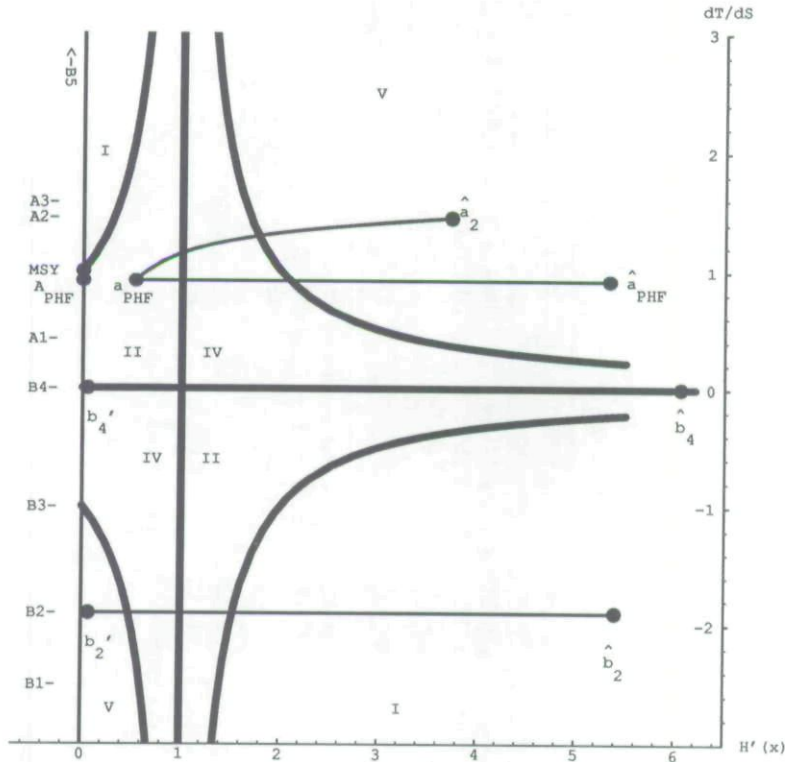
Figure 5. Contour Map of the Stability Index, dT/dX, Illustrating Stability Potential in the Pacific Halibut Fishery Model.

able supply locus or to extinction if initial stock levels are too low. In the case of $g = 3.5$, however, the entire spectrum of stability is possible for equilibria along the sustainable supply locus, ranging from Class V at B_1 through stability of Classes IV, III and II along the lower portion of the locus and back into the instability of Class I along the backward bending portion of the locus.

To illustrate the range of dynamics exhibited when $g = 3.5$ with perfectly inelastic demand, the model was run over the range of bioeconomic equilibrium harvests reflected in Figure 4 from points B_2 through B_{MSY}. This was done by shifting a perfectly inelastic demand schedule which initially passed through point B_2, over to point B_{MSY} in Figure 4. This corresponds to a movement in Figure 5 along the left-side vertical axis from B_2 to MSY. Parameter values and equilibrium variable values at the end points, B_2 and B_{MSY}, of this process are given in Table II. Using Equation (4), the growth factor and thus the stability index varies from -1.9026 (Class V) at B_2 to 1 at MSY (Class II) in Figure 5. The actual numerical results are summarized in the form of a bifurcation diagram, Figure 6[7]. In a bifurcation diagram, an initial state X_0 is chosen, and the parameter under study is then varied along the horizontal axis. At each parameter value along the horizontal axis, the levels attained by the system with the given parameter value are

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[7] A simple computer program and user guide for generating bifurcation diagrams associated with simple discrete-time renewable resource models is available from the authors upon request.

Table 2
Selected Equilibria[1] of the Pacific Halibut Fishery Model

	$A_{PHF}(a_{PHF})$	$A_{PHF}(\hat{a}_{PHF})$	$B_2(b'_2)$	$B_2(\hat{b}_2)$	$A_2(\hat{a}_2)$	$B_4(b'_4)$	$B_4(\hat{b}_4)$	B_{MSY}
g	0.58	0.58	3.5	3.5	0.58	3.5	3.5	3.5
K	1.098E + 8	1.098E + 8	1.098E + 8	1.098E + 8	1.098E + 8	1.098E + 8	1.098E + 8	1.09E + 8
q	1.15E − 6	1.15E − 6	1.15E − 6	1.15E − 6	1.15E − 6	1.15E − 6	1.15E − 6	1.15E − 6
d_1	.6	.444343	1.16968	0.26968	2.0	2.88239	.235423	.235423
d_2	1E − 8	0	3E − 8	0	1E − 8	3E − 8	0	∞
C_1	38.65	38.65	38.65	38.65	38.65	38.65	38.65	38.65
C_2	9E − 6	9E − 6	9E − 6	9E − 6	9E − 6	9E − 6	9E − 6	9E − 6
x_e	7.86672E + 7	7.86672E + 7	1.30429E + 8	1.30429E + 8	1.85465E + 7	1.58818E + 8	1.58818E + 8	1.5098E + 8
s_e	6.31015E + 7	6.31015E + 7	1.00429E + 8	1.00429E + 8	1.22391E + 7	7.05857E + 7	7.05857E + 7	5.4900E + 7
H_e	15565683	15565683	3.0E + 7	3.0E + 7	6.30741E + 6	8.82321E + 7	8.82321E + 7	9.6075E + 7
P_e	.444343	.444343	.26968	.26968	1.93692	.235423	.235423	.251295
dT/dS_t	.91335	.91335	−1.90256	−1.90256	1.4507	0	0	1
$H'(x_e)$	.528482	5.33435	0.071041	5.39849	3.7323	0.053923	6.05729	0.0

[1] Upper case labels refer to points on Figures 3, 4 and 5. Lower case labels in parentheses refer to points on Figure 5.
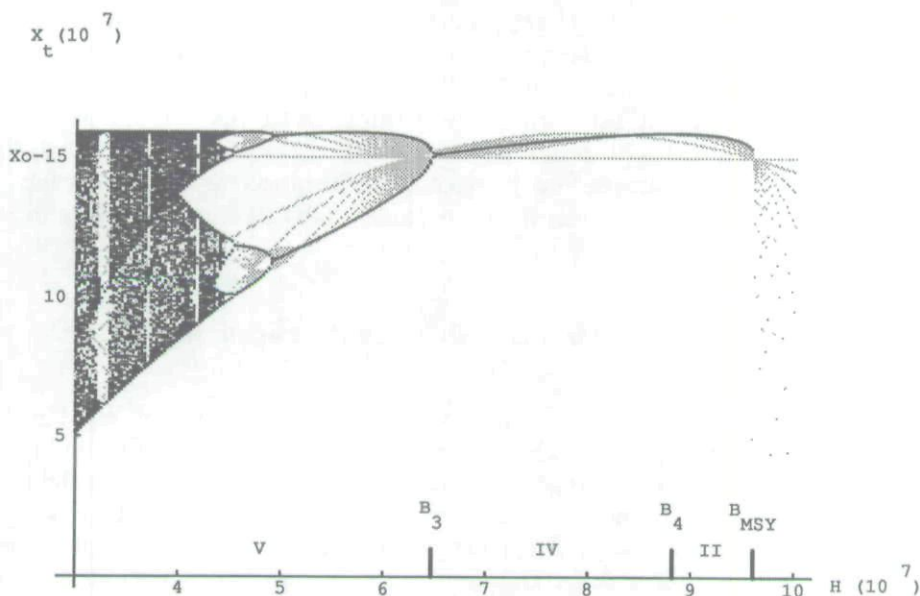
$x_t (10^7)$

Figure 6. Modified Bifurcation Diagram for the Pacific Halibut Fishery Model Over the Range: $B_0 - B_{MSY}$. Bifurcation Parameter: Invariant Harvest, $X_0 = 1.5 \times 10^8$.

plotted vertically. In this way it becomes possible to compare time series results over a wide range of values for the parameter under study, all on one compact graph. Usually bifurcation plots are restricted to long term behavior of the system. In the "modified" bifurcation diagrams presented in this paper, however, we consider the initial approach toward the bioeconomic equilibrium in addition to the states attained once the system is near the equilibrium. We plot observed stock levels X_0 through X_{100} with grey points (X_0 here being the initial value of the stock). Stock levels for X_{101} upward are plotted with black points. Consequently, values of the parameter which result in an attractive equilibrium will have a single black point plotted at the equilibrium value with a number of grey points distributed near the black point vertically which represent the approach from X_0 toward x_e. Parameter values which yield cyclic behavior will have a number of black points representing the states of the cycle plotted vertically. Parameter values which generate chaos will result in a large complicated set of black points being plotted vertically.[8]

In Figure 6, invariant harvest is the bifurcation parameter. Beginning at B_2 with harvest at 30,000,000 lbs., we observe Class V dynamics, in this case, chaos. As the harvest is increased, we observe a dramatic reduction in the range of variation in the stock. In addition, note the movements into and out of periodic behavior characteristic of chaotic dynamics. As harvest reaches 50,000,000 lbs., the stock settles into a two-cycle periodic mode. As invariant harvest is increased

[8] We chose period 101 as the cut-off point to switch from grey to black shading of the points presented in Figure 6 and period 51 as the cut-off point for Figures 7, 8, 10, 11. For some values of the bifurcation parameter, it may be that more periods are required prior to entering a periodic cycle. This means that some of the black points on our modified bifurcation diagrams should in fact be grey.

to approximately 65,000,000 lbs., the model moves into Class IV dynamics. Continued increases in the invariant harvest stabilize the model more, eventually leading to Class III dynamics at 88,232,100 lbs. (point B_4) Further increases in harvest result in Class II equilibria[9], but attempts to harvest beyond H_{MSY} = 96,075,000 lbs. (point B_{MSY}), result in extinction. This empirical test confirms the predicted systematic influence that the position of the bioeconomic equilibrium on the sustainable supply locus can have on model stability as the stability index changes.

The Harvest Response Factor and Stability in the Pacific Halibut Fishery Model

In this section we analyze the dynamics of the model when market equilibrium harvest may vary in response to changes in the stock size. For the purpose of this analysis, market harvest response is controlled by manipulation of the demand schedule. We begin with the actual equilibrium conditions for the PHF model as estimated by Cook and Copes. The actual parameter values and equilibrium variable values are given in Table II. This equilibrium is represented as point A_{PHF} in Figure 3. To study the impact of the harvest response effect, we then will allow the demand schedule D_{PHF} (Figure 3) to rotate, approaching perfectly elastic demand.[10] The link between demand rotation and harvest response may be illustrated by rotating the demand schedule in Figure 2 and observing the resulting spread in equilibria tied to $APC(X_1)$ and $APC(X_2)$, representing different stock levels x_1 and x_2. The reader may verify that the variation in market equilibrium harvest as stock level changes from x_e to x_1 or x_2 is greater when the slope of D is decreased. To isolate the dynamical impacts of the harvest response effect from the growth factor, we will rotate D_{PHF} in such a way that the position of the bioeconomic equilibrium on the supply locus is not altered. This is done by adjusting both of the inverse demand parameters d_1 and d_2 by keeping the equilibrium price P_e constant, so as to preserve the position of the bioeconomic equilibrium at point A_{PHF} in Figure 3.

While rotation of demand around this fixed point leaves the equilibrium unchanged, it has a profound impact on the stock transition dynamics and system

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[9] Although the grey points (showing the first few states attained by the system) are useful in distinguishing between oscillatory and non-oscillatory approaches to equilibria, note that no one individual bifurcation diagram can be used to precisely distinguish the transitions between the different classes of stable equilibria (Classes II through IV). In the bifurcation diagram of Figure 6, for example, it appears that the transition from oscillatory to non-oscillatory dynamics occurs at a harvest value near $8.25*10^7$, while the actual transitional Class III does not occur until harvest levels reach $8.82*10^7$. When H = $8.25*10^7$, there is short term non-oscillatory behavior of the system as seen in the figure, but the dynamics is still oscillatory once system gets close to the equilibrium. This oscillation cannot be seen in the bifurcation diagram in the Figure since the oscillations are too small for the scale of the diagram. Note also that the precise short term approaches taken by the system to the stable equilibria will depend on the particular initial value X_0 chosen to produce the diagram.

[10] Harvest response enhancement could also be achieved via control of cost parameters (and thereby influencing $APC(X_t)$) or via control of both demand and $APC(X_t)$. For this study, however, $APC(X_t)$ and its sensitivity to stock level changes is completely determined by cost-related parameters reported in Cook and Copes. See Yohe (1984).

stability. Computing the stability index for the base case bioeconomic equilibrium in this case involves both the growth factor and the harvest response effect since demand is less than perfectly inelastic. Using Equation (4), at point A_{PHF} the growth factor is 0.913. The value for $H'(x_e)$ may be calculated by taking the derivative of Equation (7) with respect to X_t:

$$H'(X_t) = \frac{(2d_1q^2X_t - C_1q)(d_2q^2X_t^2 + C_2) - (d_1q^2X_t^2 - C_1qX_t)(2d_2q^2X_t)}{(d_2q^2X_t^2 + C_2)^2} \tag{8}$$

and substituting relevant values for a_{PHF} from Table II, yielding $H'(x_e) = 0.528$. This results in an escapement response factor $(1 - H'(x_e)) = 0.472$, and a stability index of 0.431. These results may be used to locate point a_{PHF} on Figure 5. The bioeconomic equilibrium for the PHF model as specified by Cook and Copes is therefore stable/non-oscillatory, Class II. Enhancing the harvest response, $H'(X)$, while preserving the bioeconomic equilibrium at point A_{PHF} in Figure 3 implies a horizontal move to the right in Figure 5, from point a_{PHF}. The extent of this move depends on the degree of market responsiveness possible as perfectly elastic demand is approached. Given C_1 and C_2, maximum possible responsiveness occurs where $d_1 = P_e$ and $d_2 = 0$. In this case,

$$H'(x_e)|_{d2=0,\ d1=Pe} = (2P_eq^2x_e - C_1q)/C_2. \tag{9}$$

Using base case parameters, Equation (9) indicates $H'(X)_{max} = 5.33$. With the growth factor unchanged at 0.913, this indicates this extreme stability index is -3.95. This means that dynamics at the bioeconomic equilibrium point A_{PHF} of Figure 3 could range from Class II through Class V, point $â_{PHF}$ in Figure 5. Severe harvest overshoot, resulting in chaotic dynamics of Class V appears possible without disturbing the bioeconomic equilibrium point[11]. This result is confirmed by the bifurcation diagram of Figure 7 with the slope d_2 as the bifurcation parameter. The gray points in Figure 7 represent stock levels X_0 through X_{50}. The black points represent observations of longer term behavior. The model is initialized in each case at $X_0 = 98,820,000$. As $H'(x_e)$ is increased by reducing d_2, the number of periods to x_e is reduced. At $d_2 = 4.76*10^{-9}$, $H'(x_e) = 1$, and $dT/dX = 0$.

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[11] This prediction is interesting because the predicted instability is rooted in harvest overshoot and not in the growth factor. Others have noted that harvest activity can generate instability in simple discrete-time bioeconomic models. Opsomer and Conrad (1994); and Hilborne and Walters (1992), specify models which contain two difference equations: one non-linear stock equation and one linear effort equation in which current period effort adjusts in response to previous period profit. Both of these models contain a sensitivity parameter for the effort difference equation. Variation in the effort sensitivity parameter has an impact on the dynamics of the model that is similar to the effect that variation in the intrinsic growth parameter in the stock difference equation can have. Because the effort difference equation is linear in these models, however, they become explosive after achieving a periodic character.

In contrast our analysis is based on a model with only one difference equation (stock). Harvest adjusts as current period stock level changes impact on the stock dependent current period linear supply, $APC(X_t)$. Even though there is no explicit harvest or effort adjustment difference equation in our model, we still find potential instability rooted in the harvest sector.
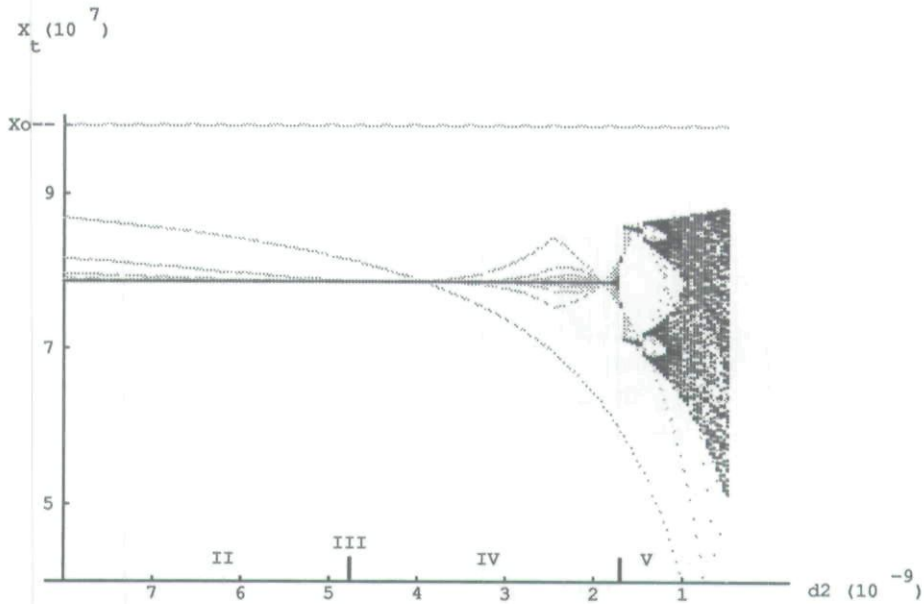
x_t (10^7)

Figure 7. Modified Bifurcation Diagram for the Pacific Halibut Fishery Model at A_{PHF}. Bifurcation Parameter: d_2, $X_0 = 9.882 \times 10^7$.

As $H'(x_e)$ is increased beyond 1, mild harvest overshoot transforms the system into the dynamics associated with a Class IV stable/oscillatory equilibrium. Notice that the approach to stable long-term bioeconomic equilibria in this range involves damped oscillation around the equilibrium stock, x_e. Notice also, the systematic adjustment in these patterns as the model moves through Class IV. The system then moves into the unstable dynamics of Class V, first via periodic and then chaotic long term behavior, ultimately resulting in extinction. Thus we confirm the result predicted from analysis of Equation (2) and Figure 5, that it is possible to drive the PHF model into chaotic vibrations and extinction via changes in market demand conditions without imposing unreasonable values for the intrinsic growth parameter, g, and without disturbing the estimated bioeconomic equilibrium point on the model's open access sustainable supply locus[12].

Examination of Figure 5 suggests the opposite effect on model stability may be possible under extreme growth factor conditions. Note that enhancing the harvest response effect beginning at point b_2' on Figure 5, (where $dT/dS_t = -1.903$, and $H'(x_e) = 0$) and moving horizontally to point $\hat{b}_2$ suggests that the same type of rotation in ex-vessel market demand (i.e. reducing the slope) as performed in moving from point a_{PHF} to $\hat{a}_{PHF}$ will instead move the system from Class V down

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[12] It should be noted that instability has been documented in commodity models which don't involve biological growth processes. Instability and deterministic chaos has been associated with commodity models with short run backward bending supply curves (Burton, 1993). But note that the backward bending sustainable supply locus of the PHF is a long term locus of equilibria. The market supply curve for any given period in the PHF is linear and upward sloping. Nonlinear cobweb models which consider adjustments over time toward market equilibrium have also been shown capable of generating chaotic dynamics (Jenson and Urban 1984), but this type of market process is not included in the PHF.

into Class I stability conditions. We test this hypothesis by modifying the PHF to have g = 3.5. We will initially assume perfectly inelastic demand (so that $H'(x_e)$ = 0) with H_e = 30,000,000 and will consider the resulting equilibrium at S_e = 100,429,000. Using (4), the growth factor will then have value −1.903, giving the point b_2' on Figure 5. Parameter values and bioeconomic equilibrium values of variables at point b_2' under these conditions are given in Table II.

Figure 8 gives a bifurcation diagram as d_2 is reduced while preserving bioeconomic equilibrium at point B_2 of Figure 4. The gray points in Figure 8 represent stock levels X_0 through X_{50}. The black points represent observations of longer term behavior. As Figure 5 predicts, the stability index is significantly altered as harvest response is enhanced at point B_2 via demand rotation. Notice how moderately increasing the market response in this case first pulls the system out of chaos from Class V through Class IV, III, and II. Further increases in harvest response forces the system into a different chaotic attractor, when the equilibrium eventually moves into Class I dynamics.

Finally, Figure 5 suggests harvest response enhancement should have no effect on model stability if growth factor is zero. Thus, a horizontal move from point b_4' to $\hat{b}_4$ in Figure 5 will have no impact on model dynamics. An intuitive explanation of this result is possible with the aid of Figure 9. Figure 9 is a graph of Equation (1) with g = 3.5. Note Class III stability requires the bioeconomic equilibrium to occur at that value of S where $T'(S) = 0$. Figure 9 implies that changes in harvest and escapement result in small changes in X_{t+1} at this equilibrium, permitting a stable approach to equilibrium despite the potential for extreme variation in $H(X_t)$. A numerical test of this special case is again summarized with a bifurcation diagram for demand slope d_2 in Figure 10. The gray points in Figure 10 represent stock levels X_0 through X_{50}. The black points represent observations of longer term behavior. Notice that, unlike Figure 8, the Class III
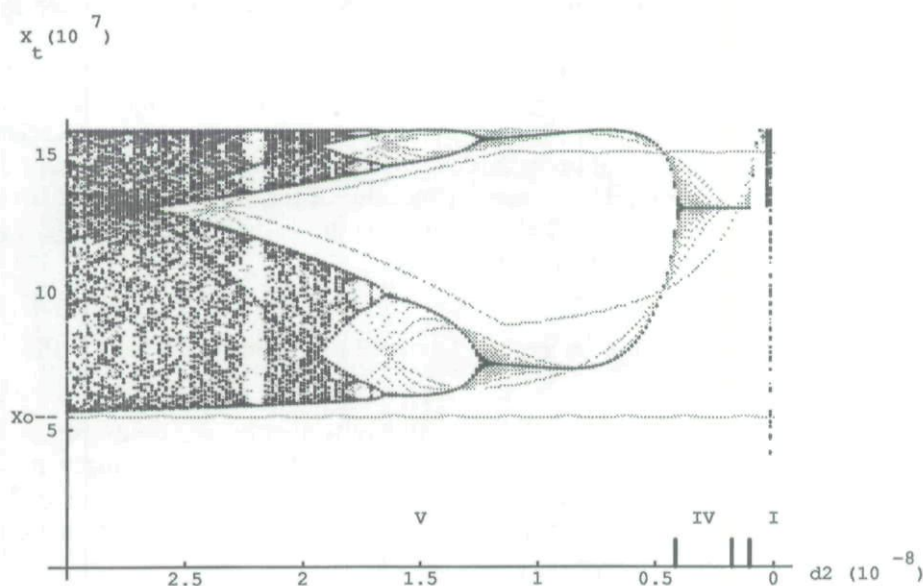

Figure 8. Modified Bifurcation Diagram for the Pacific Halibut Fishery Model at B_2. Bifurcation Parameter: d_2, $X_0 = 5.49 \times 10^7$.
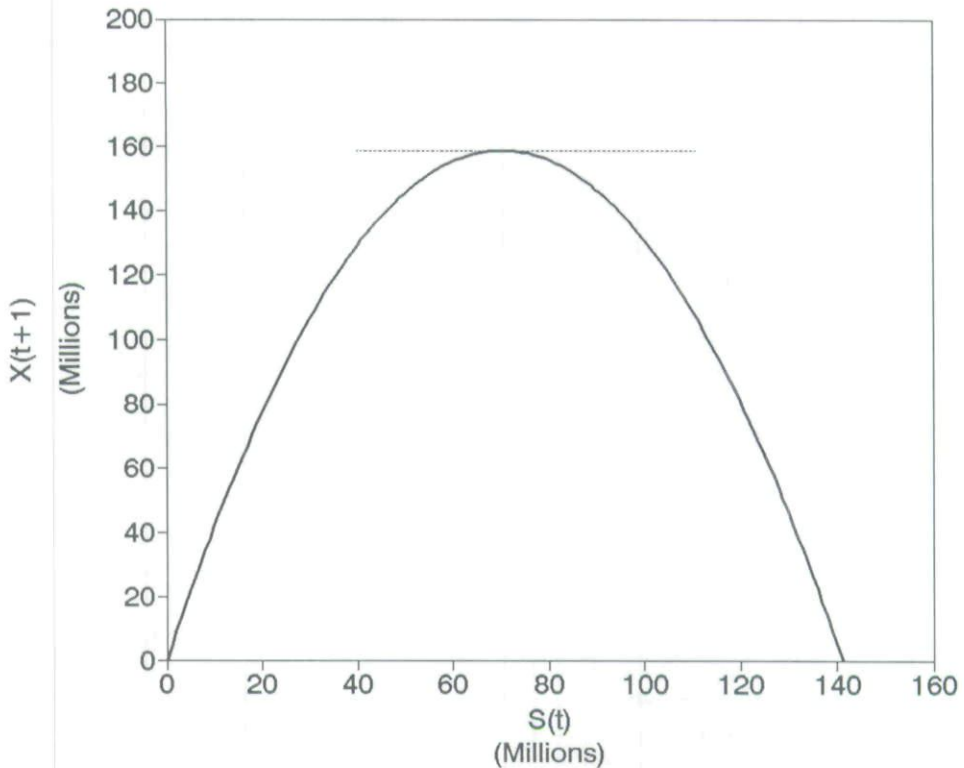
Figure 9. Discrete-Time Stock Transition for the Pacific Halibut Fishery Model with $g = 3.5$.

equilibrium in this case is perfectly preserved for all values of $H'(x_e)$ since the growth factor remains 0.[13]

Thus, we may conclude that increasing harvest response by changing the slope of the demand curve while preserving bioeconomic equilibrium may have profoundly differing impacts on stability in discrete-time renewable resource models. The actual direction and magnitude that enhanced harvest response may have on model stability depends on the accompanying value of the growth effect, which in turn is due to the value of g and the position along the sustainable supply locus at which bioeconomic equilibrium occurs.

Hybrid Impacts: Effects of Increased Demand on Stability of the Pacific Halibut Fishery Model

We have explored two distinct ways harvest activity may impact on dynamics of a harvested resource. The first of these is the growth effect tied to changes in the intersection point along the sustainable supply locus with perfectly inelastic demand. The second has been referred to as the market harvest response factor tied

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[13] Due to round-off error, the precise Class III equilibrium (with growth factor 0) could not be achieved in the numerical simulation. The actual growth factor attained in Figure 10 was on the order of $10^{\wedge}(-16)$.
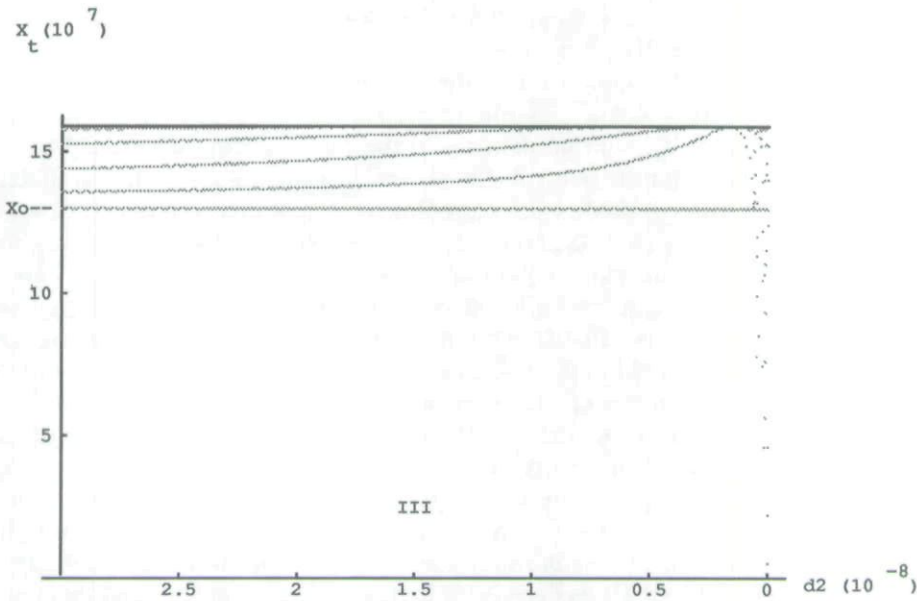
X_t (10^7)

Figure 10: Modified Bifurcation Diagram for the Pacific Halibut Fishery Model at B_4. Bifurcation Parameter: d_2, $X_0 = 13 \times 10^7$.

to variation in market supply and demand slopes through a fixed point on the sustainable supply locus.

Hybrid market response effects are possible when less than perfectly inelastic demand shifts, providing changing points of intersection with the sustainable supply locus. In this case, with demand slope held constant, any changes in market-related harvest response are more directly rooted in supply and costs. As the bioeconomic equilibrium stock is drawn down, it is possible to show that harvest tied to a Schaefer production relation becomes more responsive to stock level changes. This can be seen by taking the derivative of Equation (6) with respect to X_t:

$$\frac{dAPC}{dX_t} = \frac{-C_1}{qX_t^2} - \frac{2C_2H_t}{q^2X_t^3}.$$ (11)

For a given H_t, Equation (11) indicates $APC(X_t)$ becomes more sensitive to stock level changes as the stock is drawn down. This suggests that the harvest response effect will increase as slope-preserving right-ward shifts in demand lead to new bioeconomic equilibria along the sustainable supply locus. In addition to the impacts on market supply response, results from the fourth section suggest the growth effect also has an impact on system dynamics as the intersection point along the sustainable supply locus changes. It is generally difficult to distill the influence that each of these separate factors contributes to changes in the dynamics of the model as demand shifts to the right. However, taken together, it would be expected that this combination of influences on stability would trace a path in a northeast direction away from point a_{PHF} in Figure 5. The reader may verify that here again there is a rich potential for a wide variety of dynamics resulting from

such a transition. We test this hypothesis based on our stability index with numerical analysis of the PHF model.

We again begin with equilibrium in the Pacific Halibut model as specified by Cooke and Copes (point A_{PHF} in Figure 3, point a_{PHF} in Figure 5). We then increase the value of d_1, shifting demand to the right, leading eventually to the bioeconomic equilibrium at point A_2 in Figure 3. Table II gives the parameter values and equilibrium variable values at point A_2. Equations (1) and (3) are again used to calculate $dT/dS_t = 1.45$ and $H'(x_e) = 3.73$ respectively, for plotting point $â_2$ in Figure 5. The stability index path tied to shifts in d_1 from 0.6 to 2.0 is given by the nonlinear path $a_{PHF} \rightarrow â_2$ in Figure 5. These calculations imply that the potential harvest overshoot that is present in this model rooted in increasing cost and supply fluctuations along the backward bending portion of the supply locus may be sufficient to generate chaotic dynamics. This possibility is explored in the bifurcation analysis for this model with d_1 as the bifurcation variable presented in Figure 11. The gray points in Figure 11 represent stock levels X_0 through X_{50}. The black points represent observations of longer term behavior. As the bioeconomic equilibrium point moves away from A_{PHF} of Figure 3, in response to initial rightward shifts in demand, the equilibrium stock levels begin to fall but stability is enhanced, reaching Class III as $d_1 \approx 0.75$. Increases in demand beyond this point are destabilizing moving the model into a stable/oscillatory character of Class IV. When d_1 is increased to 1.052, the equilibria move into Class V, displaying periodic, and then chaotic oscillations. Eventually with further increases in demand, these oscillations become so great that the stock is driven to extinction. Clearly this extreme instability again has its roots in the harvest sector, not the population
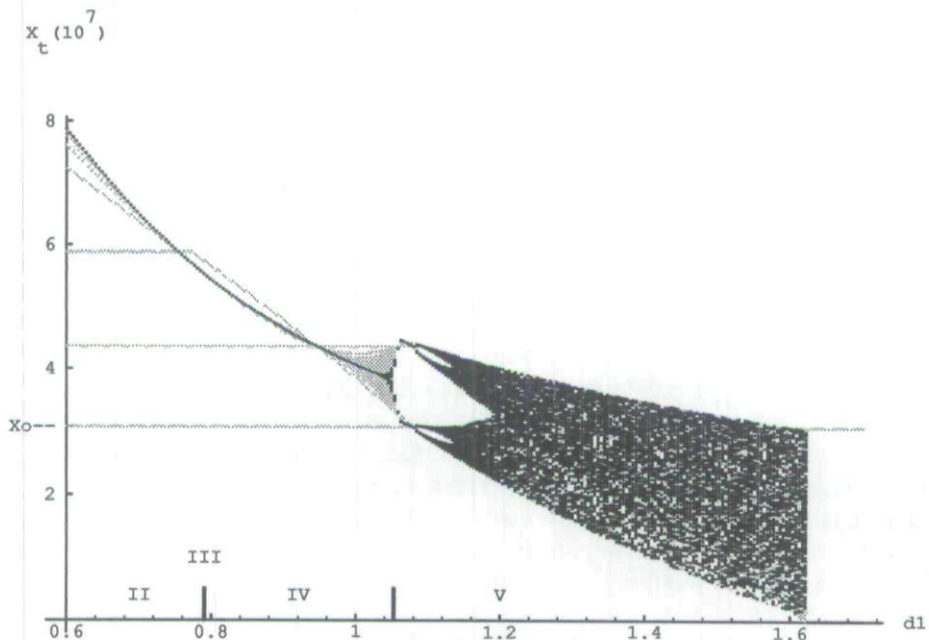

Figure 11. Modified Bifurcation Diagram for the Pacific Halibut Fishery Model Over the Range $A_{PHF} - A_2$. Bifurcation Parameter: d_1, $X_0 = 3.0744 \times 10^7$.

dynamics of the stock, and is tied to the extreme sensitivity that model supply parameters exhibit at lower stock levels.

Conclusions and Discussion

We develop a simple method for evaluating the stability of discrete-time renewable resource models. Application of the method predicts that open access market-based harvest activity may be just as influential as stock growth characteristics in determining the stability of discrete-time renewable resource models. Actual stability is shown to depend on the interplay of population dynamics and market conditions. Under certain market and growth conditions, harvest can be dramatically destabilizing. Notably, even when intrinsic growth is well within the ranges commonly observed in existing fisheries around the world, chaotic fluctuations in stock size can occur in a wide class of discrete-time renewable resource models based on market-oriented harvest. These predictions suggest that potential chaos may be lurking in unexpected places in renewable resource models when harvest is market-driven. In testing these predictions in the context of the Pacific Halibut fishery model, we find that reasonable adjustments in the slope of the demand schedule can thrust the model into instability, chaos, and extinction without changing the bioeconomic equilibrium point reported by Cook and Copes. We also show that reasonable slope-preserving increases in market demand can push the model into instability, chaos, and eventually extinction as well.

The methodology also predicts that under other growth conditions, market oriented harvest may also have a profoundly stabilizing influence, even when applied to stocks characterized by extreme intrinsic growth. In testing these predictions in the context of the Pacific Halibut fishery model, we find that reasonable adjustments in the slope of the demand schedule can stabilize a version of the Pacific Halibut Fishery model that otherwise displays highly chaotic behavior rooted in powerful intrinsic growth of the stock.

Finally the methodology predicts and our numerical analysis confirms, that under certain growth conditions, harvest activity may have little or no impact on model stability under equilibrium-preserving adjustments in market demand.

The potential for instability rooted in market-driven harvest overshoot as demonstrated in the Pacific Halibut Fishery model is particularly interesting. It is well-known that a sufficiently large increase in fishing effort may result in stock extinction in models with linear Schaefer production function components such as the PHF model. The results presented here, however suggest that myopic profit maximizing harvest activity responding to current period market incentives, is actually capable of generating such results, but prefaced with extreme instability and chaos. Most important, unlike instability rooted in the growth factor, the range of model parameters required for such results appear economically and biologically feasible. This is especially true for bioeconomic equilibria that lie along the backward-bending portion of the open access sustainable supply locus. These equilibria are associated with significantly depressed stocks levels on the one hand, and with excessive inputs available for fishing effort due to open access conditions on the other. Here, it would seem that market-driven harvest responsiveness capable of generating instability would be quite plausible over a wide range of market demand conditions.

It is important to interpret the significance of these results with care. We do not intend to suggest that the Pacific Halibut Fishery is on the verge of displaying chaotic behavior. We only assert that the model reviewed here is capable of such behavior under a variety of plausible market conditions. Severe harvest overshoot and the resulting extreme impact on stability is in part due to the linear production function, the full attainment of market equilibrium in each period, and the discrete way in which costs adjust to changes in stock conditions. We do suggest that chaotic dynamics can not be ruled out for many fisheries around the world and do not require prohibitively powerful intrinsic growth when subject to open access market-oriented harvest. Whether or not we should expect chaotic dynamics in a specific fishery will be determined at least in part by the actual responsiveness in the harvest sector to changing market incentives over time.

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Appendix I: Alternative Intra-Period Sequencing Specifications: Impact On Model Dynamics

It is also possible to specify the discrete time model discussed in the text with the intra-period sequencing *reversed:*

a) Stock, X_t is revealed,

b) Growth occurs resulting in $A(X_t)$ available for harvest phase of period t, and

c) Harvest based on $A(X_t)$ takes place.

Here, $X_{t+1} = T(X_t) = A(X_t) - H(A(X_t))$, and with logistic growth, $A(X_t) = X_t + gX_t(1 - X_t/K)$.

Ignoring the sequencing issue altogether leads to a stock transition equation that is commonly used for continuous time models but is internally inconsistent for discrete-time formulations:

$$X_{t+1} = T(X_t) = G(X_t) - H(X_t) \tag{A1}$$

In this specification, harvest is based on the stock level left over from the previous period. At the *same time* it calls for growth to take place on the stock left over from the previous period. Clearly these two discrete processes cannot depend on the *same* stock unless the time interval within which they occur becomes exceedingly small, as in a continuous time framework.

Notice that when the sequencing is reversed appropriately, then

$$dT/dx_e = (dT/dA)(dA/dx_e) = (1 - dH/dA)(dA/dx_e) \tag{A2}$$

The resulting dynamics with this sequencing specification are thus shown to be qualitatively the same as those discussed in the text based on Equation (6) for escapement-based growth. With this intra-period sequencing of harvest and growth, the first factor on the right-hand side of A2 is the *inter-period escapement response factor* while the second is the *growth factor* as in the text. The analysis of Equation (6) presented in this paper may be considered general for properly specified single stock discrete time models. When the discrete-time stock transition function is erroneously specified as in (A1), then

$$dT/dx_e = G'(x_e) - H'(x_e),$$

suggesting *different* dynamic adjustments in the model. Since this specification is internally inconsistent in the discrete-time context, the dynamics of this specification are not explored in this paper.

Appendix II: Deriving the APC schedule

Open access supply in any period depends on the level of fishing effort allocated in that period, and the initial, pre-harvest stock level:

$$H_t = qE_tX_t, \tag{A3}$$

where: q is the catchability coefficient, and E_t is fishing effort in period (t). Cook and Copes note that "in the Halibut Fishery, the unit of effort is the 'skate soak' where the standard skate consists of 1800 feet of groundline with hooks attached at 18-foot intervals. The hooks are attached by means of 'gangions' which extend approximately five feet from the ground line. On average, skates are left to fish ('soaked') for twelve hours before being hauled".

Following Anderson (1982), total pecuniary costs of fishing effort are defined

as those costs both explicit and implicit (including a normal profit) which produc-
ers must bear for the services necessary to allocate a specific amount of fishing
effort:

$$TPC(E_t) = C_1 E_t + C_2 E_t^2, \text{ for positive constants } C_1, C_2 \qquad (A4)$$

The cost of fishing can be expressed in terms of *harvest* by solving (A3) for E_t and
substituting the resulting expression into (A4):

$$TPC(H_t, X_t) = s_1(X_t) H_t + s_2(X_t) H_t^2, \text{ where} \qquad (A5)$$

$$s_1(X_t) = C_1/q X_t, \text{ and } s_2(X_t) = C_2/q^2 X_t^2$$

Equation (A5) is used to derive Equation (6) in the text.