# The Effect of Fishermen's Quotas on Expected Catch Rates 

Colin W. Clark<br>Department of Mathematics University of British Columbia Vancouver, Canada


#### Abstract

Fishermen's quotas have the effect of truncating catches at the quota limit. Hence the expected catch is smaller than the quota. A simple search model is developed that provides an estimation of the factor by which expected catches are reduced.


The use of allocated fishermen's quotas as a method of counteracting the common property externality in commercial fisheries has been suggested by several resource economists (Christy, 1973; Moloney and Pearse, 1979; Clark, 1980). It is known that such quotas, provided that they are freely transferable, are formally equivalent to landing taxes (Moloney and Pearse, 1979; Clark, 1980). "Formally equivalent" means, in particular, equivalent in a deterministic world; the nonequivalence of quotas and taxes in a stochastic setting has been demonstrated in a general context by Weitzmann (1974).

When individual quotas are recommended for actual implementation, considerable resistance on the part of fishermen is often encountered. It seems that many fishermen rely on the occasional lucky catch, and they foresee that a quota would prevent them from realizing this important irregular windfall.

In this brief note a simple search model (Mangel and Clark, 1983; Mangel and Plant, 1985) is used as a vehicle for estimating

[^0]the expected reduction in average catch rates resulting from the introduction of an individual quota system. The degree of reduction depends on several factors, including the patchiness of the distribution of fish, the time period over which quotas apply, and the extent to which groups of fishermen pool their quotas. An interesting theoretical exercise would be to model the dynamics of a short-term quota market as an alternative to the pooling of quotas (since pooling obviously brings in the possibility of the "free rider"), but we do not attempt this exercise here.

Because of the resulting reduction in average catch rates, fishermen's quotas may be less effective in improving economic efficiency than would be concluded from the deterministic theory. (The fact that quotas do reduce catch rates also establishes the nonequivalence of quotas and taxes, since catch rates would not be affected by taxes.) This is not to say that individual quotas should be thrown out of consideration for fishery management, no other feasible instrument having yet been invented to resolve the serious externality problem. It does become apparent, however, that individual quota systems must be carefully devised, with maximum flexibility. The pooling and transfer of quotas should clearly be permitted and indeed facilitated. If not, the resulting system may well prove to be more inefficient than the system it was designed to replace.

## Mathematical Analysis

Search theory has been applied to fishery problems in a series of recent papers (Swierzbinski, 1981; Mangel, 1982; Mangel and Clark, 1983; Mangel and Beder, 1984; Mangel and Plant, 1985). In these works the process of searching for fish has been modeled as a Poisson process. A more general model, which does not presuppose a uniformly random distribution of the objects of search, is provided by the negative-binomial distribution (Pielou, 1977), in which

$$
\begin{equation*}
p(n ; x, k)=\frac{\Gamma(n+k)}{k^{n} \Gamma(k)} \frac{x^{n}}{n!}\left(1+\frac{x}{k}\right)^{-(n+k)} \tag{1}
\end{equation*}
$$

Here $p(n ; x, k)$ denotes the probability of encountering $n$ objects (e.g., schools of fish) during a given period of search; $\Gamma(x)$ is the gamma function. The negative binomial distribution can be derived from a model in which search itself is a Poisson process, but the objects of search occur in clumps, or patches, of logarithmically distributed size. Both parameters $x$ and $k$ in Equation (1) are directly proportional to the search rate parameter $\lambda$ of the underlying Poisson process, with coefficients depending on the parameter $\alpha$ of the logarithmic distribution (Pielou, 1977, p. 120). The search rate in turn can easily be related to physical quantities, including vessel speed, width of the search track, and probability of detection (Mangel and Beder, 1984). In particular, if $N$ searchers are searching independently, the parameters $x$ and $k$ in Equation (1) are replaced by $N x$ and $N k$, respectively. Similarly, if the length of the search period under consideration is lengthened by a factor $T$, then the parameters $\lambda, x$, and $k$ are also all multiplied by $T$.

The variance of the negative binomial distribution is given by

$$
\begin{equation*}
\sigma^{2}=x+x^{2} / k \tag{2}
\end{equation*}
$$

so that the parameter $k$ is a measure of the extent to which this distribution differs from the Poisson distribution (for which $\sigma^{2}$ $=x$ ). Small values of the "patchiness" or "contagion" parameter $k$ correspond to a highly clumped distribution of objects of search, and vice versa; as $k \rightarrow \infty$ the negative binomial distribution approaches the Poisson distribution.

We shall base our computations on the negative binomial distribution, although the calculations could as easily be performed by using any other desired probability distribution.

Consider, then, an individual vessel searching for schools of fish. In the absence of any individual quota, the expected catch rate is $x$ schools per unit of time, and the probability of catching $n$ schools in the given time unit is $p(n ; x, k)$. (The analysis refers only to search time; time spent setting on schools that have been encountered is not included.)

In the presence of a quota of $Q$ schools per unit time period, the probability distribution of Equation (1) becomes truncated:

$$
p(n ; x, k, Q)= \begin{cases}p(n ; x, k) / C_{Q} & n \leq Q  \tag{3}\\ 0 & \text { otherwise }\end{cases}
$$

where the normalization factor $C_{Q}$ is given by

$$
\begin{equation*}
C_{Q}=\sum_{n=0}^{Q} p(n ; x, k) \tag{4}
\end{equation*}
$$

The expected catch, subject to the quota constraint, thus becomes

$$
\begin{equation*}
E\{n ; Q\}=\sum_{n=0}^{Q} n p(n ; x, k, Q) \tag{5}
\end{equation*}
$$

It is easy to see that $E\{n ; Q\}<E\{n\}=x$, and also that $E\{n ; Q\}$ $\rightarrow x$ as $Q \rightarrow+\infty$. Thus the quota constraint does decrease the individual's expected catch per unit time, the more so the smaller the quota.

To be explicit, suppose for example that the quota $Q$ is exactly equal to the average catch $x$ without quota. Let $f$ denote the fractional reduction in catch brought about by this quota constraint:

$$
\begin{equation*}
f=f(k, Q)=1-\frac{1}{Q} \sum_{n=0}^{Q} n p(n ; Q, k, Q) \tag{6}
\end{equation*}
$$

The values of this reduction factor, as a function of $Q$ and $k$, are shown in Figure 1.

As expected, the reduction factor $f$ increases as the quota $Q$ decreases, and also as the patchiness of the fish stock increases. Note also that for highly patchy distributions, $f$ does not appear to approach zero as $Q \rightarrow \infty$. (As noted above, $f$ does approach


FIGURE 1. Reduction factor $f=f(k, Q)$ as a function of quota $Q$, for various values of the patchiness parameter $k$.
zero as $Q \rightarrow \infty$, for fixed $x$, but here we are treating $x=Q$ as a single parameter).

Next, consider the possibility of pooling quotas. Suppose that $N$ fishermen, each having a quota of $Q=x$, agree to pool their quotas to obtain $Q_{\text {total }}=N x=N Q$. The resulting truncated distribution then becomes $p\left(n ; Q_{\text {total }}, N k, Q_{\text {total }}\right)$. The reduction factor $f$ can again be read from Figure 1 (or computed from Equation (6)) by using the value $N k$ in place of $k$. For example, with $k=1.0, Q=4$, and $N=5$, the values of $f$ are:

Unpooled: $f=0.61$.
Pooled: $\quad f=0.33$.
The importance of flexibility in allowing pooling or transfer of quotas is emphasized by this one example.

Analogous considerations apply to the pooling of quotas over time periods. For example, a quota of 10 tons per week might
be administered as a total quota of 100 tons over a 10 -week period. The reduction factor $f$, as computed from Equation (6) with $Q=10$ replaced by $Q=100$, and $k$ replaced by $10 k$, would be smaller as a result of this time pooling.

From this point of view it would appear that time pooling of quotas is always desirable. There may be other reasons, however, for preferring short quota periods. Long-period quotas may result in uneven deliveries of fish to processing plants or markets, for example.

In order to estimate the reduction factor $f$ corresponding to any particular quota arrangement, the management authority requires an estimate of the patchiness parameter $k$. Such an estimate can be derived from data on the actual catches of individual (similar) vessels. If $\bar{x}$ denotes the average catch per week (for example), and if $\hat{\sigma}^{2}$ is the variance, then from Equation (2) we obtain the estimate

$$
\begin{equation*}
\hat{k}=\frac{\bar{x}^{2}}{\hat{\sigma}^{2}-\bar{x}} \tag{7}
\end{equation*}
$$

Note, however, that Equation (7) can only be applied to data obtained before the imposition of quotas.

The quota $Q$ (individual or pooled) and its corresponding total catch $\bar{Q}$ are related by the equation

$$
\begin{align*}
\bar{Q} & =[1-f(k, Q)] Q \\
& =\sum_{n=0}^{Q} n p(n ; Q, k, Q) \tag{8}
\end{align*}
$$

The expression on the right side of Equation (8) is graphed in Figure 2 for various values of $k$. Given $\bar{Q}$ and $k$, the appropriate quota $Q$ can be read from this figure.

Although the curves shown in Figure 2 are not actually straight lines, they are obviously closely approximated by such. Thus a simple approximation to Equation (8) is


FIGURE 2. Expected catch $\bar{Q}$ versus quota $Q$ for various $k$.


FIGURE 3. The coefficient $\alpha(k)$; see Equation (9).

$$
\begin{equation*}
\bar{Q}=\alpha(k) Q \tag{9}
\end{equation*}
$$

The function $\alpha(k)$ is plotted in Figure 3.

## Summary

Because fishing is, to a certain degree, a random process, it follows that a quota constraint on individual vessels (or fishermen) will, by removing the chance of a large catch, reduce the expected catch per vessel to a value less than the actual quota. The greater the variability in catches, the greater will be the reduction, which may be particularly severe unless pooling or transfer of quotas (among vessels, or over time periods, or both) is freely permitted.

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