Uncertainty in Fisheries Economics: The Role of the Discount Rate

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Abstract Standard models of management of a single-species fishery generally assume that the biomass is of known size and that it is generated by a well-specified deterministic growth law. In reality the biomass is of uncertain size and usually subject to random growth. Several authors have addressed the problem of random growth assuming a known initial biomass and have shown that lowering the planning discount rate proportional to the variance is an optimal planning procedure assuming small perturbations.

In this paper we assume that the growth function is nonrandom but dependent upon a biomass stock of unknown size. We shall show that a planner should raise the discount rate relative to the certainty equivalent case by an amount related to society's distaste for risk in order to manage the biomass optimally over time. As is to be expected, the optimal steady-state biomass will be less than would occur in a situation of certainty.

Introduction

Probably the most serious problem in fish stock management is a lack of information about stock size. Seldom, if ever, are there

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attempts to obtain estimates through sampling. Yet most current models of fishery management, both deterministic and stochastic, proceed as if the initial stock size number were known with certainty. In this paper we shall address the problem of random stock size in an otherwise deterministic environment, and we shall show how to accommodate stock size uncertainty.

The Stock Problem

At the firm level in the fishing industry a competitive firm A at time t_0 will be reluctant to leave any profitably exploitable stock unharvested for two reasons: other firms may harvest this stock at time t_0, and even if the unit of stock survives until time t_1, firm A is only one of many firms in competition to harvest this resource at time t_1. Such "stock externalities" have been internalized over time in some cases by such methods as fencing and branding as part of the process of domestication. However, the usual methods of internalization are prohibitively costly for ocean and migratory fish stocks. Consequently, licensing and seasonal and quota limits have served as surrogates to appropriation.

As if the problems of management were not difficult enough, two further complications arise. One involves the existence of predators other than humans which also "harvest" an exploitable stock. A unit of stock left unharvested to grow (or reproduce) might simply fall victim to a predator such as a seal. Alternatively, a unit of stock left unharvested may be lost to commercial fisheries through a random occurrence of nature (such as an oil spill). For some species this randomness may be caused by influences as simple as an unpredicted change in water temperature. From a practical point of view, a manager may consider these two types of complications as random disturbances on the stock. Note as well that most fish stocks are unobserved. At best, estimates are made of their sizes (biomasses), and in general the only data that are available come through records of catch size.

Intergenerational Issues

Natural resource economics, like most of economics, deals with trading off the interests of one set of individuals against those of another. The usual modeling involves trading off across different generations in the process of intertemporal maximization. Thus a resource planner must address the following problems:

Is the welfare of an individual in generation t_1 to be evaluated with the same weight as an individual of generation t_2? There is a long tradition in growth economics favoring the concept of intergenerational equity. We usually associate the names of Ramsey (1928), Gale (1967), and Koopmans (1965, 1974, 1977) with this concept (to name only a few). A recent article by Solow (1974) directly addresses this equity issue in a model where an exhaustible resource is to be allocated among generations.

If a resource manager assumes intergenerational equity, several avenues of approach are available. One is the simple maximization of the integral of an undiscounted stream of utilities from the planning date to an infinite horizon. However, this approach leads to convergence difficulties. A second approach, which avoids these difficulties, is to minimize the integral of divergence of individual welfare from some steady-state (bliss) level to achieve a finite summation. This is referred to as the "overtaking principle" because one path eventually "overtakes" another. A third approach is to use the Rawlsian criterion, which requires the maximization of the minimum welfare of any individual of any generation. This approach was used by Solow (1974). In addition to the concept of intergenerational equity, there is a long history in growth economics of assuming that generations nearer the planning date will be weighted higher in the social welfare function than (equally sized) generations at more remote dates in the future. This routine of discounting the welfare of future generations is mathematically appealing and has some ethical justification.[1]

Depending upon the objective function chosen, the discount rate is chosen to be the prevailing interest rate or the social rate of time preference.[2] This social discount rate can be interpreted in terms of claims upon future goods: the higher the social rate of discount, the greater the present generation's claim upon future goods.

Deterministic Models: The Role of Discounting

In deterministic models of fishery exploitation one consideration is whether "too much" discounting will lead to species extinction. A related consideration is that high discount rates represent a lack of concern (on the part of a planner) for future generations. The welfare of future generations may not be given sufficient weight if a planner is unable to plan—for example, in an open access fishery—or where a fish stock migrates over international jurisdictions and hence property rights are not established. Although it is usual to evidence discounting in deterministic models of commercial fishing,—that is, discounting of either utilities or rents as, for example, in Plourde (1970, 1971), Clark (1976), or Clark and Munro (1975)—there are many cases where discounting does not occur. In some instances the choice seems to be made on the basis of mathematical convenience rather than ethics.

While a defense can be presented for either the mathematical or the ethical choice for discounting, a defense of any type of discounting becomes more important when uncertainty is introduced into the environment. Put another way, in an uncertain or random world, whether a planner discounts future rents or not is relatively more important for several reasons. Suppose the stock size of a replenishable resource such as fish is random. For illustrative purposes, suppose random weather may wipe out 25 percent of the stock. Then socially it may be preferable to consume more of that stock today (as compared to a nonrandom case). The alternative would be to leave that stock unharvested and have it destroyed by an unfavorable realization of nature. Alternatively, in anticipation of beneficial random events (such as favorable water temperature), it may be socially preferable to harvest less (or more) than in a completely deterministic environment. We shall attempt to show that there is theoretical validity in altering discount rates in the face of uncertainty in replenishable natural resource planning.

Some Effects of Randomness in Fishery Management

In the field of replenishable natural resources, the most commonly assumed source of randomness is in the natural reproduction process. For example, unfavorable weather conditions may interfere with the spawning process or the hatching of eggs, or predation by other species may be random. With respect to the analysis of uncertainty in resource economics, the approach has been to follow a path similar to one outlined in the literature on economic growth (as in Merton 1975) and to introduce a "white noise" term into the dynamic analysis. The consensus of the authors who have addressed uncertainty in this way seems to be that randomness can, for practical purposes, be accommodated by using a certainty-equivalent deterministic model appropriately altered. The main alteration proposed is to *lower* the discount rate by an amount directly related to the variance. This rather surprising alteration can be found in J. Barry Smith (1978), Ludwig (1979a), and Ludwig and Varah (1979). Theory suggests that in the face of uncertainty in the growth parameters it is better to err in favor of the future.

Uncertain Stock Size and Discounting: A Model

As previously stated, the major problem in fish stock management is a lack of data concerning the stock size. Let us now assume that an initial biomass N_0 is unknown but has been estimated—or guesstimated. We shall further assume, as a first iteration toward a solution, that the growth process is deterministic.

The economic literature on exhaustible resources contains many recent articles dealing with unknown stock or reserves; typical examples are Loury (1980), Gilbert (1980), and Kemp (1976). The general conclusion seems to have been that one could account for most uncertainty regarding the size of reserves (such as in mining) by using a higher discount rate than under the certainty case. However, Kemp has argued that the practice of raising the discount rate in an uncertain environment has no theoretical justification. We shall follow the spirit of Kemp's paper in our analysis.

Let N_t be the size of the biomass at time t and $\pi(s; 0)$ be the subjective probability that N_0 is least of size s at time $t = 0$. It will be assumed that $\pi(s; 0)$ possesses continuous partial derivatives $(\partial\pi/\partial s)(s; 0)$ and that

$$\pi(0; 0) = 1 \tag{1}$$

$$\lim_{s\to\infty} \pi\,(s; 0) = 0 \tag{2}$$

$$(\partial\pi/\partial s)(s; 0) \leq 0 \tag{3}$$

Furthermore, let instantaneous harvest at time t be represented as h_t, which is assumed to be produced without cost and instantly completely consumed. Thus $h_t = c_t$.

Suppose that the manager of the resource seeks to maximize an integral of discounted expected utilities, where the "instantaneous" utility at time t depends only on the rate of consumption c_t. The resulting function $U(h_t)$ is assumed to be increasing and strictly concave. In addition, the expectation of U, $E[U(h_t)]$ for any N is assumed increasing and strictly concave. Define $f(N)$ to be a deterministic growth function for the resource.

A plan formulated at time $t = 0$ to harvest at a rate $h_t > 0$ at time t will be carried out with subjective probability $\pi(N; 0)$. If the plan is carried out, the utility yield is $U_t = U(h_t)$; otherwise, the yield is $U(0)$, assumed finite, and set equal to zero. The expected utility from the plan is then

$$\int_0^\infty e^{-\rho t} E[U(h_t)]\, dt \tag{4}$$

$$\int_0^\infty e^{-\rho t} U(h_t)\pi(N_t; 0)\, dt \qquad \text{where } \pi \geq 0 \tag{5}$$

Suppose the manager now seeks the harvesting trajectory h_t^*, which maximizes equation 5 subject to the side conditions

$$\dot{N}_t = f(N_t) - h_t \tag{6}$$

and

$$h_t \geq 0 \tag{7}$$

$$\int_0^T [f(N) - h]\, dt \leq N_0 \quad \text{for all } T, N_0 \tag{8}$$

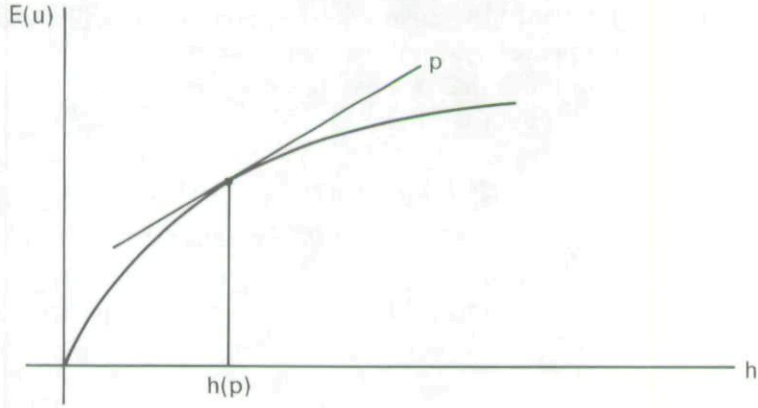

FIGURE 1. Maximization of expected utility.

If equality occurs in equation 8, then extinction is implied. Form the augmented current valued Hamiltonian $\tilde{H}$

$$\tilde{H} = U(H_t)\pi(s; 0) + p[f(N) - h_t] + \lambda \left\{ N_0 - \int_0^T [f(N) - h_t]\, \mathrm{dt} \right\} \quad (9)$$

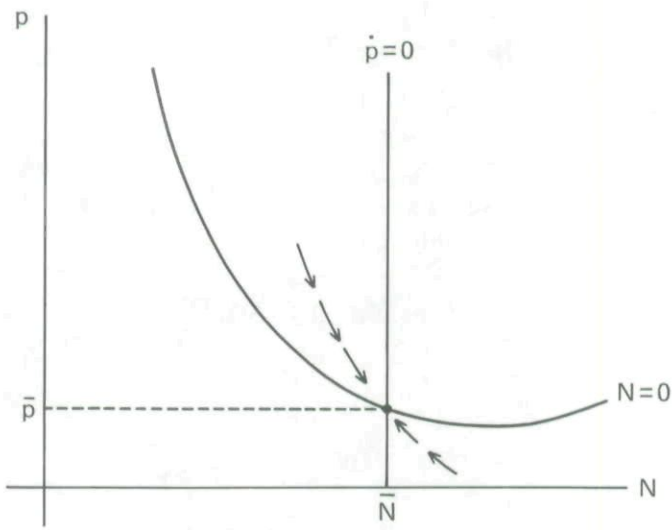

FIGURE 2. Dynamic optimal trajectories.

We consider here only the case where the manager will choose h_t to ensure no species extinction; then $\lambda = 0$.

Assume an optimal trajectory exists satisfying equations 6 and 7. Following Pontryagin we have in addition

$$\dot{p} = p\rho - [U(h_t)(\partial\pi/\partial s)(s; 0) + pf'(N)] \tag{10}$$

$$= p[\rho - f'(N)] - U(h_t)(\partial\pi/\partial s)(s; 0) \tag{11}$$

where $s = N$ and $U'(h_t)\pi(N, 0) - p = 0$. Rearranging equation 11 yields

$$p = U'(h_t)\pi(N; 0) \tag{12}$$

which states that the current-valued implicit price of a unit of fish harvest equals the expected marginal utility.

If we assume that $E(u)$ is increasing and strictly concave for any N, it follows that any p determines a unique value of h, which defines a function $h(p)$, where $h'(p) < 0$ (see Figure 1). It is then possible to write equation 10 as a function of p and N:

$$\dot{p} = F(p, N)$$

where

$$\dot{p} = p[\rho - f'(N)] - U[h(p)](\partial\pi/\partial s)(N) \tag{13}$$

and

$$\dot{N} = f(N) - h(p) = G(p, N) \tag{14}$$

Note that in the case of certainty defined by $(\partial\pi/\partial s)(N) = 0$, equations 13 and 14 collapse to

$$\dot{p} = p[\rho - f'(N)] \tag{15}$$

and

$$\dot{N} = f(N) - h(p) \tag{16}$$

the results of the Plourde (1970) deterministic model. Equations

15 and 16 can be represented in phase space as illustrated in Figure 2.

Let $(\overline{N}, \overline{p})$ be the steady-state equilibrium under certainty. Comparing equations 13 and 15, we may ask whether it is possible to adjust the discount rate ρ in equation 15 to pick up the difference $U[h(p)](\partial\pi/\partial s)(N)$. If it is, then by suitable discount rate adjustments and use of deterministic models, a planner may be able to account for the uncertainty of stock size. The required adjustment involves *adding* a correction factor of size $|(U/p)(\partial\pi/\partial s)|$ to the discount factor ρ. This factor is equivalent in absolute value to the percentage change in π divided by the percentage change in U. It approaches zero as $\partial\pi/\partial s$ approaches zero and/or as the utility function approaches linearity. The first case represents constant subjective prior probability of stock size; the second represents risk neutrality.

It is apparent that the optimal steady-state stock size $\overline{\overline{N}}$ under uncertainty is less than $\overline{N}$ (under certainty) in view of the larger discount factor. Since $U'[f(\overline{N})]\pi(N, 0) = \overline{p}$, from equation 12, it follows that

$$\left.\frac{dp}{dN}\right|_{\overline{N}} = \pi U''f' + U\frac{\partial\pi}{\partial s} < 0 \tag{17}$$

since U'' and $\partial\pi/\partial s$ are nonpositive and π, U, f' are positive. (Observe that f' may be nonpositive for situations involving production costs.) The implication of equation 17 is that the steady-state implicit evaluation of a unit of stock left unharvested has gone down in the face of uncertainty. This is as would be expected, since that unit may not be available later for consumption.[3]

Dynamic Adjustment

Consider now the linearization of $U[h(p)]$ by a Taylor expansion about $\overline{p}$:

$$U[h(p)] \doteq U[h(\overline{p})] + (p - \overline{p})U'h' \tag{18}$$

Substituting equation 18 into equation 13 and collecting terms yields

$$\dot{p} \doteqdot p\{[\rho - (\partial\pi/\partial s)U'h'] - f'(N)\} \\ - (\partial\pi/\partial s)\{U[h(\overline{p})] - \overline{p}\, U'h'\} \quad (19)$$

In equation 13 consider the $\dot{p} = 0$ curve in phase space. Taking total derivatives to find its slope in phase space yields

$$\frac{dp}{dN} = \frac{pf''(N) + U(\partial^2\pi/\partial s^2)}{[\rho - (\partial\pi/\partial s)U'h'] - f'(N)} \quad (20)$$

In Figure 2 the $\dot{p} = 0$ curve is vertical at $\overline{N}$, where $f'(\overline{N}) = \rho$. The net new recruitment of the resources was equal to the discount rate. From equation 20 it is observed that the $\dot{p} = 0$ curve will be vertical but shifted to the left to $\hat{N}$, where $f'(\hat{N}) = \rho^*$ and $\rho^* = -(\partial\pi/\partial s)(U/U'\pi)$ if one of the following conditions holds:

1. The expression $\partial\pi/\partial s = 0$ (the certainty case). But (trivially) $\hat{N} = \overline{N}$ in this case.
2. The variable U is a linear function of h.

If U is concave, it can be shown that the $\dot{p} = 0$ curve will be positively inclined.[4] This will not adversely affect the saddle-point stability properties of $\langle \hat{N}, \hat{p} \rangle$.

Summary

1. Since $\partial\pi/\partial s \leq 0$ and $h' < 0$, it follows that $p^* > \rho$. The discount rate is to be adjusted upward as expected, not downward as suggested by the results of Ludwig (1979a,b) and J. Smith (1978) as discussed earlier for the case of stochastic growth (rather than random stock size).
2. In Figure 2, since the $\dot{p} = 0$ curve shifts to the left, there results a smaller equilibrium stock with a lower imputed steady-state price.

Model with Production. Introduction of production costs complicates the analysis significantly, but in fact the conclusion is basically the same. Following the model and notation of Plourde (1971), we can define the problem as

$$\max \int_0^\infty e^{-pt}\{EU(C_1) + V(C_2)\}\, dt \tag{21}$$

subject to

$$\dot{N} = f(N) - F(N, L_1) \tag{22}$$

$$C_2 = g(L_2) \tag{23}$$

$$C_1 = F(N, L_1) \tag{24}$$

$$\overline{L} = L_1 + L_2 \tag{25}$$

The consumption of C_1, resource product, is stochastic. It is assumed that the other good in the economy is provided with certainty, as defined by equation 23.

In the absence of stock extinction, maximization of the following current-valued Hamiltonian

$$\begin{aligned} H = {} & U(C_1)\pi(s; o) + V(C_2) + p[f(N) - F(N, L_1)] \\ & + \gamma_1[g(L_2) - C_2] \\ & + \gamma_2[F(N, L_1) - C_1] + w[\overline{L} - L_1 - L_2] \end{aligned}$$

yields the following (instantaneous) first-order conditions:

$$U'\pi(N; 0) - \gamma_2 = 0 \tag{26}$$

$$V' - \gamma_1 = 0 \tag{27}$$

and

$$\gamma_1 g' = w = (\gamma_2 - p)F_2 \tag{28}$$

Observe that equation 28 is an implicit function which can be written explicitly as

$$L_1 = R(N, p) \tag{29}$$

In the same manner, the maximization of equation 21 yields

$$\dot{p} = p[\rho - f'] - (\gamma_2 - p)F_1 - U(\partial\pi/\partial s)(N; 0)$$

Certainty will occur when

$$\pi(N; 0) = 1 \quad \text{and} \quad (\partial\pi/\partial s) = 0$$

Taking the case when $f(N) = \lambda N - \epsilon N^2$ for purposes of illustration, the steady-state-stock value will be

$$\tilde{N} = \frac{1}{2\epsilon}\left[\lambda - \rho + \frac{(\gamma_2 - p)}{p} F_1 + \frac{U}{p}\frac{\partial\pi}{\partial s}\right] \tag{30}$$

$$= \frac{1}{2\epsilon}\left[\lambda - \rho^* + \frac{(\gamma_2 - p)}{p} F_1\right] \tag{31}$$

where

$$\rho^* = \rho - \frac{U}{p}\frac{\partial\pi}{\partial s} \tag{32}$$

Since $U, p > 0$ and $\partial\pi/\partial s \leqq 0$, it follows that additions to the discount factor ρ are required in the presence of uncertainty to "correct" the certainty-equivalent model and achieve optimal steady-state stock levels.[5] The steady-state stock $\tilde{N}$ will be less than under certainty, and it can be shown the implicit price will be lower.

Concluding Remarks

It has been recognized since the beginnings of fisheries economics that biomass management is similar to capital utilization and that control models from the theory of economic growth can be modified and applied to replenishable resource models. In the recent economic growth literature many differing conclusions have appeared regarding the biases in the certainty estimates of the corresponding "uncertainty" expected values. In Merton (1975) levels of variables including capital stock under uncertainty strictly exceed their respective certainty levels. The biases of Danthine and Donaldson (1981) are of opposite sign.

In the fisheries, stock uncertainty may be accommodated by increasing the discount factor, but growth uncertainty is accommodated by decreasing the discount factor. In economic terms if there is stock uncertainty, as in our model and Sutinen's (1981), discounting is increased because it would be socially wasteful to leave to a future generation the consumption of a stock which may not be realized. When there is stochastic growth, such as in Ludwig (1979a,b) or J. Barry Smith (1980), intertemporal fairness would dictate discounting the future less because future generations bear all the risk.

Perhaps the most serious weakness in present resource management practice is the absence of data on resouce stocks. We have presented an analysis of the effects of this uncertainty, and we have concluded that discounting can (theoretically) be used to reduce the effects in much the same situations as those noted above. We realize that any attempt to use discounting as a corrective device for uncertainty should be based on practicality or expediency, and where possible, we prefer that full stochastic solutions be sought for the problem rather than dressings for the symptoms.

One final comment is necessary. Throughout the literature of fishery management, the sources of uncertainty are introduced one at a time. The next logical effort toward a solution will be to introduce these sources simultaneously.

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Notes

1. See in particular Koopmans (1977).
2. See Strotz (1956) and Mendelssohn (1981).
3. We have recently received an interesting paper by J. G. Sutinen (1981), who addresses two related problems. The first involves management of a fishery where there is uncertainty about the *future* stock size. In that model Sutinen concludes that the discount rate should be increased and steady-state population decreased. His second model involves the introduction of a positive probability of overfishing, which

increases as harvesting increases. The resulting discount rate is decreased and steady-state population increased. Increased harvesting here has an increased social cost in the form of a more probable stock collapse.

4. Consider the numerator in equation 20. The variable f'' will be negative and $\partial^2\pi/\partial s^2 \leqq 0$. Thus the numerator is negative. The denominator will be zero, implying a vertical curve if and only if $[\rho - (\partial\pi/\partial s)U'h'] - f'(N) = 0$. However, along the $\dot{p} = 0$ curve,

$$p - \left(\frac{\partial\pi}{\partial s}\frac{U}{U'\Pi}\right) - f'(N) = 0$$

If U is linear,

$$U'h' = \frac{U}{p} = \frac{U}{U'\Pi}$$

and the $\dot{p} = 0$ curve is vertical. For U concave, $U'h' < U/p$ and the denominator of equation 20 is negative, resulting in a positively sloped $\dot{p} = 0$ curve.

5. Tracy Lewis (1982) discusses risk-adjusted discounting for his stochastic models of the yellow tuna fishery. He concludes that the procedure is suboptimal. His models differ from ours in that part of the randomness comes from stochastic growth. His objection to discount rate adjustments follows from the informational requirements for precise control which may vary with stock sizes and tastes. Our recipe for discount adjustment requires similar information. However, we do not attempt to achieve optimality as a practical issue but to provide a method of improving management procedures in a stochastic environment.

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