Stochastic Bioeconomics: A Review of Basic Methods and Results

Peder Andersen

Aarhus University
Aarhus, Denmark

Jon G. Sutinen

University of Rhode Island
Kingston, Rhode Island

Abstract Basic bioeconomic models which incorporate uncertainty are reviewed to show and compare the principal methods used and results reported in the literature. Beginning with a simple linear control model of stock uncertainty, we proceed to discuss more complex models which explicitly recognize risk preferences, firm and industry behavior, and market price effects. The effects of uncertainty on the results of bioeconomic analysis are rarely unambiguous, and in some instances differ little from corresponding deterministic results. This review is presented to enhance readers' appreciation of the papers to follow in this and the next issue of the journal.

1. Introduction

As with most scientific endeavors, the study of fisheries is shaped by the intellectual currents of the times. One such current in recent years has been the concern with behavior under uncertainty. After Bernoulli's (1738) paper formulating the expected utility hypothesis, the study of uncertainty languished for

Marine Resource Economics, Volume 1, Number 2
0738-1360/84/020117-00$02.00/0

two centuries before being rekindled by von Neuman and Morgenstern (1944). Their work provided the foundation for an intellectual revolution that has brought operational analyses of uncertainty to several practical endeavors in commerce, government, and law (Hirshleifer and Riley 1979).

The study of fisheries economics, or, more specifically, of bioeconomics, has not escaped this intellectual revolution. Biologists, mathematicians, and economists began developing in the 1970s the foundations for analyzing the behavior of bioeconomic systems under uncertainty. The literature in this area has grown dramatically during the last few years. Of the nearly three-score papers we examined for this review, only seven were published before 1975. Uncertainty now is clearly established in the bioeconomics literature as a principal field of inquiry.

A comprehensive review of all important aspects of bioeconomics and uncertainty is impossible in this limited space. Our modest aim here is to review the basics of bioeconomic analysis under uncertainty, especially to report and compare principal methods and results found in the literature. Where possible, we identify gaps and inconsistencies where further investigation appears fruitful.

Our review begins in section 2 with one of the simplest models of bioeconomics under uncertainty, distinguished by being linear in the control variable. In section 3 we review models which are nonlinear in the control variable, including two which take risk preferences into account. Formal analyses of firm and industry behavior under uncertainty are reviewed in section 4. Section 5 contains brief reviews of assorted analyses. We sum up our review in section 6.

2. Linear Control Models

The first model we examine is a direct extension of Clark's (1976) familiar bioeconomic framework to include stock uncertainty. First developed by Ludwig (1979), the model is in continuous time and is linear in the control variable. Stock uncertainty is incorporated by the stochastic differential equation

$$dx = [g(x_t) - h_t]\,dt + \sigma x_t\,dz \qquad (1)$$

where x_t is the size of the resource stock in period t, $g(x_t)$ is the natural growth rate of x_t, h_t is the harvest rate, σx_t reflects the level of random fluctuation in x_t, and dz is an increment of a stochastic process.[1] This specification assumes that the size of the resource stock in the current period is known without error and that the change in stock size is composed of a deterministic part, $[g(x_t) - h_t]dt$, and a random part, $\sigma x_t \, dz$. The expected rate of change in the stock size is $[g(x_t) - h_t]$.

The net revenue from harvesting the resource in each period is given by $[p - c(x_t)]h_t$, where p is the unit price of harvest, assumed constant, and $c(x_t)$ is the unit cost of harvest, $c'(x_t) < 0$. The harvest rate is constrained by upper and lower bounds, that is, $h_l(x_t) \leq h_t \leq h_u(x_t)$.[2] The expected present value of net revenue from harvest is given by

$$\epsilon\left\{\int_0^\infty e^{-\delta t}[p - c(x_t)]h_t \, dt\right\} \tag{2}$$

where ϵ is the expectation operator, and δ is the discount rate. Optimal harvest policy is found by maximizing equation 2 subject to equation 1 and $h_l(x_t) \leq h \leq h_u(x_t)$.

The basic necessary condition for this stochastic optimal control problem (see Kamien and Schwartz 1981) is

$$\max_h \left\{e^{-\delta t}[p - c(x)]h + V_x[g(x) - h] + \frac{\sigma^2 x^2}{2} V_{xx}\right\} \tag{3}$$

where $V(x_0)$ is the maximum of equation 2 at $t = 0$ and, therefore, $V_x = \partial V/\partial x$ is the marginal expected present value of (or shadow price on) the resource stock.

From equation 3 it follows that the optimal feedback policy at $t = 0$ is given by

$$h^* = \begin{cases} h_u(x_0), & \text{if } [p - c(x_0)] > V_x(x_0) \\ \bar{h}, & \text{if } [p - c(x_0)] = V_x(x_0) \\ h_l(x_0), & \text{if } [p - c(x_0)] < V_x(x_0) \end{cases}$$

The feedback policy specifies the optimal harvest rate as dependent on the current state, that is, $h_t^* = h(x_t)$.[3] The optimal policy for the stochastic linear control model has the same bang-bang feature as its deterministic counterpart. In both stochastic and deterministic models, optimal policy attempts to attain a steady state at the equilibrium stock size x^* where $p - c(x^*) = V_x(x^*)$. For the stochastic case, we denote this stock size by x_s^*, and for the deterministic case by x_d^*. In each case, harvest policy switches at the respective x^*. When $x_t > x^*$, h_t is set at its upper bound, and when $x_t < x^*$, h_t is set at its lower bound. When $x_t = x^*$, h_t is set equal to the natural growth rate, that is, $\bar{h} = g(x^*)$.[4]

Ludwig (1979) shows that $x_s^* > x_d^*$ under certain conditions.[5] Ludwig and Varah (1979) use numerical methods to show that increased noise (here, a larger σ) causes x_s^*/x_d^* to increase for large values of h_u. At low values of h_u and the discount rate, increased noise causes x_s^*/x_d^* to decrease with $x_s^* < x_d^*$ in some situations.[6] Thus the effect of stock uncertainty in this simple model is ambiguous.

Reed (1979) removes some of this ambiguity using a discrete-time, stock-recruitment model. He shows how the form of the harvest cost function $c(x)$ affects the level of x_s^* relative to x_d^*. The stochastic difference equation used is

$$x_{t+1} = z_t f(x_t - h_t) \tag{4}$$

where $f(\cdot)$ is the expected stock-recruitment function, and $\{z_t\}$ is a sequence of independent, identically distributed (iid) random variables with unit mean. Optimal policy is obtained by maximizing expected present value of net revenue,

$$\epsilon\left\{\sum_{t=1}^{\infty} \alpha^t\left[ph_t - \int_{x_t - h_t}^{x_t} c(r)\,dr\right]\right\}$$

subject to equation 4 and $0 \le h_t \le x_t$, where α is the discount factor. Note that the constraint on the harvest rate is not as general as Ludwig's, but it is this constraint that results in a constant escapement policy. That is,

$$h_t^* = \begin{cases} 0, & \text{if} \quad x_t \leq x_s^* \\ x_t - x_s^*, & \text{if} \quad x_t > x_s^* \end{cases}$$

where x_s^* is the optimal escapement level and is approximately equivalent to Ludwig's continuous time x_s^*.

Reed shows that $x_s^* \geq x_d^*$ when $xc(x)$ is strictly concave or linear and that $x_s^* \leq x_d^*$ when $xc(x)$ is strictly convex. He also asserts it can be shown that x_s^* increases with an increase in uncertainty (i.e., the spread of the z_t) when $c(x) = k/x^\theta$, $k > 0$ and $0 \leq \theta \leq 1$.[7]

The models discussed to this point allow no price effects and implicitly assume risk-neutral preferences and costless changes in the level of harvesting effort. Models which allow for risk-averse preferences and price effects are discussed in the next two sections. As reported by Ludwig (1980), costly changes in effort eliminate the bang-bang policy feature of the linear model. Using a numerical example, he demonstrates that the model with costly changes in effort yields a present value of net revenue that lies between the present values for the feedback and open-loop (constant effort) policies of the linear model.[8]

The structure of harvesting costs significantly affects the character of optimal policy. In an earlier paper, Reed (1974) develops a model which is similar to the one above but in which total costs also involve a fixed setup cost K incurred only if harvest is undertaken in the period. That is, total costs are given by

$$TC(h_t, x_t) = h_t c(x_t) + \gamma(h_t) \cdot K$$

where

$$\gamma(h_t) = \begin{cases} 0, & \text{if} \quad h_t = 0 \\ 1, & \text{if} \quad h_t > 0 \end{cases}$$

Under certain curvature properties of the growth function and the cost function, an (S, s) policy is shown to maximize the present value of net revenue. That is, optimal harvest policy is given by

$$h_t^* = \begin{cases} 0, & \text{if} \quad x_t \leq s \\ x_t - S, & \text{if} \quad x_t > s \end{cases}$$

where $S < s$. With no setup cost ($K = 0$), $S = s = x_s^*$, as above.

The intuition behind this result is straightforward. When $K = 0$, it is optimal to harvest x_t down to S whenever $x_t > S$. However, with positive setup costs, small harvests would not cover the setup costs and should not be undertaken. A harvest should be undertaken only if it is large enough to cover all costs. The level s is the smallest stock size at which positive net revenue is realized.

Spulber (1982) extends Reed's setup cost model in two ways. The first involves allowing the random disturbances in resource growth $\{z_t\}$ to follow a general Markov process. When $K > 0$, optimal harvest policy is now given by

$$h_t^* = \begin{cases} 0, & \text{if} \quad x_t \leq s(z) \\ x_t - S(z), & \text{if} \quad x_t > s(z) \end{cases}$$

where z is the last observed disturbance. Therefore, the two critical stock sizes (S, s) are not constant. Instead, expected future disturbances are taken into account which, in the Markovian framework, depend on previously observed disturbances. Spulber also shows that when $K = 0$, $S(z) = s(z)$ and when the $\{z_t\}$ are iid, S and s are constants as in Reed (1974, 1979).

Spulber's second extension is to show that the resource stock and harvest rate converge to unique, time-invariant probability distributions. He observes that the time-invariant probability distributions on harvest and stock size constitute the stochastic analogue to the steady-state equilibrium found in deterministic models. Spulber explicitly derives these probability distributions for the case $K = 0$ using the logistic model. Since his harvest cost function, $c \cdot h_t$ (c = constant), is independent of x_t, the expected stock size and harvest rate he computes for the stochastic steady state are equal to their deterministic equilibrium values.

Taken together, the analyses of Reed and Spulber show that the structure of harvest costs is a principal determinant of the expected difference between stochastic and deterministic outcomes. The lack of an expected difference does not mean that optimal stochastic policy is the same as optimal deterministic policy. Indeed, they can be quite different in any given period. Deterministic harvest policy is constant in equilibrium, while stochastic harvest policy (of the feedback type) varies with the state of the stock.

3. Nonlinear Control Models

We now consider a more complex set of models, distinguished principally by being nonlinear in the control variable. Curvature properties of the criterion function are especially important for problems involving uncertainty, so one might argue that the following models are somewhat more appropriate.

The first nonlinear model we consider was developed by Gleit (1978), who maximizes expected utility in order to account for risk preferences. The Ludwig model above is therefore modified to be

$$\max_h \epsilon\left\{ \int_0^\infty e^{-\delta t} U[ph - c(x)h] \, dt \right\}$$

subject to equation 1, where $U(\cdot)$ is the utility function, $U' > 0$, $U'' < 0$.[9] The basic necessary condition analogous to equation 3 is

$$\max_h \left\{ e^{-\delta t} U[ph - c(x)h] + J_x[g(x) - h] + \frac{\delta^2 x^2}{2} J_{xx} \right\}$$

where J_x and J_{xx} are analogous to V_x and V_{xx} in equation 3. Therefore, the optimal harvest rate h_t^* is where

$$e^{-\delta t} U'(\cdot)[p - c(x)] = J_x$$

About all we can say about optimal policy in this model is that it is not bang-bang.[10] It can also be shown that for certain conditions, h_t^* and x_t are directly related.[11] A direct relationship also exists in the linear model, but it is discontinuous. Here there is a smooth, direct relationship. To this point, it is not possible to establish any qualitative difference between stochastic and deterministic policies in the nonlinear control case.

The work of Lewis (1981, 1982) is an elaboration of the above nonlinear model. The stochastic growth relationship used by Lewis is given by

$$x_{t+1} = x_t + \eta_{1t} g(x_t) - \eta_{2t} h(E_t, x_t)$$

where

$g(\cdot)$ = expected rate of change in the stock x resulting from natural growth,
$h(E,x)$ = production function,
η_1 = random variation in growth caused by changes in recruitment, growth, and natural mortality, and
η_2 = random variation in harvest caused by changes in environmental conditions, stock distribution, catchability, etc.

Both η_1 and η_2 are nonnegative random variables, independently distributed through time.

For his calculations, Lewis uses the logistic growth law for $g(x)$ and a Cobb-Douglass production function $h(E,x) = qEx$, linear in E and x. Net returns are given by $R(E_t, x_t) = p_t h_t \eta_{2t} - C(E_t)$, where p_t = the exvessel price, a nonnegative random variable, and $C(E_t)$ = the total cost of effort. Lewis considers three specifications for $C(E)$:

$$C(E) = 0 \quad \text{(5a)}$$

$$C(E) = c_1 E + c_2 E^2, \qquad c_1, c_2 > 0 \quad \text{(5b)}$$

$$C(E) = c_3 E^{1/2}, \qquad c_3 > 0 \quad \text{(5c)}$$

He imagines a social manager who has a utility function

$U[R(x_t, E_t)]$ where $U' > 0$, $U'' \leq 0$. For a risk-neutral social manager, Lewis lets $U(R) = R$, and for the risk-averse social manager, $U(R) = \ln(R + G)$, where G is a large enough constant to insure $R + G > 0$ always. The social manager is assumed to

$$\max \sum_{t=0}^{\infty} \alpha^t \, \epsilon\{U[R(x_t, E_t)]\}$$

subject to $x_{t+1} = x_t + \eta_{1t}\, g(x_t) - \eta_{2t}\, h_t$, where α is the riskless discount factor.

Using Howard's stochastic dynamic programming algorithm, numerical solutions are generated for this model with parameter estimates from the Eastern Pacific yellowfin tuna fishery. Lewis's results are especially interesting because they describe behavior along the optimal trajectory to the steady state as well as at the steady state. We summarize Lewis's results as follows:

Effects of Risk-Bearing Attitudes. Optimal effort and harvest levels for the risk-averse manager are greater (less) than the optimal levels for the risk-neutral manager at small (large) stock sizes. Both manager's programs converge to the same steady-state stock size, but the risk-averse program converges at a slower rate.

Effects of Increased Uncertainty. For the risk-neutral manager, increased variation in price has no effect as long as expected price remains the same, but increased variation in the harvest rate parameter η_2 decreases optimal effort corresponding to each stock size. For the risk-averse manager, increased variation in price or in η_2 leads to increases or no change in optimal effort at small stock sizes and to decreases in effort at large stock sizes. Increased variation in the growth rate parameter η_1 alone has only a negligible effect on optimal effort levels, a surprising result in light of Ludwig's and Reed's analyses.

Stochastic Versus Deterministic Analysis. The difference (in present value terms) between the stochastic and deterministic results was negligible for the more conventional cost function (equation 5b) and small for the other two cost functions (5a and 5c). Therefore, in some cases solutions to deterministic problems yield good approximations for solutions to stochastic problems.

For his study of lobster, Smith (1980) also finds that stochastic growth is not significant. He uses a production function of the form $h_t = qE_t^{\beta} x_t$, where E_t is fishing effort and $0 < \beta < 1$. Like Spulber, Smith derives the time-invariant probability distribution for the resource stock for a growth process similar to equation 1. For a constant (i.e., open-loop) effort policy, Smith shows that the average, stochastic steady-state stock size is given by

$$\epsilon\{x\} = x_d - \frac{\sigma^2}{2(\rho/k)}$$

where x_d is the deterministic steady-state stock size, ρ is the intrinsic growth rate, and k is the carrying capacity for the logistic growth law. For the lobster fishery he studied, there was no significant difference between $\epsilon\{x\}$ and x_d.

Mendelssohn (1982) also examines the effects of risk preferences on optimal harvest policy. In a stochastic stock recruitment model which maximizes expected discounted utility of harvest, Mendelssohn's numerical analysis shows that risk aversion results in more harvested at small stock sizes than with a risk-neutral utility function. This same result is reported by Lewis. Mendelssohn and Lewis also conclude that adjusting the discount rate is not a satisfactory means of accounting for different attitudes toward risk.

4. Firm and Industry Analysis

In contrast to the models reviewed above, Andersen (1982) explicitly models individual firms and examines both open-access and optimal fisheries exploitation under price uncertainty. Individual firms have profits given by

$$\pi^i = p_t h_t^i - c(E_t^i)$$

where p_t is random with mean $\bar{p}$, $h_t^i = qE_t^i x_t$, $c(E_t^i)$ represents total costs, and $c'(E^i) > 0$. Each firm maximizes the expected utility of profits $\epsilon\{U(\pi^i)\}$, where $U'(\pi^i) > 0$ and $U''(\pi^i) < 0$. Since firms are risk-averse, they operate where the expected marginal value product of effort exceeds the marginal factor cost of effort. That is, their effort level is determined by the condition

$$\bar{p}qx = c'(E^i) + \gamma$$

where $\gamma = \gamma(\sigma_p^2) > 0$ is the marginal cost of risk bearing. If the variance of price σ_p^2 is zero, $\gamma(0) = 0$, firms are facing a deterministic price and apply more effort. And since $\gamma'(\sigma_p^2) > 0$, increases in price variation induce less effort to be applied. He also assumes that firms enter and exit the fishery as

$$\bar{p}qx \gtrless m + \gamma(\sigma_p^2)$$

where m is the minimum average cost of effort.

The growth of the fish resource stock is governed by

$$\dot{x} = g(x) - \sum_{i=1}^{N} h^i$$

where N is the number of firms fishing. The logistic growth model is assumed for $g(x)$.

Andersen proceeds to show that in an open-access equilibrium, total effort and the number of fishing firms are less and the stock size greater with price uncertainty than without (where the deterministic price equals $\bar{p}$). Also, increases in the variance of price reduce total effort and the number of firms and increase the stock size.

For his first-best optimum, Andersen assumes that society (in the form of a managing authority) is willing to bear risk at zero cost. That is, society is risk-neutral, attaching importance to risk only through the costs of risk borne by individuals. It is also assumed that the managing authority is able to vary the price variance (at the exvessel level) without cost and faces the same expected price, $\bar{p}$. Under these conditions, it is shown that the first-best optimum *may* have more total effort applied than would result under open access if the price variance is high. Furthermore, the only regulation method that produces the first-best optimum is proved to be a fixed price system. The second-best regulation method is a tax on revenue, which is shown to be superior to both transferable quotas and a tax on catch.

In Andersen (1981b), fishing firms' behavior and characteristics of the open-access and optimal fishery under stock uncertainty are examined. He makes the same economic assumptions as above, except price is now constant. The growth of the resource stock (with exploitation) is given by the stochastic differential equation

$$dx_t = \left[g(x_t) - \sum_{i=1}^{N} h_t^i \right] dt + \sigma x_t \, dz_t$$

where z_t is assumed to describe a white-noise stochastic process (i.e., $dz \sim N[0, \sigma]$). As before, the logistic form of $g(x)$ is used. This stochastic growth relationship is almost identical to equation 1 discussed above. Andersen shows that if the stock size is known by fishing firms at fishing time, the optimal levels of catch and effort are less than in the deterministic case. If the stock size is not known by firms at fishing time, the optimal levels of effort and catch are less than if the stock size is known at fishing time.

Comparing open access and optimum fisheries, he shows that if the stock level is known at fishing time, the optimal effort level is less than the expected effort level under open access—although, in some periods, optimal effort may be larger than under open access. If firms know only the mean and variance of the stock, effort is always larger under open access than in the optimally managed fishery.

Pindyck (1984) represents the only stochastic bioeconomic analysis which explicitly incorporates a downward-sloping demand function. Pindyck treats the case where resource markets are competitive, property rights to the resource stock are well defined and enforced, and firms are risk-neutral. The source of uncertainty is the stochastic growth of the resource stock, given by equation 1 where $\sigma(x)$, $\sigma'(x) \geq 0$, replaces σx. Equilibrium harvest (and, under these conditions, optimal harvest) is obtained by

$$\max_h \int_0^h p(r)\, dr - c(x)h + V_x[g(x) - h] + \frac{\sigma^2(x)}{2} V_{xx}$$

where $p(h)$ is the inverse market demand curve. Among several interesting results is that for a convex $c(x)$, stock uncertainty increases the expected rate of growth in price. This does not imply, however, that the harvest rate is greater than in the deterministic case, as Pindyck demonstrates with some examples. Harvest rates, for any x, can be increased, decreased, or unchanged by changes in $\sigma(x)$.

5. Additional Issues

Several issues beyond those covered in previous sections are treated in the literature. A selected few follow.

Reed (1974) and Andersen (1981a) address the issue of extinction under uncertainty. For the case $K > 0$, Reed derives two interest rate values i_1 and i_2 ($i_1 > i_2$). If the actual interest rate $i > i_1$, extinction is optimal; if $i < i_2$, survival is optimal. It is not clear what happens when $i_1 > i > i_2$. For the case $K = 0$ and a constant marginal harvest cost (i.e., $c[x] = \bar{c}$), extinction is optimal if the expected population growth rate is always less than the rate of interest. Andersen examines the implications price uncertainty has for extinction. The same assumptions are made as in his papers cited above. His principal result is that the deterministic results regarding extinction (e.g., see Clark 1973; Clark and Munro 1975) cannot be carried over to the case of price uncertainty. With price uncertainty, the conditions for extinction can be less restrictive in an optimal fishery than in an open-access fishery. That is, if extinction is optimal (in a first-best sense), it will not necessarily occur under open access. This is impossible in the deterministic case. Also, the conditions under which extinction is optimal depend on the method of regulation. For a tax on revenue, the conditions are less restrictive than with a tax on catch or individual quotas.

High variation in equilibrium (or steady-state) harvests has been established by a number of studies (e.g., Beddington and May 1977; Sissenwine 1977; and May et al. 1978), especially when harvested at high effort levels. Since the manager's utility function usually is not known, it is not clear how much variation in harvest should be permitted. Mendelssohn (1980a) approaches this problem by devising a numerical method which computes the trade-off between the mean and variance of the return. He applied this technique to the Bristol Bay, Alaska, salmon fishery to generate a trade-off schedule for the mean and standard deviation of long-run harvest.

So far in this review we have discussed results based on lumped parameter models (e.g., the logistic growth law). Two studies have examined optimal policy based on the cohort model.

Dudley and Waugh (1980) develop a numerical model for determining optimal harvest policies for a single-cohort fishery under uncertainty. Their application is to an Australian prawn fishery. A Beverton-Holt growth law is assumed where recruitment, natural mortality, and catchability are random variables. Expected net revenue for each policy, state, and period, plus the transition probabilities, were generated by simulation and used as the data in a stochastic dynamic programming model. The stochastic DP generated policies which maximize expected net revenue over the season (12 months). They find that with a high level of harvesting capacity, the optimal procedure is not to harvest at all until the biomass reaches its peak, and then to harvest at the maximum rate until the minimum profitable biomass is reached. This policy has an (S, s) character, as in Reed and Spulber. The stochastic and deterministic results differ by little and in no apparent systematic way given the tabular results presented. With a low level of harvesting capacity, optimal harvest begins a month earlier but is not applied at its maximum until the peak biomass time arrives. The presence of uncertainty results in more fishing effort in the first month and spreads it out over more months (in most, but not all, cases). A demand relationship instead of a constant price tends, as one would expect, to spread effort and catch over more of the year. As with most numerical results, one cannot confidently generalize Dudley and Waugh's results. Their results also consider the three random variables in only three combinations, and they do not present results with different levels of variation in the random parameters.

Mendelssohn (1978) develops single-species, multicohort optimal harvesting models in which recruitment and age-dependent survivorship rates are random. His two models assume perfect selection in harvesting the cohorts, or age classes. In the first model where he assumes that recruitment is independent of total stock size, Mendelssohn derives a "Fisher rule" for harvest. That is, do not harvest a cohort until it reaches an optimum age, then harvest the entire cohort. While there is no comparison with the deterministic policy, it appears qualitatively similar to the deterministic analysis of Clark, Edwards, and Friedlander (1973). In his second model, recruitment is assumed to depend on the total stock size and yields a policy that the oldest are always harvested first. Using similar methods, Mendelssohn (1980b) derives the qualitative properties of optimal policy for a stochastic multispecies fishery.

6. Concluding Remarks

In this brief review we have described the basic methods and results found in the stochastic bioeconomics literature. Where the dynamics of the resource stock are given by a stochastic differential or difference equation, stochastic dynamic programming methods are used to derive optimal policy. In several cases, optimal policy under stochastic conditions is qualitatively different from optimal policy under deterministic conditions. Such differences, however, are not unambiguous. Two empirically based studies, Lewis (1982) and Smith (1980), conclude that deterministic policies are reasonably good substitutes for stochastic policies on average. One cannot easily generalize from these results, but this does raise the question of whether uncertainty is significant. Future studies hopefully will show more clearly the significance (or insignificance) of uncertainty for policy analysis and other studies of the fishery.

Acknowledgments

An earlier version of this paper was presented to the Workshop on Uncertainty and Fisheries Economics, University of Rhode Island, November 1981. Support for this research was provided by Sea Grant, University of Rhode Island, and the Carlsberg Foundation, Denmark. Contribution No. 2191 of the Rhode Island Agriculture Experiment Station.

Notes

1. In addition, $g(x)$ is strictly concave, $\sigma > 0$, and $\epsilon\{dz\} = 0$.
2. The control variable in Ludwig's model actually is effort E in the production function $h = qEx$, and his constraint is $E_l \leq E \leq E_u$. Therefore, in the present specification, the constraint on harvest is where $h_l(x_t) = qE_l x_t$ and $h_u(x_t) = qE_u x_t$.
3. A feedback, or closed-loop, policy prescribes a rule for specifying future harvest rates when future information is given. Feedback policies are generally superior to open-loop and revised open-loop policies (cf. Ludwig 1980). An open-loop policy is one for which all present and future harvest rates are determined once and for all in the initial period. This policy is often fully appropriate in a deterministic setting where all future states are known with perfect certainty. Under a revised open-loop policy, estimates of the state and parameters of the system are periodically updated and lead to revisions in the open-loop policy.
4. Ludwig also examines the consequences of noise for depensation models with two or more switching points.
5. The conditions include a small σ^2, a large h_u, and $h_l = 0$.
6. Their model allows x_d^* to increase with σ^2 also, since they use a deterministic growth function which has σ^2 as an argument. Their justification is that this specification gives better results when formulating deterministic policy. Unfortunately, this choice seems to confound their results somewhat.
7. A number of not unreasonable conditions must be satisfied for these results to hold. Reed notes that a constant escapement policy is not optimal if the demand curve is not perfectly elastic or if the total cost function is not linear in h.

8. Clark (1979) suggests extending Ludwig's model to allow h_u to be determined optimally, thereby also determining the optimal level of excess capacity above $\bar{h} = g(x_s^*)$ that should be constructed to take advantage of stock sizes above x_s^*.

9. Gleit actually solves a finite time horizon problem; hence, we do not present his precise results.

10. Gleit derives an explicit solution for h^* where $g(x)$ is assumed linear. This case, of course, is not appropriate for fisheries problems.

11. The conditions are that $J_{xx} < 0$ and the Arrow-Pratt measure of relative risk aversion is less than unity. This result is obtained by totally differentiating the above condition.

References Cited

Andersen, P. 1981a. *Extinction and price uncertainty* (Staff Paper No. 81–09). Kingston, R.I.: University of Rhode Island, Department of Resource Economics.

———. 1981b. *The exploitation of fish resources under stock uncertainty* (Staff Paper No. 81–14). Kingston, R.I.: University of Rhode Island, Department of Resource Economics.

———. 1982. Commercial fisheries under price uncertainty. *J. Environ. Econ. Manage.* 9(1): 11–28.

Beddington, J. R., and R. M. May. 1977. Harvesting natural populations in a randomly fluctuating environment. *Science* 197: 463–465.

Bernoulli, D. 1738. Specimen theoriae novae de mensura sortis. *Commentarii Academiae Scientiarum Imperialis Petropolitanae* 5: 175–192. Translated into English by L. Sommer, 1954, Exposition of a new theory on the measurement of risk. *Econometrica* 22: 23–36.

Clark, C. W. 1973. Profit maximization and the extinction of animal species. *J. Polit. Econ.* 81: 950–961.

———. 1976. *Mathematical bioeconomics*. New York: Wiley.

———. 1979. Mathematical models in the economics of renewable resources. *SIAM Rev.* 21(1): 81–99.

Clark, C. W., G. Edwards, and M. Friedlaender. 1973. Beverton-Holt model of a commercial fishery: optimal dynamics. *J. Fish. Res. Board Can.* 30: 1629–1640.

Clark, C. W., and G. R. Munro. 1975. Economics of fishing and modern capital theory: A simplified approach. *J. Envir. Econ. Manage.* 2: 92–106.

Dudley, N., and G. Waugh. 1980. Exploitation of a single-cohort fishery under risk: A simulation-optimization approach. *J. Envir. Econ. Manage.* 7: 234–255.

Gleit, A. 1978. Optimal harvesting in continuous time with stochastic growth. *Math. Biosci.* 41: 111–123.

Hirshleifer, J., and J. G. Riley. 1979. The analytics of uncertainty and information—An expository survey. *J. Econ. Lit.* 17: 1375–1421.

Kamien, M. I., and N. L. Schwartz. 1981. *Dynamic optimization: The calculus of variations and optimal control in economics and management.* New York: Elsevier North-Holland.

Lewis, Tracy R. 1981. Exploitation of a renewable resource under uncertainty. *Can. J. Econ.* 14(3): 422–439.

———. 1982. *Stochastic modeling of ocean fisheries resource management.* Seattle: University of Washington Press.

Ludwig, D. 1979. Optimal harvesting of a randomly fluctuating resource. I. Application of perturbation methods. *SIAM J. Appl. Math.* 37: 166–184.

———. 1980. Harvesting strategies for a randomly fluctuating population. *J. Cons. Perm. Int. Explor. Mer* 39: 168–174.

Ludwig, D., and J. M. Varah. 1979. Optimal harvesting of a randomly fluctuating resource. II. Numerical methods and results. *SIAM J. Appl. Math.* 37: 185–205.

May, R. M., J. R. Beddington, J. W. Horwood, and T. G. Sherpherd. 1978. Exploiting natural populations in an uncertain world. *Math. Biosci.* 42: 219–252.

Mendelssohn, R. 1978. Optimal harvesting strategies for stochastic single-species, multiage class models. *Math. Biosci.* 41: 159–174.

———. 1979. Determining the best trade-off between expected economic returns and the risk of undesirable events. *J. Fish. Res. Board Can.* 36: 939–949.

———. 1980a. A systematic approach to determining mean-variance trade-offs when managing randomly varying populations. *Math. Biosci.* 50: 75–84.

———. 1980b. Managing stochastic multispecies models. *Math. Biosci.* 49: 249–261.

———. 1982. Discount factors and risk aversion in managing random fish populations. *Can. J. Fish. Aquat. Sci.* 39: 1252–1257.

Pindyck, Robert S. 1984. Uncertainty in the theory of renewable resource markets. *Rev. Econ. Stud.* 51:289–303.

Reed, W. J. 1974. A stochastic model for the economic management of a renewable animal resource. *Math. Biosci.* 22: 313–337.

Reed, W. J. 1979. Optimal escapement levels in stochastic and deterministic models. *J. Envir. Econ. Manage.* 6: 350–363.

Sissenwine, M. 1977. The effect of random fluctuations on a hypothetical fishery. *ICNAF Sel. Pap.* 2: 137–144.

Smith, J. B. 1980. Replenishable resource management under uncertainty: A re-examination of the U.S. northern fishery. *J. Envir. Econ. Manage.* 7: 209–219.

Spulber, D. F. 1982. Adaptive harvesting of a renewable resource and stable equilibrium. In *Essays in the economics of renewable resources*, ed. L. J. Mirman and D. F. Spulber, 117–139. New York: North-Holland.

von Neuman, J., and O. Morgenstern. 1944. *Theory of games and economic behavior.* New York: Wiley.

Related References

Allan, K. R. 1973. The influence of random fluctuations in the stock-recruitment relation on the economic return from salmon fisheries. *Cons. Perm. Int. Explor. Mer Rapp.* 164: 351–359.

Anderson, David R. 1975. Optimal exploitation strategies for an animal population in a Markovian environment: A theory and an example. *Ecology* 56: 1281–1297.

Clark, C. W. 1981. Structural uncertainty in multispecies fisheries. Background paper, University of British Columbia, Vancouver.

Doubleday, W. G. 1976. Environmental fluctuation and fisheries management. *ICNAF Sel. Pap.* 1: 141–150.

Everitt, R. R., N. C. Sonntag, M. L. Puterman, and P. Whalen. 1978. A mathematical programming model for the management of a renewable resource system: The Kemano II development project. *J. Fish. Res. Board Can.* 35: 235–246.

Gaither, N. 1980. A stochastic constrained optimization model for determining commercial fishing seasons. *Manage. Sci.* 26: 143–154.

Gatto, M., and S. Rinalch. 1976. Mean value and variability of the catch in fluctuating environments. *J. Fish. Res. Board Can.* 33: 189–193.

Goel, N. S., and N. Richter-Dyn. 1974. *Stochastic models in biology.* New York: Academic Press.

Hilborn, R. 1976. Optimal exploitation of multiple stocks by a common fishery: A new methodology. *J. Fish. Res. Board Can.* 33: 1–5.

Huang, C. C., I. B. Vertinsky, and N. J. Wilimovsky. 1976. Optimal controls for a single species fishery and the economic value of research. *J. Fish. Res. Board Can.* 33: 793–809.

Hutchinson, C. E., and T. R. Fischer. 1979. Stochastic control applied to fishery management. *IEEE Trans. Syste., Man, Cybern.* 9: 253–259.

Jaquette, David L. 1972. A discrete time population control model. *Math. Biosci.* 15: 231–252.

———. 1974. A discrete-time population-control model with setup cost. *Oper. Res.* 22: 298–303.

Ludwig, D., and C. J. Walters. 1981. Optimal harvesting with imprecise parameter estimates. *Can. J. Fish. Aquat. Sci.* 38: 711–720.

May, R. M. 1974. *Stability and complexity in model ecosystems.* Princeton, N.J.: Princeton University Press.

Mendelssohn, R. 1980. Using Markov decision models and related techniques for purposes other than simple optimization: Analyzing the consequences of policy alternatives on the management of salmon runs. *Fish. Bull.* 78: 35–50.

Mendelssohn, R., and M. J. Sobel. 1980. Capital accumulation and the optimization of renewable resource models. *J. Econ. Theory* 23: 243–260.

Reed, W. J. 1978. The steady-state of a stochastic harvesting model. *Math. Biosci.* 41: 273–307.

Sherpherd, T. G., and J. W. Horwood. 1979. The sensitivity of exploited population to environmental "noise" and the implications for management. *J. Cons. Perm. Int. Explor. Mer* 38: 318–323.

Smith, A. D. M., and Carl J. Walters. 1981. Adaptive management of stock-recruitment systems. Can. J. Fish. Aquat. Sci. 38: 690–703.

Tautz, A., P. A. Larkin, and W. E. Ricker. 1969. Some effects of simulated long-term environmental fluctuations on maximum sustained yield. *J. Fish. Res. Board Can.* 26: 2715–2786.

Turelli, M. 1977. Random environments and stochastic calculus. *Theor. Popul. Biol.* 12: 140–178.

Walters, C. J. 1975. Optimal harvest strategies for salmon in relation to environmental variability and uncertain production parameters. *J. Fish. Res. Board Can.* 32: 1777–1784.

Walters, C. J., and R. Hilborn. 1976. Adaptive control of fishing systems. *J. Fish. Res. Board Can.* 33: 145–159.