# Open-Access Fishery Models: Relaxing a Constraint and Removing an Econometric Obstacle 

C. NICHOLAS GOMERSALL<br>Department of Economics and Business<br>Luther College

Decorah IA 52101


#### Abstract

Over the past 30 years, a widely accepted model of "open-access" fisheries has been developed, yet empirical tests of the standard model have been relatively few. One difficulty is that fish stocks, the levels of which affect the rate of catch, are not directly observable. Simplifying assumptions are generally required, such as the assumption that catchability does not change over time. Estimation on the basis of the standard model also raises difficulties in specification, if contemporaneous correlation of the error term with one of the regressors is to be avoided.

This paper describes an algorithm that imposes a less restrictive pattern (than constancy) on catchability, yet does so in an econometrically acceptable fashion. It also reports on an application of this algorithm to the Flemish Cap groundfishery over the period from 1971 to 1985.


Keywords Catchability, error term, bias, Flemish Cap


#### Abstract

"We bend over computer tables, models and scenarios, and the stacks of paper grow and the fish in the seas swim and swim and have no inkling of our existence."


J. Bernlef

## Introduction and Overview

In the standard model of open-access fisheries, the "catchability coefficient" q-which links the catch per unit effort to the size of the fish stock-is assumed to remain constant. This constraint gives rise to several difficulties: (i) in a fishery where the relative use of gear types and vessel tonnage classes is frequently changing, to assume a constant level of q has little a priori likelihood; (ii) even for a specific class of vessel and type of gear, to fix $q$ ignores the existence of technical change; (iii) Hannesson (1983) has suggested that q may be dependent upon stock levels; and (iv) by fixing q over time, an complex error term is created that makes difficult the choice of a suitable estimator.

This paper describes an estimation procedure which obviates the need to assume an unchanging q , and yet remains econometrically tractable. The results of applying this algorithm to the case of a specific fishery are also reported.

The second section describes the model itself. The expectations to which it gives rise (drawn from standard theory on the dynamic behavior of fish stocks under open-access conditions) are outlined in the third section.

The fourth section describes a successful application of this model to the NorthWest Atlantic groundfishery in an area known as the Flemish Cap, using data from this fishery over the period from 1971 to 1985. Two characteristics of the least-squares solution were notable: (i) a decline in q-in contrast to the standard assumption of constancy-of $4.4 \%$ per year; and (ii) a plot of fishing effort against estimated stock levels which conformed to theoretical expectations in all respects but one-the predicted anticlockwise spiral seemed to be stretched out to form a helix.

These findings are discussed in the final section.

## An Algorithm for Estimating Dynamic Open-Access Models

The following harvest function is widely used at the fleet level:

$$
Y_{t}=q_{t} X_{t} E_{t}
$$

where: $\mathrm{q}=$ "catchability coefficient," here left unconstrained over time,
$\mathrm{X}=$ stock of fish, and
$\mathrm{E}=$ "fishing effort" (in this paper captured by the number of days spent fishing).

This functional form has a number of recognised limitations. For example, it takes account neither of gear saturation nor of congestion among fishing vessels (Clark, 1976). Hannesson's empirical work (1983) suggests, moreover that q-the catchability coefficient-may be dependent upon the stock level X. (Again, as brought out in the same paper by Hannesson, in using the concept of effort-an "intermediate output"-this functional form is unsuitable for the exploration of such issues as the optimal employment of primary inputs, capital and labor.) There is reason to suspect, then, that the true elasticities of effort and stock levels may differ from one.

On the other hand, one of the explanatory variables in the harvest functionthe stock level-is not only unobservable but also exhibits dynamic behavior (based on the biological growth rate of the fish and on the rate of harvesting, as discussed below). The parameters of this natural growth rate also need to be estimated. As a result of these complexities, elasticities other than one in the harvest function render estimation of the model intractable. Hannesson is able to avoid such difficulties by using ICES data for stock levels but, in the present study, the emphasis is on estimation of stock levels and this approach cannot be used.

This study therefore uses the harvest function $Y_{t}=q_{t} X_{t} E_{t}$, recognising its limitations.

Suppose that the growth rate of a particular fish stock, X , is assumed in the absence of harvesting to be given by the logistic growth function:

$$
F(X)=r X[1-X / K]
$$

where: $\quad r=$ biological growth rate of the species being modelled, and $K=$ environmental carrying capacity of the fishing ground.

With fishing yields being represented by Y , changes in the stock level are expressed as follows:

$$
\begin{align*}
X_{t+1}-X_{t} & =F\left(X_{t}\right)-Y_{t}+\epsilon_{t+1} \\
& =r X_{t}-r X_{t}^{2} / K-Y_{t}+e_{t}, \tag{1}
\end{align*}
$$

where $e_{t+1}$ picks up specification error in the growth function $\mathrm{F}\left(\mathrm{X}_{t}\right)$ as well as unexpected causes of mortality. It is assumed that e is i.i.d. $\mathrm{N}\left(0 ; \boldsymbol{\sigma}^{2}\right)$.

Using the harvest function discussed above, which may be rewritten as $\mathrm{X}_{\mathrm{t}}=$ $\mathrm{Y}_{\mathrm{t}} / \mathrm{q}_{\mathrm{t}} \mathrm{E}_{\mathrm{t}}$, and then substituting for $\mathrm{X}_{\mathrm{t}+1}$ and $\mathrm{X}_{\mathrm{t}}$ in Equation (1):

$$
Y_{t+1} / q_{t+1} E_{t+1}=(1+r)\left(Y_{t} / q_{t} E_{t}\right)-(r / K)\left(Y_{t} / q_{t} E_{t}\right)^{2}-Y_{t}+e_{t+1}
$$

or:

$$
\begin{align*}
\mathrm{Y}_{\mathrm{t}+1} / \mathrm{E}_{\mathrm{t}+1}= & (1+\mathrm{r})\left(\mathrm{q}_{\mathrm{t}+1} / \mathrm{q}_{\mathrm{t}}\right)\left(\mathrm{Y}_{\mathrm{t}} / \mathrm{E}_{\mathrm{t}}\right)-\left(\mathrm{r} / \mathrm{q}_{\mathrm{t}} \mathrm{~K}\right)\left(\mathrm{q}_{\mathrm{t}+1} / \mathrm{q}_{t}\right)\left(\mathrm{Y}_{\mathrm{t}} / \mathrm{E}_{t}\right)^{2} \\
& -\mathrm{q}_{\mathrm{t}+1} Y_{\mathrm{t}}+\mathrm{q}_{\mathrm{t}+1} \mathrm{e}_{\mathrm{t}+1} . \tag{2}
\end{align*}
$$

## The Nature of the Estimating Problem

Equation (2) exemplifies a difficulty that arises when estimating dynamic openaccess models. Y and E are observable, and there are three regressors. On the other hand, the parameters to be estimated include not only r and K , but also the various qs (one for each year).

This difficulty has usually been dealt with by assuming that q is constant over time. In that case, however, the yield relationship $Y_{t}=q_{t} X_{t} E_{t}$ can no longer be regarded as deterministic and now, when substituting for $X_{t}$ and $X_{t+1}$ in Equation (1), an error term associated with the yield function must be introduced, thus: $X_{t}$ $=\left(\mathrm{Y}_{\mathrm{t}} / \mathrm{q}_{\mathrm{t}} \mathrm{E}_{\mathrm{t}}\right)+\nu_{\mathrm{t}}$. Once these substitutions are made and an expression obtained that is analogous to Equation (2), part of the composite error term is found to be contemporaneously correlated with the set of regressors. Specifically:

$$
\begin{aligned}
\mathrm{Y}_{\mathrm{t}+1} / \mathrm{E}_{\mathrm{t}+1}= & (1+\mathrm{r})\left(\mathrm{q}_{\mathrm{t}+1} / \mathrm{q}_{\mathrm{t}}\right)\left(\mathrm{Y}_{\mathrm{t}} / \mathrm{E}_{\mathrm{t}}\right)-\left(\mathrm{r} / \mathrm{q}_{\mathrm{t}} \mathrm{~K}\right)\left(\mathrm{q}_{\mathrm{t}+1} / \mathrm{q}_{\mathrm{t}}\right)\left(\mathrm{Y}_{\mathrm{t}} / \mathrm{E}_{\mathrm{t}}\right)^{2} \\
& -\mathrm{q}_{\mathrm{t}+1} \mathrm{Y}_{\mathrm{t}}+\mathrm{q}_{\mathrm{t}+1} \mathrm{e}_{\mathrm{t}+1} \\
& +\mathrm{q}_{\mathrm{t}+1}\left\{(1+\mathrm{r}) v_{\mathrm{t}}-v_{\mathrm{t}+1}-(\mathrm{r} / \mathrm{K})\left[2\left(\mathrm{Y}_{\mathrm{t}} / \mathrm{q}_{\mathrm{t}} \mathrm{E}_{\mathrm{t}}\right) v_{\mathrm{t}}+v_{\mathrm{t}}^{2}\right]\right\}
\end{aligned}
$$

In such a situation, OLS is asymptotically biased. ${ }^{2}$ Instrumental variables could be used but, since these variables would have to be highly correlated with catch levels Y yet uncorrelated with the disturbance $v$, there are no obvious candidates.

The standard assumption that q is fixed cannot, therefore, be made without introducing serious econometric difficulties.

## The Algorithm

This study makes use of an algorithm that economises on the number of parameters in Equation (2), and yet allows for a deterministic harvest function: q remains defined for each period by the relationship $Y=q X E$. This can be achieved
by hypothesising that the ratio $\mathrm{q}_{\mathrm{t}+1} / \mathrm{q}_{\mathrm{t}}$ is distributed normally about a mean G . Such a hypothesis can be viewed as a forecasting model conditional on $\mathrm{q}_{\mathrm{t}}$, where:

$$
\mathrm{q}_{\mathrm{t}+1}=\mathrm{Gq}_{\mathrm{t}}+\mathrm{q}_{\mathrm{t}} \omega_{\mathrm{t}+1}
$$

Hannesson's finding (1983) that q may be partly dependent upon the stock level, X, could give rise to doubts about the validity of imposing a geometric pattern on the $q$-series. On the other hand, such a constraint is certainly less restrictive than the standard assumption of constancy in q (which it includes as a special case). Moreover, some way must be found to circumvent the estmation problem identified above-unless, as in Hannesson's study, stock levels are treated as exogenous.

Since changes in q over time reflect changes in the mix of vessel types making up the fleet, in the absence of other information one might expect that $G$ would be close to 1 and that its variance would be small.

Substituting $\mathrm{q}_{\mathrm{t}+1} / \mathrm{q}_{\mathrm{t}}=\mathrm{G}+\omega_{\mathrm{t}+1}$ into Equation (2):

$$
\begin{align*}
Y_{t+1} / E_{t+1}= & G(1+r)\left(Y_{t} / E_{t}\right)+\left(G r / q_{t} K\right)\left(Y_{t} / E_{t}\right)^{2}-G q_{t} Y_{t} \\
& +q_{t+1} e_{t+1}+(1+r)\left(Y_{t} / E_{t}\right) \omega_{t+1} \\
& -\left(r / q_{t} K\right)\left(Y_{t} / E_{t}\right)^{2} \omega_{t+1}+q_{t} Y_{t} \omega_{t+1} . \tag{3}
\end{align*}
$$

The error term is now free of any contemporaneous correlation between regressors and disturbances, since $\omega$ and Y , for example, are not linked in this way. ${ }^{3}$ On the other hand, since q is hypothesised to increase or decline geometrically over time by a factor of $G$ per period, such terms as $q_{t+1} e_{t+1}$ may suggest the presence of heteroscedasticity. Estimates generated from the model must be probed in this respect by such measures as the Breusch-Pagan test.

Before going further, an adjustment will be made which, though not essential to the analysis, focuses the discussion on the unknown stock level, X, rather than on the construct, $q$. Consider the first time period, with stock $X_{0}$. Now $q_{0}$ can be expressed in terms of $\mathrm{X}_{0}$, thus:

$$
\mathrm{q}_{0}=\mathrm{Y}_{0} / \mathrm{X}_{0} \mathrm{E}_{0}=\mathrm{m}_{0} / \mathrm{X}_{0},
$$

where $m_{0}$ is known through observation. If $E\left(q_{t+1} / q_{t}\right)=G$, an estimate of future qs is given by $\mathrm{q}_{\mathrm{t}}=\mathrm{G}^{\mathrm{t}} \mathrm{m}_{0} / X_{0}$. Substituting into Equation (3):

$$
\begin{align*}
Y_{t+1} / E_{t+1}= & G(1+r)\left(Y_{t} / E_{t}\right)-\left(G^{\prime} X_{0} / G^{t} m_{0} K\right)\left(Y_{t} / E_{t}\right)^{2} \\
& -G Y_{t}\left(G^{t} m_{0} / X_{0}\right)+G^{t+1} m_{0} e_{t+1} / X_{0} \\
& +(1+r)\left(Y_{t} / E_{t}\right) \omega_{t+1}-\left(r X_{0} / G^{t} m_{0} K\right)\left(Y_{t} / E_{t}\right)^{2} \omega_{t+1} \\
& +G^{t} Y_{t} m_{0} \omega_{t+1} / X_{0} . \tag{4}
\end{align*}
$$

The parameters to be estimated are now $\mathrm{G}, \mathrm{r}, \mathrm{K}$ and $\mathrm{X}_{0}$.
To solve Equation (4), an initial estimate of G, namely $\mathrm{G}^{*}$, is used. (Lacking other information, one plausible value might be $\mathrm{G}^{*}=1$. In the application described below, other values of $G^{*}$ were chosen in a manual search for the best
solution on least-squares grounds and other criteria shortly to be described.) G* is used only in the terms $\mathrm{G}^{\mathrm{t}}$, not where G appears alone. Regressing $\left(\mathrm{Y}_{\mathrm{t}+1} / \mathrm{E}_{\mathrm{t}+1}\right)$ on $\left(Y_{t} / E_{t}\right),\left(Y_{t} / E_{t}\right)^{2} / \mathrm{G}^{t}$ and $\mathrm{G}^{t} \mathrm{Y}_{\mathrm{t}}$ (using OLS and suppressing the intercept), these estimates are obtained:

$$
\begin{aligned}
& \hat{\beta}_{1}=\hat{\mathrm{G}}(1+\hat{\mathrm{f}}) \\
& \hat{\beta}_{2}=-\hat{\mathrm{G} \hat{\mathrm{r}}} \mathrm{X}_{0} / \mathrm{m}_{0} \hat{\mathrm{~K}} \\
& \hat{\beta}_{3}=-\hat{\mathrm{G}} \mathrm{~m}_{0} / \mathrm{X}_{0} .
\end{aligned}
$$

Then, successively:

$$
\begin{align*}
\hat{\mathrm{r}} & =\left(\hat{\mathrm{\beta}}_{1} / \hat{\mathrm{G}}\right)-1  \tag{5}\\
\mathrm{X}_{0} & =-\hat{\mathrm{G}} \mathrm{~m}_{0} / \hat{\beta}_{3}  \tag{6}\\
\hat{\mathrm{~K}} & =-\hat{\mathrm{G} \hat{r} \hat{X}_{0} / \mathrm{m}_{0} \hat{\beta}_{2}} \tag{7}
\end{align*}
$$

Since there are four parameters to be estimated and only three regressors, the process just described gives rise to a range of solutions. It is convenient to label them by the value of $\mathrm{G}^{*}$ with which they are associated; thus, the " $\mathrm{G}=0.956$ " solution refers to estimates of $\mathrm{r}, \mathrm{X}_{0}$ and K given by substituting " $\mathrm{G}=0.956$ " in Equations (5) through (7) successively.

A range of solutions $\left\{\mathbf{G}, \mathrm{r}, \mathrm{X}_{0}, \mathrm{~K}\right\}$ is thereby established. Certain of these solutions, such as those with negative r , can be eliminated on a priori grounds. The remainder may be thought of as a set of feasible solutions. (Although this is in principle an infinite set, in practice-and in the discussion that follows-its elements are a limited number of representative solutions, each associated with a particular value of $\mathrm{G}^{*}$.)

## Assumptions

The model makes certain assumptions, given below in italics.
The logistic growth function $\mathrm{rX}(1-(\mathrm{X} / \mathrm{K}))$ is appropriate for this fishery. This functional form was first proposed more than 150 years ago (Clark, 1976), and has since become widely accepted. Even so, its use in modelling aggregated species-such as the Flemish Cap groundfishery reported on in this paper-gives rise to some difficulty. Even if the parameters r and K are identical for each of the constituent species, and that for every species Equation (1) is held to be true, Equation (1) cannot be algebraically derived on the aggregate level. ${ }^{4}$ Nevertheless, as other studies have done in the past, this paper uses the logistic growth function to describe an aggregated fishery.

A related assumption is that r and K are constant over time. In the case of the Flemish Cap reported on below, the stability of water temperature, salinity and other physical characteristics suggest that, given this choice of functional form at the aggregate level, r and K may plausibly be treated as constant.

The fleet harvest function is appropriately described by the functional form Y $=\mathrm{qXE}$. Earlier, questions were raised with respect to this functional form, but those questions were set in the context of a stochastic harvest function. In the present study, however, this relationship is deterministic, by means of which $q$ is
not so much estimated as defined. The stochastic element is introduced only later, in the geometric relationship between successive qs.

## Theoretical Expectations and Empirical Tests

Before discussing further the application of the model described above to a specific case, it is worth turning to the results which might be expected.

First, parameter estimates should meet the following a priori criteria: the biological growth rate, r , and the initial stock level, $\mathrm{X}_{0}$, cannot be negative, and estimated stock levels cannot exceed estimated carrying capacity (i.e., $K / X_{0}>1$ ).

Second, using estimates for $\mathrm{X}_{0}, \mathrm{r}$ and K , and observed levels of harvest, a time series can be constructed for the level of fish stocks throughout the period under study. The relationship between this series and the associated levels of observed fishing effort could be compared with theoretical expectations, based on an extension of the standard model. This extension, due to Smith (1969), views changes in fishing effort as determined by the level of profits. A point of equilibrium ( $\mathrm{X}^{*}$, $\mathrm{E}^{*}$ ) is found where the two variables, X and E , show no tendency to change. This point may be shown on a "phase plane" diagram, such as Figure 1, in which E is plotted against X . The figure also shows the time path leading to equilibrium, in the form of a stable spiral (see, e.g., Clark, 1976).

This expected relationship between X and E has been subjected to a limited amount of empirical testing. Wilen (1976) examined the North Pacific fur seal fishery over the period from 1882 to 1900; his results were consistent with the theoretical expectations of Figure 1, although the data series was too short to tell if an approach was being made to a point of equilibrium. The North Sea herring fishery for the period from 1963 to 1977 was studied by Bjørndal and Conrad (1987); their results do show a counterclockwise movement, consistent with the pattern found in Figure 1, that might or might not have developed into an inward


Figure 1. Theoretical approach of stocks and effort to the equilibrium ( $\mathrm{X}^{*}, \mathrm{E}^{*}$ ).
spiral. Conrad (1987) examined the Western Arctic bowhead whale fishery for the period from 1848 to 1914 ; his findings were less easily reconciled to the anticlockwise loop called for by the model, though he suggested a number of reasons for this disparity.

Third, given estimates of the fish stock for each period, and observations on harvests and fishing effort, it would be possible to estimate the fleet catchability coefficient for each period. When taken together as a series, these coefficients could be examined to find the implicit $G$ (periodic change in $q$ ) which they contain. This implicit G should be identical with-or be shown to converge to-the level of $\mathrm{G}^{*}$ used to generate the initial estimates.

Fourth, the estimated stock levels could also be used to calculate a time series for q with respect to each "type" of vessel (i.e., as characterised by its gear type, class, and country of origin). The suitability of $\mathrm{Y}=\mathrm{qXE}$ as a (stochastic) functional form at the level of the vessel type could then be tested.

## Application and Results

The model previously described was applied to the Flemish Cap groundfishery from 1971 to 1985.

The Flemish Cap is a fishing ground in the NorthWest Atlantic, about 300 miles east of Newfoundland. For the purposes of this study, the Cap has three main properties of interest: its physical isolation from other fishing areas (see, e.g., Konstantinov, 1970; Lilly, 1987; and Chekova and Konstantinov, 1978); the stability of its oceanographic characteristics (see, e.g., Borovkov and Kudlo, 1980; Konstantinov, 1970; Lear, Wells and Templeman, 1981; and Lilly and Gavaris, 1982); and the fact that it lies outside the 200 -mile limit of any country's Exclusive Economic Zone, or EEZ. ${ }^{5}$

Commercial fishing on the Flemish Cap has relied on cod and redfish almost exclusively. For the aggregated model used in this study, catches of cod and redfish (as well as other groundfish) are taken together. In the specific context of the Flemish Cap, however, cod and redfish are in a predator-prey relationship to one another (see, e.g., Lilly, 1980), and the relationship between these two species is likely to be complex. ${ }^{6}$

## Data

The Flemish Cap having been fished commercially only since 1957, data on catch and effort are available for the entire period of exploitation. The current study, however, was limited to the period from 1971 to 1985, since data for that period were available in a particularly convenient form from the Northwest Atlantic Fisheries Organisation (NAFO). ${ }^{7}$

Although data on catch and effort were provided on a monthly basis, this study used annual aggregates. ${ }^{8}$

## Results

Table 1 summarises the goodness of fit that resulted from applying the model described in Section 2 to the Flemish Cap groundfishery from 1971 to 1985.

Table 1
Summary of Goodness of Fit Associated with Various Estimates from the Model Described in Section 2
$\left.\begin{array}{lcc}\hline \text { total sum of squares: } 2147.42 \\ \text { "Explained" } \\ \text { Sum of Squares }\end{array} \quad \begin{array}{cc}\text { Adjusted } \\ \mathrm{R}^{2}\end{array}\right]$

All of the values of G* used in this model explained between $95 \%$ and $98 \%$ of the variability in $Y_{t+1} / E_{t+1}$. The " $G=0.956$ ' solution had the greatest explanatory power, a distinction emphasised by the scaling in Figure 2, which shows Table 1 in graphical form. (It should be noted that in this figure, a continuous functional relationship between explained sum of squares and values of $\mathrm{G}^{*}$ may not necessarily be assumed. The line in the figure was drawn by connecting results from distinct runs of the model, and its appearance of possessing both continuity and smoothness may be deceptive.) " $\mathrm{G}^{*}=0.956$ " corresponds to a $4.4 \%$ annual rate of decline in $q$.

The following discussion considers in greater detail the results associated with $\mathrm{G}^{*}=0.956$. It will be shown that not only does this solution have the greatest explanatory power, it is also the only solution to be self-consistent when the predicted values of $q$ to which it gives rise are, in turn, used to generate an estimate of G.

The " $\mathrm{G}=0.956$ " solution is associated with the following estimates:

$$
\begin{array}{ll}
\hat{\beta}_{1}=2.45439 & (t=6.74) \\
\hat{\beta}_{2}=-0.077395 & (t=-4.19) \\
\hat{\beta}_{3}=-0.000063 & (t=-2.12)
\end{array}
$$

"explained" sum of squares
(total sum of squares: 2147.42)


G*
Figure 2. "Explained" sum of squares when estimating the model at various levels of G".

From these estimates, the following parameter values were calculated as described in Section 2:

$$
\begin{aligned}
\hat{\mathbf{r}} & =1.56735 \\
\hat{\mathrm{X}}_{0} & =229,070 \text { (tonnes) } \\
\hat{\mathrm{K}} & =292,208 \text { (tonnes) }
\end{aligned}
$$

The possibility of heteroscedasticity was investigated and, to test a null hypothesis of homoscedasticity, the Breusch-Pagan statistic was calculated. This statistic is distributed as a $\chi^{2}$ with three degrees of freedom (the number of regressors that might influence the variance; see, e.g., Judge et al., 1982); where G* $=0.956$, it had a value of 2.45 . Thus, for any reasonable level of significance, the null hypothesis of homoscedasticity could not be rejected, and the use of OLS remains justified in this respect.
[For ease of comparison with the "expected results" set out above in Section 3 , the following characteristics of the " $\mathrm{G}=0.956$ " solution are given in the same order.]

First, for the " $\mathrm{G}=0.956$ " solution (as for many others not reported here), the parameter estimates satisfied the a priori criteria mentioned above in Section 3. ${ }^{9}$

Second, the dynamic properties of the various solutions were examined. Specifically, values were generated for X throughout the 15-year period under study, rather than just for $\mathrm{X}_{0}$. The results for $\mathrm{G}^{*}=0.956$, in which X appears to be relatively stable over the last ten years of the period, are shown in Figure 3.

The results for the " $\mathrm{G}=0.956$ " solution are also given in Figure 4 as a plot of E against X. ${ }^{10}$ This figure illustrates the anticlockwise loop which, as described
estimated stock
(metric tonnes)


Figure 3. Estimated stock levels on the Flemish Cap from 1971 to 1985 (computed where $\mathrm{G}^{*}=0.956$ ).
in Section 3, was anticipated by theory and, to a limited extent, confirmed by earlier empirical work. Beyond this anticlockwise pattern, however, it may be possible to see a series of connected anticlockwise loops: (A) from "1971" to " 1974 "; (B) from "1974" to " 1979 "; and (C) from " 1977 "' to " 1985 ". The last two in particular overlap, suggesting the form of a helix, running from northwest to southeast.


Figure 4. Relationship between observed fishing effort and estimated fish stocks (computed where $\mathrm{G}^{*}=0.956$ ).

Third, having estimated a series for X , it was also possible to derive a series for q at the fleet level, by dividing $\mathrm{X}_{\mathrm{t}} \mathrm{E}_{\mathrm{t}}$ into $\mathrm{Y}_{\mathrm{t}}$. The results are shown in Figure 5, for the case where $\mathrm{G}^{*}=0.956$.

The model had assumed that successive qs were linked by the relationship:

$$
\begin{align*}
& \mathrm{q}_{\mathrm{t}}
\end{align*}=\mathrm{G}^{\mathrm{t}} \mathrm{q}_{0},
$$

As a check on the model, estimated fleet-level qs were used to determine the strength of this relationship, and to see whether the value for G implicit in the q-series was consistent with the $\mathrm{G}^{*}$ from which those qs were generated.

When the q -series derived from the " $\mathrm{G}=0.956$ " solution was used to yield the dependent variable in Equation (8), the following parameter estimates were obtained:

$$
\begin{array}{r}
\log \left(q_{t}\right)=-9.6405-0.0450 t \\
(\text { s.e.: } 0.0654 \quad 0.0079)
\end{array}
$$

from which it follows that $\hat{G}=\mathrm{e}^{-0.0450}=0.956$, an estimate that is consistent with the initial value of $\mathrm{G}^{*}$.

The mean estimate of G , obtained in this way from the series of qs, coincided with the initial value of $\mathrm{G}^{*}$ (from which the qs themselves were derived) only when $\mathrm{G} \approx 0.956$. This result is illustrated in Figure $6 .{ }^{11}$ When some other initial value of $G^{*}$ was examined, successive estimates of $G$ converged to 0.956 . ${ }^{12}$

Thus, the estimates associated with $\mathrm{G}^{*}=0.956$ not only met least-squares


Figure 5. Estimated fleet catchability coefficients on the Flemish Cap from 1971 to 1985 (computed where $\mathrm{G}^{*}=0.956$ ).

G, as calculated from the qs generated by the estimation procedure


Figure 6. Testing for the self-consistency of solutions yielded by the estimating procedure described in section 2.
criteria and a priori expectations, but were shown to be self-consistent-and apparently uniquely so.

Fourth, having determined the time path of X, it was also possible to analyse the catchability coefficient at the vessel-type level. These coefficients are shown in Figure 7 for three of the most prominent types on the Flemish Cap. ${ }^{13}$ Two comments may be made.
(1) The expected relationship among the three appears generally to hold, with stern trawlers demonstrating a higher coefficient than the side trawlers they replaced, and the less capital-intensive longliners having a productivity coefficient considerably lower than both.
(2) The hypothesis is suggested of a constant coefficient over time for each type of vessel, ${ }^{14}$ and this hypothesis was tested by applying OLS to the harvest function for each type. All but one of the equations generated high t-ratios for the catchability coefficient. However, in those (few) instances where it was possible to examine the individual qs associated with one particular country, the coefficient estimates did not relate to one another as might have been anticipated $a$ priori. Since effort was measured in terms of the number of days fished it might, for example, have been expected that larger vessels would have higher coefficients than smaller vessels, and that stern trawlers would have higher ones than the side trawlers. There were only four instance in which these expectations could be tested, but in only one case were they borne out.

On the other hand, as noted above, in almost every case $Y_{i t}=q_{i} X_{t} E_{i t}$ was strongly supported by OLS estimation. Of the 22 major vessel types (accounting in total for at least $85 \%$ of fleet effort in every year), only one exhibited a t-ratio as low as 4.3 , with all the rest being greater than 8.4. These results provided a further measure of support for the functional form used in the standard openaccess model.


Note: the typical stern trawler shown here is Russian, of tonnage class 7; the side trawler is Portuguese, of (smaller) tonnage class 6; and the longliner is from the Faeroes, of tonnage class 4

Figure 7. Estimated catchability coefficients for specific vessel types.

## Discussion

The reported results meet theoretical expectations but go beyond them in at least two ways: first, the plot of E against X shows a helical, rather than a spiral, form and, second, the fleet catchability coefficient-far from being constant, as generally assumed in the standard model-exhibits an annual rate of decline of $4.4 \%$. Both issues are discussed below.

The helical pattern may arise from the abrogation of an assumption implicit in Figure 1: that costs per unit of effort remain constant. The theoretically determined point of equilibrium in Figure 1 is ( $\mathrm{X}^{*}, \mathrm{E}^{*}$ ), where

$$
\begin{aligned}
& X^{*}=\mathrm{c} / \mathrm{pq} \\
& \mathrm{E}^{*}=\mathrm{r}(1-(\mathrm{c} / \mathrm{pqK})) / \mathrm{q}
\end{aligned}
$$

where $c$ and $p$ are measures of input and output prices. (See, e.g., Clark, 1976.)
During the second half of the 1970s fuel costs, a major component of the cost of fishing, rose steadily and substantially. All else being equal, the rise in unit costs c would cause ( $\mathrm{X}^{*}, \mathrm{E}^{*}$ ) to move in the direction of increased stock levels and lower levels of effort. It may be that the apparent helix in Figure 4 owes its origin to this effect.

Increasing fuel costs may also be connected with a changing mix of vessel types over the period under study, and this changing mix could be picked up in a
declining $q$ for the fleet as a whole. ${ }^{15}$ Rising fuel costs may have encouraged fishermen to switch to technologies, such as longlining, that were more fuelefficient in their relatively limited "drag" through the water whilst fishing. During the period under study a declining share of larger vessels in the Flemish Cap fishery was observed, and the relative decline was particularly evident in capitalintensive technologies such as the bottom trawl.

## Summary

This paper has described an estimating procedure for dealing with open-access situations in an econometrically valid way, and a specific application of this procedure has been added to the scanty file of empirical tests for open-access models, with results that were generally consistent with theoretical expectations.

It would be valuable if the model were to be applied to some other open-access situation besides the Flemish Cap. On the groundfishery of the Cap over the period form 1971 to 1985 the estimating procedure works well, yielding a leastsquares solution in which, uniquely, the starting value of $G$ is confirmed by subsequent parameter estimates. Since it has yet to be shown that a single solution of this kind is the inevitable result of using the procedure in question, it remains to be seen whether further applications will be as well-behaved in practice as the one examined in this study.

## Notes

1. I much appreciate the suggestions and comments made by two anonymous referees.
2. Such an observation has been made by others. For example, Uhler (1980) used Monte Carlo techniques to investigate the magnitude and direction of this bias.
3. I.e., the relationship between $\omega$ and Y is not such that, in regression, some of the variability in $\omega$ will be wrongly ascribed by the estimator to variability in the regressor Y .
4. $\mathrm{X}_{\mathrm{A}}=\mathrm{rX}_{\mathrm{A}}\left(1-\mathrm{X}_{\mathrm{A}} / \mathrm{K}\right)$ and $\mathrm{X}_{\mathrm{B}}=\mathrm{rX}_{\mathrm{B}}\left(1-\mathrm{X}_{\mathrm{B}} / \mathrm{K}\right) \nRightarrow \mathrm{X}_{\mathrm{A}+\mathrm{B}}=\mathrm{r} \mathrm{X}_{\mathrm{A}+\mathrm{B}}\left(1-\mathrm{X}_{\mathrm{A}+\mathrm{B}} / \mathrm{K}\right)$
5. On the other hand, one limitation of the model is that it does not recognise the introduction of EEZs by several countries in 1977. To the extent that these measures limited the options available to the fleets of certain nations, they could be expected to have had an effect on the Flemish Cap.
6. Mature cod are also known to cannibalise the young of their species.
7. I am very grateful to Joan Palmer and her staff of the National Marine Fisheries Service for making this data available. (References to the Flemish Cap in this study apply, strictly speaking, to NAFO Subarea 3M.)
8. As a result, there was insufficient data to allow Analysis of Variance (ANOVA) techniques to be used. Such an approach would attribute the variability of catch per unit effort to, respectively, the level of stocks, the country of origin, the gear employed, and the tonnage class of the vessel.
9. These observations hold for any reasonable level of significance.
10. For other "solutions" where $0.90<\mathrm{G}<0.98$, the patterns were similar to Figure 4. For $\mathrm{G} \leqslant 0.90$ the series of loops turned into a much flatter series of kinks, as when a spring is stretched beyond its limits.
11. The uniqueness of this result could not be demonstrated from first principles, and the apparently smooth curve in Figure 6 was formed merely by connecting individual points whose coordinates had been calculated separately.
12. Successive estimates of $G$ would be formed by (i) picking some value of $\mathrm{G}^{*}$; (ii) calculating the resulting $q$-series; (iii) calculating the value of $G$ which best fitted the q -series; and (iv) using that value in place of $\mathrm{G}^{*}$.
13. Each of the three types was chosen as being the representative of a particular technology (longlining, side and stern trawling) that was active on the Cap for the greatest number of years.
14. The apparent outliers in 1975 (Faeroese longliner) and 1980 (Portuguese side trawler) represent observations where the level of effort for that type was much less than in other years.
15. Such a decline is not an unprecedented result: Agnello and Anderson (1981), studying NAFO Subareas 5 Y and 5 Z with a very different model, found the "catchability" of cod, haddock and redfish to be declining over the period from 1960 to 1974. The authors associated this decline with the use of quotas, at least in the case of haddock after 1969.

## References

Agnello, R. J., and Anderson, L. G. 1981. Production responses for multi-species fisheries. Canadian Journal of Fisheries and Aquatic Sciences 38:1393-1404.
Bjørndal, T., and Conrad, J. M. 1987. The dynamics of an open access fishery. The Canadian Journal of Economics 20:74-85.
Borovkov, V. A., and Kudlo, B. P. 1980. Results of USSR oceanographic observations on Flemish Cap, 1977-78. Selected Papers Number 6: 47-52. Dartmouth, Nova Scotia: International Commission for the Northwest Atlantic Fisheries.
Chekova, V. A., and Konstantinov, K. G. 1978. Characteristics of the beaked redfish, 'Sebastes mentella Travin', in bottom and midwater trawl catches on Flemish Cap. Selected Papers Number 3: 17-21. Dartmouth, Nova Scotia: International Commission for the Northwest Atlantic Fisheries.
Clark, C. W. 1976. Mathematical Bioeconomics: the Optimal Management of Renewable Resources. New York: John Wiley \& Sons.
Conrad, J. M. 1987. Bioeconomics and the bowhead whale. Staff Paper no. 87-14. Ithaca, N.Y.: Cornell University, Department of Agricultural Economics.

Hannesson, R. 1983. Bioeconomic production function in fisheries: theoretical and empirical analysis. Canadian Journal of Fisheries and Aquatic Sciences 40:968-982.
Judge, G. G., Hill, R. C., Griffiths, W., Lütkepohl, H., and Lee, T.-C. 1982. Introduction to the Theory and Practice of Econometrics. New York: John Wiley \& Sons.
Konstantinov, K. G. 1970. On the appropriateness of the Flemish Cap cod stock for experimental regulation of a fishery. In International Commission for the Northwest Atlantic Fisheries Redbook, part III, 49-56.
Lear, W. H., Wells, R., and Templeman, W. 1981. Variation in vertebral averages for year-classes of Atlantic cod, 'Gadus morhua', on Flemish Cap. Journal of Northwest Atlantic Fishery Science 2:57-60.
Lilly, G. R. 1980. Year-class strength of redfish and growth of cod on Flemish Cap. Selected Papers Number 6: 35-39. Dartmouth, Nova Scotia: International Commission for the Northwest Atlantic Fisheries.
Lilly, G. R. 1987. Synopsis of research related to recruitment of Atlantic cod ('Gadus morhua') and Atlantic redfishes ('Sebastes sp.') on Flemish Cap. Scientific Council Studies Number 11: 109-22. Dartmouth, Nova Scotia: International Commission for the Northwest Atlantic Fisheries.
Lilly, G. R., and Gavaris, C. A. 1982. Distribution and year-class strength of juvenile redfish, 'Sebastes sp.', on Flemish Cap in the winters of 1978-82. Journal of Northwest Atlantic Fishery Science 3(2):115-122.

Smith, V. L. 1969. On models of commercial fishing. Journal of Political Economy 77:181198.

Uhler, R. S. 1980. Least squares regression estimates of the Schaefer production model: some Monte Carlo simulation results. Canadian Journal of Fisheries and Aquatic Sciences 37:1284-1294.
Wilen, J. E. 1976. Common property resources and the dynamics of over-exploitation: the case of the North Pacific fur seal. Paper \#3 in the Programme in Resource Economics. Vancouver: The University of British Columbia.

Copyright of Marine Resource Economics is the property of Marine Resources Foundation. The copyright in an individual article may be maintained by the author in certain cases. Content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.

