Irreversible Investment and Optimal Fisheries Management: A Stochastic Analysis

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Abstract In recent years, attention has been devoted to fishery management problems that arise because capital embodied in fishing fleets is often nonmalleable, having few if any alternative uses. This problem of irreversible investment was analyzed by Clark et al. (1979), using a deterministic model. In reality, however, most investment decisions must be made within an uncertain environment. This paper describes recent efforts to account for uncertainty in analyzing the problem of optimal fishery investment, where the uncertainty is caused by stochastic variability in the resource stock from year to year.

Introduction

It has been increasingly recognized in recent years that fishery management must take into account the fact that capital used in

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the exploitation of fishery resources is often nonmalleable. That is to say, the capital lacks effective alternative uses. If the capital is to be sold, one can do so only at a loss. A current example of nonmalleable capital in fisheries is provided by the excess capacity evident in many "distant-water nation" fishing fleets since the onset of Extended Jurisdiction. In this paper we summarize two recent research works in the area of optimal fishery investment, comparing in particular the differences between deterministic and stochastic analysis.

Clark et al. (1979) investigated the consequences of nonmalleability of capital for fishery management, using a continuous time deterministic model. They pointed out that if capital employed in the exploitation of a fishery resource is perfectly malleable, there is no need to be concerned with the stock of capital as such. The services of capital can be treated in exactly the same manner as the services of labor. The only stock one need be concerned with is the resource itself. Hence the only investment program of interest is the program of investment in the resource. Formally, one is confronted with a relatively simple optimal control problem involving one state variable and one control variable.

When capital used in exploiting the resource is nonmalleable, the problem becomes substantially more complicated. One then does have to be concerned with the stock of capital. Of greater importance, however, is the fact that the optimal programs of investment in the resource and investment in capital are then interdependent. Formally, we now have an optimal control problem with two state variables and two control variables.

Clark et al. (1979) confine their discussion of nonmalleability to a situation in which capital embodied in the fleet is nonmalleable. In so doing, they consider three cases with varying degrees of nonmalleability: the case of perfect nonmalleability, in which there is no positive resale value and in which the depreciation rate is zero; the case of quasi-nonmalleability, in which there is no positive resale value, but the depreciation rate is positive; and finally the case in which the resale value is positive.

Let us focus on the second case. The nature of the joint investment program can be represented by the following example.

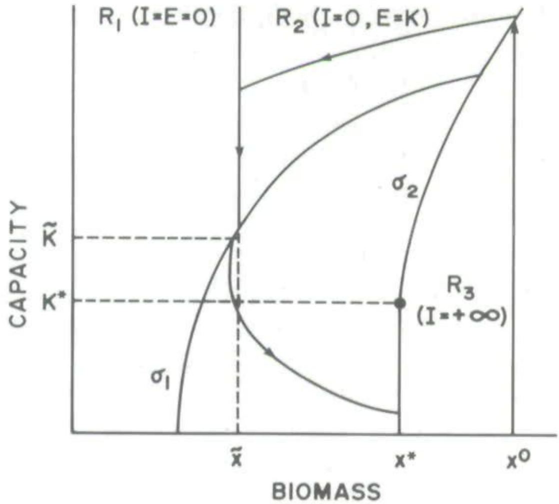

FIGURE 1. Optimal investment and harvesting policies for the deterministic continuous-time instantaneous-investment model of Clark et al. (1979). A sample trajectory is also shown. See text for details.

Figure 1 shows a state-space diagram taken directly from the Clark et al. article.

Let x^* denote the biomass level that would be optimal if capital were perfectly malleable and $\tilde{x}$ the optimal biomass level if capital were "free." The capital stocks $\tilde{K}$ and K^* are the capital stock levels required if harvesting is to take place on a sustained yield basis at $\tilde{x}$ and x^*, respectively.

Let it be supposed that we commence with the resource at its natural equilibrium level $x(0) = x^0 > x^*$. We suppose further that $K = 0$. Finally we suppose that so far it had not been profitable to harvest the resource. Now, because of a one-time change in market conditions, the resource becomes commercially exploitable.

The optimal investment in K, which is assumed to occur instantaneously ($I = +\infty$), is given by the switching curve σ_2 (the corresponding switching curve for the resource is σ_1). The optimal level of investment in K will be certain to result in a fleet of sufficient size to reduce x below x^*.

Once the investment has been undertaken, the only costs associated with the fleet that are relevant are operating costs. Optimal resource management policy thus calls for fishing the resource stock down to $\bar{x}$, if the harvesting capacity permits. The question of harvesting capacity is relevant because it will never be optimal to invest in vessels when $x(t)$ is less than x^*. Indeed, once the initial investment has been undertaken, the optimal capital investment policy will be to set gross investment equal to zero, implying that net investment $\dot{K}$ is equal to $-\gamma K$, where γ denotes the depreciation rate. It will be optimal to maintain this policy until the resource stock level has risen to x^*.

In the example given, investment in K is sufficient to reduce x to $\bar{x}$. Once $\bar{x}$ has been reached, harvesting takes place at $\bar{x}$ on a sustained yield basis. This can only be temporary, however, as the fleet is steadily declining in size. A period of enforced conservation will follow in which it is not possible to harvest the sustained yield with the existing fleet even though the fleet is used to capacity. Thus x^* will be gradually approached as shown by the trajectory. Once x^* has been reached, it will be optimal to invest in K to the extent that K is increased to K^*.

The Clark et al. (1979) model is entirely deterministic in nature. In the remaining sections of this paper we describe a related stochastic model and enquire into the consequences for resource and capital investment policy of introducing uncertainty. What will be of particular interest here is the impact of uncertainty on investment in nonmalleable capital, given that such investment is interdependent with harvesting policy. The model discussed hereafter has been analyzed more extensively by Charles (1982, 1983a, 1983b); unlike the deterministic model, it is one of discrete time and allows for fleet investment delays.

The Model

Following Clark et al. (1979), consider a fishery using an aggregated capital stock K to exploit a single-cohort fish stock. To aid exposition and add realism to the model, we shall assume a discrete-time seasonal fishery. It is assumed further that natural environmental variability causes the biomass to follow a stochastic stock-recruitment relationship. Specifically the recruitment in year n, R_n, is assumed to follow a lognormal probability distribution with mean $F(S_{n-1})$, where S_{n-1} is the previous year's escapement and $F(\cdot)$ is a deterministic stock recruitment function, assumed here to be a pure compensatory one ($F' > 0$, $F'' < 0$).

The capital stock changes from year to year through natural depreciation and possible investment. It is assumed that the resale value of the vessels is zero. Thus effectively the capital stock can be decreased only through depreciation, which is assumed to occur at the end of each season. New fleet capacity costs c_K per unit and must be paid for at the time of order, but does not become available until the season following the one in which it is ordered. This delay in investment reflects real-world behavior and tends to increase the uncertainty in capital investment planning.

The intraseasonal dynamics of the biomass x in year n are governed by the differential equation $dx/dt = -h(t) = -qE(t)x(t)$, where t is the time within the season, h is the harvest rate, q is the catchability, $x(0) = R_n$, and the instantaneous fishing effort $E(t)$ is subject to $0 \leq E(t) \leq K_n$. Hence S_n^* must be chosen subject to $R_n e^{-qTK_n} \leq S_n^* \leq R_n$, where T is the maximum feasible season length. Given S_n^*, the optimal addition to fleet capacity K desired to become available at the beginning of the next season is determined before the end of the current season.

The price of fish p, the unit cost of effort c, and the catchability q, are considered to be known, fixed constants.

The rents accruing to the fleet in year n are then given by:

$$\begin{aligned}\pi(R_n, K_n, S_n, I_{n+1}) &= \int_0^T [pqEx - cE]dt - c_K I_{n+1} \\ &= p(R_n - S_n) - (c/q)\ln(R_n/S_n) - c_K I_{n+1}\end{aligned}$$

where I_{n+1} denotes the increase in fleet capacity to come into effect at the beginning of period $n + 1$. By assumption $I_{n+1} \geq 0$.

We assume that the fishery manager is risk-neutral. Ideally we should allow for the possibility of the manager being riskaverse, and indeed this possibility will be considered in future work. However, allowing for risk aversion seriously complicates the analysis and thus shall not be attempted here.

Given our assumption of risk neutrality, it is reasonable to assume that the manager's objective is to maximize the discounted stream of expected yearly rents. Hence our problem can be summarized as follows:

$$\max_{\{S_1, I_2, S_2, \ldots\}} \left\{ \sum_{n=1}^{\infty} \alpha^{n-1} E[\pi(R_n, K_n, S_n, I_{n+1})] \right\}$$

subject to $R_{n+1} \sim \phi_{F(S_n),\, \sigma}(\cdot)$, $K_{n+1} = (1 - \gamma)K_n + I_{n+1}$, $R_n e^{-qTK_n} \leq S_n \leq R_n$, and $I_{n+1} \geq 0$, where

$$\phi_{\underline{R},\, \sigma}(R) = \left(\frac{1}{\sqrt{(2\pi)}\sigma R}\right) \exp\{-(\log \{R/\underline{R}\} - \sigma^2/2)^2/2\sigma^2\}$$

is a log normal density with mean $\underline{R}$ and uncertainty parameter σ, and where α is the annual discount factor.

The problem can be formulated in a recursive form by using Bellman's dynamic programming approach, to obtain the value V of the fishery in state (R_n, K_n) at the start of season n:

$$V(R_n, K_n) = \max_{R_n e^{-qTK_n} \leq S_n \leq R_n} \ \max_{I_{n+1} \geq 0} \{\pi(R_n, K_n, S_n, I_{n+1}) + \alpha E V(R_{n+1}, K_{n+1})\}$$

where $K_{n+1} = (1 - \gamma)K_n + I_{n+1}$ and $R_{n+1} \sim \phi_{F(S_n),\, \sigma}$.

Removing the subscripts on the variables, this can be rewritten:

$$V(R, K) = \max_{Re^{-qTK} \leq S \leq R} \ \max_{I \geq 0} \{\pi(R, K, S, I) + \alpha \underset{R' \sim \phi_{F(S),\, \sigma}}{E} V(R', (1 - \gamma)K + I)\} \quad (1)$$

It is this Equation (1) that forms the basis for the analysis presented below. For further discussion of the structure and assumptions of the model, see Charles (1982, 1983a, 1983b).

Preliminary Analysis

Performing the inner maximization in Equation (1), for fixed S we obtain the optimality equation for investment:

$$\underset{R \sim \phi_{F(S),\ \sigma}}{E} V_K(R, (1 - \gamma)K + I) = \frac{c_K}{\alpha} \tag{2a}$$

or $I = 0$ if

$$\underset{R \sim \phi_{F(S),\ \sigma}}{E} V_K(R, (1 - \gamma)K) < \frac{c_K}{\alpha} \tag{2b}$$

This indicates that unless the fleet is temporarily overcapitalized, next year's optimal capacity, $(1 - \gamma)K + I$, should be set such that the expected marginal benefit of an extra unit of capital equals its marginal cost. The term V_K is the partial derivative of V with respect to K.

Let us define $K = h(S)$ to be the solution of $E\{V_K(F(S), K)\} = c_K/\alpha$, so that $h(S)$ is next season's optimal capacity. We can observe that, substituting new terms, $h(S)$ is analogous to the switching curve σ_2 in Figure 1. It is shown in the above references that under suitable assumptions, $h(\cdot)$ is an increasing function of escapement. Thus if $(1 - \gamma)K > h(S)$, the optimal investment is $I^* = 0$ (capital is already sufficiently abundant) while otherwise I^* is chosen so that $(1 - \gamma)K + I^* = h(S)$. This can be written:

$$I^*(S, K) = \max\,[h(S) - (1 - \gamma)K, 0] \tag{3}$$

Now, performing the outer maximization, we obtain the optimality expression equating the marginal benefit and marginal cost of an incremental increase in escapement:

$$\alpha \frac{F'(S)}{F(S)} \cdot \underset{R \sim \phi_{F(S),\ \sigma}}{E} \{R \cdot V_R[R, (1 - \gamma)K + I^*(S, K)]\} = p\left(1 - \frac{x_\infty}{S}\right) \tag{4}$$

where $x_\infty = c/pq$ and the constraint $Re^{-qTK} \leq S \leq R$ has been neglected temporarily. Assuming that Equation (4) has a unique solution $S = s(K)$, the following results have been obtained:

1. If $K = h(S)$ and $S = s(K)$ intersect at $S = S_l$, then the optimal escapement is independent of fleet capacity if K is small; that is, $s(K) = S_l$ for $K < [1/(1 - \gamma)]h(S_l)$. Thus S_l represents the optimal escapement at low fleet capacities.
2. As fleet capacity K becomes very large, the optimal escapement $s(K)$ approaches (or reaches) the level $\tilde{S}$, representing the abundant-capital equilibrium and corresponding to $\tilde{x}$ in Clark et al. (1979).
3. $S_l < \tilde{S}$.
4. $s(K)$ is likely to be an increasing function of K throughout.

The function $s(K)$ is analogous, once again with appropriate substitutions, to the switching curve σ_1 in Figure 1. Incorporating the constraint $Re^{-qTK} \leq S \leq R$, we now obtain the optimal escapement function:

$$S^*(R, K) = \begin{cases} Re^{-qTK} & R > s(K)e^{qTK} \\ s(K) & s(K) \leq R \leq s(K)e^{qTK} \\ R & R < s(K) \end{cases} \tag{5}$$

Thus the two policy functions $h(S)$ and $s(K)$ are sufficient to determine optimal management for any state of the fishery. The above analysis indicates that optimal policies under uncertainty should be qualitatively similar to those deduced by assuming a deterministic world. However, to address the important questions of comparative dynamics and quantitative differences arising when uncertainty is considered, we must turn to numerical methods. Results obtained by using such methods are outlined in the following sections.

Deterministic Results

Before moving to a discussion of the stochastic results, we shall first review the corresponding deterministic results obtained by

Table 1
Parameter Values Used in the Base Case Runs of the Model for the Prawn and Whale Fisheries

Quantity	Prawn Fishery	Whale Fishery
"Fish" price (p)	0.9 A$/kg	7000 US$/BWU*
Variable cost (c)	1600 A$/week per vessel	5000 US$/catcher-day
Capital cost (c_K)	4.7×10^5 A$/vessel	10,000 US$/(catcher-day/ year)
Depreciation rate (γ)	0.15	0.15
Discount factor (α)	0.9	0.9
Catchability (q)	0.00179/week per vessel	1.3×10^{-5}/catcher-day
Maximum season length (T)	26.0 weeks	1.0 years
Maximum fecundity (a)	42.0	1.15
Maximum recruitment (b)	7.0×10^6 kg	1.186×10^7 BWU*
Natural mortality factor (M)	0.273	0.905

*Blue whale unit.

Charles (1982, 1983a). The aforementioned joint investment model was applied to each of two fisheries, namely (1) the Australian Gulf of Carpenteria banana prawn fishery (Clark and Kirkwood, 1979) and (2) the aggregated pelagic whale fishery (Clark and Lamberson, 1982). The stock-recruitment function used in the analysis was primarily the Beverton-Holt function, $R = F(S) = aS/(1 + (aS)/b)$, where values of a are reduced by an appropriate factor M to take into account natural mortality effects. We have reproduced in Table 1 the parameter values used for each fishery. The corresponding optimal policy functions for the prawn and whale fisheries are depicted in Figures 2 and 3, respectively, with sample trajectories shown in each case.

The primary difference between these results and those obtained by Clark et al. (1979) (see Figure 1) arise from discrete-time, as opposed to continuous-time, structure and from the incorporation of a delay in bringing new capacity on-line. In the continuous-time Clark et al. results, as biomass increases there exists a unique biomass level x^* at which the benefits of investment first outweigh the costs. However, in the discrete-time

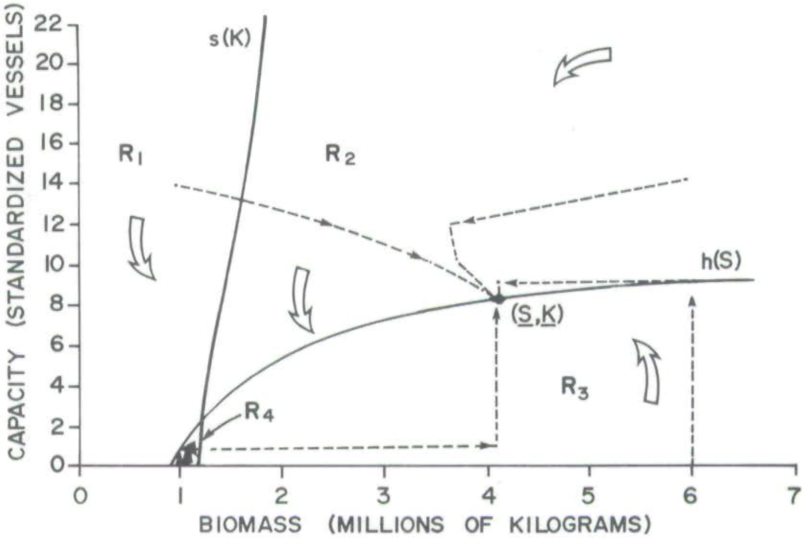

FIGURE 2. Optimal investment and escapement policy curves, $S = s(K)$ and $K = h(S)$, for the prawn fishery with delayed investment and deterministic dynamics. Large arrows indicate the overall effect of fishery dynamics. Sample trajectories and the long-run equilibrium $(\underline{S}, \underline{K})$ are also shown. (From Charles [1983a].)

fishery the intraseasonal rent structure is such that as biomass increases, the benefits of harvesting increase gradually. Even for $R < \underline{R}$, the long-run equilibrium biomass, small amounts of investment are desirable to capture the substantial rents obtainable through harvesting for part of the season. As biomass increases further, the optimal capacity also increases until at $S = \underline{S}$ we have $h(\underline{S}) = \underline{K}$, the long-run optimal capacity. This leads to the smooth concave $h(\cdot)$ policy functions shown in Figures 2 and 3, contrasting with the switching curve σ_2 of Figure 1.

For deterministic models the introduction of delays in investment, so that new capacity desired for a given season must be planned and paid for in the previous season, is essentially a change in accounting procedure. Compared with the instantaneous investment assumption used by Clark et al. (1979), the effective capital cost per unit of new capacity is increased by a

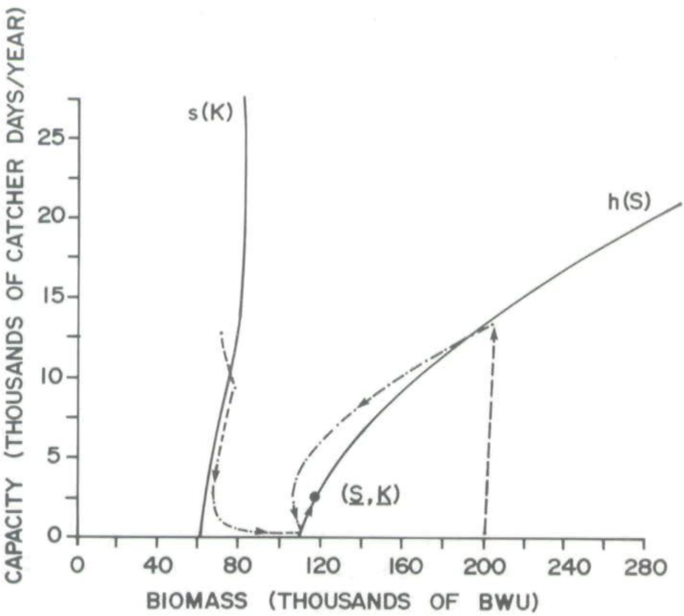

FIGURE 3. Optimal investment and escapement policy curves for the deterministic whale fishery with delayed investment. Sample trajectories approach the long-run equilibrium point ($\underline{S}$, $\underline{K}$).

factor $1/\alpha$, but the decision criterion remains the same. However, delayed investment is a realistic component of fishery models and creates additional uncertainty in a stochastic world. The effect of delayed investment on the appearance of the policy functions is to make the optimal fleet capacity dependent on escapement rather than recruitment. This produces little change for a slow-growing stock (e.g., whales) but for a fast-growing stock (e.g., prawns) the $h(\cdot)$ curve can be shifted to the left substantially and can intersect the optimal escapement curve $s(K)$. In other words it can be optimal to have positive investment during a season when no harvesting is desirable, in anticipation of a high stock size next season. This situation is depicted in Figure 2.

Having now seen typical results for the deterministic case, we turn in the next section to an examination of the effects on optimal fishery management of introducing random fluctuations in the biomass.

Stochastic Results

In considering the effect of stochastic fluctuations on optimal policies for fishery investment, there are three primary questions. First, how will investment behavior in an optimally managed stochastic model of a fishery differ qualitatively from that of the corresponding deterministic model? Second, will the optimal fleet capacity in a stochastically modeled fishery be higher or lower than that predicted by corresponding deterministic models? Finally, how does the effect of randomness interplay with economic and ecological parameters?

To examine the first question, consider a fishery with data as given for the prawn fishery in Table 1, except with capital cost $c_K = 83200$, maximum fecundity $a = 14.0$, and uncertainty parameter $\sigma = 0.58$. (This value of σ was the maximum likelihood estimate obtained by fitting a log normal distribution to prawn recruitment data [G. P. Kirkwood, personal communication, 1980].)

Figure 4 shows the resulting optimal policy functions for this stochastic ($\sigma = 0.58$) fishery, and the corresponding deterministic ($\sigma = 0$) case (together with analogous results when $a = 3.82$, not discussed here). As expected, the qualitative features of the stochastic and deterministic policies are identical. However, the behavior of the fishery itself changes considerably when stochastic fluctuations are introduced. In the deterministic model, the fishery eventually approaches a long-run equilibrium point, where $S^* = 2.5 \times 10^6$ kg and $K^* = 10.5$ vessels in this case. When stochastic effects are considered, the trend (or drift) of the fishery is again towards an equilibrium, but random fluctuations in the biomass keep the fishery from reaching this equilibrium. If environmental factors produce a higher-than-average recruitment, this will tend to result in higher escapement, greater investment, and hence higher fleet capacity. In the following seasons, both the biomass and capital stock will tend to be re-

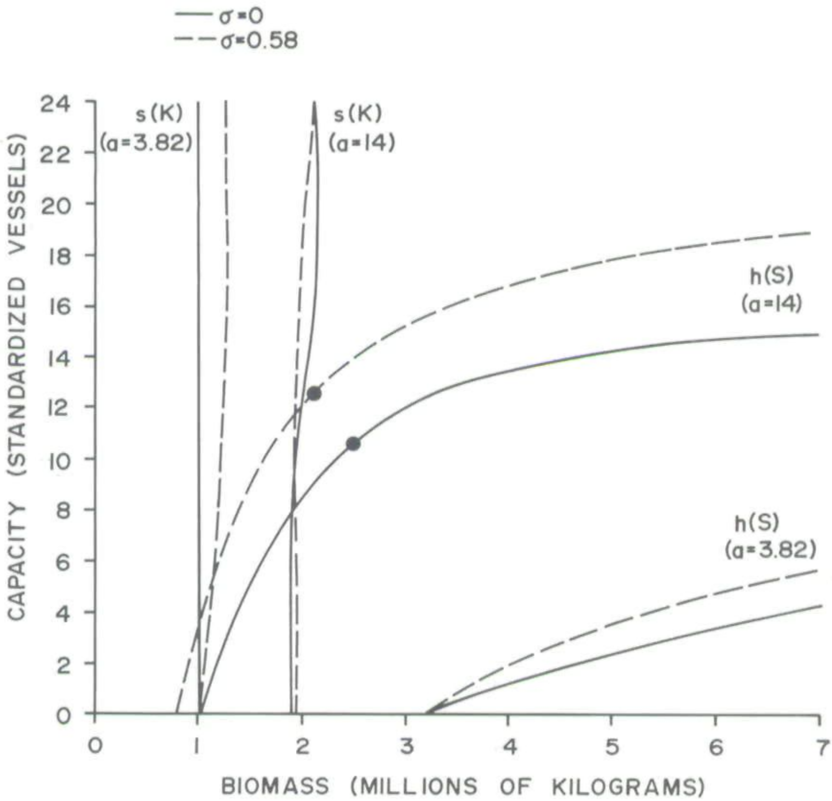

FIGURE 4. Stochastic and deterministic policy curves for prawn-type fisheries with low (a = 3.82) and moderate (a = 14) biomass growth rates, and relatively low unit cost of capital. For the a = 14 case, the deterministic equilibrium point and the stochastic "quasi-equilibrium" point are also indicated. (From Charles [1983b].)

duced towards the equilibrium, but this will again be disturbed by random environmental shocks. The fishery will be in a continual state of flux but will in fact be managed optimally. A similar result occurs if recruitment in any season is abnormally low, except that in this case the capital stock will depreciate and the biomass will tend to recover, until eventually new investment becomes desirable. (However, random fluctuations are bound to disrupt this trend to a certain extent and may hold the resource at low levels for a long period of time.) Charles (1982, 1983b) discusses this long-run behavior in more detail.

We turn now to our second and third questions, namely the effect of uncertainty in determining optimal fleet capacity and the role of economic and ecological parameters in this process. We shall concentrate on two parameters which Charles' earlier studies found to be of primary importance: the maximum biomass growth rate a and the ratio of fixed costs to operating costs $c_K/(c \cdot T)$.

Consider first the comparative dynamics when the cost ratio is changed. (Since the parameters c and T are held constant here, it suffices to vary the unit cost of capital c_K.) Figure 4 was based on a relatively low cost of capital, with $c_K/(c \cdot T) = 2.0$ in that case.

Optimal capacity under uncertainty is clearly higher than it would appear from a deterministic model. If, for example, the fishery has been previously uneconomical, with a virgin stock of $S = 7.0 \times 10^6$ kg but no fleet capacity, and a one-time change occurred in economic conditions, producing parameter values as outlined above, the desired investment would be $I^* = h(5.0 \times 10^6) \simeq 18.0$ standardized vessels. Use of the deterministic model produces an understatement of the desired investment by 22%, or 4.0 standardized vessels.

Figure 5 indicates the effect on optimal fleet capacity of varying the unit cost of capital while holding other parameters fixed. (Uncertainty has little effect on the optimal escapement curves, which have been suppressed here for the sake of exposition.) With much more expensive capital (that is, a higher ratio of fixed to variable costs) it is no longer desirable to have extra capacity to reap the upside benefits of exceptionally good years. The downside risk of being faced with a series of bad years now dominates, so that a more conservative investment strategy is preferable in the uncertain case. In other words, other things being equal, optimal capacity will be higher under uncertainty with a low cost of capital, and lower under uncertainty when capital is relatively expensive.

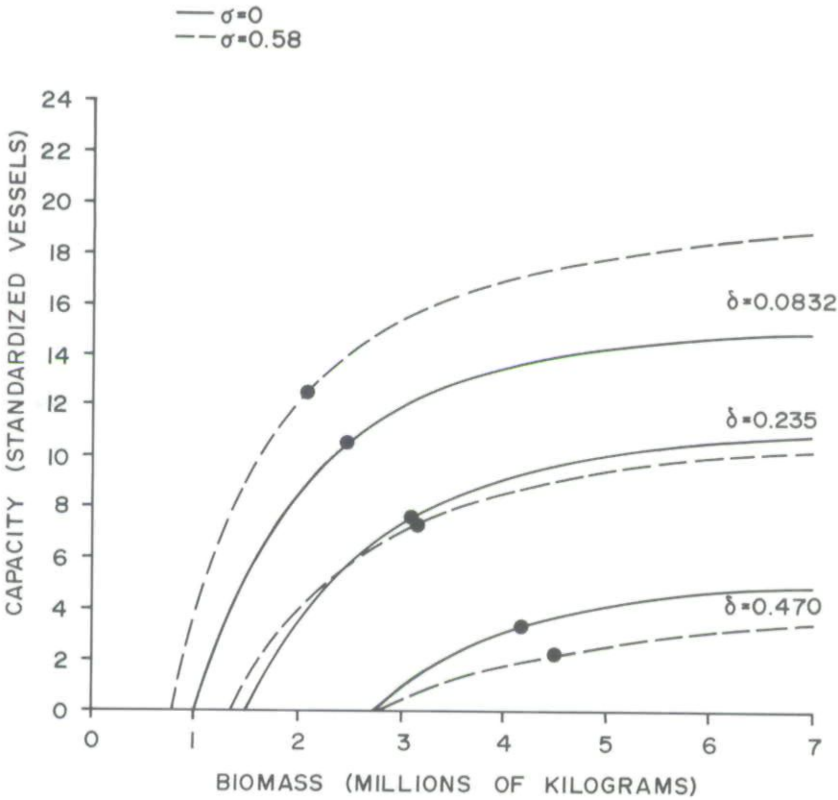

FIGURE 5. Deterministic and stochastic optimal fleet capacity curves are compared, for three values of the unit capital cost δ (with unit operating costs fixed). The effect of uncertainty on investment strategy clearly depends on the relative cost of capital. (From Charles [1983b].)

With regard to the effect of the natural biomass growth rate, Figure 6 presents a typical result showing variations in the optimal fleet capacity curve with both the intrinsic growth rate and the level of uncertainty. When the growth rate is sufficiently high (e.g., $a = 560$), the optimal fleet capacity is generally greater under uncertainty. The upside benefit of extra capacity to take advantage of occasional good years dominates the downside risk of suffering idle capacity during a series of bad years. At lower

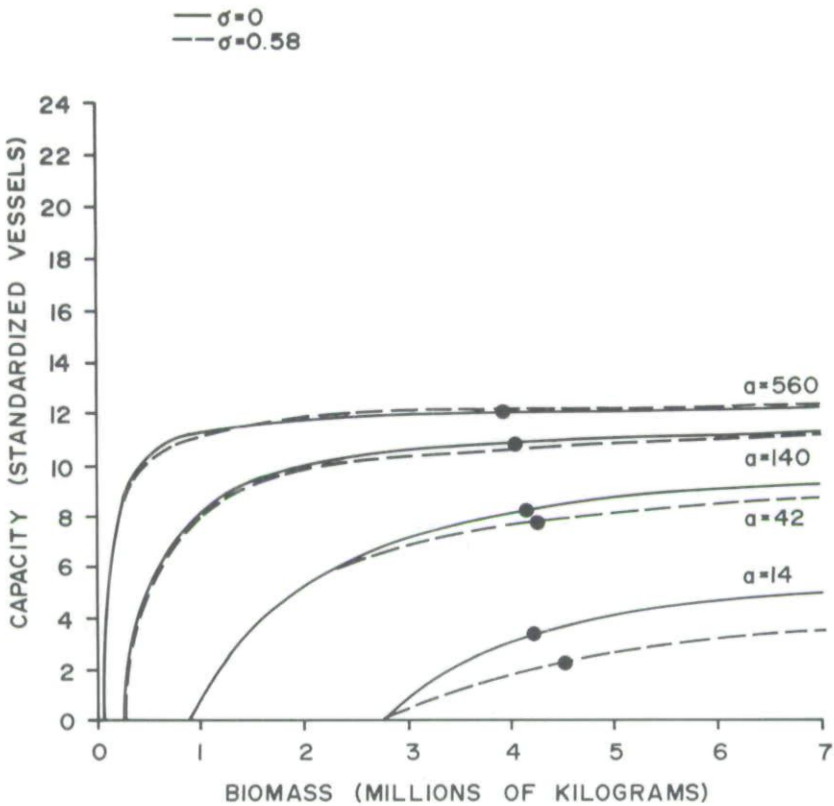

FIGURE 6. The effect of the biomass growth rate (a) on deterministic versus stochastic optimal fleet capacity curves, $K = h(S)$. The lower the intrinsic growth rate, the more likely investment is to diminish with increasing levels of uncertainty. (From Charles [1983b].)

growth rates, the "memory" inherent in the population dynamics becomes more important; both good years and bad years will tend to have repercussions farther into the future. This implies that

1. Unusually large fish stocks can likely be harvested over a number of years, at lower effort level.
2. Unusually small fish stocks are likely to persist over several years, increasing the downside risk of idle capacity.

Together these effects lead to the domination of downside risks over upside benefits in determining optimal capacity. Hence at low biomass growth rates, optimal fleet capacity will be lower with uncertainty than without.

Conclusions

The effects of uncertainty on the management of a fishery in which nonmalleable capital is employed can be summarized as follows. In most cases, but not all, optimal management of the resource itself tends to be more conservationist in the face of uncertainty. The trend found here is consistent with earlier results.

With respect to investment in the fleet, the effects of uncertainty can go either way. Having a large fleet capacity means that one can take advantage of exceptionally large recruitments. It means as well, of course, having large underutilized capacity when recruitments are exceptionally low. Whether one opts for a larger or smaller fleet capacity under uncertainty than one would have in a certain world depends on the relative cost of capital and on whether the resource stock is fast growing or slow growing.

If the resource stock is fast growing and if capital is cheap, uncertainty has the effect of causing the optimal fleet capacity to be greater than what would be optimal under conditions of certainty. If the resource stock is slow growing and capital is expensive, then the reverse is true. This phenomenon can be explained by considering the upside and downside risks involved in fishery investment decisions; see Charles (1983b) for a full discussion.

Much research, of course, remains to be done. Particularly interesting is the problem of investment in the face of parameter uncertainty, especially in regard to the price and the biological parameters. (Charles [1984] provides a preliminary analysis of this question.) In addition, issues of risk preference, together with specific policy implications, must await future research.

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