The Multicohort Fishery Under Uncertainty

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Abstract The multicohort fishery subject to random environmental disturbances is examined within a market framework. The free access problem is considered and optimal selective and proportional harvesting policies are discussed.

Introduction

Optimal harvesting of the multicohort fishery must take into account the effects of environmental disturbances on the growth and age structure of the population. Such disturbances may include predatory populations, fish species that compete for the same food supplies, changes in available food supplies, water temperature, and currents. Models of the multicohort fishery such as those developed by Spulber (1983) and Clark et al. (1973) have focused attention on optimization within a fixed price framework. The present paper explicitly introduces the market for the resource. The analysis of renewable resource markets presented by Levhari et al. (1981) and of resource markets under uncertainty by Mirman and Spulber (1983) is applied to the multicohort problem.

Multicohort population dynamics are described here within a general model in which the next period's vector of cohort bio-

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mass levels depends upon the vector of current biomass levels and the environmental disturbance. The age structure of the population will affect recruitment and cohort growth rates through cohort interaction. The size of each cohort will also affect its own growth rate, as in the standard lumped parameter model. Thus, the distribution of biomass levels across age groups will affect growth rates since marginal growth generally varies with age. By entering the growth relation in a general way, environmental disturbances are allowed to affect recruitment, cohort interaction, and cohort growth rates.

The paper begins with an examination of free access harvesting of the multicohort fishery. Myopic harvesting of a multicohort fishery creates additional problems not present in the lumped parameter case. Extinction of the population is made more likely for two reasons. First, because the total biomass harvested is selected for current market sale, there is no incentive to adjust the harvest of each cohort separately. Thus, a sustainable harvest level may lead to extinction if the total harvest is made up of excessive harvesting of recruits or spawners. This explains regulations on minimum size of capture. Second, the total harvest should be adjusted to reflect the age composition, even if the take from each cohort appears sustainable. The presence of environmental disturbances, particularly those that affect recruitment, may worsen the impact of free access harvesting.

The paper examines the optimal harvesting decision of a central planner or regulatory agency charged with managing the multicohort fishery. The optimal solution is examined for the cases of selective and nonselective harvesting. If the decision maker may selectively adjust harvest and escapement levels for each cohort, and if the net market valuations of each cohort are equal, then expected rates of return are equal across cohorts. The optimality conditions for each cohort resemble the optimality condition for the lumped parameter fishery. The rate of return to conservation includes the *i*th cohort's contribution to recruitment as well as the marginal growth of the cohort itself.

In the ocean fishery, it is unlikely that selective harvesting of individual cohorts will be feasible. While it may be possible to determine the age of first liability to capture, it may not be possible to separately harvest adult cohorts. The decision maker is assumed to choose the proportion of the total cohort biomass to be harvested. An optimality condition is obtained that differs from the lumped parameter case in that the marginal effect of harvesting on growth is a sum of the marginal effects of cohort biomass levels on cohort growth, recruitment, and interaction. Using stochastic dynamic programming we may calculate the value of the fish population to the decision maker as a function of the vector of cohort biomass levels. Because of nonselective harvesting, the marginal internal valuations of each cohort are shown not to be equal to the market valuation of a unit of fish. However, the sum of internal valuations of the cohorts equals the market valuation of the total population biomass.

The paper is organized as follows. The biological framework is presented in the next section. Free entry harvesting is then examined for some special cases, after which the optimal solution with selective harvesting is considered. Finally, the optimal solution with nonselective harvesting is examined.

The Biological Framework

Consider a single species of fish for which growth and recruitment depend on the age distribution of the population. Given a maximum age n, we may fully describe the population at date t by the vector $x_t = (x_t^1, \ldots, x_t^n)$, where x_t^i represents the biomass level of a cohort of age i at the beginning of time period t. The growth and recruitment rates for the population are assumed to depend on biomass levels as well as age structure. In addition, environmental disturbances may affect cohort recruitment, growth, and interaction. Given the biomass levels x_t and environmental disturbance $w_t \in \Omega$, the transition equation describing the growth of the fish population over a single time period is represented as follows:

$$\begin{aligned} x_{t+1} &= G(x_t, w_t) \\ G(x_t, w_t) &\equiv [G^1(x_t, w_t), G^2(x_t, w_t), \ldots, G^n(x_t, w_t)] \end{aligned} \tag{1}$$

The function G^1 describes the recruitment relation and the functions G^i, $i = 2, \ldots, n$ describe the age- and stock-dependent growth of each cohort as well as the interaction among cohorts in the years after recruitment. The interaction between adult cohorts represents competition for food sources and effects of predators on cohort mortality rates.

The growth relation G is assumed to be differentiable, increasing, and bounded so that the equilibrium population is bounded. The marginal growth rates of each cohort are assumed to be a decreasing function of their own biomass levels. Let $G^j_i(x_t, w_t) \equiv \partial G^j(x_t, w_t)/\partial x^i_t$. The environmental disturbances are independently and identically distributed according to the cumulative distribution $F(w)$.

The population growth described by G may be quite complex. Clark (1976) states that for even a single cohort in a discrete time model, "modifications in the life-history model can produce growth curves . . . of almost unlimited complexities" (p. 232). An interesting form of G that allows for spawning by the oldest cohort only is given by

$$x^1_{t+1} = g^1(x^n_t, w_t) \tag{2}$$

$$x^{i+1}_{t+1} = g^{i+1}(x^i_t, w_t) \qquad i = 1, \ldots, n-1. \tag{3}$$

This form allows us to consider the n cohorts separately. We will consider this form as a special case in the next section.

Consider now some other special forms of the growth relation G. Clark (1976) observes that certain fish populations such as the North Atlantic plaice and haddock and other demersal species have high fertility rates and highly variably recruitment (p. 218; see also Beverton and Holt [1957]). Optimal harvesting using the Beverton-Holt model is discussed by Clark et al. (1973) and Hannesson (1975). For populations where the size of the adult stock has little effect upon recruitment, the recruitment relation will depend only on the state of the environment:

$$x^1_{t+1} = g^1(w_t) \tag{4}$$

Hannesson (1975) states that there is "very little evidence of a significant stock-recruitment relation with regard to cod'' because of ``exogenous, stochastic factors controlling the survival of eggs and larvae at a critical age, resulting in wide fluctuations of recruitment'' (p. 159). If we assume no interaction between adult cohorts, the growth equations G^{i+1} for $i = 1, \ldots, n - 1$ have the form given in Equation (3). Employing Equations (3) and (4) in a harvesting model essentially assumes away all cohort interaction. Obviously, if selective harvesting is possible and the harvesting return function is additive, there is no multicohort harvesting problem since the framework breaks down into n lumped parameter problems. This framework appears, for example, in Mendelssohn (1978). Without selective harvesting, the age structure of a population described by Equations (3) and (4) does affect harvesting decisions, since age-dependent growth rates cause each cohort to have a different effect on total biomass levels in the next period.

The population biology literature has focused great attention on linear forms of the transition rule G. If we assume that G^1 has the form $x^1_{t+1} = \Sigma^n_{i=1}\sigma^i x^i_t$ and that each G^{i+1} for $i = 1, \ldots,$ $n - 1$ has the form $x^{i+1}_{t+1} = \gamma^i x^i_t$, where σ^i, γ^i are scalar parameters, then $i = 1, \ldots, n - 1$ implies that fertility rates and growth rates are age-dependent but not stock-dependent. This is the familiar Leslie matrix employed for the study of age-structured populations by Leslie (1945, 1948), Keyfitz (1968), Emlen (1973), and others. Stochastic Leslie matrices are considered by Pollard (1973), Boyce (1977), and others. The resource harvesting problem with a Leslie matrix description of growth is surveyed in Mendelssohn (1976); see also Beddington (1974) and Beddington and Taylor (1973). The Leslie model allows unlimited growth and, as Mendelssohn (1976, p. 348) points out, ``does not provide ecologically interesting solutions'' for problems of harvesting policy.

Free Entry Harvesting

The Market Framework

We assume that the market demand for fish and harvesting costs are the same in each period. The inverse demand for fish is assumed to depend only on the total biomass harvested Q and not on the age of the fish caught. The inverse demand is given by $P = D(Q)$, where P is the market price of fish. The *industry* marginal cost function is given by $C(Q)$ and is assumed to be independent of the characteristics of the harvested population. A topic for further research would be an extension of the analysis to allow costs to depend upon the biomass and age composition of the fish population. Individual firm cost functions are not examined here. For a discussion of market decentralization of the planner's problem see Levhari et al. (1981) and Mirman and Spulber (1983).

The Free Entry Problem

In the many discussions of nonoptimal harvesting in the free access fishery, beginning with Gordon (1954) and Scott (1955), it is emphasized that free entry harvest levels are not adjusted to current biomass levels because of the nonappropriability of returns to conservation. Since firms are not able to obtain biological and market returns to resource growth they pursue a myopic harvesting strategy. For the multicohort fishery, the free entry problem may be more critical for two reasons. First, if the free access harvest level is chosen on the basis of total biomass that can be sold at the market equilibrium, there is no incentive for firms to selectively harvest each cohort separately. Thus, a reasonable harvest level may cause problems if the composition of the catch involves too many new recruits or too many members of the parent stock. Second, the total harvest itself is not adjusted to the age composition of the population. These effects may be worsened by the presence of environmental disturbances.

We illustrate the problems that may arise under free access for a very simple two-cohort model. Consider first the deterministic case where $x_{t+1}^1 = g^1(x_t^2)$ and $x_{t+1}^2 = g^2(x_t^1)$, where x_t^1 are recruits and x_t^2 is the parent stock. This is similar to a two-stage model in Clark (1976, p. 212). The population dynamics are given in Figure 1. For this growth relation, we may identify two cohorts in any period.

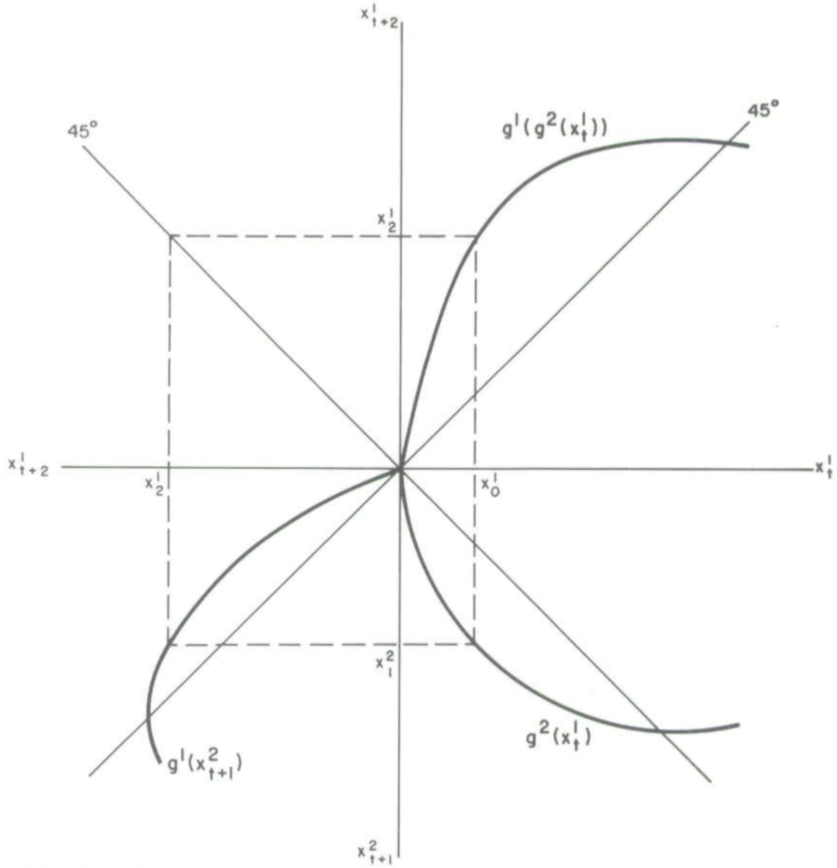

FIGURE 1. Population dynamics—two-cohort model.

Suppose now that the market clearing harvest level is given by $\overline{Q}$, where $D(\overline{Q}) = C'(\overline{Q})$ and suppose for simplicity that the same amount of recruits and parents are harvested in any period, $\overline{Q}/2$. Then the population dynamics are given by

$$\begin{aligned} a^1_{t+1} &= g^1(a^2_t) - \overline{Q}/2 \\ a^2_{t+1} &= g^2(a^1_t) - \overline{Q}/2 \end{aligned}$$

where a^1_t, a^2_t represent the escapement of recruits and parents,

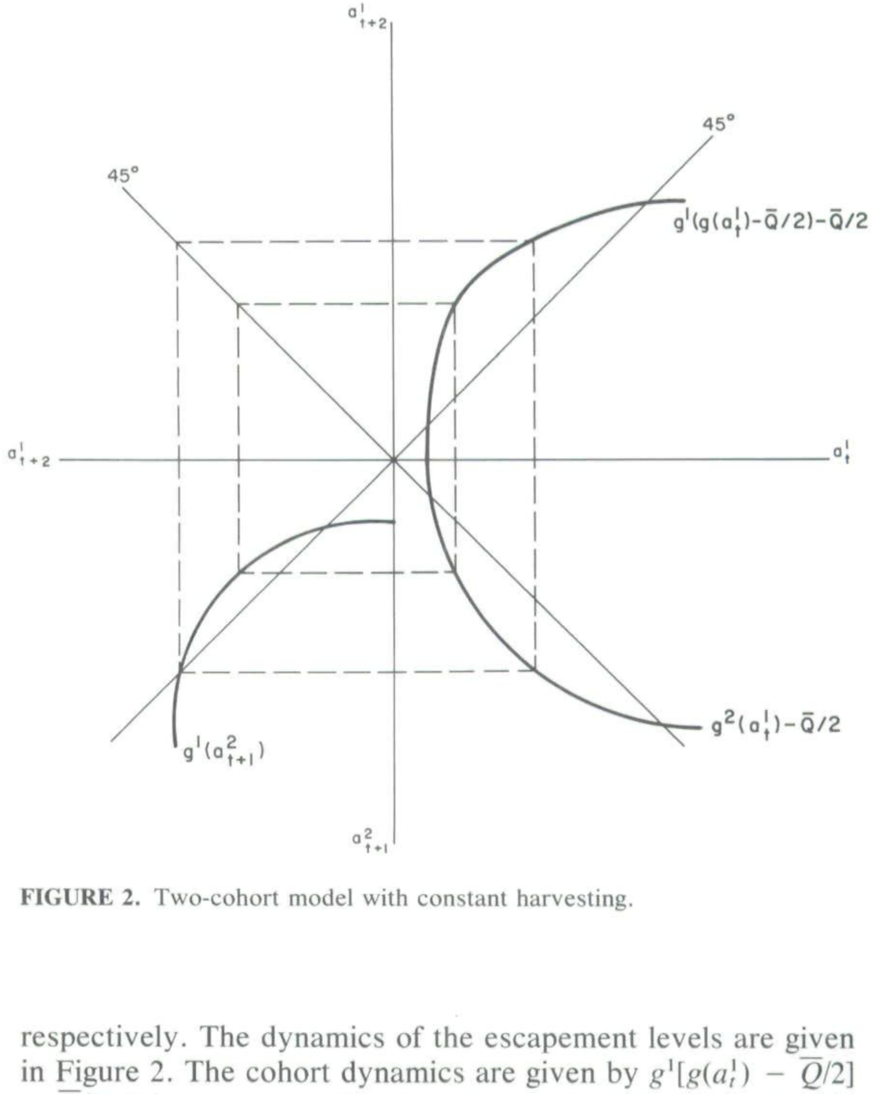

FIGURE 2. Two-cohort model with constant harvesting.

respectively. The dynamics of the escapement levels are given in Figure 2. The cohort dynamics are given by $g^1[g(a_t^1) - \overline{Q}/2] - \overline{Q}/2$. It is easy to see that for a harvest level to be sustainable at the free entry equilibrium it must allow sufficient numbers of parents and recruits. The case of $\overline{Q}$ sustainable is shown in Figure 2.

Environmental disturbances complicate the free entry problem:

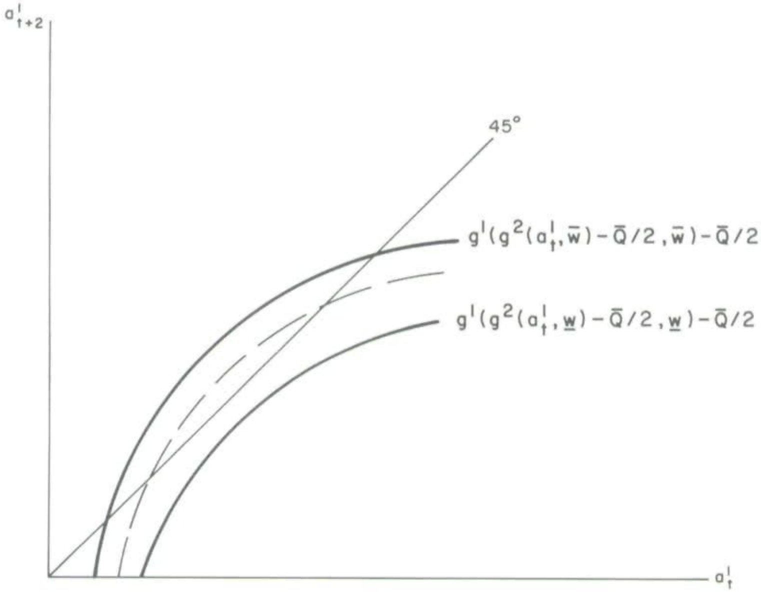

FIGURE 3. Average growth under uncertainty.

$$a^1_{t+1} = g^1(a^2_t, w_t) - \overline{Q}/2$$
$$a^2_{t+1} = g^2(a^1_t, w_t) - \overline{Q}/2$$

The dynamics of a single cohort depend on the environmental disturbances in two periods, with one disturbance affecting recruitment and the next period's disturbance affecting growth of recruits into parents:

$$a^1_{t+2} = g^1(g^2(a^1_t, w^1_t) - \overline{Q}/2, w_{t+1}) - \overline{Q}/2$$

Thus the survival of a cohort will depend on the environmental effects on recruitment and growth.

The estimation of sustainable yield by observing average growth and recruitment may be very misleading. The sustainable yield must be calculated given a succession of "bad years." Suppose w takes values in the set $[\underline{w}, \overline{w}]$ with $\partial g^1(a^2, w)/\partial w > 0$ and $\partial g^2(a^1, w)/\partial w > 0$. Then consider the range of values for a^1_{t+2} given a^1_t for $w_t = w_{t+1} = \overline{w}$ and $w_t = w_{t+1} = \underline{w}$ (see Figure 3).

It is possible that on average the free entry harvest $\overline{Q}$ may be sustainable. This is represented by the dotted line in Figure 3. However, because the "worst case" curve lies entirely below the 45° line, free access to the fishery will result in extinction of both cohorts with probability one. This occurs since there is a positive probability of a succession of low values of the environmental parameter $\underline{w}$.

Selective Harvesting

Consider now the harvesting decisions made by a central planner or regulatory agency managing the multicohort fishery. It is assumed that the planner or regulator may set harvest levels for each cohort. This is termed *selective* harvesting. The optimality conditions for the regulator's problem are shown to resemble the conditions obtained in the lumped parameter problem.

The central planner or regulatory agency chooses the total harvest levels and escapements for each cohort. Let $q_t = (q_t^1, \ldots, q_t^n)$ represent harvest levels from each cohort and let $a_t = (a_t^1, \ldots, a_t^n)$ represent escapement levels for each cohort. In vector notation, the dynamics of the harvested population satisfy

$$q_{t+1} + a_{t+1} = x_{t+1} = G(a_t, w_t) \qquad t = 1, 2, \ldots, \qquad (5)$$

Let $Q_t = \Sigma_{i=1}^n q_t^i$ represent the total biomass of fish harvested at date t. The regulator chooses harvest and escapement levels to maximize the expected consumer's surplus discounted with a factor δ. The regulator's problem is then

$$\max_{\{q_t,\, a_t\}} E \sum_{t=1} \delta^{t-1} \int_0^{Q_t} [D(s) - C'(s)]\, ds \qquad (6)$$

subject to x_1 given, $q_1 + a_1 = x_1$ (5) and $Q_t = \Sigma_{i=1}^n q_t^i$, $t = 1, 2, \ldots$. Using stochastic dynamic programming (see Easley and Spulber [1981] and Spulber [1982]), we may obtain stationary, valued functions $V: \mathbb{R}_+^n \times \Omega \rightarrow E$ that are differentiable on $\mathbb{R}_+^n$ and measurable on Ω. The value functions satisfy the wellknown recursive equation of dynamic programming:

$$V(x_t) = \max_{q_t, a_t} \left\{ \int_0^{Q_t} [D(s) - C'(s)]\, ds + \delta \int_\Omega V[G(a_t, w_t)]\, dF(w_t) \right\} \tag{7}$$

subject to

$$q_t^i + a_t^i = x_t^i \qquad i = 1, \ldots, n \tag{8}$$

We may rewrite the problem stated in Equations (8) and (9) as a Lagrangean problem:

$$L = \int_0^{Q_t} [D(s) - C'(s)]\, ds + \sum_{i=1}^{n} \lambda_t^i [x_t^i - q_t^i - a_t^i] + \delta \int_\Omega V[G(a_t, w_t)]\ dF(w_t) \tag{9}$$

where λ_t^i represents the shadow price on the stock constraint for cohort i. The first-order conditions are then

$$D(Q_t) - C'(Q_t) = \lambda_t^i \qquad i = 1, \ldots, n \tag{10}$$

$$\delta \int_\Omega \sum_{j=1}^{n} V_j[G(a_t, w_t)] G_i^j(a_t, w_t)\, dF(w_t) = \lambda_t^i \qquad i = 1, \ldots, n \tag{11}$$

and Equation (8), where $V_j \equiv \partial V / \partial x_j$. From Equation (10) we see that since the market values of all cohorts are equal and since marginal harvesting costs are also equal, *the shadow prices for* *each cohort stock constraint will be equal*. Furthermore, we may show that by the envelope theorem,

$$\partial L/\partial x_t^i = \lambda_t^i = V_i(x_t) \tag{12}$$

Thus we may rewrite Equations (10) and (11) to obtain

$$D(Q_t) - C'(Q_t) = \lambda_t \tag{13}$$

$$\sigma \int_{\Omega} \lambda_{t+1} \sum_{j=1}^{n} G_i^j(a_t, w_t)\, dF(w_t) = \lambda_t \qquad i = 1, \ldots, n \tag{14}$$

Given Equations (13) and (14), we may obtain stationary policy functions that give the shadow price harvest levels and escapement levels as a function of the current vector of cohort biomass levels, $\lambda_t = \lambda(x_t) = \lambda[G(q_{t-1}, w_{t-1})]$, $q_t^i = q^i(x_t) = q^i[G(a_{t-1}, w_{t-1})]$ and $a_t^i = a^i(x_t) = a^i[G(a_{t-1}, w_{t-1})]$ for $i = 1, \ldots, n$. Note that since $a_t^n = 0$, $q_t^n = x_t^n$ for all t.

Suppose that the regulator's discount rate equals the market rate of interest, $\delta = 1/(1 + r)$. Then, Equation (14) may be rewritten as

$$1 + \frac{\int_{\Omega} \lambda_{t+1} \sum_{j=1}^{n} G_i^j(a_t, w_t)\, dF(w_t) - \lambda_t}{\lambda_t} = 1 + r \qquad i = 1, \ldots, n \tag{15}$$

Thus, *expected rates of return are equal for each cohort*. Equation (15) is very similar to the lumped parameter case under uncertainty described in Mirman and Spulber (1983). The difference with the lumped parameter case is that the marginal effect of additional escapement on growth is the sum of the marginal recruitment, marginal growth of the cohort itself, and marginal cross-effects on other cohorts.

Applying the special form of the growth function given in Equation (3) and letting $g^1(a_t, w_t)$ represent the recruitment function, then Equation (15) becomes

$$\left[1 + \frac{\int_\Omega \lambda_{t+1} g_i^1(a_t, w_t)\, dF(w_t) - \lambda_t}{\lambda_t}\right] + \left[1 + \frac{\int_\Omega \lambda_{t+1} g_1^{i+1}(a_t^i, w_t)\, dF(w_t) - \lambda_t}{\lambda_t}\right] = 1 + r \qquad i = 1, \ldots, n \quad (16)$$

Thus when there is no interaction among adult cohorts, we may identify two rates of return to conservation: the rate of return to recruitment and the rate of return to cohort growth. The rate of return to cohort growth has the same form as is observed in the lumped parameter model (Mirman and Spulber, 1983).

If we eliminate the dependence of recruitment on the adult population and assume that given some minimum adult population, recruitment is essentially random as in Equation (4), Equation (16) then reduces to

$$1 + \frac{\int_\Omega \lambda_{t+1} g^{i+1'}(a_t^i, w_t)\, dF(w_t) - \lambda_t}{\lambda_t} = 1 + r \qquad (17)$$

The shadow price λ_{t+1} is random even though the cohort growth is deterministic since $\lambda_{t+1} = \lambda[g^1(w_t), g^2(a_t^1, \mathrm{w}_t), \ldots, g^n(a_t^{n-1}, w_t)]$. The occurrence of large fluctuations in recruitment should be sufficient to cause variations in the resource rent, since the shadow price λ_t must reflect future stock effects.

Nonselective Harvesting

For the ocean fishery, it is generally not possible to harvest individual age groups from a multicohort population. Thus, the optimal selective harvesting solution obtained in the previous section may not be attainable. Without selectivity, a given harvest level may involve taking different proportions of each age group depending upon their liability to be captured and the fishing techniques employed. For ease of presentation, we assume that a unit of fish harvested may come from any cohort with equal likelihood. Thus, the central planner or regulatory agency faces the problem of choosing the *proportion* of the current stock to be harvested.

It is assumed that gear selectivity is fixed. Also, no distinction is made between the age of recruitment and the age of first liability to capture. Clark (1976, p. 286) states that while this distinction is commonly made in the fishery literature, it is not needed in theory when gear selectivity is fixed.

As before, let $Q_t = \Sigma_{i=1}^n q_t^i$ represent the total harvest. Also, let $A_t = \Sigma_{i=1}^n a_t^i$ and $X_t = \Sigma_{i=1}^n x_t^i$ represent the total biomass of the fish that escape and of the current stock, respectively. Then, given the total harvest and the current stock, the proportion of each cohort harvested is given by β:

$$\beta = Q_t/X_t \tag{18}$$

Thus, since $Q_t + A_t = X_t$, the escapement from cohort i equals

$$a_t^i = (1 - \beta)x_t^i = (1 - Q_t/X_t)x_t^i = (A_t/X_t)x_t^i \tag{19}$$

With nonselective harvesting the regulator must choose only the total harvest and escapement levels. Thus, the planner's problem is given by

$$\max_{\{Q_t,\ A_t\}} E \sum_{t=1} \delta^{t-1} \int_0^{Q_t} [D(s) - C'(s)]\, ds \tag{20}$$

subject to x_1 given, $q_1 + a_1 = x_1$, $Q_t = \Sigma_{i=1}^n q_t^i$, $A_t = \Sigma_{i=1}^n a_t^i$, $Q_t + A_t = X_t$, and

$$q_{t+1} + a_{t+1} = x_{t+1} = G[(A_t/X_t)\, x_t,\ w_t] \tag{21}$$

The value function for the planner's problem is again obtained by using stochastic dynamic programming:

$$V(x_t) = \max_{Q_t,\ A_t} \left(\int_0^{Q_t} [D(s) - C'(s)]\, ds + \delta \int_\Omega V\{G[(A_t/X_t)x_t,\ w_t]\}\, dF(w_t) \right) \qquad (22)$$

subject to

$$Q_t + A_t = X_t \qquad (23)$$

Rewriting the planner's problem as a Lagrangean one, we obtain

$$L = \int_0^{Q_t} [D(s) - C'(s)]\, ds + \lambda_t[X_t - Q_t - A_t] + \delta \int_\Omega V\{G[(A_t/X_t)x_t,\ w_t]\}\, dF(w_t) \qquad (24)$$

where λ_t is the shadow price on the total biomass constraint. The first-order conditions are then

$$D(Q_t) - C'(Q_t) = \lambda_t \qquad (25)$$

$$\delta \int_\Omega \sum_{j=1}^{n} V_j \sum_{i=1}^{n} G_i^j[(A_t/X_t)x_t,\ w_t](x_t^i/X_t)\, dF(w_t) = \lambda_t \qquad (26)$$

and Equation (23) for $t = 1, 2, \ldots$. Equations (23), (25), and (26) may be solved for stationary harvesting, escapement, and shadow price functions:

$$\begin{aligned} Q_t &= Q(x_t) = Q\{G[(A_{t-1}/X_{t-1})x_{t-1},\ w_{t-1}]\} \\ A_t &= A(x_t) = A\{G[(A_{t-1}/X_{t-1})x_{t-1},\ w_{t-1}]\} \\ \lambda_t &= \lambda(x_t) = \lambda\{G[(A_{t-1}/X_{t-1})x_{t-1},\ w_{t-1}]\} \end{aligned}$$

These policy functions depend on the vector of cohort biomass levels. This emphasizes that even though harvesting is not selective, the age structure of multicohort populations should be taken into account in choosing harvest and escapement levels.

From the envelope theorem and Equations (21) to (26), one may derive

$$\begin{aligned}\partial L/\partial x_t^i &= \lambda_t(1 - A_t/X_t) + (A_t/X_t)\,\delta \int_\Omega \sum_{j=1}^n V_j G_i^j \, dF(w_t) \\ &= \beta\lambda_t + (1 - \beta)\,\delta \int_\Omega \sum_{j=1}^n V_j G_i^j \, dF(w_t) \\ &= V_i(x_t) \end{aligned} \tag{27}$$

Multiplying both sides of Equation (27) by x_t^i and summing over i, one obtains

$$\sum_{i=1}^n (\partial L/\partial x_t^i)x_t^i = \lambda_t X_t = \sum_{i=1}^n V_i(x_t)x_t^i \tag{28}$$

The last equality follows from Proposition 4 in Spulber (1983).

With selective harvesting we saw in Equation (13) that the marginal valuation of *each* cohort equals the current market valuation of the last unit harvested. Clearly, Equation (25) resembles Equation (13). However, in Equation (25), the shadow price λ_t is the shadow price on the *total* biomass constraint and represents the rent on the entire resource stock. The marginal internal valuation of cohort i given by Equation (27) represents a weighted average of the market valuation λ_t and the expected marginal future return to conservation of cohort i. The current and future valuations are weighted by β and $(1 - \beta)$ where β equals the proportion of the total current stock that is harvested. Thus, the marginal interval value of cohort i does not equal its market value. However, we see from Equation (28) that the sum of internal marginal valuations of each cohort sum to the total market value $\lambda_t X_t$. This implies that some cohorts are internally valued above their market value and others are valued below their market value. The optimal proportional harvest policy assures that the marginal return to investment in the entire resource stock equals its current market value, as shown in Equation (26). Clearly, selective harvesting is preferable since the decision maker may then adjust each individual cohort so that its expected marginal return equals its current market value. The analysis presented here points out the complexities of harvesting decisions, which are often hidden in lumped parameter models.

References

Beddington, J. R. 1974. Age structure, sex ratio and population density in the harvesting of natural populations. *J. Applied Ecol.* 11: 915–924.

Beddington, J. R., and D. B. Taylor. 1973. Optimal age-specific harvesting of a population. *Biometrics* 29: 801–809.

Beverton, R. J. H., and S. J. Holt. 1957. *On the dynamics of exploited fish populations*. Fisheries Investigations Series 2 (19). London: Ministry of Agriculture, Fisheries, and Food.

Boyce, M. S. 1977. Population growth with stochastic fluctuations in the life table. *Theoretical Pop. Biol.* 12: 366–373.

Clark, C. W. 1976. *Mathematical bioeconomics*. New York: Wiley.

Clark, C. W., G. Edwards, and M. Friedlaender. 1973. Beverton-Holt model of a commercial fishery: optimal dynamics. *J. Fish. Res. Board Can.* 30: 1629–1640.

Easley, D., and D. F. Spulber. 1981. "Equilibrium and optimality with rolling plans. *Inter. Econ. Rev.* 22 (Feb.): 79–103.

Emlen, J. M. 1973. *Ecology: an evolutionary approach*. Reading, MA: Addison-Wesley.

Gordon, H. S. 1954. "Economic theory of a common-property resource: the fishery. *J. Polit. Econ.* 62 (April): 124–142.

Hannesson, R. 1975. Fishery dynamics, a North Atlantic cod fishery. *Can. J. Econ.* 8: 151–173.

Keyfitz, N. 1968. *Introduction to the mathematics of population*. Reading, MA: Addison-Wesley.

Leslie, P. H. 1945. "The use of matrices in certain population mathematics. *Biometrika* 33: 183–212.

Leslie, P. H. 1948. Some further notes on the use of matrices in population mathematics. *Biometrika* 35: 213–245.

Levhari, D., R. Michener, and L. J. Mirman. 1981. Dynamic programming models of fishing: competition. *Amer. Econ. Rev.* 71 (Sept.): 649–661.

Mendelssohn, R. 1976. Optimization problems associated with a Leslie matrix. *Amer. Naturalist* 110: 339–349.

Mendelssohn, R. 1978. Optimal harvesting strategies for stochastic single-species, multiage class models. *Math. Biosci.* 41: 159–174.

Mirman, L. J., and D. F. Spulber. 1983. Renewable resource markets under uncertainty. Economics Working Paper 8302. Los Angeles: University of Southern California.

Pollard, J. H. 1973. *Mathematical models for the growth of human populations*. Cambridge: Cambridge University Press.

Scott, A. D. 1955. The fishery: the objectives of sole ownership. *J. Polit. Econ.* 63/4 (April): 116–124.

Spulber, D. F. 1982. Adaptive harvesting of a renewable resource and stable equilibrium. In *Essays in the economics of renewable resources*, ed. L. J. Mirman and D. F. Spulber, pp. 117–139. Amsterdam: North-Holland.

Spulber, D. F. 1983. Pulse fishing and stochastic equilibrium in the multicohort fishery. *J. Econ. Dynamics Control* 6: 309–332.