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Optimal Intra- and Interseasonal Harvesting Strategies when Price Varies with Individual Size

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Abstract *A major concept in fisheries management is the optimal age for first capture. Because there can be separate market categories for fish of different sizes and different costs for their harvest, a more rational statement of the problem would be to find the optimal range of harvest sizes in any given year. Two models for solving this problem are presented. The shrimp model discusses optimal harvest of a single cohort of shrimp as it grows through a season. The lobster model discusses optimal simultaneous harvest of several cohorts over several seasons. The difficulty of defining a cost per fish in the lobster model makes it a much more complex undertaking.*

Keywords Optimal harvesting, intraseason, interseason, variable price, fisheries management.

Introduction

Because of their joint effect on total weight of harvest and stock recruitment relationships, fishing mortality and age at first capture are both important in determining management controls. However, the latter has received relatively little coverage in the fisheries economics literature. This is somewhat surprising because price per pound can sometimes vary with the size of the individual fish, and hence for economists there is a third reason why age at capture is important.

This is not to say that the age size relationship and its effects on optimal utilization have been ignored. One approach focuses mainly on the effect of age at first capture on total physical yield. Turvey (1964) presents a general statement on the optimal combination of mesh and effort controls. Clark et al. (1973), Hannesson (1975), and Clark (1976, p. 276 ff) present a more rigorous analysis of the optimal harvest timing of a single cohort fishery to take advantage of the growth of the individuals in the stock.

Coming at the problem from a slightly different angle, Gates (1974) showed how the relationship between fish size and the price of fish makes it difficult to correctly specify the demand curve and demonstrated the adverse effects this can have on policy derivation. More recently, Conrad (1982) derived the optimal utilization of a multiple cohort fishery when price varies with size of individual.

The purpose of this article is to formalize the issues in the size price conceptualization of the problem. Admittedly, it may apply to only a small subset of active fisheries, but where it is applicable it can be very important. A general model will be derived to present the essence of the management problem in this case. Obviously practical applications will call for adaptations to this model. Some of them will call for detailed specifications of the benefit function, but at the same time, others will require more restrictive assumptions and economic and biological simplifications to take into account data limitations and institutional constraints. The results of the model are only modest extensions of the standard dynamic fisheries model, but they do provide a template against which suggested management regimes for these types of fisheries can be compared. The template is useful because when price does vary with individual size, the goal is not necessarily to find the optimal age at first capture as it is when the focus is on the amount of the physical catch, but rather to find the optimal range of harvest sizes.

The discussion will focus on two very important fisheries where price varies with individual size, the gulf shrimp and the northern lobster, although the results can easily be generalized. The two have distinct differences, which make it worthwhile to construct a separate submodel for each. Gulf shrimp is a one-cohort, single-year fishery where the stock migrates over the season such that restrictions on time of harvest are effectively restrictions on size of capture. The lobster fishery, on the other hand, has many cohorts each of which lives several years and each of which is subject to the same gear. Hence, there is no easy way to focus effort on individuals of a particular size.

The principal regulation in the New England lobster fishery is the prohibition of taking lobsters with a carapace of less than a specified length (see Acheson 1985, Acheson and Reidman 1982, Botsford et al. 1986, Richardson and Gates 1986, Wang and Kellogg 1984, 1986). The principal regulation in the Gulf shrimp fishery is the closure of inshore fishing grounds so that more of the annual crop (which gradually migrates offshore as the season progresses) may be taken later in the year at larger sizes, see Poffenberger (1984).

The implicit assumption behind these regulations is that each and every larger individual caught at a later time will be more valuable than the same individual harvested earlier. However, this ignores the possibility that these fisheries can each simultaneously supply several interrelated markets. There is a downward sloping demand curve for each market-defined size classification and although those for larger individuals may be higher than those for smaller ones, the last unit of "big" lobsters in the next period may have a lower present value than the first unit of "little" ones in the current period. In maximizing the net present value of harvest it is necessary to consider the full range of markets available.

The article will proceed as follows. The recruitment of an annual crop that can be harvested in distinct locations with directly attributable marginal costs as in the shrimp fishery will be discussed in the first section. The basics of the intertemporal allocation by size can be easily introduced in the context of this relatively simple case. The more complex case where different cohorts are simultaneously harvested by the same effort will be discussed in the second section. The relationship between recruitment and size of the parent stock (and hence on the amount of harvest), a topic of theoretical and empirical controversy, will be ignored in the formal analysis. This makes the problem tractable with little loss of authenticity, especially for shrimp fisheries. The policy implications of this assumption will be discussed, however. For notational simplicity and ease in interpretation of first-order conditions, the number of cohorts and/or time periods under consideration will be limited, but the discussion will generalize the results. The final section will provide a summary and a set of conclusions.

Optimal Utilization of a Single Cohort: The Shrimp Fishery

The fundamentals of optimal interperiod utilization of a cohort of a given size can be presented in terms of a model that closely mimics the workings of a typical shrimp fishery. Consider a fishery with a dichotomous market for two sizes of shrimp where the value of each is dependent on the quantity consumed of both. At the beginning of each year, a certain stock enters the fishery and the season can be conveniently divided into periods where exclusive or predominant catches of small and large individuals, respectively, may be made. As with other fisheries, open-access utilization will be suboptimal in that too much effort will be produced because the firms will use average rather than marginal valuation to make private decisions (Anderson 1986) and in addition (subject to constraints on fleet size) the shrimp will all be exploited at the minimum market size (Smith 1968).

Because there is no stock-recruitment relationship in shrimp fisheries, the size of the annual crop is essentially a random variable and is not known until the beginning of the year. Optimal utilization, therefore, reduces to maximizing the net value of production in both periods for each year. The optimal utilization in any year can be posed as a constrained maximum problem.

Let $B(N_0, N_1)$ represent the gross benefits from harvesting, where N_i represents the number of individuals harvested in the ith period. Because the cross-elasticities of demand between the different sized individuals are likely to be quite high, this specification, rather than using individual demand curves, simplifies matters without any loss of substance. The first partial derivatives of the gross benefit function can be interpreted as the ceteris paribus gross marginal willingness to pay. Further, let the cost in each period be a function of the amount harvested and the size of the fish stock: $C_i = C_i(N_i, S_i)$.

Given an initial stock size of S_0, and a natural mortality rate between periods of m, the stock size at the beginning of the next period will be $S_1 = (1 - m)(S_0 - N_0)$. Given the above, the constrained maximum problem can be formally stated as

$$\begin{gathered}\text{Maximize} \quad B(N_0, N_1) - C_0(N_0, S_0) - C_1(N_1, S_0)\\ \text{St} \quad S_0 - N_0 \geq 0\\ (1 - m)\,(S_0 - N_0) - N_1 \geq 0\end{gathered}$$

In addition, the catch in each period is constrained to be nonnegative.

The appropriate Lagrangian and the two first-order maximizing conditions are as follows:

$$\begin{aligned}L &= B(N_0, N_1 - C_0(N_0, S_0) - C_1(N_1, S_1)\\ &+ \lambda_0(S_0 - N_0) + \lambda_1[(1 - m)(S_0 - N_0) - N_1]\end{aligned} \tag{1}$$

$$\frac{\partial L}{\partial N_0} = \underbrace{\frac{\partial B}{\partial N_0} - \frac{\partial C_0}{\partial N_0} - \frac{\partial C_1}{\partial S_1}\frac{\partial S_1}{\partial N_0}}_{\text{MNB}_0} - \lambda_0 - \lambda_1\,(1 - m) \leq 0 \tag{2}$$

$$\frac{\partial L}{\partial N_1} = \underbrace{\frac{\partial B}{\partial N_1} - \frac{\partial C_1}{\partial N_1}}_{\text{MNB}_1} - \lambda_1 \leq 0 \tag{3}$$

The first three terms in (2) are the marginal net benefit from harvesting an individual small fish in period 0, MNB_0. It is the algebraic sum of gross marginal willingness to pay for that fish minus the cost of its harvest minus the marginal interperiod stock externality cost. The latter is the increase in cost in period 1 due to the decreased stock size in that period because of harvest in period 0. The first two terms in (3) are the marginal net benefit from harvesting an individual large fish in period 1, MNB_1. Because this is the last period, there is no interperiod stock effect.

For practical purposes, only one of the constraints can be binding, because if the first constraint is binding (i.e., $N_0 = S_0$) there will be no stock in the second period. Therefore, the constraint in period 1 is no longer relevant. Formally, this means that in the solution to the basic problem, Equations 2 and 3 will both hold as equalities. Further λ will be positive and λ_1 will be negative. From the Kuhn–Tucker conditions, the latter means that the associated constraint is not a limitation and the solution can be obtained by maximizing the original objective function after removing the period 1 constraint. Therefore, there are only three relevant possible alternatives for the signs of the multipliers. These are listed below with the corresponding relationships of harvest to stock size in each period.

(a) $\lambda_0 = 0,\ N_0 < S_0$
$\quad\lambda_1 = 0,\ N_1 < S_1$
(b) $\lambda_0 = 0,\ N_0 < S_0$
$\quad\lambda_1 > 0,\ N_1 = S_1$
(c) $\lambda_0 > 0,\ N_0 = S_0$
$\quad N_1 = 0$

A full graphical explanation of each case follows.

(a) If neither of the constraints are binding, which is to say there is more than enough fish than can be profitably harvested in either period, then by the Kuhn–Tucker theorem, both of the λ's will be zero. The optimal solution for this case is depicted in Figure 1*a*. The left and right graphs depict Equations 2 and 3, respectively. In the left graph S_0 is the beginning stock size, while S_1 in the right graph is the stock size at the beginning of period 1. As indicated above, it is a function of S_0 and N_0 in the left graph.

In this case, the marginal net benefit curves of the two periods both intersect the horizontal axis at a point to the left of the stock constraint for that period. Although catch could be increased in either period, the extra fish would yield negative benefits. Therefore, these intersection points represent the optimal level of harvest in each period. Recall that MNB_0 has the negative interperiod stock externality term and is therefore lower at each N_0 than it otherwise would be. This is to say that to keep the cost of harvesting in period 1 at the optimal level, the optimal catch in period 0 is lower than it otherwise would be.

The relative sizes of N_0 and N_1 depend on the relative values of harvest in each period. For example, the lower the relative value of smaller shrimp, the lower is the MNB_0, and the smaller will be N_0. Therefore, S_1 and the potential harvest in period 1 will be higher. Further, the larger the value of large shrimp, the higher is MNB_1, and the more of the remaining stock that will be harvested.

(b) Carrying the example of a higher MNB_1 to the extreme results in the second general case where the constraint is binding in period 1. In this case λ_1 will be positive. The λ_1 term appears in both Equations 2 and 3, and the optimal allocation is depicted in Figure 1*b*. In this case, the MNB_1 curve intersects the S_1 constraint curve, which is determined by the amount of harvest in period 0. Because of the constraint, harvest must

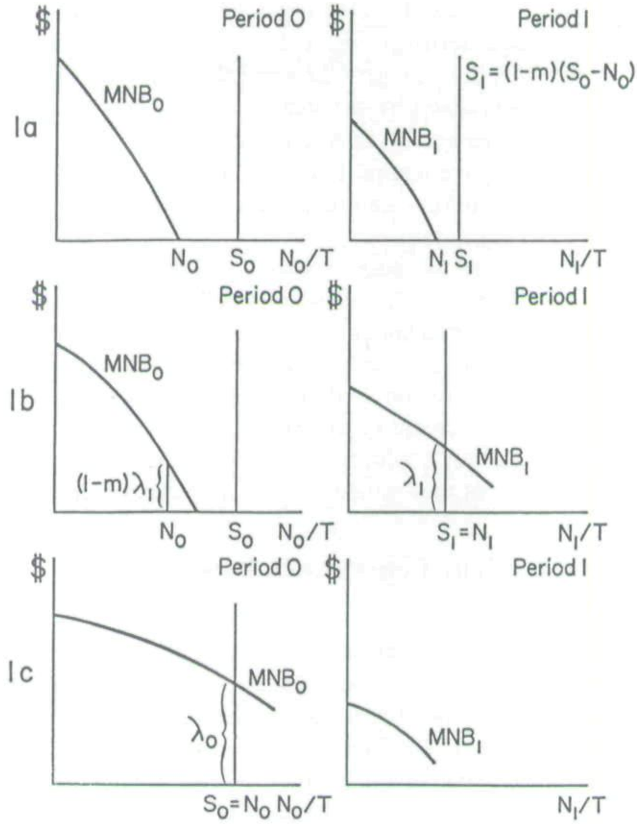

Figure 1 Shrimp fisheries: When the stock constraint is not binding, the marginal net benefit of the last fish in each period will be zero. When it is binding, the time pattern of harvest must be such that no switch will increase net harvest value.

stop in this period before MNB_1 falls to zero. If more fish were available, it would make sense to harvest them. The net value of the last fish harvested is λ_1, the shadow price of an extra unit of fish in period 1.

The optimal harvest of small fish in period 0 is where MNB_0 equal $\lambda_1(1 - m)$. This has a rather straightforward interpretation. Assume that fish are measured in units of 100 and m equals 0.1; therefore 100 small shrimp at the end of period 0 will result in 90 large shrimp at the beginning of period 1. Because of natural mortality, this is the rate at which the potential for harvest can be transferred between size classes by foregoing harvest of small shrimp. The marginal net value of the last small shrimp should only be 90% of the marginal net value of the last large shrimp because only 90% of the time will that small shrimp be available for harvest as a large shrimp. Put differently, the optimum allocation occurs where $MNB_0 = (1 - m)\ MNB_1$, see Equations 1 and 2.

Although there is no constraint in terms of what is available to catch in period 0, the optimal harvest pattern necessitates that production stop before MNB_0 falls to zero. There would be positive benefits from expanding harvest beyond N_0, but there are even greater benefits from allowing the extra shrimp to grow to offer the potential that those that are still alive to be harvested the next period. The shadow price of an extra fish in period 1, λ_1, corrected for natural mortality, becomes a user cost for harvest in period 0.

Therefore, when the stock constraint is binding in the last period, there are two interperiod costs effects: the stock externality cost and a user cost.

An extreme form of this case is where the vertical intercept of the MNB_0 is less than $\lambda(1 - m)$, where λ_1 is determined by the intersection of the MNB_1 curve and the S_1 constraint curve which is generated by an N_0 equal to zero. In that case, N_0 should equal zero, and all shrimp that survive natural mortality should be harvested in period 1. In terms of the formal Kuhn–Tucker conditions, this is where Equation 2 holds as an inequality and so N_0 must equal zero.

(c) The final case is where the stock constraint is binding in period 0. In this case only λ_0 is positive, see Figure 1*c*. The value of small shrimp is high enough that the MNB_0 intersects the S_0 stock constraint curve and it is optimal to harvest the entire stock in period zero. The value of the last fish harvested that period will be equal to λ_0. Since all fish are harvested in the first period, both S_1 and N_1 must equal zero. If this is truly to be an optimum, the vertical intercept of the MNB_1 curve must be less than λ_0. Otherwise, the total value of harvest could be increased by allowing some shrimp to go unharvested in period 0 so that they would be available in period 1.

Optimal Utilization of Many Cohorts Simultaneously: The Lobster Fishery

The analysis of the lobster fishery is different from that of the shrimp fishery in several respects, although there are some obvious similarities. The main differences are that different cohorts are harvested simultaneously by the same effort and that cohorts can live from one season to the next. The first difference is the most fundamental because it means that it is not possible to define a separable cost function for lobsters of a particular size. Costs must be defined in terms of fishing effort but there is no way to apportion them to catches of various sized individuals. It is for this reason that *effort* rather than *catch* must be used as the main control variable in this section. The analytical effects of this change in the control variable are discussed at various points in this section and in the appendix.

Because of the problem of defining separable cost functions and the possibility that the optimal catch of each of the sizes in any period will not be obtained by the same level of effort, the statement of the problem is very complex.[1]

Perhaps a better appreciation of these complexities, the different issues they raise, and the way in which they may be examined can be obtained by a closer examination of the differences between the lobster model and the shrimp model. In the first place, if lobsters of different ages could be harvested independently of each other, the problem would be nothing more than a simultaneous, but independent, operation of the shrimp model (expanded to a multiyear horizon) on the various lobster cohorts.

However, the harvest of different-aged lobsters is an interdependent operation. When pots are put in the water, there is no way to strictly control the age of the individuals that are captured. Fishing effort will obtain output from the various cohorts roughly in proportion to the total cohort sizes. However, if lobster fishing is truly a joint-product operation with no control over the proportions, the analysis boils down to a simple problem of determining the optimal amount of effort to produce in any year. The potential to decide the optimal amount of harvest by size is precluded.

But since lobsters can be returned to the sea with relatively low discard mortality, there is some flexibility in the makeup of the total harvest. Indeed, the current mode of regulation is to prohibit the landing of small lobsters. Let directed effort at a particular sized cohort be defined as that part of nominal effort when catch of that cohort is kept rather than returned to the sea. For example, in the current situation, directed effort at legal lobsters is equal to nominal effort, whereas directed effort at undersized lobsters is presumably zero. This is not to say that small lobsters do not come up in traps, but that whenever they do, they are to be returned to the sea.

For simplicity assume a lobster fishery where the individuals live 2 years after recruitment and there is a distinct market for 1- and for 2-year-olds. Assume also that there is a known and constant recruitment each year.

The object is to find the optimum catch of 1- and 2-year-old lobsters in any year where catch is a function of directed effort and cohort size. That is, catch of the jth cohort in the ith year is $N_{ij} = N_{ij}(E_{ij}, S_{ij})$. E_{ij} is the directed effort at the jth cohort in the ith year. It is that proportion of total effort in the ith period when lobsters of age j which are taken may be kept. If E_{ij} is the amount of effort produced in the ith period and E_{ij} is less than E_i, then j-year-old lobsters must be returned to the sea for part of the season. Cost in each period will be a function of effort only. The stock externality enters through the production function when effort is the control variable.

In terms of the constrained maximization problem, the control variables for each period are E_i (total effort in period i), E_{i1} (effort in period i during which 1-year-old lobsters are retained), and E_{i2} (effort in period i during which 2-year-old lobsters are retained). It is a formal necessity to include the constraints that both E_{i1} and E_{i2} must be less than or equal to E_i. Also the catch from any cohort in any year cannot be larger than that cohort at that time. The same rule for natural mortality between periods will hold; i.e., $S_{12} = (1 - m)(S_{01} - N_{01})$. The formal maximization problem, stated in terms of a 2-year horizon, is

$$\text{Max } B_0(N_{01}, N_{02}) - C_0(E_0) + \rho[B(N_{11}, N_{12}) - C_1(E_1)]$$

St $N_{01} \leq S_{01}$ where S_{01} is recruitment in period 0

$N_{02} \leq S_{02}$ where S_{02} is determined from recruitment and catch the previous year, but is given as far as this problem is concerned

$N_{11} \leq S_{11}$ where S_{11} recruitment in period 1

$$\begin{aligned}
N_{12} &\leq (1 - m)(S_{01} - N_{01}) \\
E_{01} &\leq E_0 \\
E_{02} &\leq E_0 \\
E_{11} &\leq E_1 \\
E_{12} &\leq E_1
\end{aligned}$$

All other terms are as analogously defined in the shrimp model except that ρ is the discount factor.

The appropriate Lagrangian and first-order conditions are as follows. For ease of interpretation, the multiplier for the constraint on effort in period 1 has been made a current value shadow price by including the discount term.

$$\begin{aligned}
L = {} & B_0[N_{01}(E_{01}, S_{01}), N_{02}(E_{02}, S_{02})] - C_0(E_0) \\
& + \rho B_1[N_{11}(E_{11}, S_{11}), N_{12}(E_{12}, S_{12})] - \rho C_1(E_1) \\
& + \lambda_{01}(S_{01} - N_{01}) \\
& + \lambda_{02}(S_{02} - N_{02}) \\
& + \lambda_{11}(S_{11} - N_{11})
\end{aligned}$$

$$
\begin{aligned}
&+ \lambda_{12}[(1 - m)(S_{01} - N_{01}) - N_{12}] \\
&+ \phi_{01}(E_0 - E_{01}) + \phi_{02}(E_0 - E_{02}) \\
&+ \rho\phi_{11}(E_1 - E_{11}) + \rho\phi_{12}(E_1 - E_{12})
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial E_{01}} &= \underline{\frac{\partial B_0}{\partial N_{01}} \frac{\partial N_{01}}{\partial E_{01}} + \rho \frac{\partial B_1}{\partial N_{12}} \frac{\partial N_{12}}{\partial S_{12}} \frac{\partial S_{12}}{\partial N_{01}} \frac{\partial N_{01}}{\partial E_{01}}} \\
&- \lambda_{01} \frac{\partial N_{01}}{\partial E_{01}} \lambda_{12} \frac{\partial N_{01}}{\partial E_{01}} \left[(1 - m) + \frac{\partial N_{12}}{\partial S_{12}} \frac{\partial S_{12}}{\partial N_{01}} \right] \\
&- \phi_{01} \le 0
\end{aligned} \tag{5}
$$

$$\frac{\partial L}{\partial E_{02}} = \underline{\frac{\partial B_0}{\partial N_{02}} \frac{\partial N_{02}}{\partial E_{02}}} - \lambda_{02} \frac{\partial N_{02}}{\partial E_{02}} - \phi_{02} \le 0 \tag{6}$$

$$\frac{\partial L}{\partial E_{02}} = \phi_{01} + \phi_{02} - \frac{dC_0}{dE_0} \le 0 \tag{7}$$

$$\frac{\partial L}{\partial E_{11}} = \rho \underline{\frac{\partial B_1}{\partial N_{11}} \frac{\partial N_{11}}{\partial E_{11}}} - \lambda_{11} \frac{\partial N_{11}}{\partial E_{11}} - \rho\phi_{11} \le 0 \tag{8}$$

$$\frac{\partial L}{\partial E_{12}} = \rho \underline{\frac{\partial B_1}{\partial N_{12}} \frac{\partial N_{12}}{\partial E_{12}}} - \lambda_{12} \frac{\partial N_{12}}{\partial E_{12}} - \rho\phi_{12} \le 0 \tag{9}$$

$$\frac{\partial L}{\partial E_1} = \rho\phi_{11} + \rho\phi_{12} - \rho \frac{dC_1}{dE_1} \le 0 \tag{10}$$

The first-order conditions are subject to the normal interpretations and full description of their economic significance will be provided below. For the moment note that the terms preceding the Lagrangian multipliers in Equations 5 and 6 and Equations 8 and 9 represent the marginal gains from efforts directed at the various cohorts during periods 0 and 1 respectively. Equation 5, which shows the marginal benefits of effort directed at the entering year cohort in period 0, has a term to show the stock externality affects on the harvest of that cohort in period 1. Because this problem has been defined over only two periods to keep the notation as simple as possible, there is no analogous term in Equation 8 that shows the marginal benefits of effort directed at the entering year cohort in period 1. The interpretation below will be modified to correct for this omission, however.

By solving Equations 5, 6, 8, 9 for the ϕ terms, expressions for the marginal revenues of the various directed efforts can be obtained. Viewed in this light, the interpretation of Equations 7 and 10 is quite simple. The sum of the marginal revenues of the directed efforts in both years must equal the marginal cost of producing effort. As will be pointed out below, however, there is more to this interpretation than first meets the eye.

By stating the problem in these stark simplistic terms, it is easy to see the main difference between the shrimp and the lobster models. In the lobster model, in addition to allocating effort at the entering cohort in this and the coming year, simultaneously it is necessary to allocate effort at the remainder of the cohort that entered the previous year and at the cohort that will enter in the next year. Therefore, instead of the three possible

Table 1
Relevant Solutions to Lobster Problem

	(a) $N_{01} < S_{01}$, $N_{12} < S_{12}$		(b) $N_{01} < S_{01}$, $N_{12} = S_{12}$		(c) $N_{01} = S_{01}$, $N_{12} = 0$	
(1) $N_{02} < S_{02}$	$\lambda_{02} = 0$	$\lambda_{01} = 0$	$\lambda_{02} = 0$	$\lambda_{01} = 0$	$\lambda_{02} = 0$	$\lambda_{01} > 0$
$N_{11} < S_{11}$	$\lambda_{11} = 0$	$\lambda_{12} = 0$	$\lambda_{11} = 0$	$\lambda_{12} > 0$	$\lambda_{11} = 0$	
(2) $N_{02} = S_{02}$	$\lambda_{02} > 0$	$\lambda_{01} = 0$	$\lambda_{02} > 0$	$\lambda_{01} = 0$	$\lambda_{02} > 0$	$\lambda_{01} > 0$
$N_{11} < S_{11}$	$\lambda_{11} = 0$	$\lambda_{12} = 0$	$\lambda_{11} = 0$	$\lambda_{12} > 0$	$\lambda_{11} = 0$	
(3) $N_{02} < S_{02}$	$\lambda_{02} = 0$	$\lambda_{01} = 0$	$\lambda_{02} = 0$	$\lambda_{01} = 0$	$\lambda_{02} = 0$	$\lambda_{01} > 0$
$N_{11} = S_{11}$	$\lambda_{11} > 0$	$\lambda_{12} = 0$	$\lambda_{11} > 0$	$\lambda_{12} > 0$	$\lambda_{11} > 0$	
(4) $N_{02} = S_{02}$	$\lambda_{02} > 0$	$\lambda_{01} = 0$	$\lambda_{02} > 0$	$\lambda_{01} = 0$	$\lambda_{02} > 0$	$\lambda_{01} > 0$
$N_{11} = S_{11}$	$\lambda_{11} > 0$	$\lambda_{12} = 0$	$\lambda_{11} > 0$	$\lambda_{12} > 0$	$\lambda_{11} > 0$	

types of solutions in the shrimp model, there are 12 in the lobster model, see Table 1. Cases a–c are analogous to those described for the shrimp model. For each of these three ways to use an entering cohort, there are four possible ways to use the end of last years cohort and the first of next years cohort: (1) part of both can be used, (2), (3), part of one and all of the other can be used, and (4) all of both can be used. Which of the 12 possible types of solutions will actually occur depend in part on the relative values of young and old lobsters.

An economic understanding of the complete problem of the dynamically optimal utilization of a multicohort stock can best be obtained by graphically examining some of the possible solutions listed in the table. Because Equations 5, 6, and Equations 8, and 9 are subsumed in Equations 7 and 10, respectively, this can be done by studying the plots of the latter two equations.

Consider first case al, the simplest situation where the catch of all cohorts in both years is never constrained by cohort size. Then by the Kuhn-Tucker theorem, all of the λ's will equal zero. In this instance, assuming the marginal cost of effort in both years is constant, Equations 7 and 10 can be plotted as in Figure 2. Ignore for the moment the dotted curve. The optimal amounts of aggregate effort in periods 0 and 1 are E_0 and E_1, respectively. At those points, the sum of marginal revenues from both directed efforts is equal to marginal cost of effort in each year. Because both directed efforts are needed to cover production costs in each year, the directed effort for both cohorts is equal to aggregate effort. In this situation, although there are more fish from every cohort available for harvest, it does not make economic sense to produce the effort to catch them.

In calculating the optimal levels of effort, note that revenues and costs in period 1 are discounted to period 0 terms. In addition, the stock externality effect of the harvest of N_{01} on the harvest of N_{12} is included in ϕ_{01}.

It is not always the case, however, that the directed efforts at both cohorts will equal aggregate effort in this situation. For example, if ϕ_{01} is represented by the dotted curve, but the sum of the two ϕ expressions remains the same, aggregate effort and the directed effort for S_{02} will equal E_0. Directed effort at S_{01} will equal E'_{01}. This is because the return from harvesting the entering cohort in period 0 will fall to zero when aggregate effort gets as high as E'_{01}. This could occur if there were a very limited market for small lobsters and when catch gets too high the price falls dramatically. However, directed

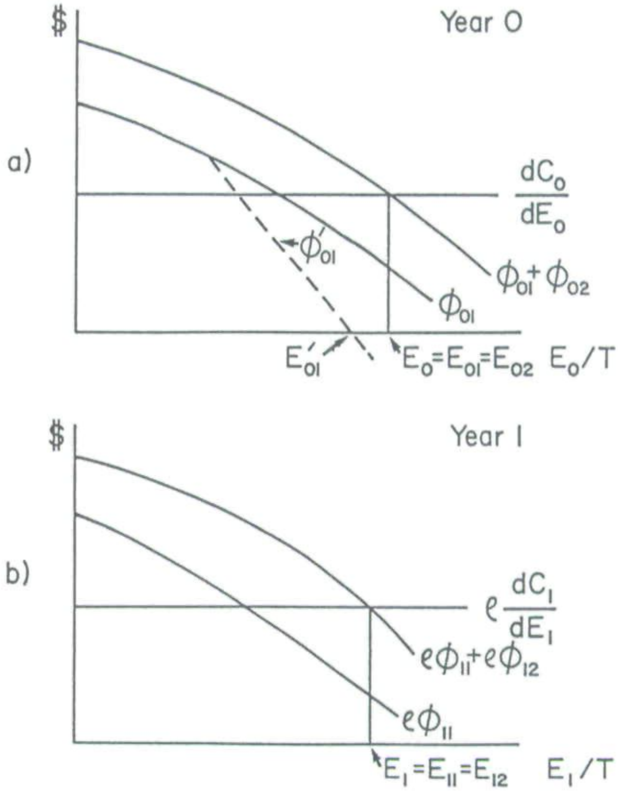

Figure 2 Lobster fisheries case a1; When the stock constraints are not binding, the optimal amount of aggregate effort is where the returns from both cohorts equal the marginal cost of effort.

effort at last year's cohort can, by itself, cover the expenses of an increase in aggregate effort to E_0.

It may be argued that the cost of catching the last individual from any particular cohort is very high, and hence this is the only relevant case. However, it has been estimated that as much as 90% of all northern lobsters are caught. (Townsend 1988, personal communication). Therefore, some of the other cases may be of more than mere theoretical interest.

For a slightly more complicated circumstance, consider case a2 where in period 0 the size of the remaining stock of the cohort that entered the previous year is an absolute constraint on its harvest. In this case, of all the multipliers only λ_{02} will be positive. A possible depiction of this situation is found in Figure 3. In the period 0, when aggregate effort reaches E_{02}, all of the fish in S_{02} will have been captured. Therefore, this is the total amount of directed effort at this cohort. As depicted here, directed effort at the stock entering in period 0 will by itself cover the costs of increasing effort to E_0. Stated more formally, the optimal amounts of aggregate effort in the two periods will be E_0 and E_1, respectively. Directed effort for both cohorts in period 1 will both equal the aggregate effort, but in period 0 only E_{01} will equal aggregate effort. Aggregate effort will not be a binding constraint on the production of E_{02} and, hence, in the final solution the multiplier λ_{02} will be zero. At the point where the stock of old fish becomes a binding constraint on the production of E_{02}, the ϕ_{02} expression is eliminated and the level of aggregate effort is determined exclusively by ϕ_{01}, the marginal revenue from the catch of young fish. Aggregate effort in period zero occurs where ϕ_{01} equals the marginal cost of effort. At E_{02}, the point where S_{02} runs out, the marginal return for directed effort at S_{02} is equal to λ_{02} $(\partial N_{02}/\partial E_{02})$, see Equation 6. The first component, λ_{02}, is the shadow price of another unit of S_{02}, and the entire term is the shadow price for the ability to productively use another unit of E_{02}.

Analogous to the situation in Figure 2, it is also possible that E_{01} can be less than aggregate effort. If the ϕ_{01} curve is so low that it intersects the marginal cost curve at a point to the left of E_{02} but the sum of the ϕ terms remains the same, aggregate effort would equal E_{02} and E_{01} would occur where the ϕ_{01} curve intersected the marginal cost curve. Under these circumstances the difference between ϕ_{02} and the marginal cost of effort would be equal to λ_{02} $(\partial N_{02}/\partial E_{02})$. This situation could occur if the market for S_{01} individuals is saturated in the season, but the market for S_{02} individuals will support the cost of aggregate effort until the stock constraint is reached.

Case bl is slightly more complicated and allows for a direct comparison of optimal allocations of fish across years. In this case, only S_{12} acts as a binding constraint on production. Because the stock of 2-year-old fish in period 1 is a constraint on harvest, there is obviously a trade-off with 1-year-old fish in period 0. An example of an optimal solution of this type is pictured in Figure 4. The markets for S_{02} and S_{11} are strong enough by themselves to support aggregate levels of effort in the two periods of E_0 and

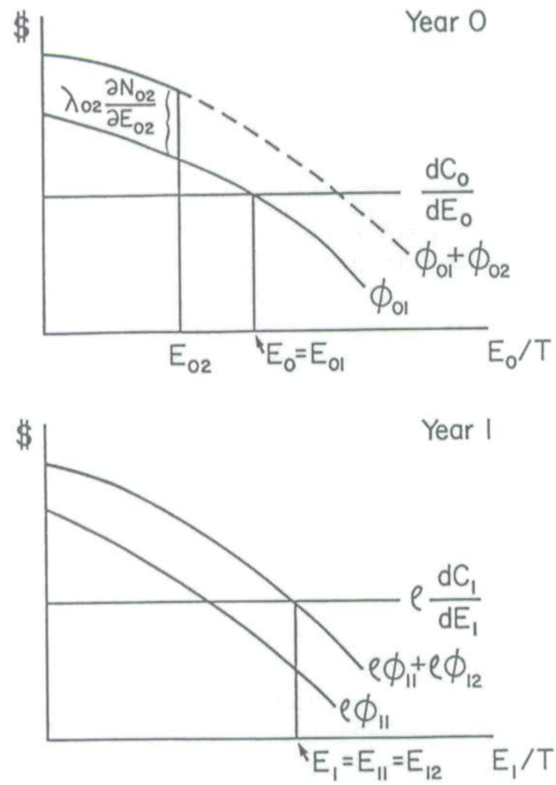

Figure 3 Lobster fisheries case a2: When there is a constraint on older fish in period 0, directed effort at that stock ceases when all are harvested, but optimal directed effort at the other steock may be higher if returns can cover the marginal effort costs.

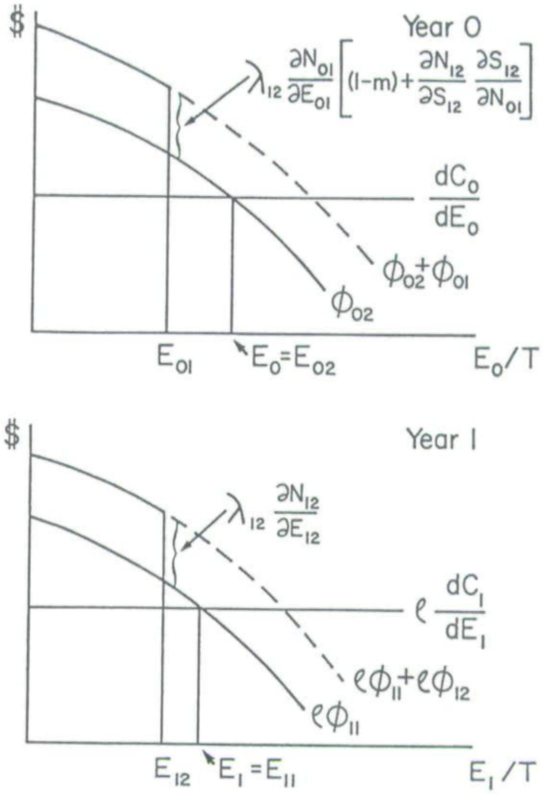

Figure 4 Lobster fisheries case bl: When there is a constraint on older fish in period 1, t here will be a user cost on the harvest of young fish in period 0.

E_1, respectively. In both years, however, the optimal level of directed effort at the other stock is less than this. In year 1, the optimal level of effort directed at S_{12} is E_{12}, because at that point the last unit of this stock is harvested. The value that would be generated by the effort that would be possible if more 2-year-old fish were available in period 2 is equal to λ_{12} $(\partial N_{12}/\partial E_{12})$, see equation 9. In period 0, although there are still unharvested individuals in the S_{01} stock, it is optimal to cease directed effort at that stock at E_{01}, because the remaining units of fish will be more valuable as a potential harvest in period 1. At this point the marginal value of E_{01} directed effort is

$$\lambda_{12}\frac{\partial N_{01}}{\partial E_{02}}\left[(1-m)+\frac{\partial N_{12}}{\partial S_{12}}\frac{\partial S_{12}}{\partial N_{01}}\right]$$

See Equation 5. This amount is the user cost in terms of lost production next year from producing another unit of E_{01} this year. As is shown in the appendix, the optimal relationship between the marginal values of directed effort in the two periods is analogous to that of the marginal values of fish in different periods in case b of the shrimp model.

As in the previous cases, the relationship between the two directed efforts in both years and the determination of which, if either, will be less than aggregate effort will depend upon the height of ϕ_{02} and ϕ_{11} curves, respectively.

It is not necessary to go over the other cases in detail. They are similar to, or combinations of, the cases described above. However, the basic results and economic conclusions from this section can be summarized as follows.

The dynamic optimal utilization of a multicohort fish stock involves an annual simultaneous decision on how much directed effort to allocate to each of the available cohorts. Except for the oldest cohort, it is necessary to consider both a stock effect on the productivity of (i.e., the cost of harvesting) that cohort in future years, and the possibility that all available fish will be harvested in some year such that there is a user cost of taking a fish. Because it is not possible to attribute aggregate effort costs to the directed effort at any cohort when more than one is being fished, it is necessary to compare the marginal gains from all active directed efforts against aggregate effort costs. Therefore, the optimal amount of a particular directed effort in any year will depend in part on the markets for, and the stock constraints imposed by, all cohorts.

Summary and Practical Conclusions

The analysis has shown that optimal utilization of fish stocks where value varies with individual size requires a specific pattern of harvest by size. It does not necessarily call for a single optimum age at first capture. The nature of the pattern depends on the relative net values by size, the natural mortality rate, and the discount rate. The policy implications are that the prohibition on inshore shrimp fishing and a single size limit on lobsters may not be taking full advantage of the potential gains from considering the price–size relationship. An optimal shrimp policy may require specified catch both inshore and offshore. Similarly, the optimal lobster policy may be annual quotas by size.

The above conclusions, of course, must be tempered by the realities of institutional constraints, enforcement costs, efficiency of producing effort, and recruitment. Whereas the optimal pattern of harvesting has been described, the process of moving to it from the existing harvest pattern has been ignored. For example, current lobster utilization has left very few large individuals in the water. The imposition of annual quotas of various size lobsters could place severe restrictions on the industry until there are sufficient large lobsters to make up for the restricted catches of small ones.

The practicalities of enforcing quotas by size may be quite difficult especially in the lobster fishery where the different sizes would be harvested simultaneously. Preventing the continued harvest of one size after its quota is filled would require surveillance of individual fishermen. This is already required for the existing size limits to some extent, but the problems will likely increase. Because harvest by size is separated by time and space, the enforcement problems will be less severe in the shrimp fishery. General seasonal or area restrictions would likely suffice.

The exact method of achieving the catch limits by size is also important because of the effect certain regulations can have on the efficiency of producing effort (Crutchfield 1961, Anderson 1986, chapter 6). Transferable individual quotas by size may eliminate these side effects, however.

Recruitment poses a problem in two ways. First, it is necessary to know the size of the recruiting cohorts so that the quotas by size can be set for the next periods (shrimp) or years (lobster). It may be difficult and expensive to obtain estimates of sufficient accuracy. In addition, because recruitment can sometimes vary significantly from year to year, the optimal policy in any one year may appear silly to industry participants. For example, industry support and voluntary compliance may suffer if the quota on, say, medium-sized lobsters is absolutely and relatively smaller compared to other sizes.

Recruitment is also important to the extent that it is dependent on stock size and hence on the size of harvest. Whereas this is not the case for shrimp, at least over the relevant range of harvest, stock size may well be an important determinant of lobster recruitment. Unfortunately, the exact relationship has not been identified. The problem is made more difficult because other things may affect recruitment as well. For example, it may well be that seawater temperature is a very important variable (Townsend 1986). In any event, to the extent that a program to optimize utilization through time and across sizes increases the average size and age of the stock, it will have serendipitous effects on recruitment relative to existing management.

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Appendix

The purpose of this appendix is to show the equivalency of the marginal trade-off rules between two periods in the multicohort lobster model where, of necessity, effort is the control variable, and the more simple shrimp model where output is the control variable. The rule is applicable only when the stock size of a cohort is a binding constraint on harvest (i.e., when all of the fish either die or are caught). The basic trade-off rule can be derived in the shrimp model. By solving Equations 2 and 3 for λ(remember if $\lambda_1 > 0$, then $\lambda_0 = 0$) it can be shown that

$$\mathrm{MNB}_0 = (1 - m)\mathrm{MNB}_1 \tag{A1}$$

See above for the exact formulations of the marginal net benefit (MNB) of a unit of fish in both periods. In essence, condition A1 says that the marginal net benefit of the last unit of fish of a particular cohort in period 0, including any stock externality costs, is equal to the natural mortality corrected marginal net benefit of a unit of fish of that same cohort in period 1.

The optimal trade-off in the multicohort lobster model depends on which, if either, of the directed efforts are not constrained by the amount of aggregate effort produced. The mathematics will be reduced by starting with the most general case and then showing how the basic formulation is modified in other instances.

The equations in the multicohort model that describe the optimal harvest of fish of the same cohort in the different years (i.e., stocks S_{01} and S_{12} are (5) and (9). In the most general case where all directed efforts in both periods are equal to the aggregate efforts, all of the first-order conditions are necessary to derive the optimal trade-off rule. Because the S_{12} stock constraint is binding, it follows that $\lambda_{12} > 0$ and $\lambda_{01} = 0$. To keep things uncluttered, assume that the S_{02} and S_{11} stock constraints are not binding and so $\lambda_{02} = \lambda_{01} = 0$.

Because aggregate effort in both periods is a binding constraint on all directed efforts, all of the ϕ terms will be positive.

By substituting (5) and (6) into (7), noting that

$$\frac{\partial S_{12}}{\partial N_{01}} = -(1 - m)$$

and (8) and (9) into (10), we obtain

$$\frac{\partial B_0}{\partial N_{01}}\frac{\partial N_{01}}{\partial E_0} + \frac{\partial B_0}{\partial N_{02}}\frac{\partial N_{02}}{\partial E_0} - \rho\,(1 - m)\,\frac{\partial B_1}{\partial N_{12}}\frac{\partial N_{12}}{\partial S_{12}}\frac{\partial N_{01}}{\partial E_0} - \frac{dC_0}{dE_0}$$

$$= \lambda_{12}\frac{\partial N_{01}}{\partial E_0}\left(1 - \frac{\partial N_{12}}{\partial S_{12}}\right) \tag{A2}$$

$$- \rho\,\frac{\partial B_1}{\partial N_{11}}\frac{\partial N_{11}}{\partial E_1} + \rho\,\frac{\partial B_1}{\partial N_{12}}\frac{\partial N_{12}}{\partial E_1} - \rho\,\frac{dC_1}{dE_1} = \lambda_{12}\frac{\partial N_{12}}{\partial E_1}$$

Because all directed efforts are assumed to always be equal to aggregate effort, the distinction between aggregate and direct effort is dropped.

Dividing equations (A2) and (A3) by $\partial N_{01}/\partial E_0)$ and $\partial N_{12}/\partial E_{12}$, respectively, will express the marginal conditions in terms of units of fish from stocks S_{01} and S_{12}, respectively, rather than in units of directed efforts. By performing these operations and solving each for λ_{12}, we obtain

$$\frac{\left(\dfrac{\partial B_0}{\partial N_{01}}\right) - NMC_{N_{02}} - \rho\,(1 - m)\,\dfrac{\partial B_1}{\partial N_{12}}\dfrac{\partial N_{12}}{\partial S_{12}}}{(1 - m)\left[1 - \dfrac{\partial N_{12}}{\partial S_{12}}\right]} = \lambda_{12} \tag{A4}$$

$$\rho\,\frac{\partial B_1}{\partial M_2} - \rho\,\mathrm{NMC}_{N_{12}} = \lambda_{12} \tag{A5}$$

where NMC_i represents the net marginal cost of fish from the ith cohort as follows:

$$\mathrm{NMC}_{N_{01}} = \frac{\left(\dfrac{\partial B_1}{\partial N_{02}}\right)\left(\dfrac{\partial N_{02}}{\partial E_0}\right) - \left(\dfrac{dC_0}{dE_0}\right)}{\left(\dfrac{\partial N_{01}}{\partial E_0}\right)}$$

$$\mathrm{NMC}_{N_{12}} = \frac{\left(\dfrac{\partial B_1}{\partial N_{11}}\right)\left(\dfrac{\partial N_{11}}{\partial E_1}\right) - \left(\dfrac{dC_1}{dE_1}\right)}{\left(\dfrac{\partial N_{12}}{\partial E_1}\right)}$$

That is, the net marginal cost of another unit of N_{01} is the loss due to the marginal cost of producing the effort to obtain it plus the gain of the benefits that are obtained from selling the units of N_{11} that will be jointly produced with it.

Equilibrating Equations A4 and A5 obtains

$$\frac{\partial B_0}{\partial N_{01}} - \mathrm{NMC}_{N_{01}} + \frac{\partial N_{12}}{\partial S_{12}} \frac{\partial S_{12}}{\partial N_{01}} \mathrm{NMC}_{N_{12}} = \rho\,(1 - m) \left(\frac{\partial B_1}{\partial N_{12}} - \mathrm{NMC}_{N_{12}}\right) \tag{A6}$$

Close observation will reveal that the left side is analogous to MNB_0 in Equation 2. The first two terms are the net benefit of the last unit of N_{01} harvested in period 0. The third term is the stock externality effect. It shows the cost of keeping the harvest of N_{12} the same with the lower initial stock size. Similarly the right side is analogous to MNB_1 in Equation 3. Therefore, (A6) is the multicohort equivalent of (A1).

The other potential cases can be handled in short order. The only difference is the nature of the NMC_i terms. If the directed efforts for N_{02} and N_{11} are less than their aggregate efforts (that is, fishing for those stocks ceases before the year ends) then ϕ_{02} and ϕ_{11} are both zero. Therefore, only Equation 5 is used to derive Equation A2 and only Equation 7 is used to derive Equation A3. The only effect of tracing these changes through all the calculations is that the NMC_i terms are replaced with a more traditional marginal cost expression

$$\mathrm{MC}_{N_{01}} = \frac{\dfrac{\partial N_{01}}{\partial E_{01}}}{\dfrac{dC_0}{dE_0}}$$

$$\mathrm{MC}_{N_{12}} = \frac{\dfrac{\partial N_{12}}{\partial E_1}}{\dfrac{dC_1}{dE_1}}$$

Because there are no direct efforts for N_{02} and N_{11} at the margin, all effort costs can be allocated to N_{01} and N_{12}, respectively.

The final case is where the directed efforts for N_{01} and N_{12} are less than aggregate effort. In this case the trade-off conditions can be derived from Equations 5 and 9 directly. Because of the nonbinding constraint on the production of directed effort at S_{01} and S_{12}, the marginal costs are zero. Therefore, all cost terms, including the stock externality term, drop out, and the relevant expression is

$$\frac{\partial B_0}{\partial N_0} = \rho\,(1 - m)\,\frac{\partial B_1}{\partial N_{12}}$$

The gross marginal benefits of a unit of fish in each year, properly discounted and corrected for natural mortality that must be equal.

Notes

1. Clark (1976) states "We come now to the dynamic optimization problem for a multicohort fish population. Even with the simplifying assumption to be made here that recruitment is independent of stock size, an analytic solution for the general problem seems completely unattainable. We therefore make an additional assumption that the costs of fishing are negligible."

It is the direct cognizance of costs in the current model that provides most of the complexity. The difficulty of analytical solutions notwithstanding, the presentation does allow for a rigorous description of the economic issues involved in the dynamic utilization of multicohort fishery, something that has heretofore not appeared in the literature.

2. For example, suppose total effort in the present year, E_0, is 200 pot lifts, which, under existing conditions, would yield 600 one-year-old lobsters and 100 two-year-old lobsters. However, it may be optimal to return 300 of the 1-year-old lobsters to the sea for potential harvest next year.

Assuming that catch is spread evenly throughout the fishing season, directed effort at 1-year-olds, E_{01}, would be 100 pot lifts. During the first 100 pot lifts all lobsters would be retained, but for the second 100 lifts only the 2-year-olds would be kept.

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