Marine Resource Economics, Volume 22, pp. 137–154
Printed in the U.S.A. All rights reserved

0738-1360/00 $3.00 + .00

Sharing Rules and Stability in Coalition Games with Externalities

LONE GRØNBÆK KRONBAK
University of Southern Denmark

MARKO LINDROOS
University of Helsinki

Abstract *This paper examines cooperative sharing rules in fisheries coalition games and develops a new sharing rule that takes into account the stability of cooperation when externalities are present. We contribute to existing knowledge by introducing a connection between cooperative games (sharing rules) and non-cooperative games (stability). As an illustrative example, we describe a discrete-time, deterministic, coalition game model of the major agents who exploit the cod stock in the Baltic Sea.*

Key words Baltic Sea cod, characteristic function, coalition game, cooperation, fisheries, nucleolus, Shapley value, sharing rules, stability of cooperation.

JEL Classification Codes C62, C70, Q22, Q28.

Introduction

The feature that distinguishes non-cooperative from cooperative games is the ability to make binding agreements. The non-cooperative game is often referred to as the competitive game, where the players act out of rational self-interest and cannot communicate prior to the game. In a cooperative game, the players have a binding agreement and the objective is to maximize the joint payoff of the game. The coalition game allows for a group that is smaller in number than the total number of players (a coalition) to cooperate.

Non-cooperative games have the advantage that if players adopt their best response strategies, no one has incentives to deviate from the adopted strategy. For cooperative and coalition games, stability has to be evaluated after the solution to the game is determined. The stability is affected by the way in which benefits within the cooperation are shared among players. To evaluate whether a solution is stable requires determining the distribution of benefits within the coalitions. This is performed using the characteristic function game and different sharing rules, but it also requires determining the solution to the non-cooperative game. When externalities are present, it is not fully satisfactory to work with characteristic function games (Yi 2003). This means that the action available to a coalition is typically assumed to be independent of the actions chosen by non-members (Greenberg 1994). The present

---

Lone Grønbæk Kronbak is an assistant professor in the Department of Environmental and Business Economics, University of Southern Denmark, Niels Bohrs Vej 9-10, DK-6700 Esbjerg, Denmark, email: lg@sam.sdu.dk. Marko Lindroos is a lecturer in the Department of Economics and Management, P.O. Box 27, 00014 University of Helsinki, Finland, email: marko.lindroos@helsinki.fi.

Valuable comments and suggestions from Trond Bjørndal, Hans Frost, Henning Jørgensen, and two anonymous referees are acknowledged.

paper deals with the extraction of a renewable resource by large numbers of agents, where externalities are present. Externalities are present in a game in coalition form if and only if there is at least a merger of coalitions that changes the payoff of a player belonging to a coalition not involved in the merger.

The main motivation for this paper is a merger between non-cooperative (stability) and cooperative games (sharing rules). We define this merger in a coalition game applying a γ characteristic function. When positive externalities are present, stand-alone stability is hard to achieve due to strong free-rider incentives. This is clear from previous studies, where the stability of the grand coalition applying sharing rules as nucleolus or the Shapley value is ambiguous.[1] In fisheries coalition games, this link between non-cooperative and cooperative games has received insufficient attention since the externalities are not receiving attention in the cooperative games.

The traditional cooperative game approach is based on the fundamental assumption that players have already agreed to cooperate and that the model allows for transferable utility. A key reference in establishing a model of a cooperative fisheries game is in Kaitala and Lindroos (1998). They established a cooperative game in characteristic function[2] game framework and determined different one-point cooperative solution concepts; however, the model does not take into account externalities in the fisheries game and is a purely theoretical work.

Previous empirical work in fishery economics has included applications of the coalition game approach to the Norwegian spring-spawning herring (Lindroos and Kaitala 2000; Arnason, Magnusson, and Agnarsson 2000) and the Northern Atlantic bluefin tuna (Duarte, Brasão, and Pintassilgo 2000). However, these empirical studies do not consider the important connection between the applied cooperative sharing rules and the stability of cooperation when externalities are present; therefore, none of the determined sharing rules actually satisfy all of the players, and stand-alone stability is not achieved. Brasão, Costa-Duarte, and Cunha-e-Sá (2000) applied a coalition game to the Northern Atlantic bluefin tuna fishery and recognized the instability of the Shapley value due to free-rider incentives. They found a stable, non-cooperative feedback Nash equilibrium with side payments, but were unable to determine the connection between the joint solution and its stability.

Pintassilgo (2003) demonstrates that the grand coalition is only stand-alone stable if no player is interested in leaving the cooperative agreement to adopt free-rider behaviour. Thus Pintassilgo sets the prerequisite for the stand-alone stability when externalities are present but does not study the sharing of benefits inside the grand coalition.

Eyckmans and Finus (2004) also recognize the problems with stability of grand coalitions when externalities are present. They propose a sharing scheme for the distribution of the gains from cooperation where any particular solution belonging to this scheme leads to the set of stable coalitions. Their work is purely theoretical. Weikard (2005) suggest a sharing rule that distributes the coalition payoff proportional to the outside-option payoff.[3]

The main contribution of our paper is to highlight the difficulty of how the coalition payoffs are divided among the members in a characteristic function approach. It develops a new sharing rule that takes into account the stability of cooperation when externalities are present and players are heterogeneous. This paper also contributes to the literature by allowing all members of a coalition to be active in the

[1] Stability is defined as not having incentives to free ride.
[2] There exists also a non-transferable utility version of the characteristic function.
[3] This paper was first published as a working paper in March 2004 (Kronbak 2004). The working papers by Eyckmans and Finus (2004) and Weikard (2005) are thus worked out independently and simultaneously to (or after) the first version of this paper.

fishery until the marginal benefits of the different technologies in the coalition are identical. In a number of previous studies, cost functions have been linear, and only the most efficient coalition members have been active.

The first section describes the underlying bioeconomic model and applied parameter values, while the following section introduces the theoretical setup of the game, solves the game, and arrives at a solution for the inadequacies of the Shapley value and the nucleolus. Furthermore, we discuss the stability of the sharing functions and define a core that is stable in the face of free riding. Then follows a section that contains details of a sensitivity analysis, and the last section concludes the paper.

Bioeconomic Model

Baltic Cod

The Baltic Sea is shared among members of the European Union (EU) (Denmark, Finland, Germany, Sweden, Estonia, Latvia, Lithuania, and Poland) and the Russian Federation. The Baltic Sea consists of the central Baltic Sea, the Gulf of Bothnia, the Gulf of Finland, the Sound, and the Danish Straits. It is a fairly remote area and it contains no international waters.

The most valuable fishery in the Baltic Sea is the cod fishery, which used to be managed by the International Baltic Sea Fishery Commission (IBSFC).[4] All parties that exploit the cod stock are members of the IBSFC, which sets the total allowable catches (TACs) for the fishery.[5] This arrangement appears to represent a coalition, because TAC measures are agreed upon by all contracting parties in the IBSFC. The fact is, however, that TAC measures are often exceeded; thus, free riding exists within the coalition. As all countries exploiting the Baltic Sea cod fishery are contracting parties of the IBSFC, and the main objective of the fishery commission is 'to cooperate closely',[6] we find it therefore natural to apply a cooperative approach for our analysis.

We limit our model to three players and assume that groups of countries can represent the countries around the Baltic Sea. We call our three players Country Group 1, 2, and 3, respectively.

Population Dynamics

Population dynamics are described by a discrete time age-structured model. This is a standard type of cohort-model, where the numbers are determined as follows:

$$
\begin{aligned}
&N_{2,y} = R_y && y > y_1 \\
&N_{a+1,y+1} = N_{a,y}e^{-m_a - S_a f_y} && a \quad \{2, 3, \ldots, 7\}, \\
&N_{a,y_1} \ known && a \quad \{2, 3, \ldots, 8\}
\end{aligned}
\tag{1}
$$

---

[4] The IBSFC as an organisation ceased to function in January 2006. Thereafter all main exploiters of the cod stock are members of the EU. Within the EU the members will, however, still act as different players. Even though the example discusses the previous setup with the IBSFC, the lessons to be learned are sufficiently general to apply also to the EU.

[5] The TACs are the main regulatory tool for the Baltic Sea cod fishery.

[6] Source: Article 1 of the Gdansk Convention, IBSFC (2003).

where R_y describes recruitment into the stock in year y, m is the natural mortality, f is the total fishing mortality, and S_a is the fishing gear selectivity, such that if an age class is not harvested, then the selectivity is zero; otherwise, it is one. We assume that the initial abundance for all age classes in year y_1, N_{a,y_1}, is known. The population dynamics are determined by seven age classes: $a = \{2,3,...,8\}$. These age classes are chosen in accordance with measures of the International Council for the Exploration of the Sea (ICES) (ICES 2000), in which recruits are two years of age before they become part of the stock. y_1 is the initial year for the simulation model. The biomass is determined as the total number of fish multiplied by their stock weights at age, over all age classes:

$$B_y = \sum_{a=2}^{8} SW_a N_{a,y}, \tag{2}$$

where SW_a is the stock weight at age. B_y is the biomass in year y. The total spawning stock biomass (SSB) is given by the sum of mature fish over all age classes:

$$SSB_y = \sum_{a=2}^{8} MO_a SW_a N_{a,y}, \tag{3}$$

where MO_a is the proportion of mature fish in age class a and SSB_y is the spawning stock biomass in year y. We assume a Beverton-Holt stock-recruitment relationship, identical to the one used by ICES (2000), which is defined as follows:

$$R_y = \frac{cSSB_{y-1}}{1 + bSSB_{y-1}}, \tag{4}$$

where c and b are biological recruitment parameters. c is the maximum number of recruits per spawner in a low-spawning stock size, while c/b is the maximum number of recruits when the spawning stock biomass is very large.[7] The biological parameters of the stock recruitment relationship and other parameter values are summarized in table 1. Table 2 provides the initial biological parameters for the year classes.

Yield

The catch measured in numbers of fish for country i and for a specific cohort is given by:

$$C^i_{a,y} = \frac{S_a f^i_y}{m_a + S_a f_y}(N_{a,y} - N_{a+1,y+1}), \tag{5}$$

---

[7] The stock recruitment estimated by ICES assumes that recruits are not entering the population before age two. Therefore, the spawning stock biomass (SSB) lags two years behind the Beverton-Holt recruitment function applied by ICES (2000). For reasons of simplicity, we apply only a one-year lag in our simulation model. We do not see this as a critical assumption because the SSB biomass is similar for successive years.

Table 1
Biological Parameter Values

Parameter	Value
Mortality parameter	
$m_{2,3,\ldots,8}$	0.2
Stock-recruitment (B-H) parameters	
c	0.9814216
b	0.000002340

Source: ICES (2000).

Table 2
Initial Biological Parameters for our Model

	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8+
MO	0.14	0.32	0.84	0.94	0.98	0.96	1
SW	0.244	0.548	1.230	1.595	2.963	4.624	5.417
CW	0.662	0.773	1.127	1.448	2.337	3.485	4.647
*N*0	136,493	71,852	37,621	15,421	4,332	2,026	1,452

MO=Proportion Mature at the Start of the Year, *SW*=Mean Weight in Stock (kg), *CW*=Mean Weight in Catch (kg), *N*0=Initial Abundance (thousands).
Source: 1998 estimates (ICES 2000).

where f_y^i is the fishing mortality by country i, f_y is the total fishing mortality, and $C^i_{a,y}$ is the catch in numbers of fish by country i during year y, of a specific cohort a. The catch function is defined as the numbers of fish that do not survive to the next year and are not subject to natural mortality.

The yield (harvest) for a single country is defined by inserting the number of fish (1) into the catch in numbers of fish (5) multiplied by the catch weights at age:

$$Y_y^i = \sum_{a=2}^{8} CW_a N_{a,y} \frac{S_a f_y^i}{m_a + S_a f_y} (1 - e^{-m_a - S_a f_y}), \tag{6}$$

where Y_y^i is the total yield in kg for country i in year y.

The Cost Function

The cost function is assumed to follow the cost function for harvesting cod in the North Sea for Denmark, Iceland, and Norway (Arnason *et al.* 2000):

$$Q_y(t) = \alpha^i \frac{Y_y^{i2}}{B_y}, \tag{7}$$

where α^i is a cost parameter and B_y is the total biomass in year y. The dependent variable, costs, is defined as total costs less depreciation, interest payments, and skipper wages. This may be regarded as an approximation of the total variable costs. The cost function is defined such that if the total biomass is increased, the cod are easier to locate, and costs therefore decrease. The effect of other players also exploiting the stock is included in changes in the biomass. It is also important to note that the costs increase quadratically with yield. Therefore, in a coalition it is most likely that all participating countries are active, otherwise the coalition is not competitive against countries acting alone.

Net Present Value

The net present value is defined as a function where the control variable is the fishing mortality for player i, f^i;[8] the state variable is the total number of fish in the stock, N; and π^i is the instantaneous profit for player i:

$$\pi_y^i = pY_y^i - Q_y^i, \tag{8}$$

where π_y^i is the instantaneous profit for country i in year y. The net present value of all future profit for a single player i is defined by the functional:

$$J^i(f^i, N) = \sum_{y=y_1}^{y_2} \frac{\pi_y^i}{(1+r)^{y-y_1}}. \tag{9}$$

The players choose their optimal fishing mortality or strategy by maximising the functional J. The model is assumed to have a finite horizon from y_1 to y_2. We chose the period from 1997 to 2046, yielding a running period of 50 years. The start and end points of the time horizon are of no importance to the model; the important factor is the total time horizon of the model (*i.e.*, the length of the running period).

Economic Parameter Values

We assume that there is an open market for raw fish where the fishermen all command the same price for their landings.[9] Furthermore, the Baltic Sea is a comparatively small supplier of cod to a global white fish market in which there are many substitute species and thus the price that fishermen face is approximately constant. The price applied in our model is 10.74 Dkr/kg,[10] which was the average price received for landed cod on the Island of Bornholm during 1998 and 1999 (Fiskeridirektoratet 1999, 2000). Bornholm is located in a central position within the Baltic Sea.

---

[8] In the way that this model is defined, there is a direct link between fishing mortality and yield. Therefore, it is appropriate for the control variable to be the yield. Fishing mortality would then be determined as a residual.

[9] Since there are only three players and they are playing cooperative games, the price could be endogenous. If the price is allowed to be variable, coalition formation would likely be more appealing.

[10] The exchange rate is approximately 1 USD = 5.7 DKr.

Cost parameters are calibrated for the year 1998. This is done by finding cost parameters that yield the fishing effort and a total biomass population equivalent to the arithmetic mean over the period 1966 to 1999, when fishermen engaged in non-cooperative behaviour.[11] This calibration reveals cost parameters at 9, 14, and 15 Dkr/kg for the Country Groups 1, 2, and 3, respectively. Thus, Country Group 1 is assumed to be the most efficient, and Country Group 3 the least efficient. As the calibration method involves some uncertainty, the cost parameter is subject to a sensitivity analysis discussed later in the paper. We assume that there is no technological progress, *etc.*, for the simulated 50-year period and thus cost parameters and prices remain unchanged throughout the lifetime of the model. Functional relationships also remain unchanged, and we assume that there are no stochastic jolts to the system.

It is possible beforehand to conclude that with this type of cost function all countries in a coalition will apply effort until their marginal costs are equivalent. Parameter values for the economic parameters are summarised in table 3.

The fishing mortality applied by the three groups of countries is assumed to be constant over the simulation period.[12] Fishermen are committed to their strategies only at the beginning of the game; this is a sort of open-loop control. The open-loop controls allow the players less rationality and flexibility compared to closed-loop control, but computing open-loop solutions are much easier. There is a tendency in previous studies to resort to the use of open-loop solution concepts (Sumaila 1999). The game is played under complete information because all fishermen know all pay-off functions, but imperfect information because the fishermen are moving simultaneously. In the first year, there are two stages of the game. The first stage involves deciding upon which coalition to join. In the second stage, players determine which fishing mortality to apply. When the model is solved backwards, this endogenizes the coalition formation.

Table 3
Economic Parameter Values for our Model

Parameter	Value
First fishing age, a_1	3
Selectivity S_2	0
Selectivity $S_{3,\ldots,8}$	1
Cost parameter, country 1: α^1	9 Dkr/kg
Cost parameter, country 2: α^2	14 Dkr/kg
Cost parameter, country 3: α^3	15 Dkr/kg
Discount rate, r	2%
Price, p	10.74 Dkr/kg
Max. fishing mortality, $f^1_{\max}$	0.35
Max. fishing mortality, $f^i_{\max}$, i = 2,3	0.3

Source: ICES (2000); Fiskeridirektoratet (1999, 2000).

[11] There have been fluctuations in the size of the biomass over the years. To moderate these fluctuations, the arithmetic mean over the period 1966 to 1999 is applied in the analysis.
[12] The constant fishing mortality over the entire model period is a limitation of the model. The players have no chance to adapt to changes or fluctuations in, for instance, stock. One way to cope with this would be to allow for renegotiations in the model.

The following section solves the game by determining the characteristic function and some of the corresponding one-point solution concepts. The stability of the sharing rules is also discussed further.

Solving the Game

In this paper we take the coalition game approach. The normalised characteristic function is defined and applied for determining two, one-point sharing rules: the Shapley value and the nucleolus. The benefits from applying these sharing rules are compared to the free-rider values. A cooperative solution that applies a sharing rule is only stand-alone stable if there are no free-rider incentives (Pintassilgo 2003). This condition does not always hold for the Shapley value and the nucleolus. Therefore, a stable sharing rule is developed by the model that endogenizes coalition formation and thus searches for equilibrium cooperation structures. This is done by taking the free-rider incentives into account when developing a new sharing rule. It thus represents a merger of the non-cooperative and cooperative games.

The Characteristic Function

The characteristic function (c-function) is determined by applying the definition of the characteristic function described by Mesterton-Gibbons (1992) that is the benefits of cooperation associated with the coalition. This is the difference between the benefits when members form a coalition and the sum of benefits of individual members; *e.g.*, individual players' threat points. We define the characteristic function as follows:

$$\bar{v}(i) = J^i(F^i, N) - J^i(F^i, N) = 0, \ i \in \{1, 2, 3\} \tag{10a}$$

$$\bar{v}(i, j) = J^{i,j}(F^{i,j}, N) - \sum_{i,j} J^i(F^i, N), i \in \{1, 2, 3\}, j \in \{1, 2, 3\}, i \neq j$$

$$\bar{v}(1, 2, 3) = J^{1,2,3}(F^{1,2,3}, N) - \sum_{i=1}^{3} J^i(F^i, N),$$

where $\bar{v}(i)$ is the value of singletons, $\bar{v}(i, j)$ is the value of a two-player coalition, remarks $\bar{v}(i, j) = \bar{v}(j,i)$ and $\bar{v}(1,2,3)$ are the value of the grand coalition: the maximum payoff by the joint action of all players. The strategies are denoted by a cap, F^K, indicating strategies chosen when coalition K plays a Nash game (Nash 1951) against players outside the coalition [see equation (9)]. K refers to the seven possible coalitions,[13] $K = \{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$, while $F^{1,2,3}$ denotes the full cooperative strategy, and J is the functional. We apply a γ-type c-function, as we assume that players outside the coalition adopt Nash strategies against the coalition (Chander and Tulkens 1997). The function is normalized by dividing the characteristic function by the benefits of the grand coalition:

[13] We ignore the empty coalition, in which we assume the benefits are zero. The seven coalitions correspond to only five coalition structures since there is only one coalition structure comprised only of singletons.

$$v(i) = \frac{J^i(F^i, N) - J^i(F^i, N)}{\bar{v}(1, 2, 3)} = 0, i \in \{1, 2, 3\}, \tag{10b}$$

$$\bar{v}(i, j) = \frac{J^{i,j}(F^{i,j}, N) - \sum_{i,j} J^i(F^i, N)}{\bar{v}(1, 2, 3)}, i \in \{1, 2, 3\}, j \in \{1, 2, 3\}, i \neq j$$

$$v(1, 2, 3) = \frac{J^{1,2,3}(F^{1,2,3}, N) - \sum_{i=1}^{3} J^i(F^i, N)}{\bar{v}(1, 2, 3)} = 1.$$

The normalized characteristic function, v, has the properties that the value for a grand coalition is one and the value for a singleton is zero. The optimal fishing mortalities are determined for all possible coalition structures to determine the characteristic function.

When fishermen form a coalition, we assume that they are rational and therefore distribute effort between participants until the marginal profits of members of the coalition are identical. This distribution yields the highest possible benefits from the coalition. As we assume that fishermen face an identical and constant price for landed fish, this implies it is optimal to distribute effort in the coalition such that marginal costs from applying all technologies in the coalition are identical. This requires that a perfect redistribution of effort is possible.[14] The redistribution of effort within the coalition is an extension of Lindroos and Kaitala (2000), where, due to the cost function, it is only the most efficient player in the coalition that harvests. Solving the systems such that marginal costs are identical yields constant harvest shares in each year for members of the coalition (Kronbak 2004). The benefits of different possible coalitions and optimal strategies are summarised in table 4.

Benefits from the grand coalition exceed the sum of benefits from free riding; therefore, there are enough benefits from the grand coalition to be distributed in a

Table 4
Benefits of the Seven Possible Coalition Structures and Optimal Strategies

Coalition	Strategy, f	Net Benefit (Dkr)	Free-rider Value (Dkr)
1	0.35	$2.30694 * 10^{10}$	NA
2	0.29	$1.67376 * 10^{10}$	NA
3	0.27	$1.56076 * 10^{10}$	NA
1,2	0.457	$4.25624 * 10^{10}$	$2.02757 * 10^{10}$ (f^3=0.264)
1,3	0.457	$4.12502 * 10^{10}$	$2.10943 * 10^{10}$ (f^2=0.279)
2,3	0.407	$3.35437 * 10^{10}$	$2.84559 * 10^{10}$ (f^1=0.35)
1,2,3	0.351	$7.47167 * 10^{10}$	$6.98259 * 10^{10}$ (sum of the above)

Note: Numbers have been rounded.

[14] Individual transferable quotas (ITQ) are a possible solution to the fact that an enormous amount of information is required to perfectly distribute effort.

satisfactory way such that the grand coalition is stable (Pintassilgo 2003). By studying the benefits from the seven possible coalitions, we clearly observe the technological advantage of Player 1, as this player receives significant higher benefits than Players 2 and 3, both when acting as a singleton and as a free rider. If the players are all playing a non-cooperative game, then they will each choose a strategy (fishing mortality) that optimizes their own payoff as the best response to the fishing mortalities of the other two players. This yields a Nash equilibrium, where the aggregate fishing mortality is at its highest level. The optimal strategies clearly indicate that overall fishing mortality is reduced with increasing numbers of members within the coalition.

The average population of the cod stock in the Eastern Baltic Sea for the period 1966 to 1999 was 500,000 tonnes. Our model suggests a long-run population of approximately 550,000 tonnes in the non-cooperative scenario and approximately 1,200,000 tonnes in the cooperative scenario. The initial population of the cod stock in this model is set at the 1998 level, which is a very low level of only 174,000 tonnes. Therefore, each of the presented scenarios begins with a period of rebuilding the population before the long-run equilibrium is reached after around 10 years (Kronbak 2004). In years with a high abundance of cod, the population reached 1,023,000 tonnes at its peak.[15] The cooperative equilibrium population might seem unreasonably high compared to the population levels in the record year, but the stock has not been exploited in a way that corresponds to cooperative behaviour. The non-cooperative simulation has a fishing mortality equivalent of 0.91, while the average fishing mortality between 1966 and 1999 was 0.89 (ICES 2000). The total population in the estimations is similarly close to the average population estimated by researchers (ICES 2000).

The characteristic function and the normalised characteristic function are then determined. Their values are provided in table 5. From the characteristic function in table 5, it is apparent that the two-player coalitions yield relatively small benefits compared to those of the grand coalition. It is also clear that it is relatively important, from an economic viewpoint, to have Player 1 join the coalition.

Shapley Value and the Nucleolus

The Shapley value for a single player is defined as the expected marginal contribution. The Shapley value for player i is defined as follows (Aumann and Dréze 1974):

$$\varphi_i = \sum_{s \subset K, i \in s} \frac{(n - |s|)!\,(|s| - 1)!}{n!} \left[v(s) - v(s - \{i\})\right], \tag{11a}$$

where K is the seven possible coalitions, n is the number of players in the game, and $|s|$ is the number of players in coalition s. Equation (11a) shows that the Shapley value is determined by the probability of the different coalitions multiplied by the marginal contribution to the coalition by player i.

In our specific case, with three players, and because we apply a normalised characteristic function, the Shapley value becomes:

$$\varphi_i = \frac{1 - v(j,k)}{3} + \frac{v(i,j)}{6} + \frac{v(i,k)}{6}, i \in \{1, 2, 3\}, j \in \{1, 2, 3\}, k \in \{1, 2, 3\}, i \neq j \neq k. \tag{11b}$$

[15] We consider only the years on record, 1966–99 (ICES 2000).

Table 5
Characteristic Function and Normalised Characteristic Function

Coalition	Strategy, *f*	Characteristic Function (Dkr)	Normalized Char. Function
1	0.35	0	0
2	0.29	0	0
3	0.27	0	0
1,2	0.457	$2.7554*10^9$	0.142751
1,3	0.457	$2.5732*10^9$	0.133312
2,3	0.407	$1.1985*10^9$	0.062092
1,2,3	0.351	$1.93021*10^{10}$	1

Note: Numbers have been rounded.

Equations (11a) and (11b) describe player *i*'s expected marginal contribution.

The idea of the nucleolus is to minimize the dissatisfaction of the most dissatisfied coalition. This is done by finding the 'lexicographic center' of the core, which is the imputation that maximizes the minimum gains to any possible coalition. The nucleolus has the advantage that it always lies in the core, if the core exits. To determine the nucleolus, we define the reasonable set, the excess function, and the core.

The reasonable set is defined as imputations that satisfy three equations. First, a player receives no more than what the player contributes to the coalition. Second, the imputation is individually rational; that is, all players should be better off with cooperation. Third, the imputation is Pareto-optimal or group rational; that is, all benefits are distributed among players. The reasonable set determines the set of fair distributions of the benefits.

The excess is defined as the difference between the fraction of the benefits of cooperation that s can obtain for itself and the fraction of benefits of cooperation that the imputation allocates to s:

$$e(s, x) = v(s) - \sum_{i \in s} x_i, \tag{12}$$

where $x = (x_1,x_2,x_3)$ is a three-dimensional vector that describes different shares (imputations) and x_i describes the share to player i. The core is defined by the excess being negative as an addition to the reasonable set; thus, the core in our case, with three players, takes the following form:

$$\begin{aligned}
&v(i) - x_i \leq 0 \quad \forall i \in \{1, 2, 3\} && \text{(individual rationality)} \\
&x_1 + x_2 + x_3 = 1 && \text{(group rationality)} \\
&v(1,2) - x_1 - x_2 \leq 0 \\
&v(1, 3) - x_1 - x_3 = v(1, 3) - x_1 - (1 - x_1 - x_2) = x_2 - 1 + v(1, 3) \leq 0 \\
&v(2, 3) - x_2 - x_3 = v(2, 3) - x_2 - (1 - x_1 - x_2) = x_1 - 1 + v(2, 3) \leq 0.
\end{aligned} \tag{13}$$

The individual rationality is always satisfied because $v(i)$ is predefined to be zero according to equation (10b). The core thus ensures that each player receives at least the payoff that it would have received from playing singleton (the individual rationality). The core also ensures that all the cooperative benefits are shared among the players (the group rationality). And finally, the core ensures that the players receive at least what they would have received by joining a two-player coalition [the last three constraints of equation (13)]. In this specific case, the core does not narrow down the number of imputations because the core and the reasonable set coincide. The reason for this is that the contribution from all three players to the grand coalition is relatively high; or to put it another way, the two-player coalitions have relatively low payoffs. Thus, the gains from a grand coalition are significantly higher than the gains from a two-player coalition. In a two-player coalition, the players within and outside the coalition play a Nash game against each other.

The rational ε-core is determined by shrinking the boundaries of the core at the same rate until it collapses into either a line or a single point. The rational ε-core consists of the imputations $x = (x_1,x_2,x_3)$ that satisfy:

$$-x_1 \leq \varepsilon, -x_2 \leq \varepsilon, -x_3 \leq \varepsilon \tag{14a}$$

$$v(1,2) - x_1 - x_2 \leq \varepsilon,$$

$$x_2 - 1 + v(1,3) \leq \varepsilon,$$

$$x_1 - 1 + v(2,3) \leq \varepsilon,$$

$$x_1 + x_2 + x_3 = 1.$$

Applying the values from the normalised characteristic function in table 5 yields a lower bound of ε, which in this case is –0.3333. Setting ε to this lower bound yields the following specific inequalities:

$$0.333333 \leq x_1 \leq 0.604475 \tag{14b}$$

$$0.333333 \leq x_2 \leq 0.533355$$

$$0.476084 \leq x_1 + x_2 \leq 0.66667.$$

Clearly, the only imputation that satisfies these equations is $x_1 = x_2 = x_3 = 0.3333$, which is the least rational core. As the boundary of the core collapses to a single point, this is identical to the nucleolus.

The Shapley value is determined by applying the characteristic function values to equation (11b). Upon determining the Shapley values, we easily confirm that in our case the Shapley values lie within the core. Table 6 summarises the distribution shares determined by the two, one-point solution concepts and the shares received by free riding relative to cooperative benefits.

None of the two-player coalitions has a very high value determined by the normalised characteristic function; therefore in this case, the nucleolus distributes benefits equally among the players. The boundaries of the reasonable set, that are identical to those of the core, are determined mainly by individual rationality. The centre of this set then reveals an equal share to the players. The Shapley value is based on the average contribution to the coalition of players who join or leave it. As Player 1 has the lowest cost parameter, it contributes more to the coalitions than the other players, on average; therefore, the Shapley value for Player 1 exceeds the Shapley value for other players. The results of the game in table 6 clearly illustrate

Table 6
Sharing Solutions

Player	Shapley	Nucleolus	Free-rider Shares
1	35.9%	33.3%	38.1%
2	32.3%	33.3%	28.2%
3	31.8%	33.3%	27.1%

Note: Numbers have been rounded.

the difference between the two, one-point solution concepts. Both results are in the core and are characterised as fair sharing rules. There is, however, still a problem because Player 1 is not satisfied with any of the two sharing rules. With the nucleolus sharing rule, Player 1 receives 33.3% of the cooperative benefits, but when free riding on the grand coalition, Player 1 can receive what corresponds to 38.1% of the cooperative benefits. Player 1 is clearly better off by free riding. Therefore, the grand coalition that applies this nucleolus sharing rule is not a stable cooperative solution. With the Shapley value, Player 1 receives 35.9% of the cooperative benefits, which is also below the free-rider value.

Currently, the Baltic Sea cod fishery does not have a cooperative harvest solution. This can be explained as instability by our model if the benefits in a cooperative solution are distributed according to the Shapley or the nucleolus sharing rules. Some players have an incentive to free ride, and as such the cooperative agreement collapses. The problem is that the sharing rules do not take the stability of cooperation into consideration when externalities are present. This is a problem that has also arisen in previous empirical studies [Lindroos and Kaitala (2000); Arnason, Magnusson, and Agnarsson (2000); Duarte, Brasão, and Pintassilgo (2000)], but it has not been recognized.[16] Brasão, Duarte, and Cunha-e-Sá (2000) identified this problem, but did not propose a cooperative solution to it.

The Satisfactory Nucleolus

Given that the previous section demonstrated that the Shapley values and the nucleoli are not necessarily stable against free-rider incentives, we suggest an alternative distribution, which has not previously been applied, of the cooperative benefits solution concept. We define a new set: *the satisfactory core*. This is done by redefining the core by applying the concept of *individual satisfaction*. The *individual satisfaction* ensures players are at least as well off as when free riding. This is a parallel to the individual rationality which ensures the players are as well off as when playing as singletons. The breaking point is that players have already agreed to cooperate, and if they should stick to this agreement, they must not be tempted to deviate. Hence, the sharing rule should ensure all players receive at least their free-rider value. Let us define the *satisfactory core* as follows:

[16] One could argue that free riding on the grand coalition is shortsighted because the consequences are that the coalition formed by the non free riders is likely to break down in the long term. If this is the case, then stability should be discussed in light of Trigger strategies. We prefer instead to search for a distribution of benefits among members, which is also stable compared to the free-rider value.

$$e(s, x) = v(s) - \sum_{i \quad s} x_i \leq 0 \tag{15}$$

$$x_i \geq \frac{v(freerider)}{v(M)} \text{ (Individual Satisfaction)}$$

$$x_1 + x_2 + x_3 = 1.$$

Here v(*freerider*) denotes the benefit of leaving the grand coalition, M, and $v(M)$ denotes the full cooperative benefits. Individual players should, therefore, receive at least what they would gain by free riding. Note that the existence of a stable grand coalition is determined by the comparison of grand coalition net benefit and the total free-rider value. If the grand coalition net benefit is larger, then a stable sharing rule becomes easier to find. The point sharing rule hinges on the coalition's bargaining power.

The satisfactory core deviates from the ordinary core by the individual satisfaction constraint. This constraint ensures that each player receives at least the amount the player would receive by free riding on the grand coalition. In our specific case, the free-rider values are credible threats because all two-player coalitions are stable. This means that if one player leaves the grand coalition, the equilibrium will be such that there is a two-player coalition and a singleton. The individual satisfaction, applying values from the normalised characteristic function, takes the following form:

$$x_1 \geq 0.3809 \tag{16}$$

$$x_2 \geq 0.2823$$

$$1 - x_1 - x_2 \geq 0.2714.$$

When comparing the sharing rules from table 6 with equation (16), it is evident that both the Shapley value and the nucleolus violate the individual satisfaction of Player 1.

We define another sharing rule, the *satisfactory nucleolus*, which is similar to the nucleolus in the sense that it is defined as the lexicographic center of the satisfactory ε-core. The results of the satisfactory nucleolus are summarised in table 7. This table clearly shows that the satisfactory nucleolus is stable against free riding.

A graphic illustration, in a Player 1, Player 2 diagram of the difference between the core and the satisfactory core and the three applied sharing rules is provided in figure 1. It should be emphasised that the proportions in the figure are not correct.

In figure 1 the core is bounded by the lines ensuring that all two-player coalitions receive their joint profit; that is, $v(1,2)$ as a lower bound, $[1 - v(2,3)]$ and $[1 - v(1,3)]$ as upper bounds. In addition it is recognized that the no more than the total profit can

Table 7
Values for the Satisfactory Nucleolus

Player	Satisfactory Nucleolus	Free-rider Shares
1	40.3%	38.1%
2	30.4%	28.2%
3	29.3%	27.1%

Note: Numbers have been rounded.

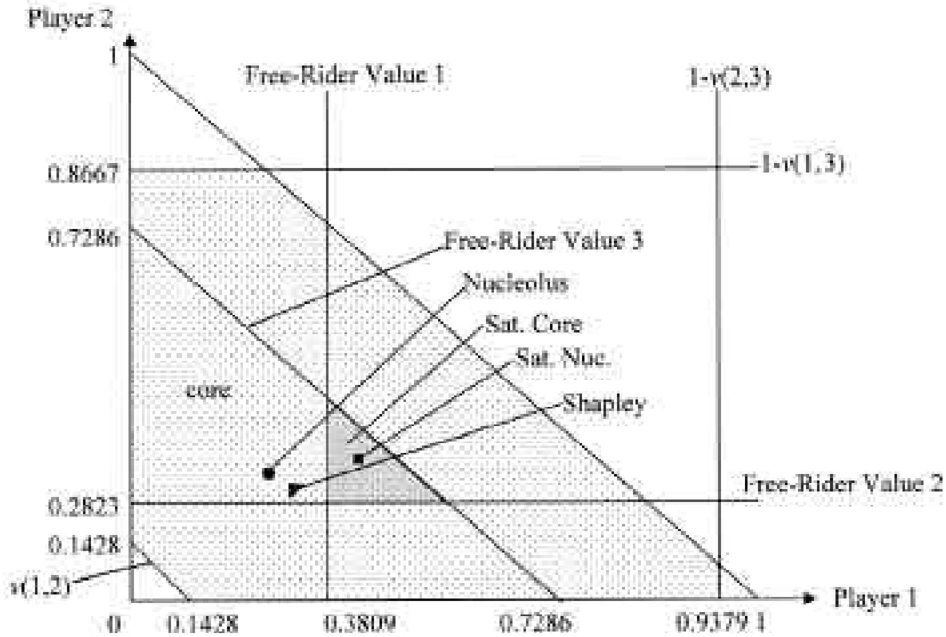

Figure 1. The Reasonable Set, Core, and Satisfactory Core in a Player 1, Player 2 Diagram
Note: Also shown are the sharing rules: the nucleolus, Shapley value, and satisfactory nucleolus.

be shared among the players (the line from 1 to 1). The satisfactory core is bounded by the free-rider values; that is, Players 1 and 2 should receive at least their free-rider values and can receive no more than one minus free-rider values to Player 3. Figure 1 again underlines the fact that the nucleolus and the Shapley values do not lie within the satisfactory core. The satisfactory nucleolus is a cooperative sharing imputation that is stable in the face of free-rider values, as it is defined with the aim of taking stability into account. The satisfactory nucleolus establishes a connection between non-cooperative and cooperative games. The satisfactory nucleolus takes into account the fact that Player 1 can make relatively large gains by free riding on the grand coalition; therefore, this player receives a larger share of the cooperative benefits compared to the other two players. The results of our model suggest that the IBFSC should consider the satisfactory nucleolus as a sharing rule to be administered for reaching a stable cooperative solution in the Baltic Sea cod fishery.

Sensitivity Analysis

We tested the robustness of the results by varying different economic and biological parameters. In particular, we focused on economic parameters such as cost parameters, the discount rate, and simulation length. Cost parameters were increased, decreased, and their mutual proportions changed. None of these changes affected the theory result that a grand coalition is a possible stable solution. The mutual proportions cost parameters and their levels do, however, affect the stability of the coalition structures when applying different sharing rules. If the cost parameters are all very low, then the players and the partial coalitions will apply full effort, and the free-rider values coincide with the singleton benefits. The core and the satisfactory core are identical and both the Shapley and nucleolus are located in the core and are, therefore, stable sharing rules. Also, if countries have relatively similar cost parameters, then they receive similar benefits by free riding; this makes it more likely that the existing sharing rules are also located in the satisfactory core. However, if one country is more efficient than the others, then it receives relatively large benefits from free riding, which can diminish the satisfactory core significantly, making it more unlikely that applying the Shapley value and the nucleolus will yield stable grand coalitions. Increasing the discount rate to 5% or 8% does not change the fact that it is possible to find a stable cooperative solution. Applying the Shapley value and the nucleolus does, however, yield unstable grand coalitions. Reducing the simulation length from 50 to 25 years still provides a stable cooperative solution, but again we have to search among solutions other than the Shapley value and the nucleolus to reach stability of the grand coalition.

We shifted the Beverton-Holt stock-recruitment curve up and down by increasing and decreasing the maximum recruits per spawner at low spawning stock size [the parameter c in equation (4)]. It is still possible to find a stable cooperative solution, but the Shapley values and the nucleoli are again not among these stable solutions.

We also varied the initial conditions, including the initial abundance of cod, the stock weight at age, the catch weight at age, and the proportion of mature stock (see table 2 for initial conditions). The original scenario is based on data from the 1998 level, which is a year along with other more recent years, that had a low abundance of cod. For the purpose of sensitivity analysis, the initial level was set to the 1982 level, which is the year with the highest abundance of cod on record (ICES 2000). These simulations do, however, show the same trend as the other results. There exists a stable grand coalition, but neither the Shapley value nor the nucleolus are stable solutions.

We can thus conclude that the grand coalition formed by our model is a rather robust solution, because we can find a stable sharing rule in all the analysed cases. Whether or not the Shapley values and the nucleoli are among these sharing rules is more parameter specific.

Conclusions

Our model shows that there are sufficient benefits to make all players better off in the grand coalition compared to a non-cooperative or partly cooperative solution. This result is in stark contrast with previous more pessimistic empirical coalitional game models. The critical point is how the benefits received in a grand coalition are shared among the players in the game. Two different known one-point sharing rules, the Shapley value and the nucleolus, do not take into account the stability of the coalition even though they are both located in the core and are both characterised as fair sharing rules. If the benefits in the Baltic Sea cod fishery are shared according to these rules, it is shown to be an unstable solution that does not satisfy all players of the grand coalition, as one player receives more benefits by free riding. We, therefore, suggest a new sharing rule that connects the cooperative and non-cooperative games. The main contribution of our paper is the development of a sharing rule that makes the grand coalition stand-alone stable dealing with three heterogeneous players. The satisfactory core, as we call the core compromising this sharing rule, takes into account the stability of the grand coalition by including the free-rider values as threat points. The corresponding satisfactory nucleolus sharing rule ensures that all players receive a share of the cooperative benefits that is at least as large as their free-rider value; this yields a stable sharing imputation. A cooperative solution can be stable, but the Achilles' heel of such a model is the distribution of the benefits; one should be aware that all players are satisfied compared to their free-rider values. We show that if the satisfactory nucleolus sharing rule is applied to the Baltic Sea cod fishery, a stable solution can be achieved.

The current model is limited to three players. It can be argued that in some countries fishermen are members of producer organizations (POs) and that these organizations act as a single group. The assumption might, however, be critical because not all countries have a high degree of membership in POs. If the number of players in a coalitional game increases, it most likely becomes more difficult to achieve a grand coalition solution. Olson (1965) discusses this point as a general problem to collective goods, while Hannesson (1997) discusses it as a problem in fishery models, where he defines the critical number of fishermen for a full cooperative solution. The number of players in a fishery can be reduced by supporting memberships of POs and supporting the organisation of POs in developing countries.

One reason for not establishing a grand coalition in the Baltic Sea fisheries might be that having all fishermen join a grand coalition may generate higher transaction costs involved in planning and organising the grand coalition. It also decreases the likelihood of a stable grand coalition. Organizing a grand coalition should be performed by existing commissions, such as the IBSFC, and a solution to the redistribution problem might prove to be the introduction of individual transferable quotas (ITQs).

Although cooperation is achieved on the country level, the fishermen in each country could still play a competitive game. Therefore, individual incentive incompatibility in each country group may also be the reason why the Baltic Sea cod fishery does not have a cooperative harvest solution. This issue would need a two-level game structure such as in Kronbak and Lindroos (2006).

The Shapley value, the nucleolus, and the newly developed satisfactory nucleolus only consider the sharing of benefits at the end of the game. It is beyond the scope of these sharing rules to discuss how the benefits of the grand coalition should be shared among members over time; this is, however, a very relevant point that should be subject to future research. A related issue that should be developed further is the open-loop control that may not match the reality of the fishery game being played. With feedback control, some unstable sharing rules could become stable.

The model results are stable to changes in both economic and biological parameter values, and for many of the tested scenarios the pattern is preserved, namely that the nucleoli and the Shapley values are not stable solutions. This conclusion can also be drawn from previous studies [Lindroos and Kaitala (2000); Arnason, Magnusson, and Agnarsson (2000); Duarte, Brasão, and Pintassilgo (2000)], and we, therefore find it particularly important to recognize the connection between the sharing rule and the free-rider values, which is accommodated in the development of the satisfactory nucleolus.

References

Arnason, R., G. Magnusson, and S. Agnarsson. 2000. The Norwegian Spring-Spawning Herring Fishery: A Stylized Game Model. *Marine Resource Economics* 15(4):293–319.

Arnason, R., L. Sandal, S.I. Steinshamn, N. Vestergaard, S. Agnarsson, and F. Jensen. 2000. *Comparative Evaluation of the Cod and Herring Fisheries in Denmark, Iceland and Norway*. TemaNord 2000:526. Ekspressen Tryk og

Kopicenter, Denmark.
Aumann, R.J., and J.H. Dréze. 1974. Cooperative Games with Coalitions Structures. *International Journal of Game Theory* 3:217–37.
Brasão, A., C.C. Duarte, and M.A. Cunha-e-Sá. 2000. Managing the Northern Atlantic Bluefin Tuna Fisheries: The Stability of the UN Fish Stock Agreement Solution. *Marine Resource Economics* 15(4):341–60.
Chander, P., and H. Tulkens. 1997. The Core of an Economy with Multilateral Environmental Externalities. *International Journal of Game Theory* 26:379–401.
Duarte, C.C., A. Brasão, and P. Pintassilgo. 2000. Management of the Northern Atlantic Bluefin Tuna: An Application of C-Games. *Marine Resource Economics* 15(1):21–36.
Eyckmans, J., and M. Finus. 2004. An Almost Ideal Sharing Scheme for Coalition Games with Externalities. *Working Paper* 155. Trieste, Italy: Fondazione Eni Enrico Mattei.
Fiskeridirektoratet. 1999. *Fiskeristatistisk Årbog* 1998 (*Yearbook of Fishery Statistics* 1998). Danmarks Statistiks Trykkeri, København (in Danish).
___. 2000. *Fiskeristatistisk Årbog* 1999 (*Yearbook of Fishery Statistics* 1999). Danmarks Statistiks Trykkeri, København (in Danish).
Greenberg, J. 1994. Coalition Structures. *Handbook of Game Theory with Economic Applications*, Vol. 2, R.J. Aumann and S. Hart, eds., Chapter 37. Amsterdam: Elsevier.
Hannesson, R. 1997. Fishing as a Supergame. *Journal of Environmental Economics and Management* 32:309–22.
IBSFC. 2003. International Baltic Sea Fishery Commission website. The Commission ceased to function by January 2007 and the website no longer exists.
ICES. 2000. Cooperative Research Report No. 236. *Report of the ICES Advisory Committee on Fishery Management*, 1999, Part 2.
Kaitala, V., and M. Lindroos. 1998. Sharing the Benefits of Cooperation in High Seas Fishery Game: A Characteristic Function Game Approach. *Natural Resource Modeling* 11:275–99.
Kronbak, L.G. 2004. A Coalition Game of the Baltic Sea Cod Fishery. *IME Working Paper No.* 55/03. Department of Environmental and Business Economics, University of Southern Denmark.
Kronbak, L.G., and M. Lindroos. 2006. An Enforcement-Coalition Model: Fishermen and Authorities Forming Coalitions. *Environmental and Resource Economics* 35:169–94.
Lindroos, M., and V. Kaitala. 2000. Nash Equilibria in a Coalition Game of the Norwegian Spring-Spawning Herring Fishery. *Marine Resource Economics* 15(4):321–40.
Mesterton-Gibbons, M. 1992. *An Introduction to Game-theoretic Modelling*. Redwood, CA: Addison-Wesley.
Nash, J.F. 1951. Non-Cooperative Games. *Annals of Mathematics* 54:286–95.
Olson, M. 1965. *The Logic of Collective Action*. Cambridge: Cambridge University Press.
Pintassilgo, P. 2003. A Coalition Approach to the Management of High Seas Fisheries in the Presence of Externalities. *Natural Resource Modeling* 16(2):175–97.
Sumaila, U.R. 1999. A Review of Game-theoretic Models of Fishing. *Marine Policy* 23(1):1–10.
Weikard, H.-P. 2005. Cartel Stability under an Optimal Sharing Rule. *Working Paper* 77.2005, Trieste, Italy: Fondazione Eni Enrico Mattei.
Yi, S.-S. 2003. Endogenous Formation of Economic Coalitions: A Survey of the Partition Function Approach. *The Endogenous Formation of Economic Coalitions*, Carlo Carraro, ed. Cheltenham, UK: Edward Elgar.